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r.1

for RC

$$y(s) = k_\infty s + k_0 + \sum_{i=1}^{N} \frac{k_i s}{s+\omega_i^2}$$

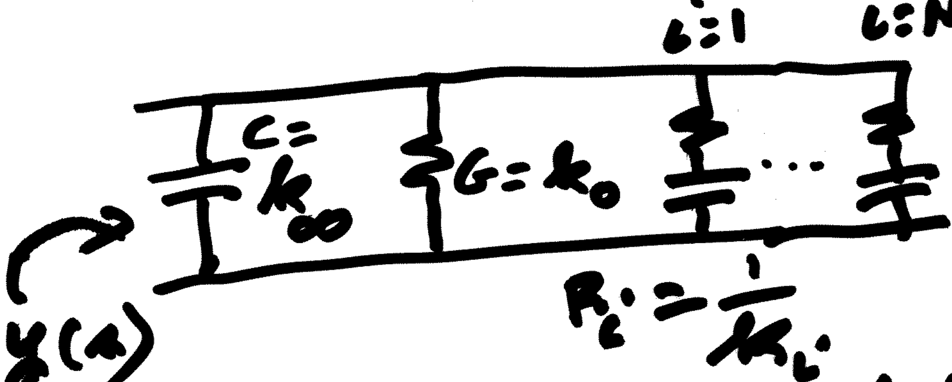
note $$\frac{y(s)}{s} = k_\infty + \frac{k_0}{s} + \sum_{i=1}^{N} \frac{k_i}{s+\omega_i^2}$$

here k_i are residues at poles of $\frac{y(s)}{s}$

$$F(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{\prod (s+p_i)^{r_i}}$$

a_1 = residu

$$= \frac{a_{r_i}}{(s+p_i)^{r_i}} + \frac{a_{r_i-1}}{(s+p_i)^{r_i-1}} + \dots \frac{a_1}{(s+p_i)} + \underbrace{F_1(s)}_{\text{no pole at } s=-p_i}$$



$$Z_i = \frac{s+\omega_i^2}{k_i s} = \frac{1}{k_i} + \frac{\omega_i^2/k_i}{s}$$

$$R_i = \frac{1}{k_i}$$

$$C_i = k_i/\omega_i^2$$

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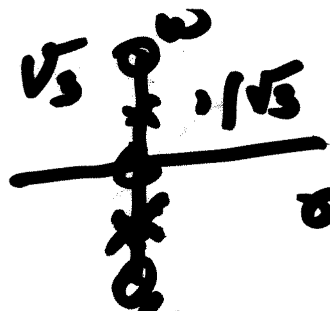
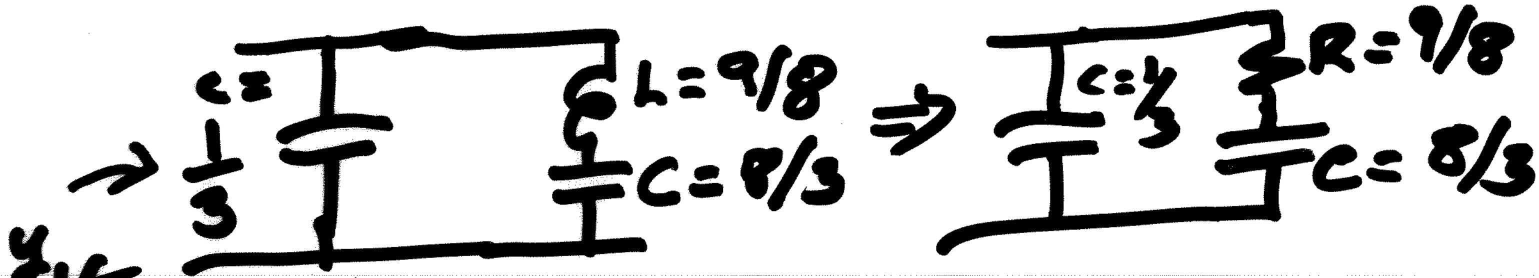
r. 2

$$y_{LC}(s) = \frac{s(s^2+3)}{3(s^2+1/3)}$$



$$= \frac{1}{3}s + \frac{k_1 s}{s^2+1/3} \quad ; \quad k_1 = \frac{s^2+3}{3}\Big|_{s^2=-\frac{1}{3}} = \frac{-\frac{1}{3}+3}{3} = 8/9$$

