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Richards function, p. 361

Hurwitz polynomial:
$P(s)$, has no zeros
in $\sigma \geq 0$
↑ = makes strictly Hurwitz

o.c. nat. freq. or s.c. nat
of lossless circuit
terminated in R are
only in $\sigma < 0$

$P(s) = Ev(s) + Od(s)$

$2Ev(s) = P(s) + P(-s) = P + P_*$

$2Od(s) = P(s) - P(-s) = P - P_*$

lower $*$ means $s \to -s$ and
call Hurwitz conjugate

$$P(s) = Ev(s)\left[1 + \frac{Od(s)}{Ev(s)}\right]$$

$\frac{Od(s)}{Ev(s)}$ = $y(s)$ of a lossless circuit.

$$s^2 - 1 = (s+1)(s-1)$$

Given $P(s)$, is it Hurwitz

form $y(s) = \frac{Od(s)}{Ev(s)}$

if $\delta[P(s)] = \delta[y(s)]$ then

δ = degree

if $y(s)$ is lossless then

$P(s)$ is Hurwitz

Ex
$$P(s) = s^3 + 3s^2 + 3s + 1$$
$$= \underbrace{(3s^2 + 1)}_{Ev} + \underbrace{s(s^2 + 3)}_{Od}$$

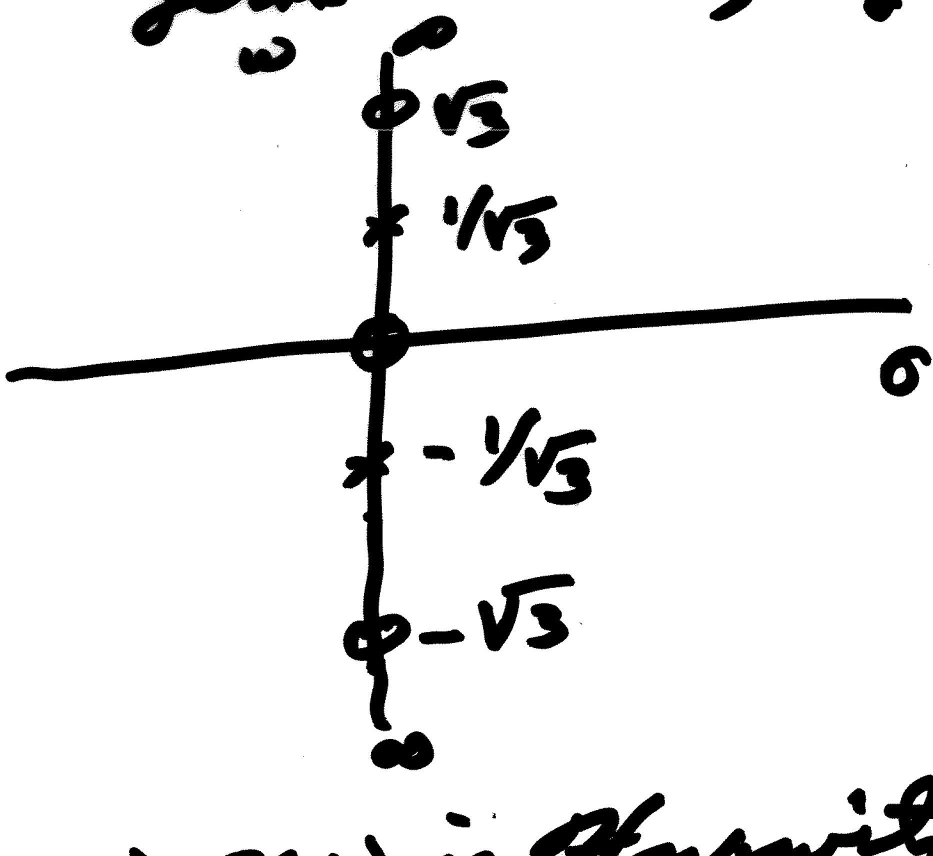
$$y(s) = \frac{s(s^2+3)}{3s^2+1} = \frac{\text{od}}{\text{Ev}}$$

