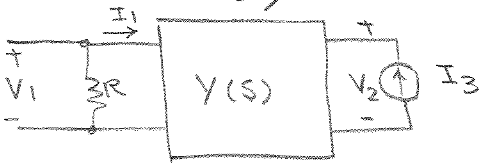
ENEE610 Homework #2 Solution

1) (P2, consider 5)



$V_1 = -I_1 R$

$$\begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 \\ I_3 = y_{21} V_1 + y_{22} V_2 \end{cases}$$

10 points
$\begin{cases} 5 \rightarrow \text{correct basic equation} \\ 5 \rightarrow \text{simplification} \end{cases}$

$$\Rightarrow I_1 = -\frac{V_1}{R} = y_{11} V_1 + y_{12} V_2 \Rightarrow V_2 = \frac{-V_1/R - y_{11} V_1}{y_{12}}$$

$$I_3 = y_{21} V_1 + y_{22} V_2 = y_{21} V_1 + y_{22} \frac{(-V_1/R - y_{11} V_1)}{y_{12}}$$

$$= \frac{(y_{21} y_{12} - y_{22} y_{11})}{y_{12}} V_1 - \frac{y_{22}}{y_{12}} \times \frac{1}{R} V_1 \Rightarrow \boxed{\frac{V_1}{I_3} = \frac{-R y_{12}}{y_{22} + R(y_{22} y_{11} - y_{21} y_{12})}}$$

2) (P3, Consider 5)

5 points $\begin{cases} 3 \rightarrow R>0 \\ 2 \rightarrow R<0 \end{cases}$

The condition of $T(S) = \frac{V_1}{I_2}$ (for previous problem) to be realizable is the condition of synthesis of the transfer function:

$$T(S) = \frac{V_1}{I_2} = \frac{-R y_{12}}{y_{22} + R\Delta y} = \frac{-y_{12}/y_{22} \times R}{1 + R\,\Delta y/y_{22}}$$

a) Y(S) is a lossless ladder & R>0

* Y(S) lossless $\Rightarrow \begin{cases} Y(s) \text{ positive real} \Rightarrow \Delta \geq 0 \\ Y(s) = -Y(-s) \Rightarrow \begin{cases} y_{11}(s) = -y_{11}(-s)\ ;\ y_{12}(s) = -y_{12}(-s) \\ y_{21}(s) = -y_{21}(-s)\ ;\ y_{22}(s) = -y_{22}(-s) \end{cases} \end{cases}$

* $\Delta Y(s) = y_{11}(s) y_{22}(s) - y_{12}(s) y_{21}(s)$

$= y_{11}(-s) y_{22}(-s) - y_{12}(-s) y_{21}(-s)$

$= \Delta(-s)$

Denominator $1 + \frac{R \Delta y(s)}{y_{22}} = 0 \Rightarrow y_{22} = -R\Delta y \leq 0$

$\Rightarrow$ if $R>0 \Rightarrow$ poles of T(S) are not in the right half plane.

if $R<0 \Rightarrow$ poles of T(S) could be in the right half plane in which case T(S) wouldn't be positive real or lossless anymore.

15 { 5 → poles, zeros
10 → driving admittance

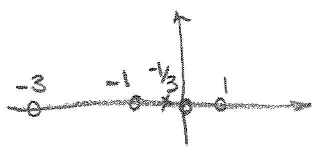
up to 5 bonus points for Richard function

3) (consider 6, P3)

$$Y(s) = \frac{s(s+3)}{3(s+1/3)} \quad \text{poles: } s = -1/3, \quad \text{zeros: } s=0, \; s=-3$$

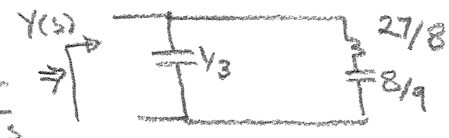
$$Ev\{Y(s)\} = \frac{Y(s)+Y(-s)}{2} = \frac{8s^2}{9s^2-1}, \quad \text{zeros: } s=0$$

$$Odd\{Y(s)\} = \frac{Y(s)-Y(-s)}{2} = \frac{3s(s^2-1)}{9s^2-1}, \quad \text{zeros: } s=0, \; s=\pm 1.$$



pole-zero plot of Y(s)

$$Y(s) = \frac{s(s+3)}{3(s+1/3)} = 1/3\, s + \frac{8/9\, s}{3(s+1/3)} = 1/3\, s + \frac{1}{27/8 + \frac{1}{8/9\, s}} \Rightarrow$$



RC realization

note: $Z_{LC}(s)$ of the form: $\begin{cases} \dfrac{H(s^2+\omega_1^2)(s^2+\omega_3^2)\cdots}{s(s^2+\omega_2^2)(s^2+\omega_4)^2\cdots} \\ \text{or} \\ \dfrac{Hs(s^2+\omega_2^2)(s^2+\omega_4^2)\cdots}{(s^2+\omega_1^2)(s^2+\omega_3^2)\cdots} \end{cases}$

to get Z_{RC} from Z_{LC}, replace Ls by R, so $Z_{LC} = Ls + \frac{1}{sC}$ becomes $R + \frac{1}{sC}$, or replace s^2 by s in the above expression for Z_{LC}.

