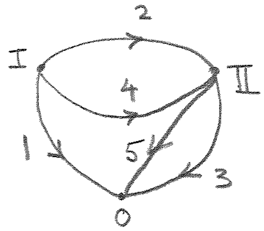
Homework #1 solution

10/20/02
P1/6
AD

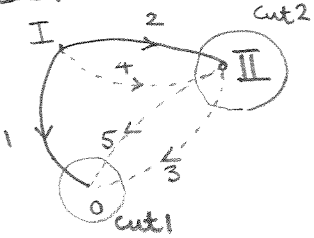
1-



$V_b = \begin{bmatrix} V_t \\ V_\ell \end{bmatrix}$ $\quad i_b = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}$

25 { 10 → case1
10 → case2
5 → transfer func.

case1:



$V_b = \begin{bmatrix} V_1 \\ V_2 \\ \hline V_3 \\ V_4 \\ V_5 \end{bmatrix}_b$ $\quad i_b = \begin{bmatrix} i_1 \\ i_2 \\ \hline i_3 \\ i_4 \\ i_5 \end{bmatrix}_b$

$$\text{KCL:} \quad \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix}}_{\mathcal{C} \text{ (cutset matrix)}} \cdot i_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad k_i = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

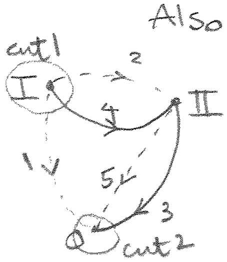
note: Each tree branch makes a cutset with some tree links.
(i.e. (t_2 with l_3, l_4, l_5) and (t_1 with l_3, l_5))

$$\text{KVL:} \quad \underbrace{\begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\mathcal{T} \text{ (tieset matrix)}} \cdot V_b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad k_v = \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ -1 & 1 \end{bmatrix}$$

note: Each tree link makes a loop with some tree branches.
(i.e. (l_4 with t_2) and (l_5 with t_1, t_2) and (l_3 with t_1, t_2))
These 3 loop KVL equations are gauranteed to result in independant equations.

note: $k_i = -k_v^T$

note: The current source and R1 can be assumed to be in two different branches.
Also since $i_4 = 0$, we can simplify the graph by deleting b_4



$i_b = \begin{bmatrix} i_3 \\ i_4 \\ \hline i_1 \\ i_2 \\ i_5 \end{bmatrix}_b$ $\quad V_b = \begin{bmatrix} V_3 \\ V_4 \\ \hline V_1 \\ V_2 \\ V_5 \end{bmatrix}_b$

$$\text{KCL:} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \cdot i_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\text{KVL:} \begin{bmatrix} -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot V_b = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

case2:

$T(s) = \frac{V_{II}}{I_1}$

Setup the state equations with $y = V_{II}$ as the output and $x = V_{II}$ as the state variable. Find A, B, C such that:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

$\dot{x} = \dot{V}_{II} = \dot{V}_{c3} = \frac{i_3}{C_3} \Rightarrow sV_{II} = \frac{i_3}{C_3}$

KCL @ 0: $i_3 = -i_1 - i_5 = -(I_1 + \frac{V_I}{R_1}) - g_m(V_I - V_{II})$

$\Rightarrow -sC_3 V_{II} = V_I(\frac{1}{R_1} + g_m) - g_m V_{II} + I_1$

KCL @ I: $I_1 + I_{R1} = -I_{R2} \Rightarrow I_1 + \frac{V_I}{R_1} = \frac{V_{II} - V_I}{R_2} \Rightarrow V_I(g_1 + g_2) + I_1 = V_{II}\, g_2$

$(g_1 = \frac{1}{R_1},\ g_2 = \frac{1}{R_2})$

$\Rightarrow -sC_3 V_{II} = I_1 - g_m V_{II} + (g_m + g_1)\left[\frac{g_2 V_{II} - I_1}{g_1 + g_2}\right]$

$\Rightarrow sC_3 V_{II} = (g_m - \frac{(g_m + g_1)g_2}{g_1 + g_2}) V_{II} - (1 - \frac{g_m + g_1}{g_1 + g_2}) I_1$

$$\Rightarrow \begin{cases} sV_{II} = \underbrace{\frac{1}{C_3}\left(\frac{g_m g_1 - g_1 g_2}{g_1 + g_2}\right)}_{A} V_{II} \underbrace{- \frac{1}{C_3} \cdot \frac{g_2 - g_m}{g_1 + g_2}}_{B} I_1 \\ V_{II} = \underset{C}{1} \cdot V_{II} \end{cases}$$