$$y(1) = \frac{1(4)}{4} = +1 > 0$$

$$y(s) + y(-s) = 0$$

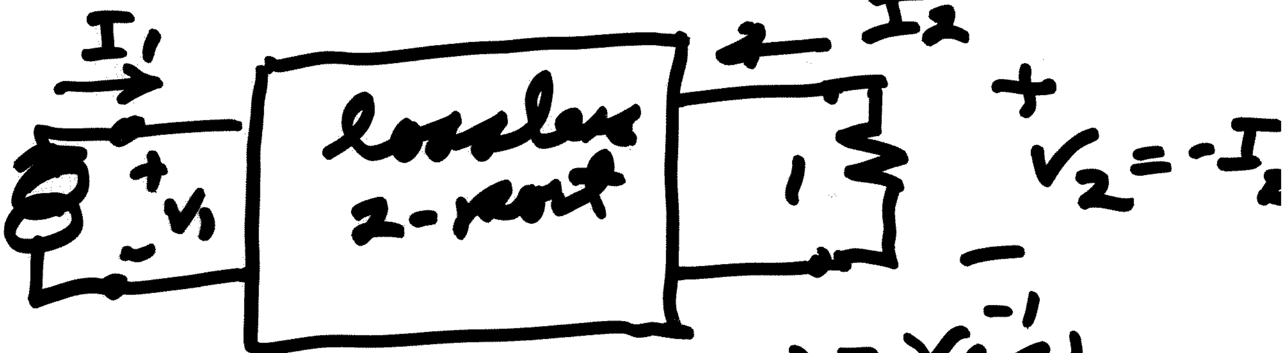
poles: $s = \pm j\,1/\sqrt{3},\ \infty$

zeros: $s = 0,\ \pm j\sqrt{3}$



$\Rightarrow P(s)$ is Hurwitz

Synthesis of transfer functions



$Y(s)$, $Z(s) = Y^{-1}(s)$

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$V_2 = z_{21} I_1 - z_{22} V_2$$

$$(1 + z_{22}) V_2 = z_{21} I_1$$

$$\frac{V_2}{I_1}(s) = \frac{z_{21}}{1 + z_{22}}$$

} Hurwitz if as above

z_{21} must be odd in s

$$Z(s) = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

$v^T Z v = z(s)$ must be PR if v real & for any v.

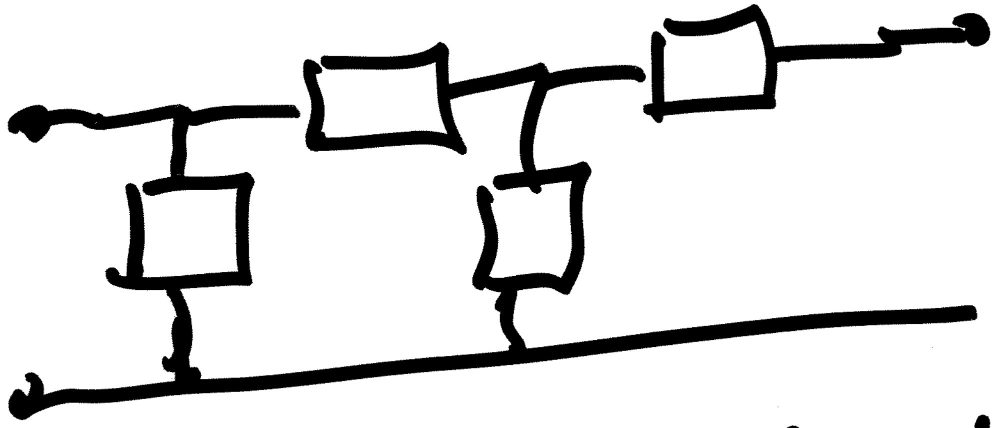
$\Rightarrow$ requires all poles in z_{12} and z_{21} must be in z_{11}, z_{22} as z_{11} & z_{22} are lossless

$\Rightarrow$ all poles of z_{12} and z_{21} are on $j\omega$ axis; make partial fraction expansion of

$$z_{21} = \frac{k_0}{s} + k_\infty s + \sum \frac{k_i}{s+j\omega_i} + \frac{k_i^*}{s-j\omega_i}$$

some coeff can be negative but still $z_{21}(s) = -z_{21}(-s)$

synthesis for lossless ladders:



zeros of transmission are where series arms are open circuits ($Z = \infty$) and shunt arms are short circuits ($Z = 0$)

$\therefore$ interested in poles of Z in series & poles of Y in shunt arms.

