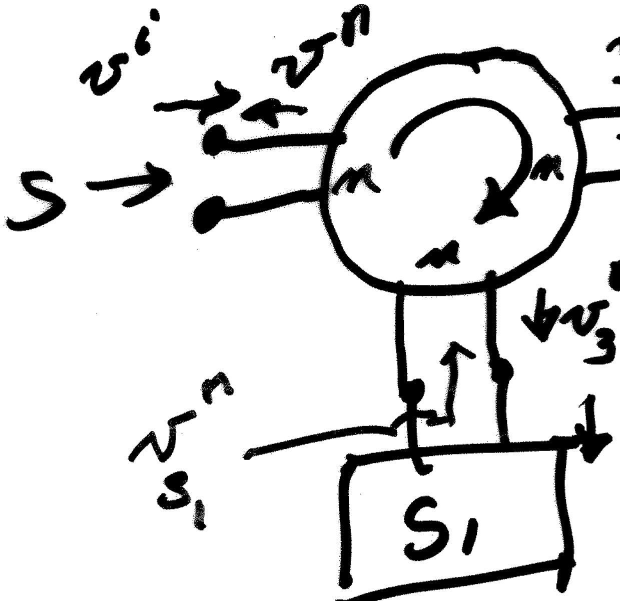
EE610 10/02/02

v.1

1. Given a circulator can make

$S_1 \cdot S_2 = S$

Circulator → 3n-port



$v^r \rightarrow v_{S_2}^i \Rightarrow v_{S_2}^r = S_2 \cdot v_{S_2}^i$

$v_3^r = v_2^i = v_{S_2}^r$

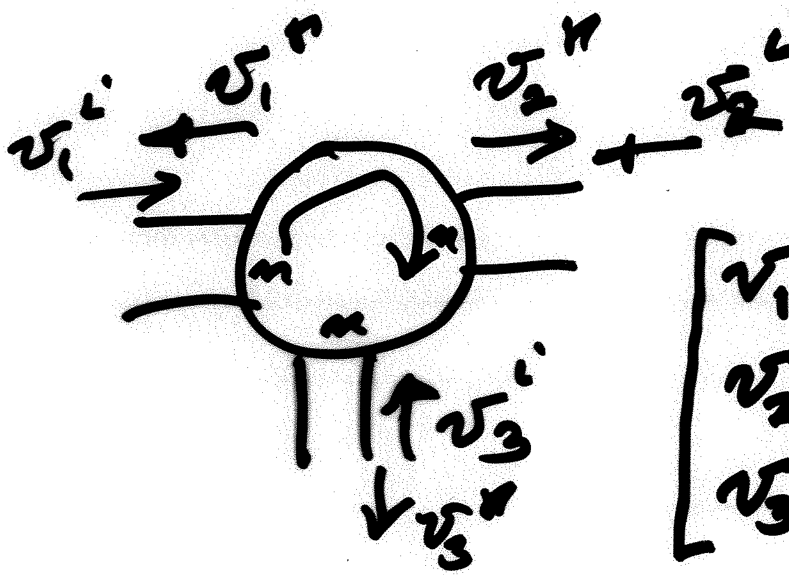
$v_{S_1}^i = v_3^r \Rightarrow$

$v_{S_1}^r = S_1 \cdot v_{S_1}^i$

$v^i, v_{S_2}^r$, etc. are n-vectors

$$v^r = v_{S_1}^r = S_1 v_{S_1}^i = S_1 v_3^r = S_1 v_2^i = S_1 \cdot v_{S_2}^r$$
$$= S_1 \cdot S_2 v_{S_2}^i = S_1 \cdot S_2 v_2^r$$
$$= S_1 \cdot S_2 v^i \Rightarrow v^r = S_1 \cdot S_2 v^i$$
$$= S \cdot v^i$$

$$\Rightarrow S = S_1 S_2$$



$$\begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = S_{circ} \cdot \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$$

$$\begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = \begin{bmatrix} 0_n & 0_n & 1_n \\ 1_n & 0_n & 0_n \\ 0_n & 1_n & 0_n \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$$

3n-vector $3n \times 3n$ S_{circ} 3n-vector

$$S^T_{circ} S_{circ} = \begin{bmatrix} 0_n & 1_n & 0_n \\ 0_n & 0_n & 1_n \\ 1_n & 0_n & 0_n \end{bmatrix} \begin{bmatrix} 0_n & 0_n & 1_n \\ 1_n & 0_n & 0_n \\ 0_n & 1_n & 0_n \end{bmatrix}$$