$V_2 = C(sI - A)^{-1}B = \left(s - \frac{1}{C_3}\frac{g_m g_1 - g_1 g_2}{g_1 + g_2}\right)^{-1}\left(-\frac{g_2 - g_m}{C_3(g_1 + g_2)}\right)$

$$= \frac{g_m - g_2}{sC_3(g_1 + g_2) + g_1 g_2 - g_m g_1}$$

note: Could have defined x more complex as $x = \begin{bmatrix} V_t \\ \hline i_\ell \end{bmatrix}$

for case 1:

$$Y_b = \begin{bmatrix} 1/R_1 & 0 & 0 & 0 & 0 \\ 0 & 1/R_2 & 0 & 0 & 0 \\ 0 & 0 & sc_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_m \end{bmatrix} \quad j = \begin{bmatrix} I_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} V_1 \\ V_2 \\ \hdashline i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

$$\begin{bmatrix} Y_b C^T & -C^T \end{bmatrix} \begin{bmatrix} V_t \\ \hdashline i_\ell \end{bmatrix} = j$$

$$Y_b C^T = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_2 \\ sc_3 & -sc_3 \\ 0 & 0 \\ g_m & -g_m \end{bmatrix} \Rightarrow \left[\begin{array}{cc:ccc} 1/R_1 & 0 & 1 & 0 & 1 \\ 0 & 1/R_2 & -1 & 1 & -1 \\ sc_3 & -sc_3 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ g_m & -g_m & 0 & 0 & -1 \end{array}\right] X = \begin{bmatrix} I_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left\{ \begin{array}{l} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ c_3 & -c_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dfrac{dx}{dt} = \begin{bmatrix} -1/R_1 & 0 & -1 & 0 & -1 \\ 0 & -1/R_2 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -g_m & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \end{bmatrix} x \end{array} \right.$$

$$[C Y_b C^T]^{-1} C j = \begin{bmatrix} g_1 + sc_3 & -sc_3 + g_m \\ -sc_3 & g_2 + sc_3 - g_m \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ 0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = V_t$$

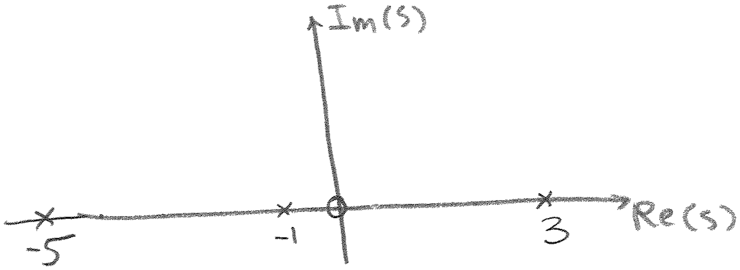
$$\Rightarrow V_{\text{II}} = V_1 - V_2 = \frac{g_m - g_2}{g_1(g_2 - g_m) + sc_3(g_1 + g_2)}$$

$$25 \begin{cases} 5 \to a \\ 5 \to \text{each case in b} \\ \quad (\text{total } 4) \end{cases}$$

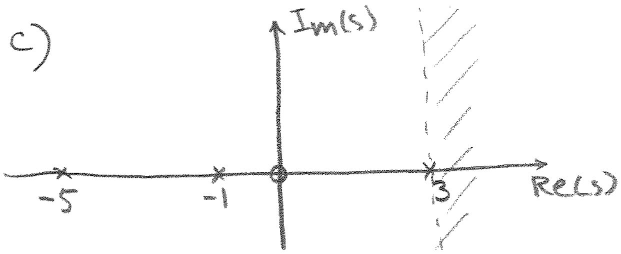
$$\frac{a}{s-p} \xrightarrow{\mathcal{L}^{-1}} \begin{cases} e^{pt}\, 1(t) & Re\{s\} > p \\ -e^{pt}\, 1(-t) & Re\{s\} < p \end{cases}$$

2- $F(s) = \dfrac{2}{s+1} + \dfrac{3}{s-3} - \dfrac{5}{s+5}$

a)

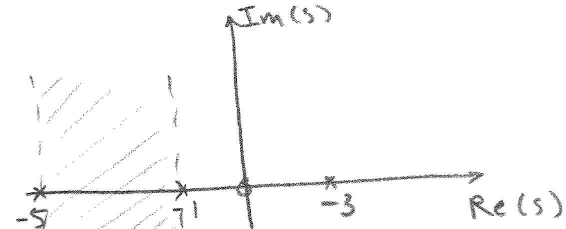


b) c)



