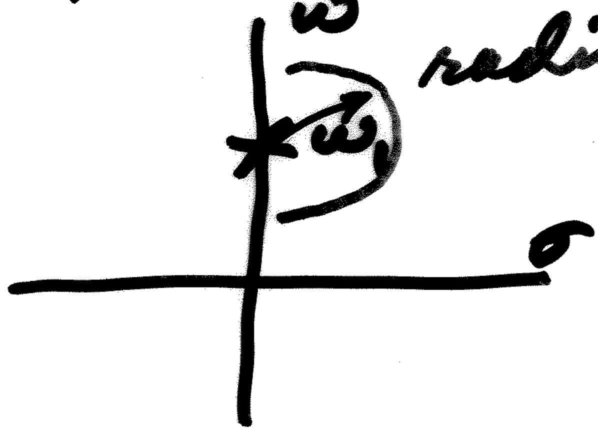
lossless 1-port passive synthesis $\Rightarrow$ $y(s)$ rational, positive coefficients

$$y(s) = N(s)/D(s)$$

$$y(s) + y(-s) = 0 \quad \text{lossless}$$

poles are all on the $j\omega$ axis



near a pole at $j\omega_1$

$y(s)$ near $j\omega_1$

$$\sim \frac{k}{(s-j\omega_1)^m}$$

$$2\,\mathrm{Re}\,y(s) \approx \frac{k}{(s-j\omega_1)^m} + \frac{k^*}{(s^*+j\omega_1)^m}$$

near $j\omega_1$

$$\overset{\text{near}}{\approx} 2\,\mathrm{Re}\left(\frac{|k|\, e^{j(\angle k - m \angle s - j\omega_1)}}{|s-j\omega_1|^m}\right)$$

$$\approx \frac{|k|}{r^m} \cos\left(\angle k - m\underbrace{(\angle s-j\omega_1)}_{-90 < \angle s-j\omega_1 < 90}\right)$$

as sweep $\measuredangle s - j\omega_1$

the $\cos(\measuredangle k - m\measuredangle[s-j\omega_1])$

will be negative if $\measuredangle k \neq 0$

and $m \neq 1$

$\therefore$ $y(s)$ near $j\omega_1 \Rightarrow \dfrac{|k| = k}{(s-j\omega_1)^1}$

$$y(s) = \frac{k}{s-j\omega_1} + \frac{k}{s+j\omega_1} + \cdots$$

$$= \frac{2ks}{s^2+\omega_1^2} + \cdots$$

$$\therefore\ y(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2}$$

where $k_0 \geq 0$, $k_\infty \geq 0$, $k_i \geq 0$

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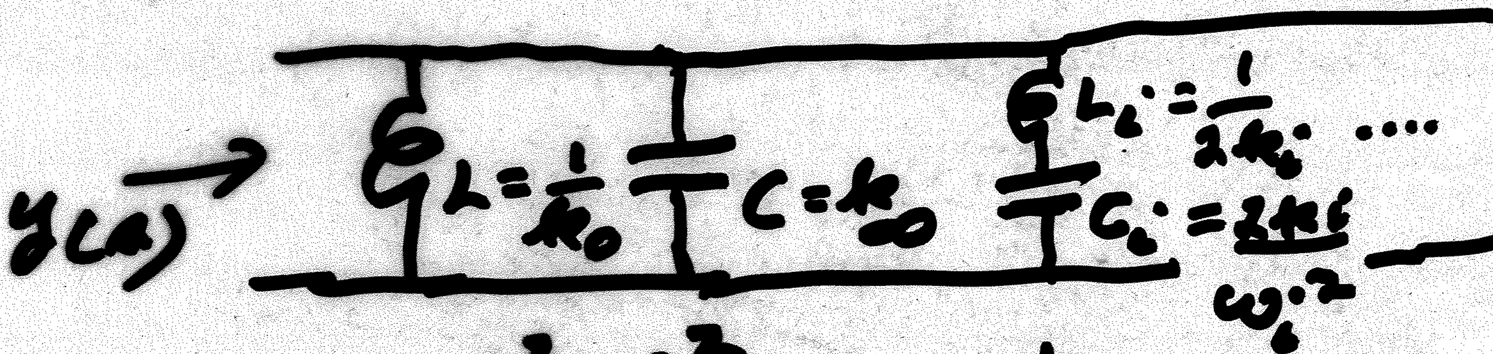
synthesis



$$Z_i(s) = \frac{s^2+\omega_i^2}{2k_i s} = \frac{s}{2k_i} + \frac{1}{\frac{2k_i}{\omega_i^2} s}$$