$Z_{RC}(s)$ of the form: $\begin{cases} \dfrac{H(s+\alpha_1)(s+\alpha_3)\times\cdots}{s(s+\alpha_2)(s+\alpha_4)\times\cdots} \\ \dfrac{H(s+\alpha_2)(s+\alpha_4)\times\cdots}{(s+\alpha_1)(s+\alpha_3)\times\cdots} \end{cases}$

$\Rightarrow Y_{RC}$ of the form: $\begin{cases} \dfrac{1}{H}\, \dfrac{s(s+\alpha_2)(s+\alpha_4)\cdots}{(s+\alpha_1)(s+\alpha_3)\cdots} \\ \dfrac{1}{H}\, \dfrac{(s+\alpha_1)(s+\alpha_3)\cdots}{(s+\alpha_2)(s+\alpha_4)\cdots} \end{cases}$

Our function $Y(s) = \frac{s(s+3)}{3(s+1/3)}$ has the Y_{RC} configuration. (can use any of Foster I, Foster II, Cauer I, II synthesis methodes for solution).

3 cont.)

Richard function Synthesis:

$$y_L = y_{in}(k) \times \frac{k\,y_{in}(k) - s\,y_{in}(s)}{k\,y_{in}(s) - s\,y_{in}(k)}$$

$$y_L = \frac{k(k+3)}{3(k+1/3)} \times \frac{k\,\frac{k(k+3)}{3(k+1/3)} - s\,\frac{s(s+3)}{3(s+1/3)}}{k\,\frac{s(s+3)}{3(s+1/3)} - s\,\frac{k(k+3)}{3(k+1/3)}}$$

eliminate $(k-s)$ from numenator & denominator

$$= \frac{k(k+3)}{3(k+1/3)} \times \frac{k^3 s + 1/3\,k^3 + 3k^2 s + k^2 - s^3 k - 1/3\,s^3 - 3s^2 k - s^2}{\cancel{k^2 s^2} + \underline{1/3\,ks^2} + \underline{3k^2 s} + ks - \cancel{k^2 s^2} - 3s^2 k - \underline{1/3\,k^2 s} - ks}$$

$$= \frac{\cancel{k}(k+3)}{3(k+1/3)} \times \frac{\cancel{(k-s)}\,(sk^2 + ks^2 + 1/3\,k^2 + 1/3\,ks + 1/3\,s^2 + 3ks + k + s)}{\frac{8}{3}\,\cancel{k}s\cancel{(k-s)}}$$ eliminate k as well

$$= \frac{1}{8}\,\frac{k+3}{(k+1/3)\times s}\,(sk^2 + ks^2 + 1/3\,k^2 + 1/3\,ks + 1/3\,s^2 + 3ks + k + s)$$

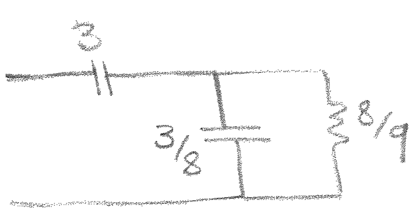
now let $k=0$

$$y_L\big|_{k=0} = \frac{3}{8} \times \frac{1}{s/3} \times s\left(\frac{s}{3}+1\right) = \frac{s+3}{8/3} = 3/8\,s + 9/8$$

$$g = y_{in}(k) = 0$$

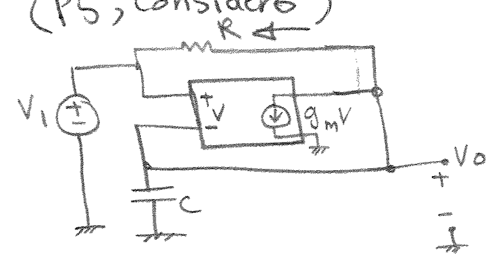
$$C = \frac{k}{y_{in}(k)}\bigg|_{k=0} = \frac{0}{0} \xrightarrow{\text{take derivitive}} C = \frac{1}{\frac{1}{3}\,\frac{(2k+3)(k+1/3) - (k^2+3k)}{(k+1/3)^2}}\Bigg|_{k=0} = 3$$

