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p. 1

adjoint, p. 406–410

optimization, p. 404

$$[v^T \; i^T]\begin{bmatrix} i^a \\ -v^a \end{bmatrix} = 0$$

if Y exists

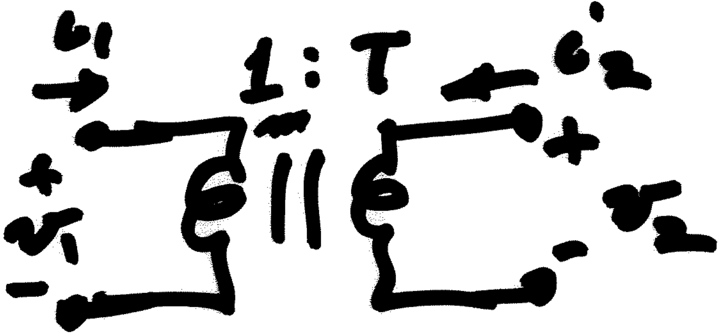
$$[v^T \;\; (Yv)^T]\begin{bmatrix} Y^a v^a \\ -v^a \end{bmatrix} = 0$$

$$v^T[1_n \;\; Y^T]\begin{bmatrix} Y^a \\ -1_n \end{bmatrix}v^a = 0_{1\times 1}$$

$$Y^a - Y^T = 0_{n\times n} \Rightarrow Y^a = Y^T$$

If no Y use the more general definition

Transformer:



let v_1 & i_1 be m vectors

i_2 & v_2 be n vectors

$$P_{in} = 0 = v^T i = [v_1^T, v_2^T]\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = 0_{1\times}$$

$$1_m \cdot i_1 + \underbrace{T}_{m\times n} \cdot i_2 = 0_{m\times 1}$$

$$[v_1^T, v_2^T]\begin{bmatrix} -T\cdot i_2 \\ i_2 \end{bmatrix} = 0_{1\times 1}$$

transpose

turns ratio

$$= [v_1^T, v_2^T]\begin{bmatrix} -T \\ 1_n \end{bmatrix} \cdot i_2 = 0_{1\times 1}$$

$$\Rightarrow -v_1^T T + v_2^T = 0_{1\times n}$$

$$\left.\begin{aligned} v_2 &= T^T v_1 \\ i_1 &= -T i_2 \end{aligned}\right\}$$ law for transformer

m terminal input
n terminal output

T is m × n

$$A v = B i$$

$$\begin{array}{c c} n\{ & \\ m\{ & \end{array}\begin{bmatrix} T^T & -1_n \\ 0 & 0 \end{bmatrix} v = \begin{bmatrix} 0 & 0 \\ 1_m & T \end{bmatrix} i$$

$$\begin{array}{cc} \underbrace{}_{m} & \underbrace{}_{n} \end{array}$$

to find the adjoint

$$\begin{bmatrix} v^T & i^T \end{bmatrix} \begin{bmatrix} i^a \\ -v^a \end{bmatrix} = 0_{1\times 1}$$

$$= \begin{bmatrix} v_1^T & v_1^T T \,\vdots\, (-T i_2)^T & i_2^T \end{bmatrix} \begin{bmatrix} i^a \\ -v^a \end{bmatrix} = 0$$

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$$\begin{bmatrix} v_1^T & v_1^T T & -i_2'^T T^T & i_2'^T \end{bmatrix} \begin{bmatrix} i_1^a \\ i_2^a \\ -v_1^a \\ -v_2^a \end{bmatrix} = 0$$

$\Rightarrow$

$$v_1^T i_1^a + v_1^T T i_2^a$$
$$- i_2'^T T^T(-v_1^a) + i_2'^T(-v_2^a) = 0$$

$$= v_1^T[i_1^a + T i_2^a]$$
$$+ i_2'^T[T^T v_1^a - v_2^a] = 0$$

$\Rightarrow$ need coefficients = 0 as can freely choose v_1 & i_2'

$$i_1^a = -T i_2^a$$

$$v_2^a = T^T v_1^a$$

∴ The adjoint of a transformer is another transformer.