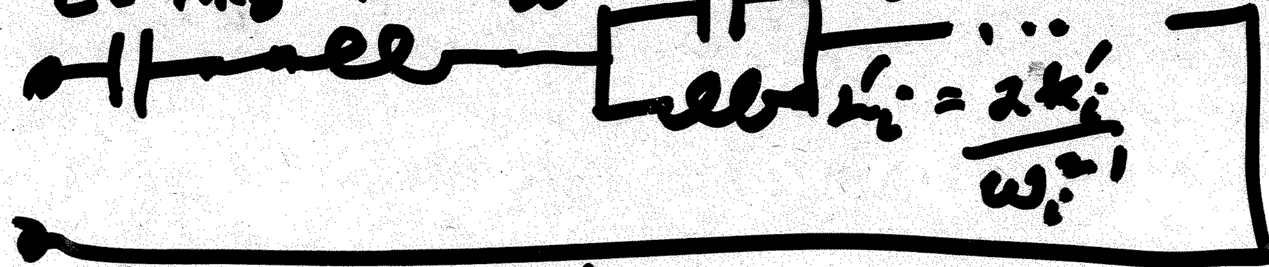
this is a 2nd Foster form

1st Foster:

assume $Z(s)$ given

$$Z(s) = \frac{k_0'}{s} + k_\infty' s + \sum_{i=1}^{n'} \frac{2k_i' s}{s^2+\omega_i'^2}$$

$C' = 1/k_0$ $\quad L' = k_\infty'$ $\quad C_i' = 1/2k_i'$



$$Y_i'(s) = \frac{s^2+\omega_i'^2}{2k_i' s} = \frac{s}{2k_i'} + \frac{1}{\frac{2k_i'}{\omega_i'^2} s}$$

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poles of $z(s)$ are
zeros of $y(s) = 1/z(s)$

for lossless, all poles and all zeros are on the $j\omega$ axis and poles and zeros alternate.

$$z(j\omega) = j\omega k_\infty + \frac{k_0}{j\omega} + \sum_{i=1}^{n} \frac{2k_i \cdot j\omega}{\omega_i^2 - \omega^2}$$

