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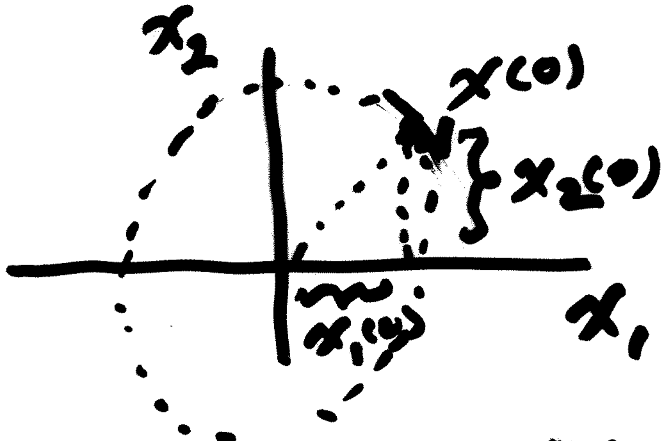
Van der Pol

$\dot{x}_1 = x_2$ $\quad\cdot = d/dt$

$\dot{x}_2 = -x_1$ $\quad x(0)$ given, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\Rightarrow \ddot{x}_1 = \dot{x}_2 = -x_1 \Rightarrow \ddot{x}_1 + x_1 = 0$

$(d^2+1)x_1(t) = 0$





$$\frac{dx_2}{dx_1} = \frac{dx_2/dt}{dx_1/dt} = \frac{-x_1}{x_2} = \frac{-1}{x_2/x_1}$$

$$= \frac{-1}{\text{slope of vector (from origin)}}$$

gives circles as limit cycles which depend on initial

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conditions. These are structurally unstable

If add positive resistance makes it structurally stable but the limit cycle is the origin.

$$\frac{dx_1}{dt} = x_2 - f(x_1)$$ ; if $f(x_1) = x_1$

$$\frac{dx_2}{dt} = -x_1$$

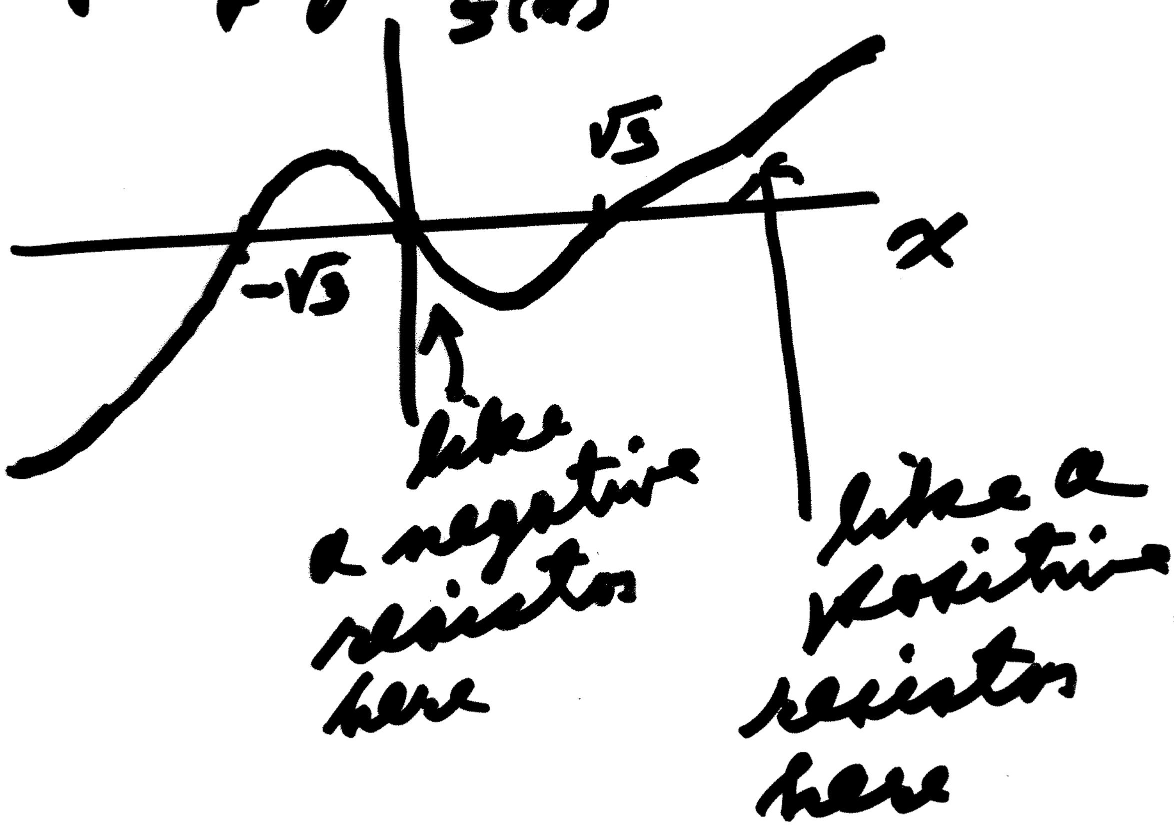
This damper forces to $x(\infty)=0$

By adding $f(x_1)$ can make a structurally stable limit cycle ($\neq 0$).