From num. & denum. with respect to k



rcT/6

4) (P5, consider 6)



$$\frac{V_o - V_1}{R} + g_m(V_1 - V_o) + sCV_o = 0$$

$$\Rightarrow T(s) = \frac{V_o}{V_1} = \frac{1/R - g_m}{1/R - g_m + sC}$$

15 points: 5 points for each case
- 3 points for adjoint from sensitivity
- 2 points for direct method

1- Direct calculation:

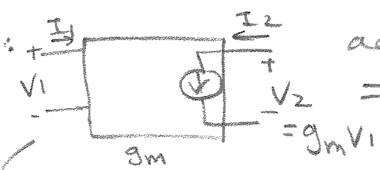
$$S^T_{g_m} = \frac{g_m}{T}\frac{\partial T}{\partial g_m} = \frac{-(1/R - g_m + sC) + (1/R - g_m)}{(1/R - g_m + sC)^2} \times \frac{g_m}{\frac{(1/R - g_m)}{1/R - g_m + sC}} = \frac{-sCg_m}{(-g_m + \frac{1}{R})(\frac{1}{R} - g_m + sC)}$$

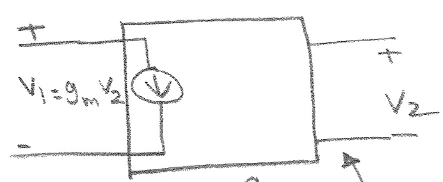
$$S^T_C = \frac{C}{T}\frac{\partial T}{\partial C} = \frac{-s(1/R - g_m)}{(\frac{1}{R} - g_m + sC)^2} \times \frac{C}{\frac{1/R - g_m}{1/R - g_m + sC}} = \frac{-sC}{1/R - g_m + sC}$$

$$S^T_R = \frac{R}{T}\frac{\partial T}{\partial R} = \frac{-1/R^2(1/R - g_m + sC) + 1/R^2(1/R - g_m)}{(1/R - g_m + sC)^2} \times \frac{R}{\frac{1/R - g_m}{1/R - g_m + sC}} = \frac{-sC}{(1 - g_m R)(\frac{1}{R} - g_m + sC)}$$

2- Using adjoint circuit:

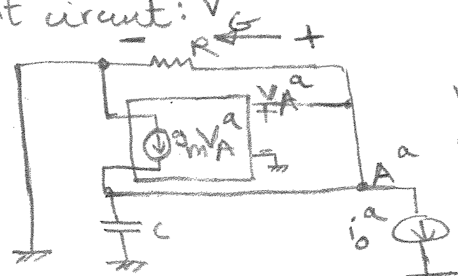
Adjoint circuit:

Consider: 

adjoint $\Longrightarrow$ 

① $Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} \xrightarrow{②} Y^a = \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix}$ ③

adjoint circuit: V_G^a



$|i_o^a| = |V_1| = 1$

notice, the graph of the original and adjoint circuit stays the same. (if one branch for input and one branch for output is considered.)

$$\frac{\partial T}{\partial R} = \frac{\partial T}{\partial G} \times \underbrace{\frac{\partial G}{\partial R}}_{\frac{\partial 1/R}{\partial R} = -1/R^2} = -1/R^2 \times (V_G \times V_G^a) = \frac{-1}{R^2}\underbrace{(V_o - V_1)}_{V_G}\underbrace{\left(\frac{-1}{sC + \frac{1}{R} - g_m}\right)}_{V_G}$$

KCL @ A^a: $i_o^a = V_A^a(-sC - \frac{1}{R} + g_m) \Rightarrow V_A^a = V_G^a = \frac{-1}{sC + \frac{1}{R} - g_m}$

$|i_o^a| = 1$

$$V_G = V_O - V_I = T - 1 = \frac{1/R - g_m}{1/R - g_m + sC} - 1 = \frac{-sC}{1/R - g_m + sC}$$

$|V_I| = 1$

$$\Rightarrow S^T_R = \frac{R}{T} \times \frac{\partial T}{\partial R} = \frac{\cancel{R}}{\frac{1/R - g_m}{1/R - g_m + sC}} \times \frac{-sC}{\cancel{1/R - g_m + sC}} \times \frac{-1}{sC + \frac{1}{R} - g_m} \times \frac{-1}{R^2} = \frac{-sC}{(1 - Rg_m)(\frac{1}{R} - g_m + sC)}$$

$$\frac{\partial T}{\partial C} = \frac{\partial T}{\partial Y_C} \times \frac{\partial Y_C}{\partial C} = s\,\frac{\partial T}{\partial Y_C} = s \times V_{Y_C} \times V^a_{Y_C} =$$

