

ENEE 610 Fall 2002

Problems to consider #3

1. Determine which of the following are positive real and which are bounded real; give your reasons

a)  $F(s) = 0.5$

b)  $F(s) = \frac{2}{s+1} + \frac{3}{s-3} - \frac{5}{s+5}$

c)  $F(s) = I_2 + \frac{1}{s} \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

d)  $F(s) = I_2 + \frac{1}{s+8} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

e)  $F(s) = \sqrt{9(s+4)}$

f)  $F(s) = \text{ctanh}(9[s+4])$

2. Show that the following  $F(s)$  can be a lossless admittance and give four different syntheses as an admittance. Repeat if it is considered as an impedance. What if it is a scattering coefficient? Assuming it is an admittance find the corresponding scattering coefficient; find the poles and zeros and check that  $S(s)S(-s)=1$ .

$$F(s) = \frac{5s(s^2+9)(s^2+25)}{(s^2+1)(s^2+16)}$$

3. Consider the voltage out over voltage in transfer function

$$T(s) = \frac{10s}{(s+5)(s^2+5s+25)}$$

Give several means to realize this through a 2-port structure. Can it be realized by a resistor loaded RC 2-port? Check one of your realizations by setting up an indefinite matrix considering all internal nodes and then eliminating internal nodes.

4. Given real  $n$ -vectors  $x$ ,  $y$  with the scalar product  $\langle y, x \rangle = y^T x$  where  $*$  = complex conjugate (which can be ignored at this point), show that the following is a bounded real matrix. If this is a scattering matrix, if possible find the corresponding admittance  $Y_x$ . Give a circuit that realizes this  $S_x$  as a scattering matrix.

$$S_x = I_n - \frac{2}{\langle x, x \rangle} x x^T$$

5. For  $S_x$  of 4. above find  $S = S_y S_x$  and show that this product of two scattering matrices is bounded real. Is it equal to  $S_{yx}$ ?

6. Consider similar questions to 4. above for

$$S_{x,y} = I_n - \frac{2}{\langle x, y \rangle} x y^T$$

6. For what  $f(s)$  is the following bounded real?

$$S_x(s) = I_n - 2f(s) \frac{x x^T}{\langle x, x \rangle}$$