Choose $\epsilon x\left(\frac{x^2}{3} - 1\right) = f(x)$

gives positive damping

for large $x$ & negative damping for small $x$; $\epsilon > 0$




$$\frac{dx_1}{dt} = x_2 - \epsilon x_1 \left(\frac{1}{3}x_1^2 - 1\right)$$

$$\frac{dx_2}{dt} = -x_1$$

$$\ddot{x}_1 = \dot{x}_2 - \epsilon\left[x_1^2 - 1\right] \cdot \frac{dx_1}{dt}$$

$$= -x_1 - \epsilon(x_1^2 - 1)\dot{x}_1$$

$$\ddot{x}_1 + \epsilon(x_1^2 - 1)\dot{x}_1 + x_1 = 0$$

need $x_1(0)$, $\dot{x}_1(0)$

state space trajectory:


$$\frac{dx_1}{dt} = x_2 - f(x_2)$$

$x(0)$ given

$$\frac{dx_2}{dt} = -x_1$$

$$\frac{dx_2}{dx_1} = \frac{dx_2/dt}{dx_1/dt} = \frac{-x_1}{x_2 - f(x_1)}$$

$$= -\frac{1}{[x_2 - f(x_1)]/x_1}$$

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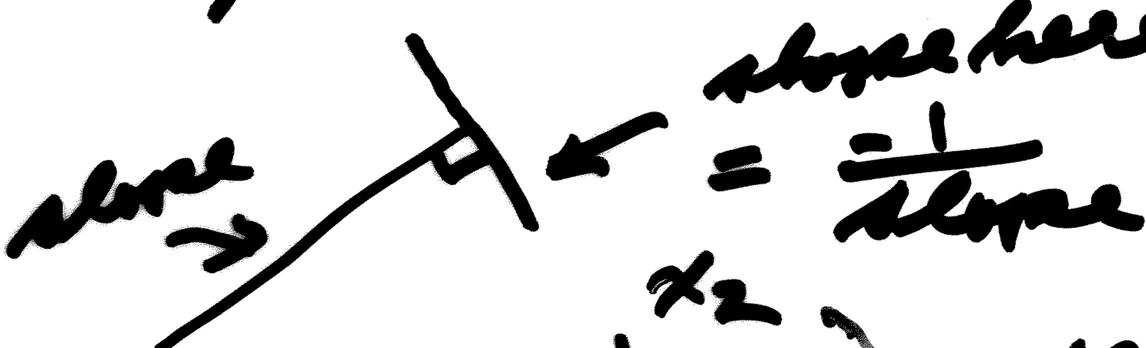
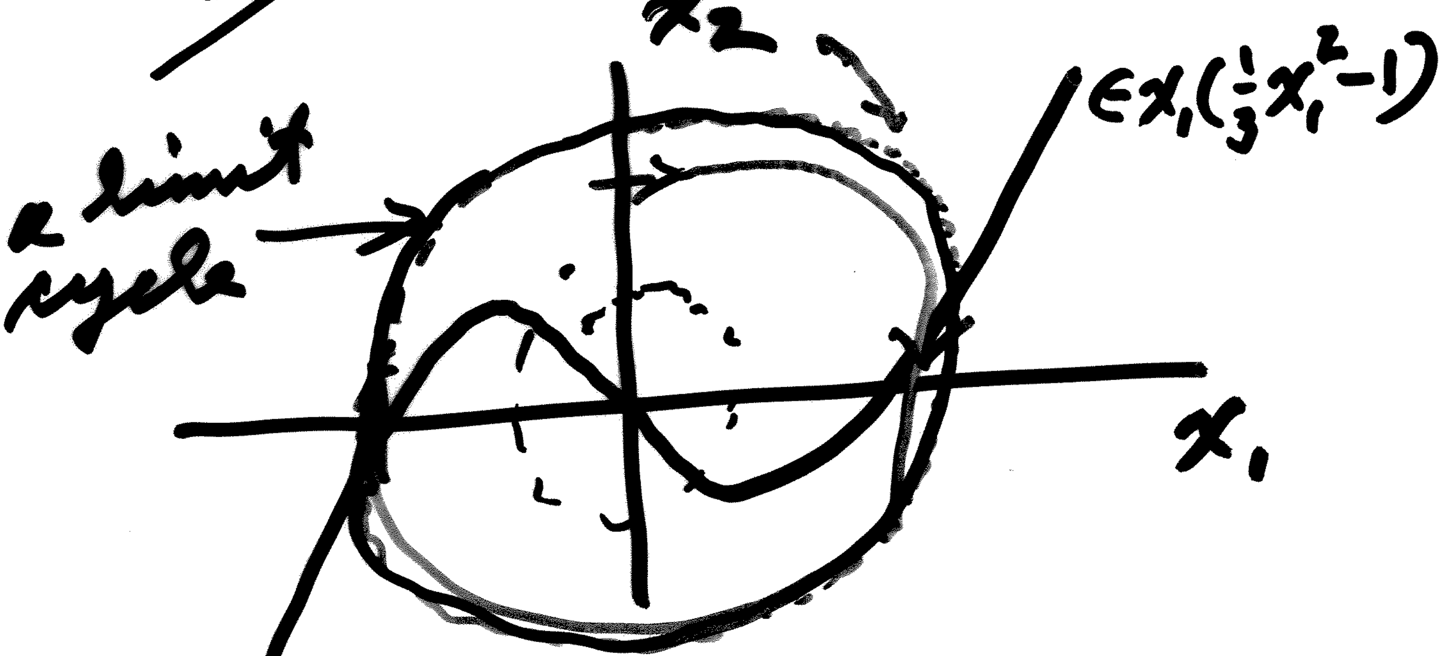
note: The slope of a ⊥ to line is −1/slope of the line