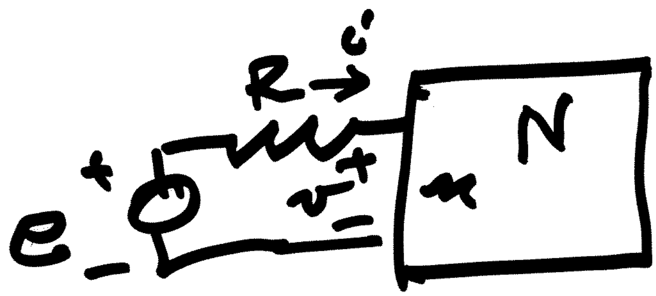
$$= \begin{bmatrix} 1_n & 0_n & 0_n \\ 0_n & 1_n & 0_n \\ 0_n & 0_n & 1_n \end{bmatrix}$$

3n-port circ

$$1_{3n} - S^T_{circ} \cdot S_{circ} = 0_{3n} \Leftarrow$$ is lossless

$2v^r = v - Ri$

$e = 2v^i = v + Ri$



if N has an admittance $i = Yv$

$$2v^r = 2Sv^i$$
$$= v - RYv = (1_n - RY)v$$
$$= S[v + Ri] = S[v + RYv]$$
$$= S[1_n + RY]v$$

assume $(1_n - RY)^{-1}$ exists

$$1_n v = v = (1_n - RY)^{-1} S (1_n + RY) \cdot v$$

as Y exists (assumed) any v can be used

$$1_n = (1_n - RY)^{-1} S (1_n + RY)$$

or $\boxed{(1_n - RY)(1_n + RY)^{-1} = S}$

$$\equiv (1_n + RY)^{-1}(1_n - RY)$$

now $(1_m - X)(1_m + X)^{-1}(1_m + X)$

$$= 1_m - X$$

~~better~~ $(1_m + X)(1_m - X) = 1_m + X - X - X^2$

$$= (1_m - X)(1_m + X) = 1_m - X + X - X^2$$

times $(1_m + X)^{-1}$ on right & left

$$(1_m - X)(1_m + X)^{-1} = (1_m + X)^{-1}(1_m - X)$$

to get Y given S

$$(1_m - RY) = S(1_m + RY)$$

$$= 1_m - RY = S + SRY$$

$$\Rightarrow 1_m - S = RY + SRY = (1_m + S)RY$$

$$\text{or } RY = (1_m + S)^{-1}(1_m - S)$$

$$= (1_m - S)(1_m + S)^{-1}$$

$$\text{or } Y = G(1_m + S)^{-1}(1_m - S) \; ; \; G = R^{-1}$$

EE610 10/02/02

p. 5

$$P_{av}(j\omega) = \mathrm{Re}\left(V^{T*}(j\omega)\, I(j\omega)\right]$$

in S.S.S. sinusoidal steady state

$$= \frac{V^{T*}(j\omega)\, I(j\omega) + I^{T*}(j\omega)\, V(j\omega)}{2}$$

given $Y(s)$ exists:

$$2P_{ave}(j\omega) = V^{T*}Y(j\omega)V + V^{T*}Y^{T*}(j\omega)V$$

$$= V^{T*}\left[Y + Y^{T*}\right]\Big|_{s=j\omega} V \geq 0 \text{ if passive}$$

if lossless

$$P_{ave}(j\omega) \equiv 0 \Rightarrow Y(j\omega) + Y^{T*}(j\omega) = 0_n$$

$$Y(j\omega) + Y^{T}(-j\omega)\Big|_{\omega = \frac{s}{j}} = 0$$

$$Y(s) + Y^{T}(-s) = 0_n$$ in all of s-plane if rational

EE610 10/02/02 poles of $Y(s)$, × p. 6

$s = \sigma + j\omega$

a pole of $Y^T(-s)$ if p_1 is a pole of $Y(s)$

can't be as a pole of $Y^T(-s) = -Y(s) \Rightarrow$ a pole of $Y(s)$

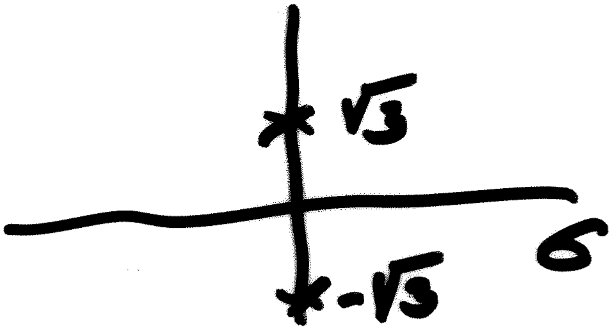
$\Rightarrow$ all poles of a lossless $Y(s)$ are on $j\omega$ axis

also true any pole is simple with positive semi-definite residue matrix.

