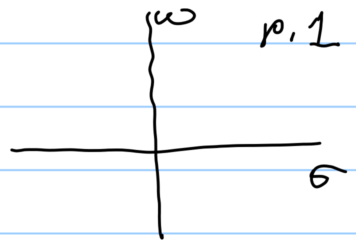
EE 610 09/15/03

09/15/03
p. 1

Lossless $Z(s) \Leftarrow P_{ave}(\omega) = 0$

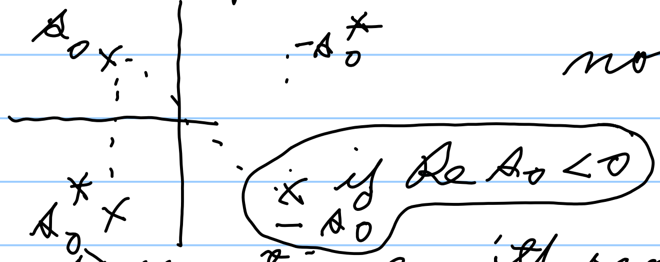
$$Z(s) + Z^T(-s) = 0_{n\times n}$$



$s = \sigma + j\omega$

if passive elements, the system is stable

$\Rightarrow$ no poles in $\sigma > 0$



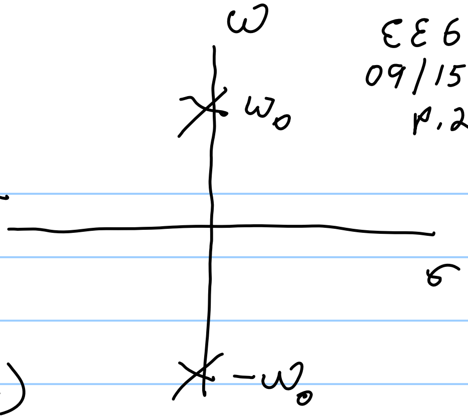
if $\mathrm{Re}\, s_0 < 0$

↑ if rational with real coefficients

no poles in $\sigma < 0$ if lossless

$\Rightarrow$ all poles are on $j\omega$ axis

lossless



$Z(s)$ all poles are simple & on $j\omega$ axis

also true for $1/Z(s) = Y(s)$

$\Rightarrow$ all zeros and poles of $Z(s)$ are on the $j\omega$ axis

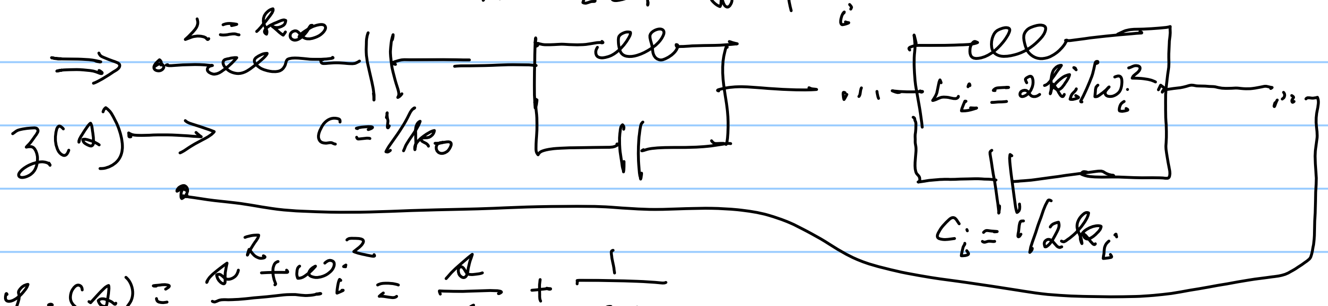
$$Z(s) = k_\infty s + \frac{k_0}{s} + \underbrace{\frac{k_1}{s+j\omega_0} + \frac{k_2}{s-j\omega_0}} + \cdots$$

$$\frac{k_1 s - j\omega_0 k + k_2 s + j\omega_0 k_2}{s^2+\omega_0^2} = \frac{(k_1+k_2)s + j\omega_0(k_2-k_1)}{s^2+\omega_0^2}$$

$\Rightarrow$ require $k_2 = k_1$ & these are real

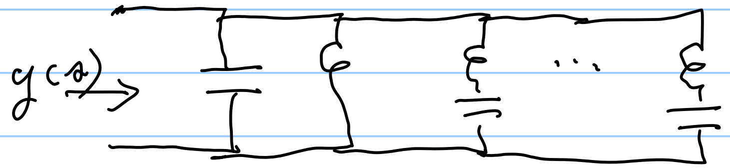
EE610
09/15/03
p.3

$$Z(s) = k_\infty s + \frac{k_0}{s} + \sum_{i=1}^{m} \frac{2k_i s}{s^2+\omega_i^2}$$ here $Z(-s) = -Z(s)$



$$y_i(s) = \frac{s^2+\omega_i^2}{2k_i s} = \frac{s}{2k_i} + \frac{1}{\frac{2k_i}{\omega_i^2} s}$$

$$y(s) = \frac{1}{Z(s)} = \hat{k}_\infty s + \frac{\hat{k}_0}{s} + \sum_{i=1}^{\hat{m}} \frac{2\hat{k}_i s}{s^2+\hat{\omega}_i^2}$$