Use a piecewise linear curves

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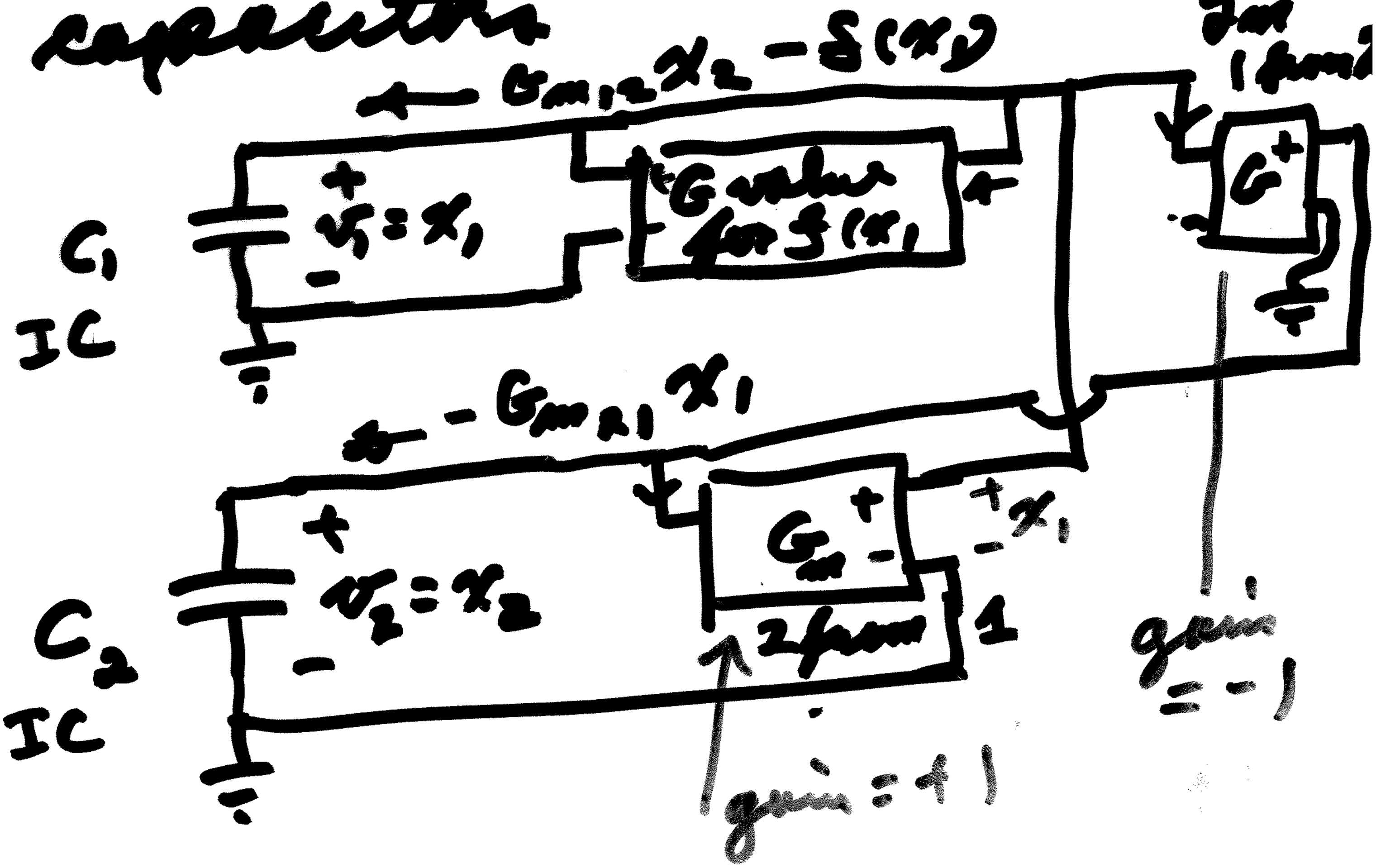
$$C_1 \frac{dx_1}{dt} = G_{m12} x_2 - \underbrace{S(x_1)}_{\text{a conductance (current out voltage } x_1 \text{ in)}}$$