$$= jX(\omega)$$

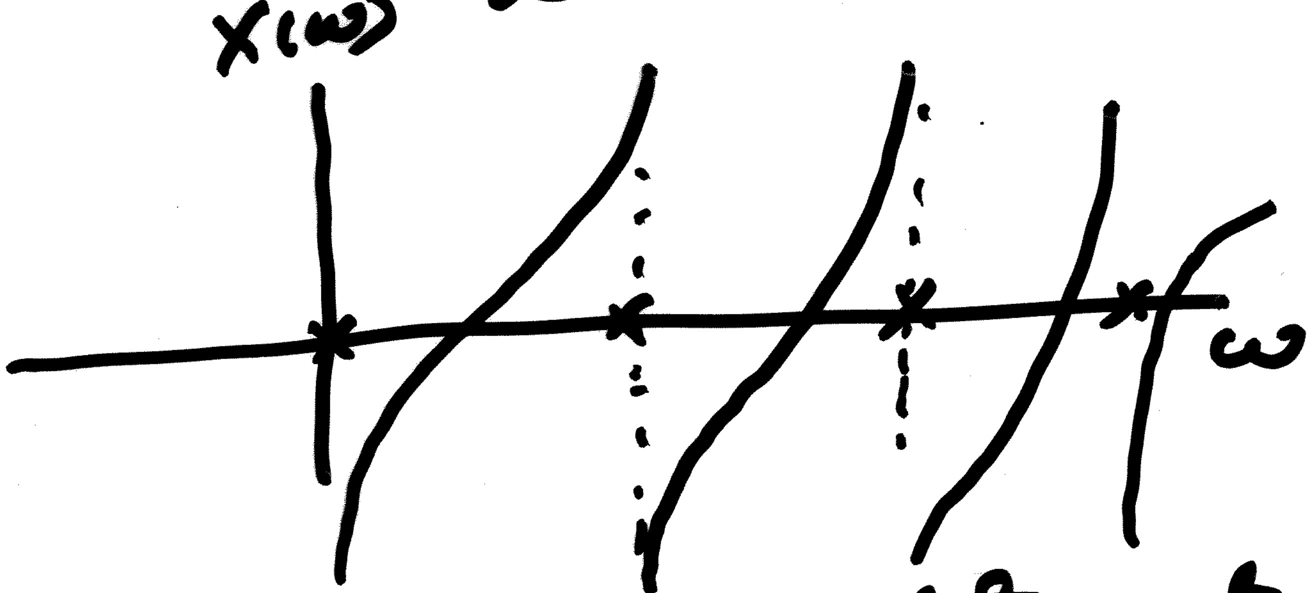
$$\frac{dX(\omega)}{d\omega} = k_\infty + \frac{k_0}{\omega^2} + \sum_{i=1}^{n} \frac{2k_i(\omega_i^2 + \omega^2)}{(\omega_i^2 - \omega^2)^2}$$

$$\frac{d \frac{\omega}{\omega_i^2 - \omega^2}}{d\omega} = \frac{1}{\omega_i^2 - \omega^2} - \frac{\omega(-2\omega)}{(\omega_i^2 - \omega^2)^2}$$

$$= \frac{\omega_i^2 - \omega^2 + 2\omega^2}{(\omega_i^2 - \omega^2)^2} = \frac{\omega_i^2 + \omega^2}{(\omega_i^2 - \omega^2)^2}$$

$$\geq 0$$

$\therefore$ slope $\dfrac{dX(\omega)}{d\omega} \geq 0$ or ∞



$\therefore$ zeros of $X(\omega)$ alternate with the poles these are zeros of $Z(s)$

$\therefore$ poles and zeros of lossless positive real rational function alternate and are on the $j\omega$ axis.

$$y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+2)}$$

this is a lossless admittance

1st Foster synthesis

$$z(s) = \frac{1}{y} = \frac{s(s^2+2)}{(s^2+1)(s^2+4)}$$

$$= \frac{2k_1 s}{s^2+1} + \frac{2k_2 s}{s^2+4}$$

To find k_1 multiply by $\left.\frac{s^2+1}{s}\right|_{s^2=-1}$

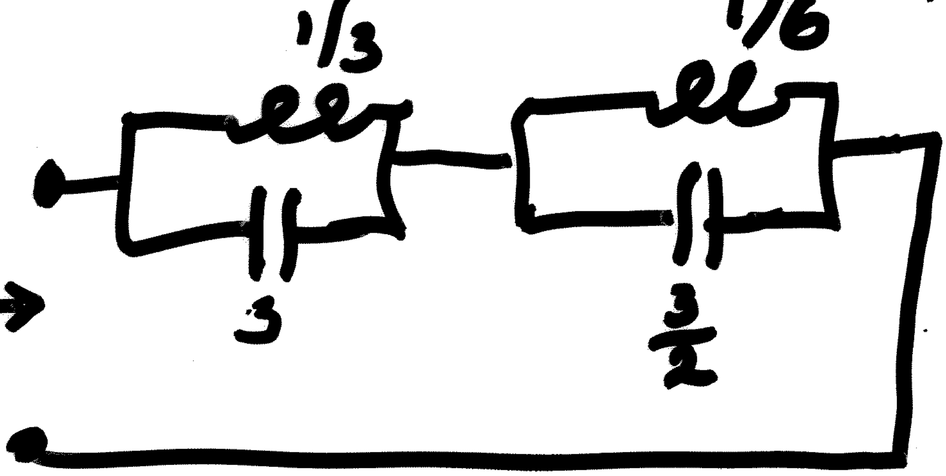
$$\left.\frac{s^2+2}{s^2+4}\right|_{s^2=-1} = 2k_1 + \left.\underbrace{\frac{s^2+1}{s^2+4} \cdot \frac{s}{s} \cdot 2k_2}_{0}\right|_{s^2=-1}$$