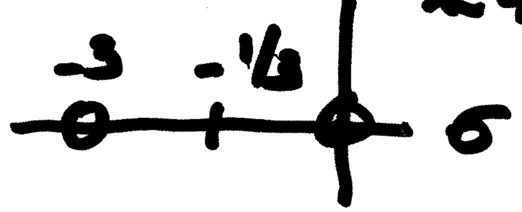
$$= \frac{1}{3}s + \frac{\frac{8}{9}s}{s^2+1/3}$$



$$y_{LC}(s) \to y_{RC} = \frac{1}{3}s + \frac{1}{\frac{9}{8} + \frac{3}{8s}}$$

$$= \frac{1}{3}s + \frac{8s}{9s+3} = \frac{1}{3}s + \frac{\frac{8}{9}s}{s+1/3}$$

$$= \frac{\frac{1}{3}s^2 + \frac{9}{9}s}{s+1/3} = \frac{s(s+3)}{3(s+1/3)}$$



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$$y_{RC} = \frac{s(s+3)}{3(s+1/3)} = \frac{1}{3}s + \frac{k_1}{s+1/3}$$

$$k_1 = \left[\frac{s(s+3)}{3} - \frac{1}{3}s(s+1/3)\right]_{s=-1/3}$$

$$= -\frac{1}{3}\left(-\frac{1}{3}+3\right)\cdot\frac{1}{3}$$

$$= -\frac{1}{27}(8) = -\frac{8}{27} < 0$$

but

$$\frac{y_{RC}}{s} = \frac{(s+3)}{3(s+1/3)} = \frac{1}{3} + \frac{\hat{k}_1}{s+1/3}$$

$$\hat{k}_1 = \left.\frac{s+3}{3}\right|_{s=-1/3} = \frac{-1/3+3}{3} = \frac{8}{9}$$

$$\frac{y_{RC}}{s} = \frac{1}{3} + \frac{8/9}{s+1/3} \Rightarrow y_{RC} = \frac{1}{3}s + \frac{\frac{8}{9}s}{s+1/3}$$

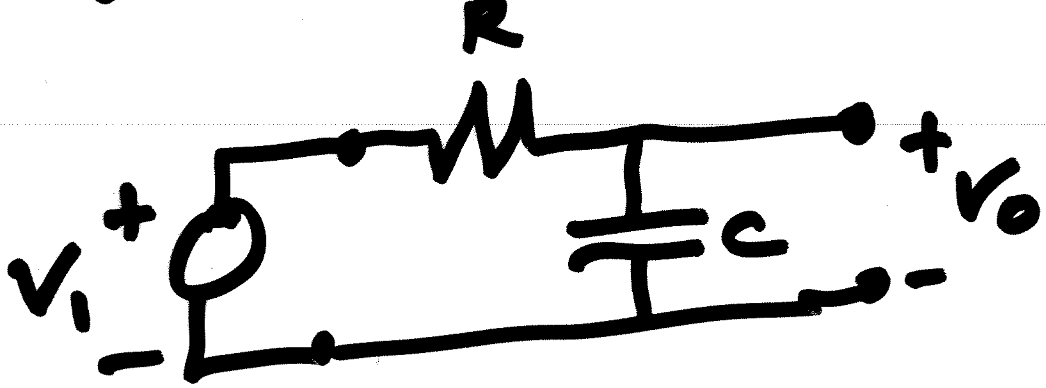
which can be synthesized

Sensitivity

$$S_x^{T(s)} = \text{sensitivity of } T(s) \text{ to } x$$

$$= \frac{x}{T(s)} \frac{\partial T(s)}{\partial x}$$

will use the adjoint circuit to find $\partial T(s)/\partial x$:



$$\frac{V_o}{V_1}(s) = T(s) = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{1 + RCs}$$

$$= \frac{G}{G + Cs}, \quad G = 1/R$$

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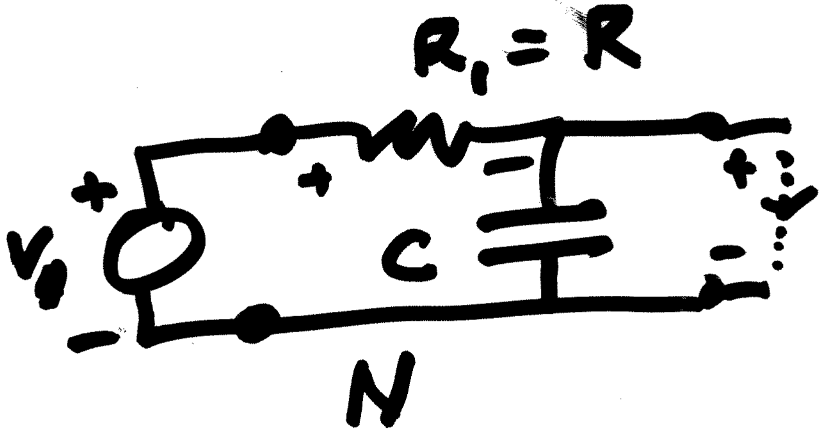
$$S_G^{V_0/V_1} = \frac{G}{\left(\frac{G}{G+CA}\right)} \cdot \frac{\partial T}{\partial G}$$

