ENEE610_09/08/03 with corrections and revisions of

Note Title

9/8/2003

P. 1

$$Z = \begin{bmatrix} \frac{1}{cs} & \frac{1}{cs} + r \\ \frac{1}{cs} - r & \frac{1}{cs} \end{bmatrix} = \begin{bmatrix} kz(k)/s & \frac{kz(k)}{s} + z(k) \\ \frac{kz(k)}{s} - z(k) & \frac{kz(k)}{s} \end{bmatrix}$$

at $s = k \Rightarrow z_{21}(k) = 0 \Rightarrow$ zero of transmission

have $R(s) = \dfrac{kz(s) - sz(k)}{kz(k) - sz(s)}$

if $z(k) = -z(-k) \Rightarrow$

Ⓐ $s = -k$ also get a cancellation

$$R(-k) = \frac{k\,Z(-k) - (-k)\,Z(k)}{k\,Z(k) - (-k)\,Z(-k)} = \frac{-k\,Z(k) + k\,Z(k)}{k\,Z(k) - k\,Z(k)} \Bigg|_{Z(k) = -Z(-k)}$$

$\therefore$ $\alpha = -k$ will also cancel if $Z(k) + Z(-k) = 0 \Rightarrow$ if α is a zero of the even part of $Z(\alpha)$

By def: $\mathcal{E}v\,Z(\alpha) = \frac{1}{2}\left(Z(\alpha) + Z(-\alpha)\right)$ ie $\mathcal{E}v(Z(\alpha)) = \mathcal{E}v(Z(-\alpha))$

$\mathcal{O}d\,Z(\alpha) = \frac{1}{2}\left(Z(\alpha) - Z(-\alpha)\right)$

$Z(\alpha) = \mathcal{E}v\,Z + \mathcal{O}d\,Z$

Ex: $Z(\lambda) = \dfrac{3\lambda+1}{\lambda+2}$ let $k=2$; $Z(k) = 7/4$

$$Z_L = r \cdot R(\lambda) = Z(k) \cdot \frac{k Z(\lambda) - \lambda Z(k)}{k Z(k) - \lambda Z(\lambda)} = \left(\frac{\cancel{1}\,4}{7}\right)\left(\frac{2\left(\frac{3\lambda+1}{\lambda+2}\right) - \lambda\, 7/4}{\frac{7}{2} - \frac{\lambda(3\lambda+1)}{\lambda+2}}\right)$$

$$= \frac{7}{4}\left(\frac{6\lambda+2-\frac{7}{4}\lambda^2-\frac{7}{2}\lambda}{\frac{7}{2}\lambda+7-3\lambda^2-\lambda}\right) = \frac{7}{8}\left(\frac{-7\lambda^2+10\lambda+8}{-6\lambda^2+5\lambda+14}\right)$$

$$\begin{array}{r|l} & -7\lambda \quad -4 \\ \hline \lambda-2 & -7\lambda^2+10\lambda+8 \\ & -7\lambda^2 \underline{+14\lambda} \\ & \qquad\quad -4\lambda \end{array}$$

$$= \frac{7}{8}\left(\frac{\cancel{(\lambda-2)}(-7\lambda-4)}{\cancel{(\lambda-2)}(-6\lambda-7)}\right)$$

$$\begin{array}{r|l} & -6\lambda \quad -7 \\ \hline \lambda-2 & -6\lambda^2+5\lambda+14 \\ & -6\lambda^2 + \underline{12\lambda} \\ & \qquad\quad -7\lambda \end{array}$$

$$= \frac{7}{8}\left(\frac{7\lambda+4}{6\lambda+7}\right); \quad \delta[Z_L] = \delta[Z] = 1$$