$$2k_1 = \frac{-1+2}{-1+4} = \frac{1}{3}$$

$$2k_2 = \left.\frac{s^2+2}{s^2+1}\right|_{s^2=-4} = \frac{-2}{-3} = \frac{2}{3}$$

$$z(s) = \frac{\frac{1}{3}s}{s^2+1} + \frac{\frac{2}{3}s}{s^2+4}$$

$z(s) \rightarrow$



1st Foster

2nd Foster

$$y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+2)}$$

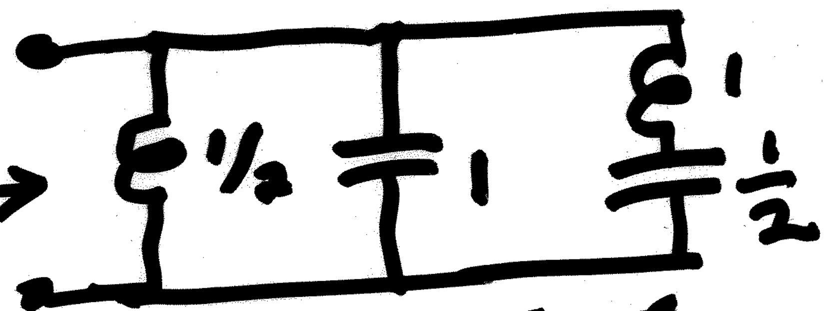
$$= \frac{k_0}{s} + 1 \cdot s + \frac{2k\,s}{s^2+2}$$

$$= \frac{2}{s} + s +$$

$$2k = \left.\frac{(s^2+1)(s^2+4)}{s^2}\right|_{s^2=-2} = \frac{(-1)(2)}{-2} = 1$$

$$y(s) = \frac{2}{s} + s + \frac{s}{s^2+2}$$

$Z = \frac{1}{Y}$ →



2nd Foster

1st Cauer: removes poles at ∞

$$Y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+2)} = \frac{s^4+5s^2+4}{s^3+2s}$$

$$\begin{array}{r|l} & s \\ s^3+2s & s^4+5s^2+4 \\ & s^4+2s^2 \\ \hline & 3s^2+4 \end{array}$$

$$\begin{array}{r|l} & \frac{1}{3}s \\ 3s^2+4 & s^3+2s \\ & s^3+\frac{4}{3}s \\ \hline & \frac{2}{3}s \end{array}$$