$$z(s) = \frac{3s(s^2+4)}{(s^2+1)(s^2+6)} \quad ; \quad \delta[z] = 4$$

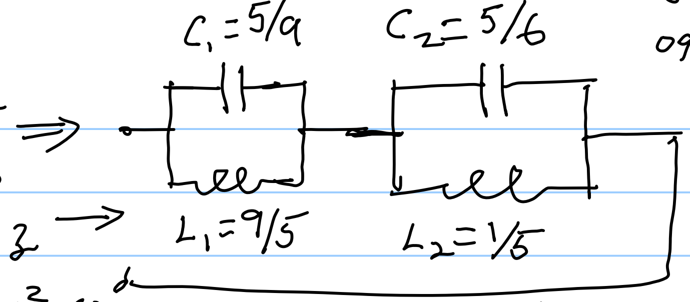
$$= \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+6} = \frac{\frac{9}{5}s}{s^2+1} + \frac{\frac{6}{5}s}{s^2+6}$$

$$2k_1 = \frac{(s^2+1)}{s} z(s)\Big|_{s^2=-1} = \frac{3(s^2+4)}{s^2+6}\Big|_{s^2=-1} = \frac{3(3)}{5} = 9/5$$

$$2k_2 = \frac{s^2+6}{s} \cdot z(s) - 2k_1\left(\frac{s^2+6}{s^2+1}\right) = \frac{3(s^2+4)}{s^2+1}\Big|_{s^2=-6} - 2k_1 \frac{s^2+6}{s^2+1}\Big|_{s^2=-6} = \frac{3(-2)}{-5} = \frac{6}{5}$$

EE 610
09/15/03
P.5

$z(s) = \frac{\frac{9}{5}s}{s^2+1} + \frac{\frac{6}{5}s}{s^2+6} \Rightarrow$

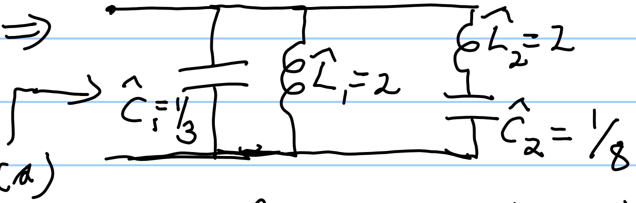


1st Fosters

$y(s) = \frac{1}{z(s)} = \frac{(s^2+1)(s^2+6)}{3s(s^2+4)} = \frac{1}{3}s + \frac{6/12}{s} + \frac{2\hat{k}_1 s}{s^2+4}$

$2k_1 = \frac{(s^2+1)(s^2+6)}{3s^2}\Big|_{s^2=-4} = \frac{(-3)(2)}{-12} = 1/2$

$y(s) = \frac{1}{3}s + \frac{1/2}{s} + \frac{\frac{1}{2}s}{s^2+4} \Rightarrow$



2nd Fosters

Both use $\delta[z] = \delta[y]$ reactive elements (C's & L's)

EE 610
09/15/03
p. 6

$$z(s) = \frac{3s(s^2+4)}{(s^2+1)(s^2+6)} = \frac{3s^3+12s}{s^4+7s^2+6}$$

removal of poles at ∞ (1st Cauer); of z or of y

$$y(s) = \frac{s^4+7s^2+6}{3s^3+12s} = \frac{1}{3}s + \frac{3s^2+6}{3s^3+12s} = \frac{1}{3}s + \cfrac{1}{\cfrac{3s^3+12s}{3s^2+6}}$$

$$\begin{array}{r|l} & \frac{1}{3}s \\ 3s^3+12s & \overline{s^4+7s^2+6} \\ & \underline{s^4+4s^2} \\ & 3s^2+6 \end{array}$$

$$\begin{array}{r|l} & s \\ 3s^2+6 & \overline{3s^3+12s} \\ & \underline{3s^3+6s} \\ & 6s \end{array}$$

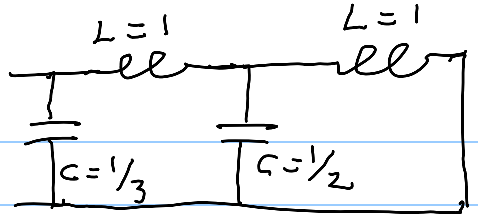
$$= \frac{1}{3}s + \cfrac{1}{s + \cfrac{1}{\cfrac{3s^2+6}{6s}}}$$

$$\begin{array}{r|l} & \frac{1}{2}s \\ 6s & \overline{3s^2+6} \\ & \underline{3s^2} \\ & 6 \end{array}$$

$$\begin{array}{r|l} & s \\ 6 & \overline{6s} \end{array}$$

$$= \frac{1}{3}s + \cfrac{1}{s + \cfrac{1}{\frac{1}{2}s + \cfrac{1}{s}}}$$

$$z(s) = \cfrac{1}{\frac{1}{3}s + \cfrac{1}{s + \cfrac{1}{\frac{1}{2}s + \frac{1}{s}}}} \longrightarrow$$



1st Cauer – continued fraction expansion about $s = \infty$