$\delta[\cdot] =$ degree

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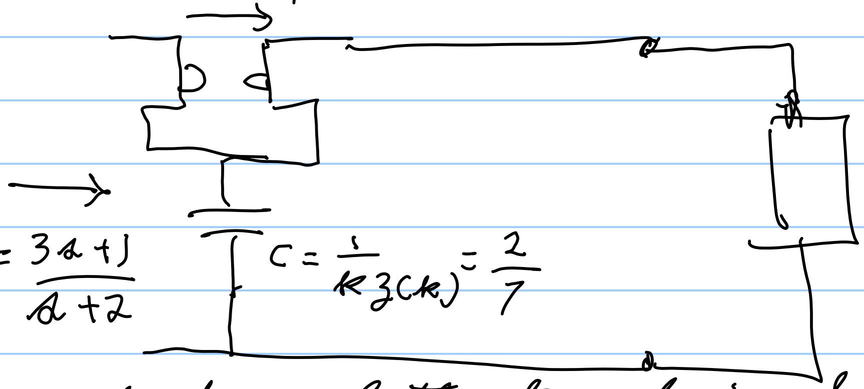
$r = 7/4 = z(k)$



$z = \frac{3s+1}{s+2}$

$C = \frac{1}{k\,z(k)} = \frac{2}{7}$

$z_L = \frac{49}{48}\left(\frac{s+4/7}{s+6/7}\right)$

look for a better k: desire k = zero of $\mathcal{E}v\, z(s)$

$$2\,\mathcal{E}v\,z(s) = z(s)+z(-s) = \frac{3s+1}{s+2} + \frac{-3s+1}{-s+2}$$

$$\Rightarrow \frac{2}{3}\,\mathcal{E}v\,z(s) = \frac{(s+1/3)(-s+2)+(-s+1/3)(s+2)}{(s+2)(-s+2)} = \frac{-2s^2+4/3}{D(s)D(-s)}$$

where $D(s) = s+2$

$$\Rightarrow -s^2+2/3 \Rightarrow s^2-2/3 = 0$$

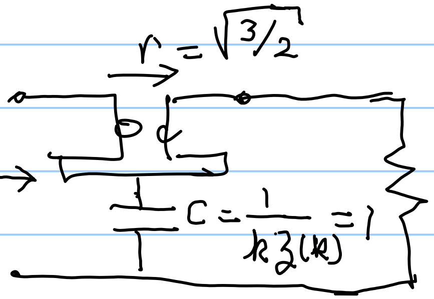
$$s_{zero} = \pm\sqrt{2/3} \Rightarrow k = \sqrt{2/3}$$

$z(s) = \frac{3s+1}{s+2}$; $k = \sqrt{2/3}$; $z(k) = \frac{\sqrt{6}+1}{2+\sqrt{2/3}} = \sqrt{\frac{3}{2}}$

$R = \frac{\sqrt{2/3}\cdot\frac{3s+1}{s+2} - s\sqrt{3/2}}{\sqrt{2/3}\cdot\sqrt{3/2} - s\left(\frac{3s+1}{s+2}\right)}$ = constant as $(s-\sqrt{2/3})(s+\sqrt{2/3})$ $= s^2 - 2/3$ will cancel

$= \frac{-\sqrt{3/2}\,(s^2-2/3)}{-3\,(s^2-2/3)} = \frac{1}{\sqrt{6}}$

$z = \frac{3s+1}{s+2} \rightarrow$



$R_{load} = rR = \frac{1}{2}$

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p. 6

Lossless circuits - p. 342 (LC = reactance)

$$P_{ave}(\omega)\Big|_{s=j\omega} = 0 \Rightarrow P_{ave} = \text{average power}$$

$$= \mathrm{Re}\, V^{T*} I = \frac{V^{T*} I + I^{T*} V}{2}$$

assume $Z(j\omega)$ exists; $V = Z I$

$$2 P_{ave}(\omega) = I^{T*} Z^{T*}(j\omega) I + I^{T*} Z(j\omega) I$$

$$= I^{T*} [Z^{T*}(j\omega) + Z(j\omega)] I$$

lossless $\Rightarrow Z(j\omega) + Z^{T*}(j\omega) = 0_{n\times n}$ if V & I are n-vectors