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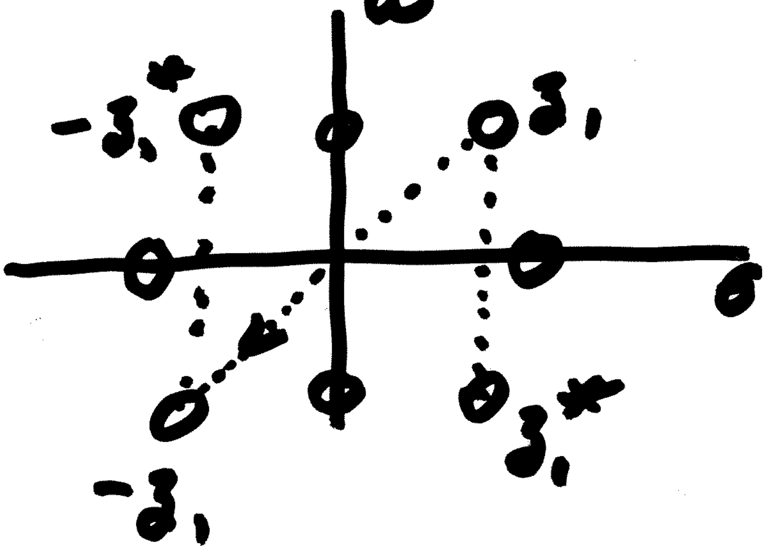
factoring polynomials

zeros of even parts

$f(s)$; $Ev\, f(s) = \dfrac{f(s)+f(-s)}{2}$

$= Ev(f(-s))$



Given a polynomial $P(s)$ with real coefficients factor out degree two real coefficient terms

$$P_1(s) = s^2 + a_1 s + a_0$$

$$P(s) = P_1(s) \cdot P_2(s) \,; \quad \delta[P] > 2$$

Ex: $P(s) = s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0$

$$P_1(s) = s^2 + a_1 s + a_0$$

$$P_2(s) = s^2 + d_1 s + d_0$$

here $P_1 \cdot P_2 = P$

$$(s^2 + a_1 s + a_0)(s^2 + d_1 s + d_0)$$

$$= s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0$$

Philosophy: Do a division, get remainder & force these to zero

$$s^4 + (a_1 + d_1)s^3 + (d_0 + a_1 d_1 + a_0)s^2 + (a_1 d_0 + a_0 d_1)s + a_0 d_0$$

$$= s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0$$

$c_3 = a_1 + d_1$

$c_2 = d_0 + a_1 d_1 + a_0$

$c_1 = a_1 d_0 + a_0 d_1$

$c_0 = a_0 d_0$

[illegible] given choose initial values for a_1 & a_0

Then

this gives

$$d_1 = c_3 - a_1$$

$$d_0 = c_2 - a_1 d_1 - a_0$$

but need not satisfy

$$c_1 = a_1 d_0 + a_0 d_1$$

$$c_0 = a_0 d_0$$

method 1: solves these for a_1 & a_0 and repeat

$$\begin{bmatrix} c_1 \\ c_0 \end{bmatrix} = \begin{bmatrix} d_0 & d_1 \\ 0 & d_0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$$

$$\Rightarrow \text{new} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = \frac{1}{d_0^2} \begin{bmatrix} d_0 & -d_1 \\ 0 & d_0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_0 \end{bmatrix}$$

may or may not converge

method 2:

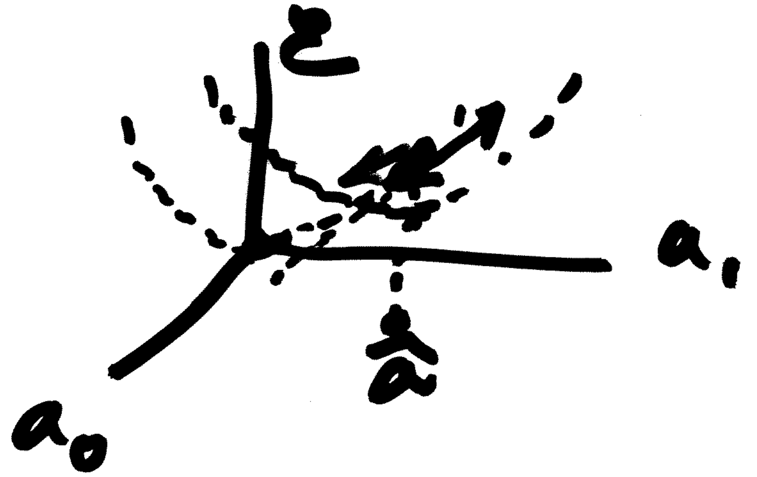
use p. 404

set up an "energy" function take its gradient with respect to a_1 and a_0

By long division get a remainder $r_1 \lambda + r_0$

set $\mathcal{E}(a_1, a_0) = r_0^2 + r_1^2$

$$= (r_0 - 0)^2 + (r_1 - 0)^2$$



$$\mathcal{E}(a_1, a_0) = \mathcal{E}(a) = \mathcal{E}(\hat{a})$$

$$+ \nabla_a^T \mathcal{E}(a)\Big|_{\hat{a}} (a - \hat{a}) + (\ldots$$

$$\mathcal{E}(a) = \mathcal{E}(\hat{a}) + \left[\frac{\partial \mathcal{E}}{\partial a_i}\right]^T (a-\hat{a})$$

$$+ [a-\hat{a}]^T \underbrace{\left[\frac{1}{2}\frac{\partial \mathcal{E}}{\partial a_i \, \partial a_j}\right]}_{\text{Hessian}} (a-\hat{a}) + \cdots$$

desire to force $\mathcal{E}(a) = 0$

as p. 404 shows – choose

$$\Delta a = -\mu \nabla_a \mathcal{E}\Big|_{\hat{a}} \; ; \; \mu = \frac{\alpha}{\|\nabla \mathcal{E}\|}$$

choose α small, say 0.1

(α = convergence factor)

To get remainder:

choose $P_1(s) = s^2 + a_1 s + a_0$

for a given

$$P(s) = \sum_{i=0}^{\delta} c_i \cdot s^i, \quad c_\delta = 1$$

then need to form

$$P_2(s) = \sum_{j=0}^{\delta-2} d_j s^j$$

$j+1=i$

$$P_1 \cdot P_2 = \sum_{j=0}^{\delta-2} d_j \cdot s^{j+2} + \sum_{j=0}^{\delta-2} a_1 d_j s^{j+1} + \sum_{j=0}^{\delta-2} a_0 d_j s^j$$

$j+2=i$

$i = 2$ to δ

$j = i$

$$= \sum_{i=2}^{\delta} d_{i-2} s^i + \sum_{i=1}^{\delta-1} a_1 d_{i-1} s^i + \sum_{i=0}^{\delta-2} a_0 d_i s^i$$

$i = 0 \qquad c_0 = a_0 d_0$

$i = 1 \qquad c_1 = a_1 d_0 + a_0 d_1$

$i > 1$

$$c_i = d_{i-2} + a_1 d_{i-1} + a_0 d_i$$

this gives the d_j down to 0&1

the remainders are

$$r_0 = c_0 - a_0 d_0$$

$$r_1 = c_1 - (a_1 d_0 + a_0 d_1)$$

Choose

$$\mathcal{E}(a_1, a_0) = \mathcal{E}(a) = r_0^2 + r_1^2$$

$$= (c_0 - a_0 d_0)^2 + (c_1 - a_1 d_0 - a_0 d$$

$$\nabla E = \begin{bmatrix} \frac{\partial E}{\partial a_0} \\ \frac{\partial E}{\partial a_1} \end{bmatrix} =$$

$$= \begin{bmatrix} 2 r_0 \cdot (-d_0) + 2 r_1 (-d_1) \\ 0 + 2 r_1 \cdot (-d_0) \end{bmatrix}$$

choose

$$a_{new} = \hat{a} + \left(- \frac{\alpha \nabla E}{\| \nabla E \|^2} \right)$$

$$= \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} - \begin{bmatrix} \quad \end{bmatrix} \alpha$$