$\frac{2}{3}s \,\big)\, 3s^2+4$

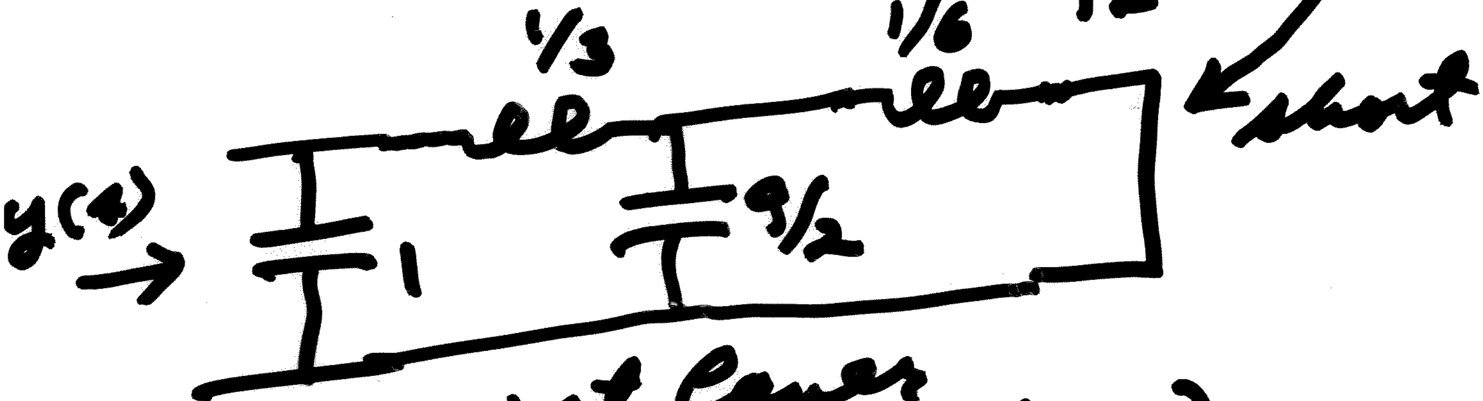
$Y(s)$

$$\begin{array}{r|l} & \frac{9}{2}s \\ \frac{2s}{3} & 3s^2+4 \\ & 3s^2 \\ \hline & 4 \end{array}$$

$$\begin{array}{r|l} & \frac{2}{12}s \\ 4 & \frac{2}{3}s \\ & \frac{2}{3}s \\ \hline & 0 \end{array}$$

$$Y(s) = s + \frac{3s^2+4}{s^3+2s} = s + \frac{1}{\dfrac{s^3+2s}{3s^2+4}}$$

$$y(s) = s + \cfrac{1}{\underset{z_1 \rightarrow}{\tfrac{1}{3}s} + \cfrac{1}{\underset{y_2 \rightarrow}{\tfrac{9}{2}s} + \cfrac{1}{\tfrac{2}{12}s + 0}}}$$



1st Cauer
(removes poles at ∞)

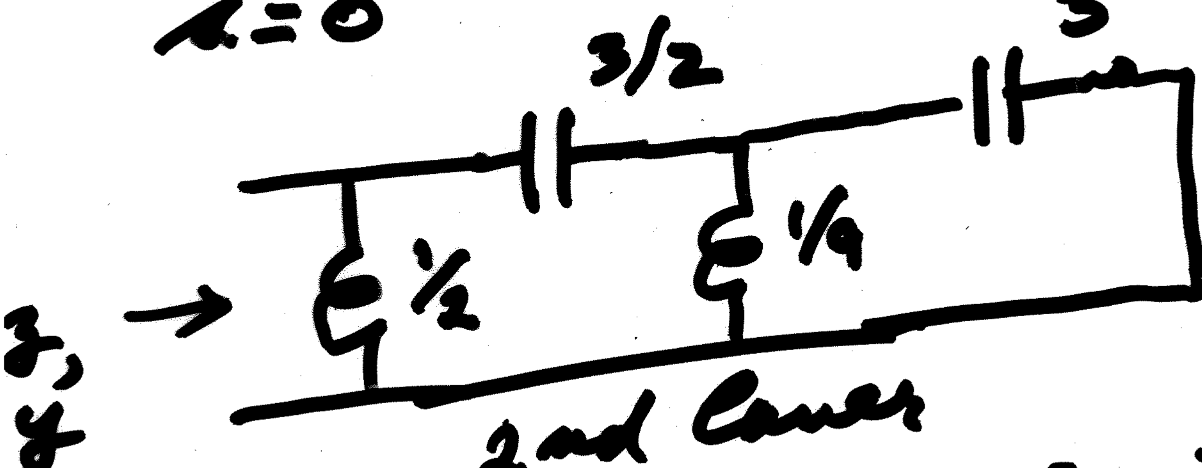
2nd Cauer — remove poles at zero

$$y(s) = \frac{4 + 5s^2 + s^4}{2s + s^3}$$

$$2s + s^3 \,\overline{\big)\, 4 + 5s^2 + s^4} \quad \frac{4}{2s}$$
$$\underline{4 + 2s^2}$$
$$3s^2 + s^4 \,\overline{\big)\, 2s + s^3} \quad \frac{2}{3s}$$
$$\underline{2s + \tfrac{2}{3}s^3}$$
$$\tfrac{1}{3}s^3 \,\overline{\big)\, 3s^2 + \cancel{s^4}} \quad \frac{9}{s}$$
$$3s^2 \cancel{s}$$
$$s^4 \,\overline{\big)\, \tfrac{1}{3}s^3} \quad \frac{1}{3s}$$