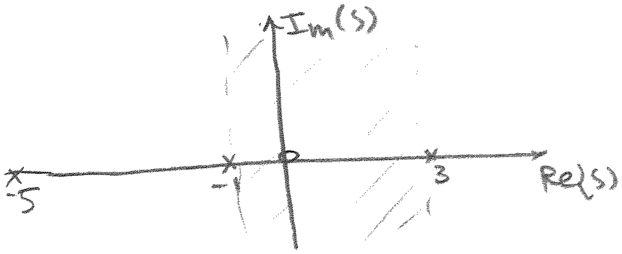
$Re\{s\} > 3$

$f(t) = (2e^{-t} + 3e^{3t} - 5e^{-5t})\, 1(t)$



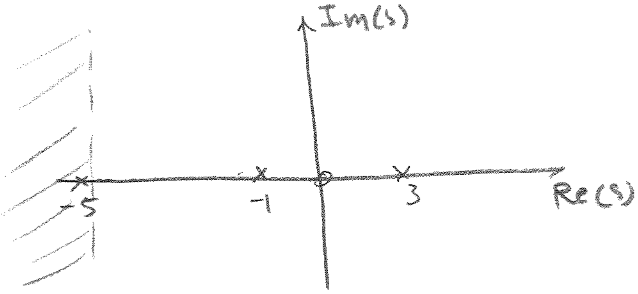
$-5 < Re\{s\} < -1$

$f(t) = -2e^{-t}\, 1(-t) - 3e^{3t}\, 1(-t) - 5e^{-5t}\, 1(t)$



$-1 < Re\{s\} < 3$

$f(t) = 2e^{-t}\, 1(t) - 3e^{3t}\, 1(-t) - 5e^{-5t}\, 1(t)$



$Re\{s\} < -5$

$f(t) = -(2e^{-t} + 3e^{3t} - 5e^{-5t})\, 1(-t)$

3- $T(s) = C(sE - A)^{-1} B$

15 points

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s & 0 & -1 \\ 1 & s & 0 \\ 0 & 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2s^2} \begin{bmatrix} 2s & 0 & s \\ -2 & 2s & -1 \\ 0 & 0 & s^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/s & 1/2s \\ -1/s^2 & -1/2s^2 \\ 0 & 1/2 \end{bmatrix}$$

4-

$$30 \begin{cases} 3 \rightarrow \text{part1} \\ 10 \rightarrow \text{part2} \\ 10 \rightarrow \text{part3} \\ 3 \rightarrow \text{part4} \\ 4 \rightarrow \text{part5} \end{cases}$$

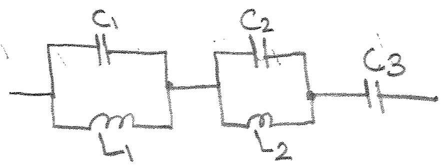
1- If F(s) admittance: poles are on jw axis

$$F(s)+F(-s)=\frac{5s(s^2+9)(s^2+25)}{(s^2+1)(s^2+16)}+\frac{5(-s)(s^2+9)(s^2+25)}{(s^2+1)(s^2+16)}=0$$

$\Rightarrow$ lossless

2- $Y(s)=\dfrac{5s(s^2+9)(s^2+25)}{(s^2+1)(s^2+16)}=F(s)$

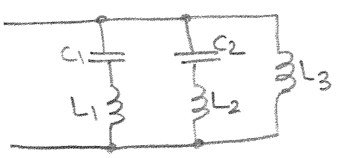
Foster1: $Z(s)=\dfrac{1}{F(s)}=\dfrac{(s^2+1)(s^2+16)}{5s(s^2+9)(s^2+25)}=\dfrac{.108s}{s^2+25}+\dfrac{.078s}{s^2+9}+\dfrac{.0142}{s}$



$Z_1=\dfrac{sL \times \frac{1}{sC_1}}{sL+\frac{1}{sC_1}}=\dfrac{s \times \frac{1}{C_1}}{s^2+\frac{1}{L_1C_1}}$

$$\begin{cases} C_1=\frac{1}{.108}, \; L_1=\frac{.108}{25}=.004 \\ C_2=\frac{1}{.078}, \; L_2=\frac{.078}{9}=.01 \\ C_3=\frac{1}{.0142}=71 \end{cases}$$

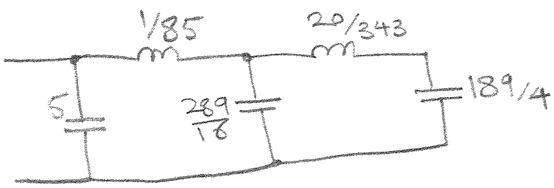
Foster2: $Y(s)=F(s)=\dfrac{21s}{s^2+16}+\dfrac{64s}{s^2+1}+5s$