EE610 10/02/02

Ex: $y(s) = \dfrac{2s}{s^2+3}$ is PR lossless

$$y(s) + y(-s) = \frac{2s}{s^2+3} + \frac{-2s}{(-s)^2+3} = 0$$



$$y(s) = \frac{r_1}{s+j\sqrt{3}} + \frac{r_2 = r_1^*}{s-j\sqrt{3}} = \frac{1}{s+j\sqrt{3}} + \frac{1}{s-j\sqrt{3}}$$

$$r_1 = (s+j\sqrt{3})\, y(s) - \frac{r_1^*(s+j\sqrt{3})}{s-j\sqrt{3}}$$

$$= \frac{2s}{s-j\sqrt{3}} - \frac{r_1^*(s+j\sqrt{3})}{s-j\sqrt{3}}$$

set $s = -j\sqrt{3}$

$$r_1 = \frac{2(-j\sqrt{3})}{-j\sqrt{3}-j\sqrt{3}} = 1$$

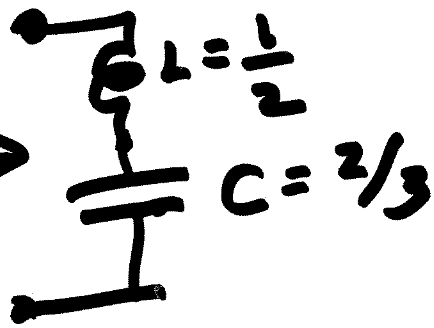
synthesis of

$$y(s) = \frac{2s}{s^2+3} \; ; \; z(s) = \frac{1}{y(s)} = \frac{s^2+3}{2s}$$

$$= \frac{s}{2} + \frac{3}{2s}$$

$$= \frac{s}{2} + \frac{1}{\frac{2}{3}s}$$

$$\frac{1}{z(s)} = y(s) \rightarrow$$



$$y(s) = \frac{2s}{s^2+3}$$

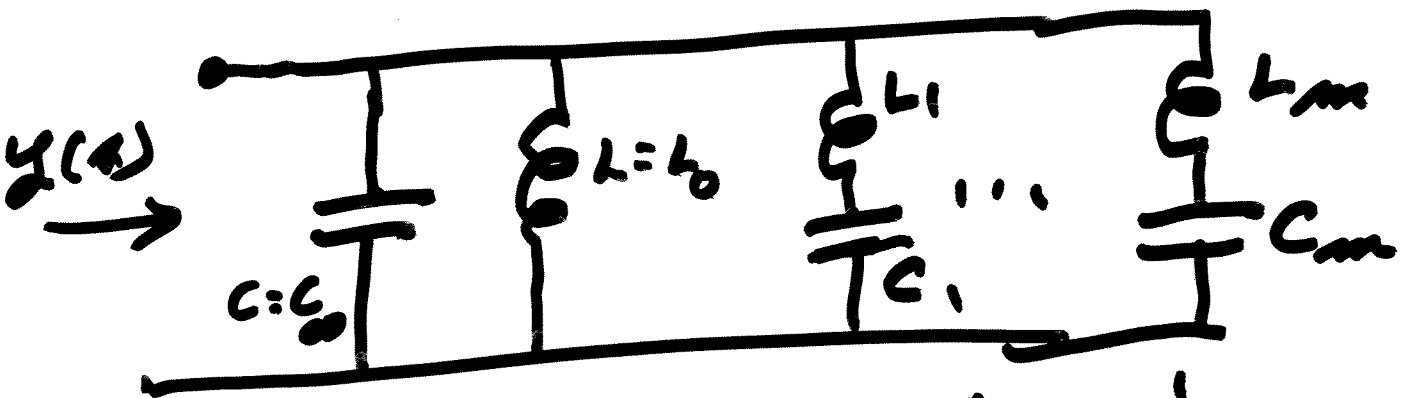
Given any 1×1 lossless $y(s)$, rational w/ real residue in 1×1

$$y(s) = \left[\sum_{i=1}^{m} \frac{r_i}{s+j\omega_i} + \frac{r_i^*}{s-j\omega_i}\right] + L_\infty s + \frac{1}{C_0 s} \qquad r_i > 0, \; r_i^* = r_i$$

$$= \sum_{i=1}^{m} \frac{2 r_i s}{s^2+\omega_i^2} + L_\infty s + \frac{1}{C_0 s}$$

$$y(s) = C_\infty s + \frac{1}{L_0 s} + \sum_{i=1}^{m} \frac{2r_i s}{s^2+\omega_i^2}$$

by partial fraction expansion



$$y_i = \frac{2r_i s}{s^2+\omega_i^2}$$

$$L_i = \frac{1}{2r_i}$$

$$C_i = \frac{2r_i}{\omega_i^2}$$

$$L_i C_i = \frac{1}{\omega_i^2}$$

$$\omega_i = \frac{1}{\sqrt{L_i C_i}}$$