$$y(s) = \frac{4}{2s} + \cfrac{1}{\cfrac{2}{3s} + \cfrac{1}{\cfrac{9}{s} + \cfrac{1}{\cfrac{1}{3s + 0}}}}$$

a continued fraction expansion about $s = 0$



2nd Cauer

$$y(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+1)}$$

all 4 synthesize this

$\delta[y] = 4 =$ degree of $y(s)$

all are minimal in the number of "reactive" elements

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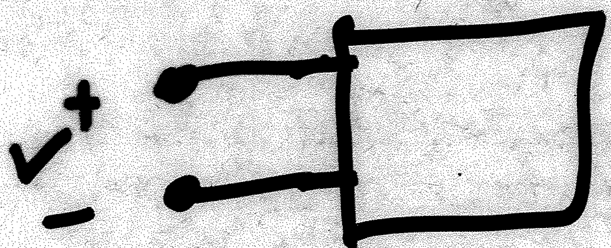
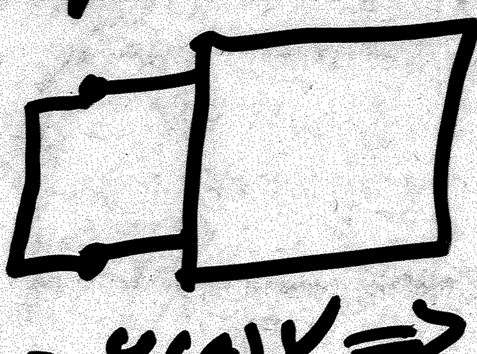
natural frequencies



$V = Z(s) \cdot I$ if $I = 0$ & $V \neq 0$
$\Rightarrow$ s is at a pole of $Z(s)$

$\Rightarrow$ poles of $Z(s)$ are open circuit $(I = 0)$ natural frequencies.



$I = Y(s) V \Rightarrow$ if $V = 0$, $I \neq 0$
then have $Y(s) \to \infty$ or
poles of $Y(s)$ are short circuit natural frequencies

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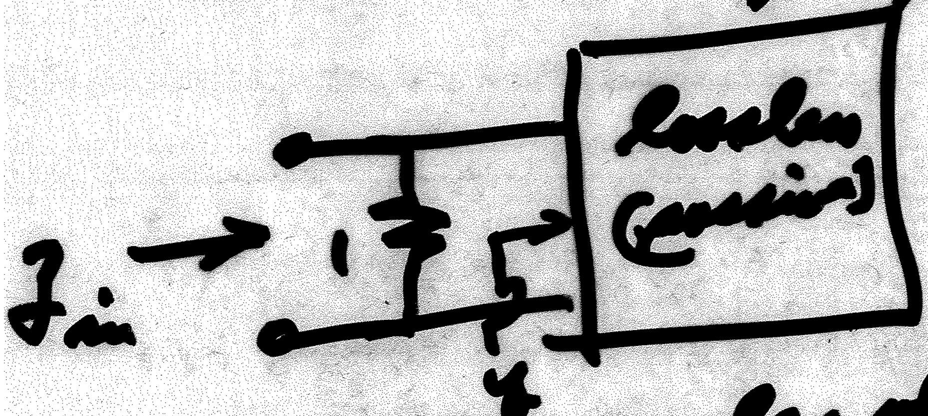
o.c. nat. freq. of



desire poles of $Z_{in} = \frac{1}{y_{in}}$

$$= \frac{1}{1 + y(s)}$$

∴ o.c. nat. freq. where $y(s) \to -1$

can't be in RHP if $y(s)$ is P.R.

can't be on $j\omega$ axis if lossless

as $y(j\omega) = jB(\omega)$

∴ no poles in $\sigma \geq 0$

let $y(s) = N/D$

$Z_{in} = \frac{D}{N + D}$ has $N+D$ as an Hurwitz polynomial