Ex:

$$\frac{V_2}{I_1}(s) = \frac{c}{s^3+3s^2+3s+1}$$

c = real constant (has 3 zeros of transmission at $s=\infty$)

$$= \frac{z_{21}}{1+z_{22}}$$

need to divide out the odd part of the denominator to make z_{21} odd

$$\frac{V_2}{I_1} = \frac{c/(s^3+3s)}{1+\dfrac{3s^2+1}{s^3+3s}}$$

$$z_{21} = \frac{c}{s(s^2+3)} \;;\; z_{22} = \frac{3(s^2+1/3)}{s(s^2+3)}$$

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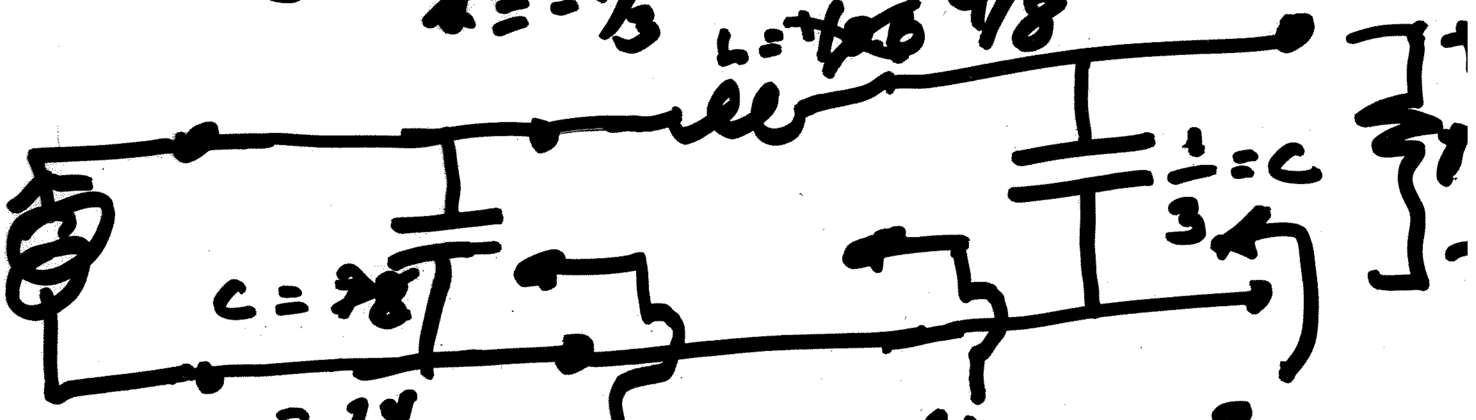
synthesize $z_{22}(s)$ so shunt arms have $y = \infty$ at $s = \infty$, series arms have $z = \infty$ at ∞.

$$\frac{1}{z_{22}} = y'_{22} = \frac{s^3+3s}{3s^2+1} = \frac{s}{3} + \frac{\overset{c\ 8/9}{k_1} s}{s^2+1/3}$$

$$k_1 = \left.\frac{s^2+3}{3}\right|_{s^2=-1/3} = \frac{-1/3+3}{3} = \cancel{\frac{25}{27}}\ 8/9$$



$$z''' = \frac{1}{\cancel{\quad} s\ 9/8}$$

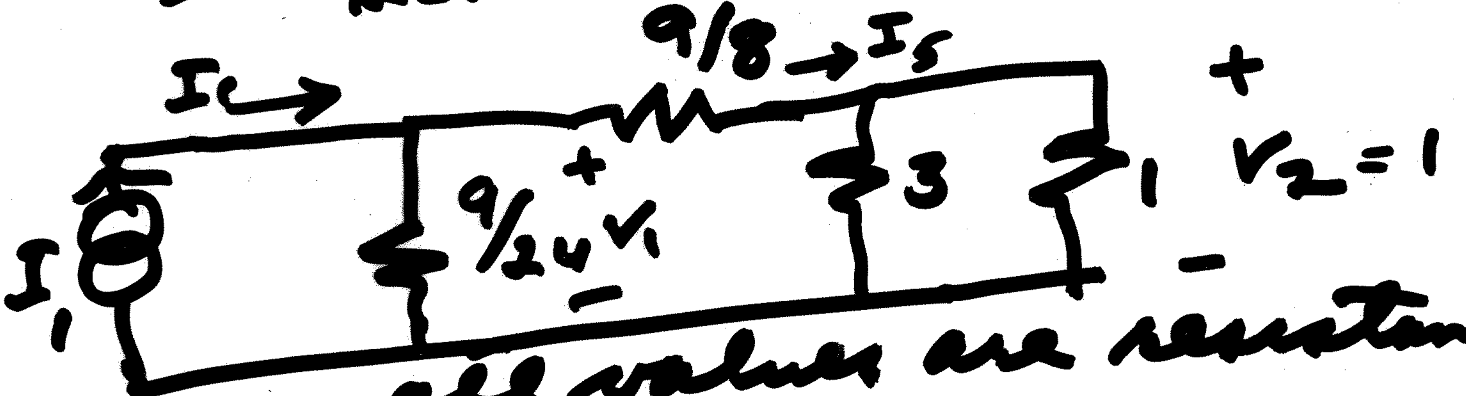
$$y''_{22} = \frac{\cancel{26}\, s\ 8/9}{s^2+1/3}$$