$$C_2 \frac{dx_2}{dt} = -G_{m21} x_1$$

let $x_1$ & $x_2$ be voltages on capacitors



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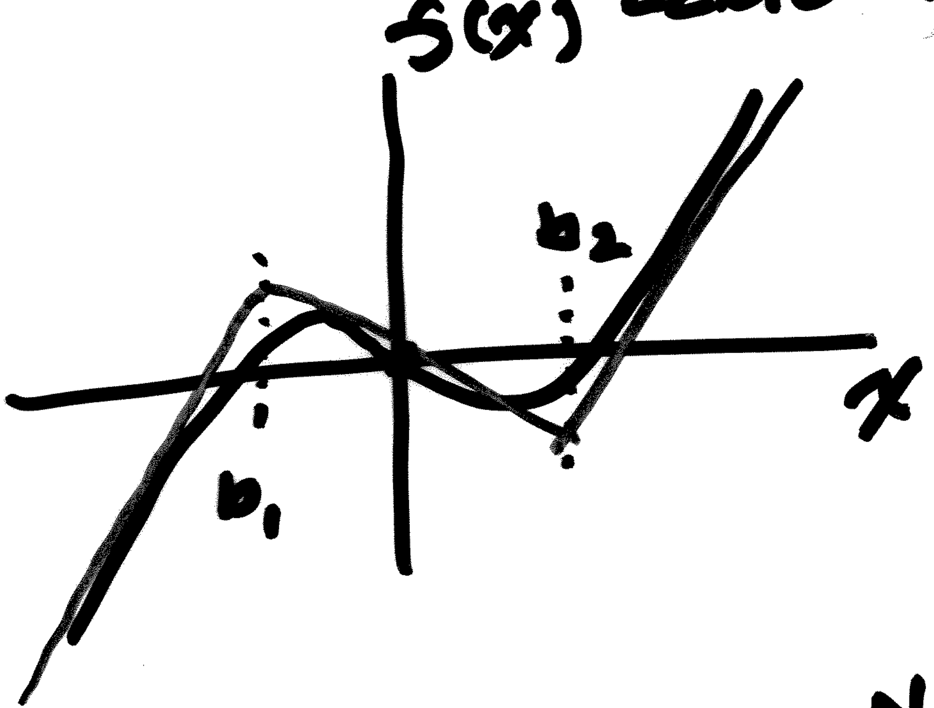


use

$$y(x) = a_\infty x + b_\infty + \sum_{i=1}^{N} a_i |x - b_i|$$

here $b_i$ are the break points

$b_1 < b_2 < b_3 \cdots$

N = # of break points

Take derivatives

$$x < b_1 ; \quad y_1(x) = a_\infty x + b_\infty - \sum_{i=1}^{N} a_i (x - b_i)$$

$$y_1'(x) = a_\infty - \sum_{i=1}^{N} a_i = a_\infty - a_1 - a_2 \cdots - a_N$$

$b_1 < x < b_2$

$$y_2(x) = a_\infty x + b_\infty + a_1(x - b_1) - \sum_{i=2}^{N} a_i (x - b_i)$$

$$\frac{d y_2(x)}{dx} = a_\infty + a_1 - a_2 - \cdots - a_N$$

take $y_2'(x) - y_1'(x) = 2a_1 + 0$

or $2a_1 = y_2'(x_2) - y_1'(x_1)$

$x_1 < b_1$, $b_1 < x_2 < b_2$

Continue, as go through a break point get a new $a_i$

now $y(0) = b_\infty + \sum_{i=1}^{N} a_i |-b_i|$

$$\Rightarrow b_\infty = y(0) - \sum_{i=1}^{N} a_i |0 - b_i|$$

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still need $a_\infty$:

$$y(x_0) = a_\infty x_0 + b_\infty + \sum a_i |x_0 - b_i|$$

gives $a_\infty$

end result

given a piecewise linear curve can write

$$y(x) = a_\infty x + b_\infty + \sum_{i=1}^{N} a_i |x - b_i|$$