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ENEE 610 Fall 2002
Problems to consider #7

1. a)Factor into degree two factors using only real numbers

$$Pa(s) = s^4 + 5s^3 + 4s^2 + 3s + 2$$

b) Repeat on

$$Pb(s) = s^4 + 2s^3 + 3s^2 + 4s + 5$$

Note that the Pa has some real roots while Pb(s) does not. As a first guess one can plot P(s) versus real s and note where it comes the closest to zero.

c) Develop a similar method with highest powers of s as remainders by using P(1/s) and carry this out on the above two examples.

2. For the state variable equations

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 5 \\ -10 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 3 \end{bmatrix} x$$

a) Is the system lossless?
b) Plot x_2 versus x_1 for the zero input case.
c) Set up a PSpice schematic and run Spice to check your result in b).

3. For the van der Pol system described by

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \varepsilon \begin{bmatrix} f(x_1) \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \; f(x) = x\left(\frac{x^2}{3} - 1\right), \; x(0) \text{ given}$$

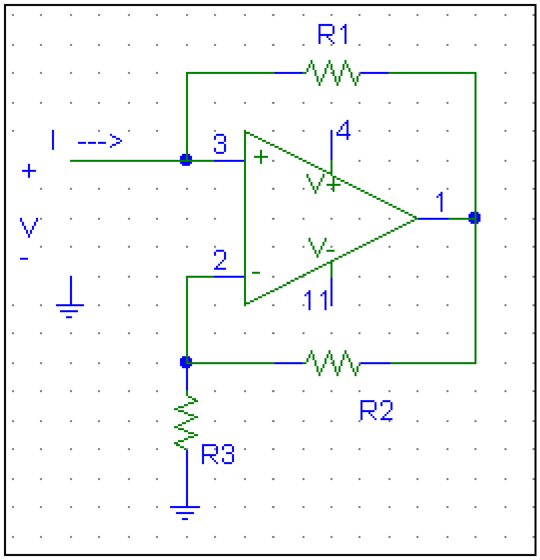
a) Sketch f(x) and find its local extrema. Use that to sketch x_2 versus x_1 for the following two values of ε: $\varepsilon=100$ and $\varepsilon=0.001$.

b) Set up the system as a PSpice schematic with x_1 and x_2 as capacitor voltages and using G and Gvalue components. Let x(0) and ε be parameters. Check your sketches by running PSpice. Place the plot of $f(x_1)$ on the same plot as x_2 versus x_1.

4. Repeat 3. with f(x) replaced by a piecewise linear (=PWL) approximation, $f_{PWL}(x)$ with break points at the same values as for the cubic f(x).

a) Give an equation for $f_{PWL}(x)$.
b) Run PSpice and compare the results with those of 3.
c) For this PWL van der Pol oscillator obtain analytic equations for the solutions in the three regions of x_1. Note though that determination of the values at transitioning between any two of the three regions yields a transcendental equation which is best solved numerically. Develop a means to perform this numerical "boundary matching."

d) Give a circuit which will practically realize this PWL version of the van der Pol oscillator. For this consider the following (saturating) op-amp circuit to give $\varepsilon f_{PWL}(x)$; give equations for the three slopes and two breakpoints in terms of the five circuit elements.



5. For the design of various systems it is convenient to have circuits which will realize, for constants a and time-domain signals x and y: a) ax, b) (x+a), c) x^2, d) exp(x), e) tanh(x), f) ln(x), g) |x|, h) 1/x, i) $x^{1/2}$, j) x*y with *=multiplication k) x*y with *=convolution, l) sin(x), m) cos(x).

Consider various means of making as many of these as you can in both voltage mode and current mode.

6. Consider that from 5. one can make x^2 and x+a.

a) Show that then one can make with vlsi hardware any polynomial P(x) in x with real coefficients.

b) Discuss how you would make a system for the (scalar) state variable equations

$$\frac{dx}{dt} = P(x) + bu$$
$$y = cx$$

c) Set up and run in PSpice the Riccati system

$$\frac{dx}{dt} = x^2, \qquad x(0) = 1$$
$$y = x$$

d)Repeat c) for

$$\frac{dx}{dt} = x^3, \qquad x(0) = 1$$

e) Show how to extend your result of b) to semistate equations if x is an n-vector.