$Y_C = sC$

$$V_{Y_C} = V_O = T = \frac{1/R - g_m}{1/R - g_m + sC}$$

$$V^a_{Y_C} = V^a_G = \frac{-1}{sC + \frac{1}{R} - g_m}$$

$$\Rightarrow S^T_C = \frac{C}{T} \times \frac{\partial T}{\partial C} = \frac{C}{\frac{1/R - g_m}{1/R - g_m + sC}} \times s \times \frac{1/R - g_m}{1/R - g_m + sC} \times \frac{-1}{sC + \frac{1}{R} - g_m} = \frac{-sC}{sC + \frac{1}{R} - g_m}$$

$$\frac{\partial T}{\partial g_m} = V_A \times V_A^a = (V_1 - V_o) \times V_G^a = (1 - T)(V_G)^a$$

$$\Rightarrow S^T_{g_m} = \frac{g_m}{T} \times \frac{\partial T}{\partial g_m} = \frac{g_m}{\frac{1/R - g_m}{1/R - g_m + sC}} \times \left(1 - \frac{1/R - g_m}{1/R - g_m + sC}\right) \times \frac{-1}{sC + 1/R - g_m} = \frac{-g_m sC}{(-g_m + \frac{1}{R})(\frac{1}{R} - g_m + sC)}$$

5) (P6, consider 6)

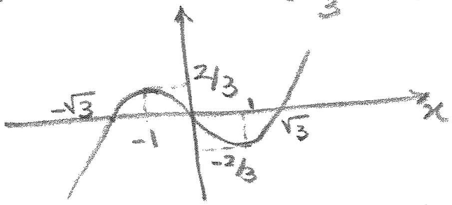
To get the adjoint circuit, the input voltage source is replaced with short in adjoint circuit and the output "open" voltage is replaced by a current source. The graph of the original and adjoint circuit stays the same. Basic rule is : $V_b^T i_b^a - V_b^{aT} i_b = 0$ (for all branches), this implies $Y_b^a = Y_b^T$, which suggests that all passive 1 port elements like R, C, L, G stay the same in both circuits. For n-ports in general, we use $Y_b^a = Y_b^T$ to extract its adjoint as we did for the G component in previous problem.

For the adjoint of the previous problem, please refer to the previous page.

10 points; 6 → how to build
4 → contruction on prev. example

5) $f(x) = x\left(\frac{x^2}{3} - 1\right)$

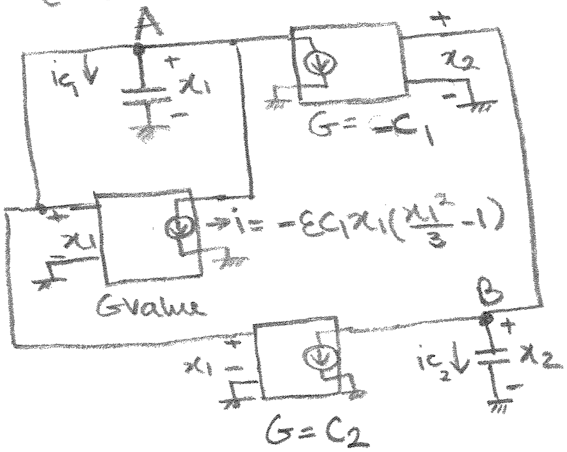
30 points $\begin{cases} 20 \text{ part a (limit cycles} + f(x)) \\ 10 \text{ part b PSpice setup} \end{cases}$ 16/



exterma: $(1, -\frac{2}{3})$, $(-1, \frac{2}{3})$

$$\begin{cases} \dot{x}_1 = x_2 + \varepsilon x_1\left(\frac{x_1^2}{3} - 1\right) \\ \dot{x}_2 = -x_1 \end{cases} \Rightarrow \begin{cases} C_1\dot{x}_1 = C_1 x_2 + \varepsilon C_1 x_1\left(\frac{x_1^2}{3} - 1\right) = i_{C_1} \\ C_2\dot{x}_2 = -C_2 x_1 = i_{C_2} \end{cases} \rightarrow$$

2 KCLs should be realized at points A & B.



* Shape of limit cycle (x_2 versus x_1) sharpens as ε increases.

(for $\varepsilon = .001$, it's oval, for $\varepsilon = 1000$ it becomes a square).