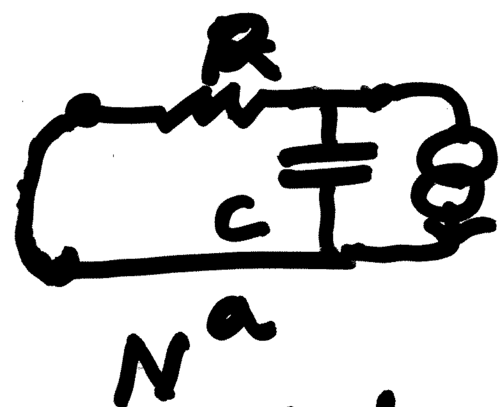
$$\frac{\partial T}{\partial G} = \frac{1}{G+CA} - \frac{G}{(G+CA)^2}$$

$$= \frac{CA}{(G+CA)^2}$$

$$S_G^{V_0/V_1} = (G+CA) \cdot \frac{CA}{(G+CA)^2} = \frac{CA}{G+CA}$$

. . . .





= adjoint

$$v_b^T i_b^a - v_b^{Ta} i_b = 0$$

$$v_b = v_2 = \begin{bmatrix} v_1 \\ v_0 \\ \hat{v}_b \end{bmatrix}, \quad v_b^a = \begin{bmatrix} v_1^a \\ v_0^a \\ \hat{v}_b^a \end{bmatrix}; \quad i_b = \begin{bmatrix} i_1 \\ i_0 \\ \hat{i}_b \end{bmatrix} = i_2$$

$$i_b^a = \begin{bmatrix} i_1^a \\ i_0^a \\ \hat{i}_b^a \end{bmatrix}$$

$$v_1 i_1^a - v_1^a i_1$$
$$+ v_0 i_0^a - v_0^a i_0$$
$$\hat{v}_b^T \hat{i}_b^a - \hat{v}_b^{Ta} \hat{i}_b = 0$$

if $\hat{Y}_b$ exists

where

$$\hat{v}_b^T \hat{i}_b^a - \hat{v}_b^{Ta} \hat{i}_b =$$
$$\hat{v}_b^T \hat{Y}_b^a \hat{v}_b^a - \hat{v}_b^{Ta} \hat{Y}_b \hat{v}_b$$
$$= \hat{v}_b^T \left[\hat{Y}_b^a - \hat{Y}_b^T \right] \hat{v}_b^a$$

$$\frac{\partial v_o}{\partial G} = \frac{\partial v_o}{\partial \hat{g}_{11}}$$

fix $V_1 = v_1$, $\frac{\partial i_1^a}{\partial G} = 0$, $\frac{\partial v_1^a}{\partial G} = 0$
& N^a

$$\frac{\partial i_o^a}{\partial G} = 0, \quad \frac{\partial v_o^a}{\partial G} = 0$$

$$\hat{Y}_b = \begin{bmatrix} G & 0 \\ 0 & sC \end{bmatrix}$$

Take $\frac{\partial}{\partial G}$ of the $v_n^T i_n^a - v_n^{aT} i_n = 0$

$$0 - v_1^a \cdot \frac{\partial i_1}{\partial G} + \frac{\partial v_o}{\partial G} \cdot i_o^a + 0 - 0$$

$-$ $\overset{\shortparallel}{0}$ as part
a short

$$+ \frac{\partial \hat{v}_b^T}{\partial G} \cdot \hat{Y}_b^a \hat{v}_b^a + \hat{v}_b \cdot \underset{\shortparallel\, 0}{\frac{\partial \hat{Y}_b^a}{\partial G}} v_b^a + \hat{v}_b^T \hat{Y}_b^a \overset{0\, \shortparallel}{\frac{\partial \hat{v}_b^a}{\partial G}}$$

$$- \frac{\partial \hat{v}_b^T}{\partial G} \cdot \hat{Y}_b^T \hat{v}_b^a - \underset{G \text{ branch}}{\hat{v}_b} \cdot 1 \underset{G \text{ branch}}{\hat{v}_b^a} - \hat{v}_b^T \hat{Y}_b^T \overset{0\, \shortparallel}{\frac{\partial \hat{v}_b^a}{\partial G}}$$

or $\frac{\partial v_0}{\partial G} \cdot i_0^a - \hat{v}_{0_1} \hat{v}_{0_1}^a$

$+ \frac{\partial \hat{v}_0}{\partial G}^T \underbrace{[\hat{y}_0^a - \hat{y}_0^T]}_{\text{force} = 0} \hat{v}_0^a = 0$

$y^a \triangleq y^T$

$\Rightarrow \frac{\partial v_0}{\partial G} \cdot i_0^a = \hat{v}_{0_1} \cdot \hat{v}_{0_1}^a$

if $V_1 = 1$ $\frac{\partial v_0}{\partial G} = \frac{\partial\, v_0/V_1}{\partial G} = \frac{\partial T}{\partial G}$

$\therefore$ fix $i_0^a = 1$ say analyze N & N^a to get $\hat{v}_{0_1}$ & $\hat{v}_{0_1}^a$