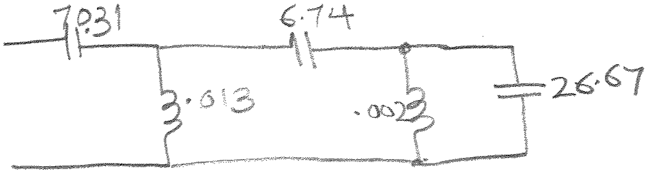
$Y_1=\dfrac{1}{sL_1+\frac{1}{sC_1}}=\dfrac{sC_1}{1+L_1C_1s^2}$

$$\begin{cases} C_1=\frac{21}{16} \quad L_1=\frac{1}{21} \\ C_2=64 \quad L_2=\frac{1}{64} \\ L_3=5 \end{cases}$$

Cauer1: $Y(s)=F(s)=5s+\cfrac{1}{\frac{1}{85}s+\cfrac{1}{\frac{289}{16}s+\cfrac{1}{\frac{20}{3213}s+\cfrac{1}{\frac{189}{4}s}}}}$

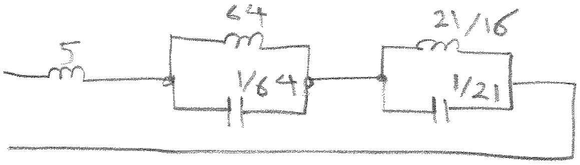


Cauer2: $Z(s)=\dfrac{1}{Y(s)}=\dfrac{16+17s^2+s^4}{1125s+170s^3+5s^5}=\dfrac{1}{70.31s}+\cfrac{1}{\frac{1}{.013s}+\cfrac{1}{\frac{1}{6.74s}+\cfrac{1}{\frac{1}{.002s}+\frac{1}{26.67s}}}}$

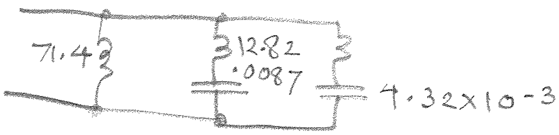


3- $F(s) = Z(s)$

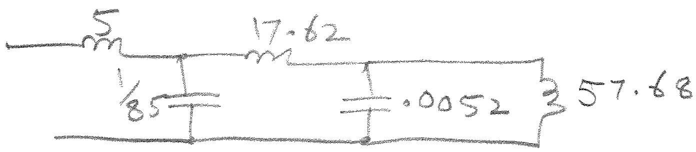
Foster1: $Z(s) = 5s + \frac{64s}{s^2+1} + \frac{21s}{s^2+16}$



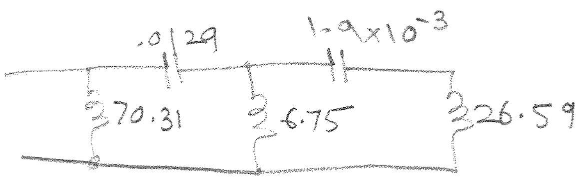
Foster2: $Y(s) = \frac{1}{F(s)} = \frac{.07}{5s} + \frac{.078s}{s^2+9} + \frac{.108s}{s^2+25}$



Cauer1: $Z(s) = 5s + \cfrac{1}{\frac{1}{85}s + \cfrac{1}{17.62s + \cfrac{1}{.0052s + \cfrac{1}{57.68s}}}}$



Cauer2: $Y(s) = \frac{16}{1125s} + \cfrac{1}{\frac{77.11}{s} + \cfrac{1}{\frac{.148}{s} + \cfrac{1}{\frac{523.6}{s} + \cfrac{1}{\frac{.376}{s}}}}}$



4- If $F(s)$ is scattering ⇒ cannot be lossless since it has poles on the $j\omega$ axis (also $F(s) \times F(-s) \neq 1$)

5- $F(s) = Y(s) \Longrightarrow S(s) = \frac{1 - Y(s)}{1 + Y(s)} = \frac{(s^2+1)(s^2+16) - 5s(s^2+9)(s^2+25)}{(s^2+1)(s^2+16) + 5s(s^2+9)(s^2+25)}$

Poles: $-.054 \pm 5j$, $-.0389 \pm 3j$, $-.0142$

Zeros: $.054 \pm 5j$, $.0389 \pm 3j$, $.0142$

$S(s) \times S(-s) = 1 \Rightarrow$ lossless

5- $\sum_{i=1}^{3} V_i = V_1 i_1 + V_2 i_2 + V_3 i_3 = V(i_1 + \cancel{i_2} + i_{out}) = 0 \Rightarrow$ passive & lossless

KCL: 0

$V_1 = V_2 = V_3 = V$

5 points