$$\frac{V_2}{I_1} = \frac{k}{s^3+3s^2+3s+1}$$

to find k: let $s = 1$

then $Ls = L$, $Cs = C$ (as resistors)

$$\left.\frac{V_2}{I_1}(s)\right|_{s=1} = \frac{k}{1+3+3+1} = \frac{k}{8}$$



all values are resistances

$I_5 = 1\cdot(1+1/3) = 4/3$ amp

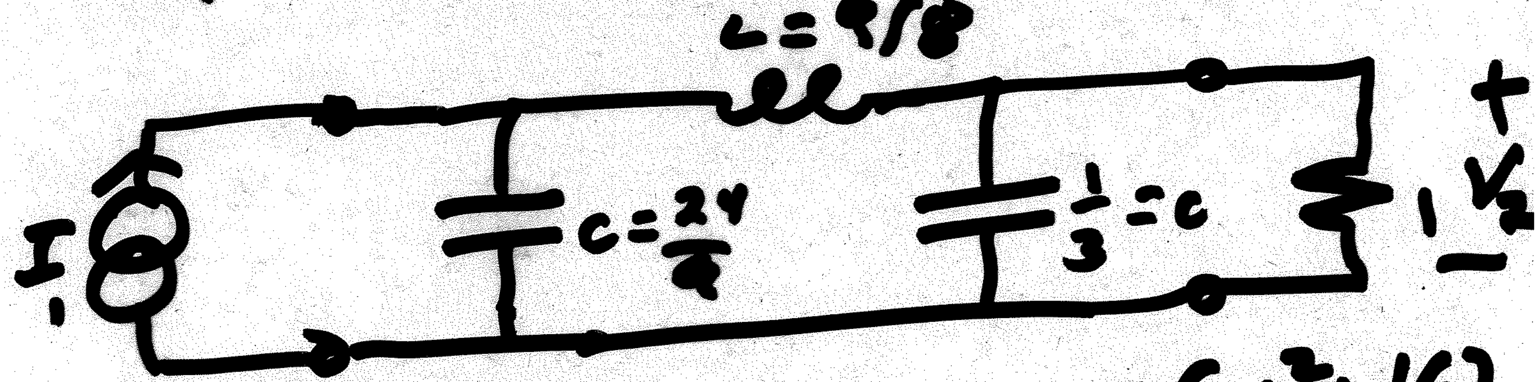
$V_1 = V_2 + I_5 \cdot 9/8$
$= 1 + 4/3 \times 9/8 = 1 + 3/2 = 5/2$ volt

$I_1 = \frac{24}{9}\cdot V_1 + I_5 = \frac{24}{9}\times\frac{5}{2} + \frac{4}{3} = \frac{60}{9} + \frac{12}{9}$

$= \frac{72}{9} = 8$ amp or $\frac{V_2}{I_1} = \frac{1}{8} = \frac{k}{8}$

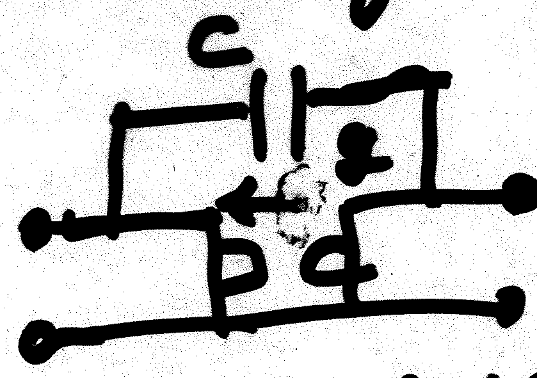
$k = 1$

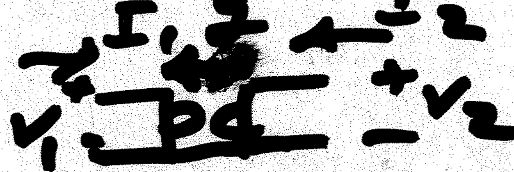
$$\frac{V_2}{I_1}(s) = \frac{1}{s^3+3s^2+3s+1}$$



$$z_{21} = \frac{k=1}{s(s^2+3)}, \quad z_{22} = \frac{3(s^2+1/3)}{s(s^2+3)}$$

Richards' function:





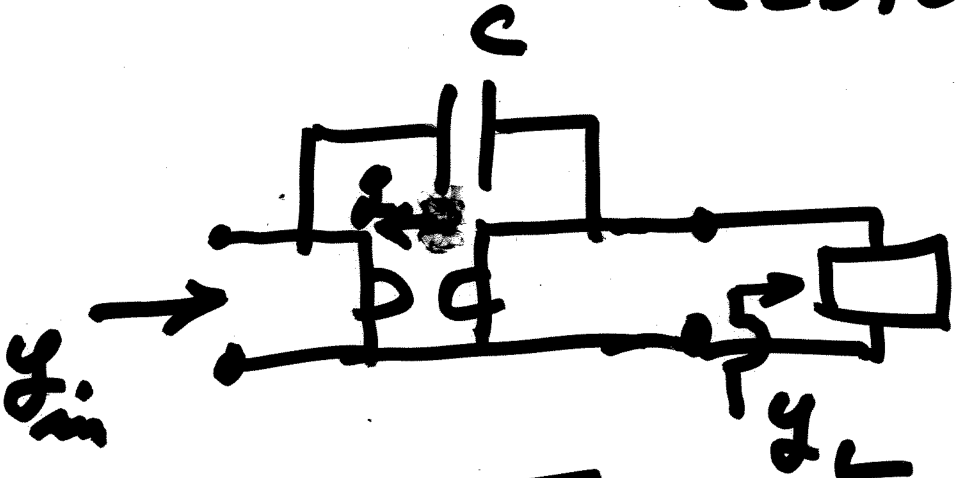
$$Y_{gy} = \begin{bmatrix} 0 & -g \\ +g & 0 \end{bmatrix}$$

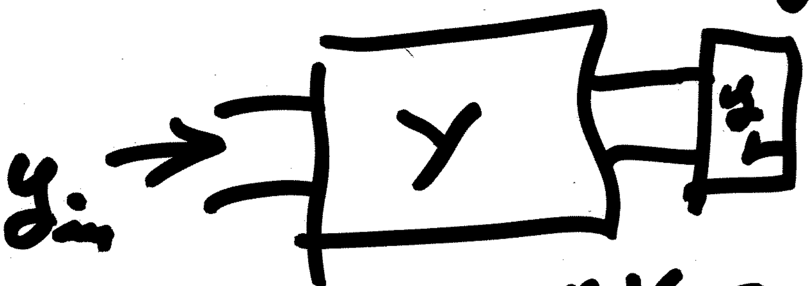
$$Y(s) = \begin{bmatrix} Cs & -(g+Cs) \\ g-Cs & Cs \end{bmatrix}$$

$$Y_g + Y_{gy}^T = 0_2$$

$$Y_{cap} = \begin{bmatrix} Cs & -Cs \\ -Cs & Cs \end{bmatrix}$$

a lossless
2-port





$$I_2 = -y_L V_2 =$$

$$-y_L V_2 = I_2 = y_{21} V_1 + y_{22} V_2$$

$$-(y_L + y_{22})^{-1} y_{21} V_1 = V_2$$

$$I_1 = y_{in} V_1 = y_{11} V_1 + y_{12} V_2$$

$$= \left\{ y_{11} + y_{12} \left[-(y_L + y_{22})^{-1} y_{21} \right] \right\} V_1$$

$$y_{in} = \frac{\Delta_y + y_{11} y_L}{y_L + y_{22}}$$

$$\Delta y = y_{11} y_{22} - y_{12} y_{21}$$

$$= (Cs)^2 + (g + Cs)(g - Cs)$$

$$= g^2$$

$$\therefore \; y_{in} = \frac{g^2 + Cs \cdot y_L}{y_L + Cs}$$ is PR if y_L is PR

to get y_L of y_{in}:

$$y_{in} \cdot y_L + Cs \cdot y_{in} = g^2 + Cs\, y_L$$

$$(y_{in} - Cs)\, y_L = g^2 - Cs \cdot y_{in}$$

$$y_L(s) = \frac{g^2 - Cs \cdot y_{in}(s)}{y_{in}(s) - Cs}$$ here y_L is PR if

identify with a Richards' y_{in} is PR function p. 361

$$= \frac{k\, y(k) - s\, y(s)}{k\, y(s) - s\, y(k)}$$