$$\int_{-\infty}^{\infty} v^{n*} v^{i} \, dx = \int_{-\infty}^{\infty} v^{i*} v^{n} \, dx$$

$$E(\infty) = 2\langle v^{i}, v^{i}\rangle - \langle v^{n}, v^{n}\rangle = \|v^{i}\|^{2} - \|v^{n}\|^{2}$$

≥ 0 if passive

Now $V^{n}(a) = S(a)\, V^{i}(a)$, $S(a)$ = scattering matrix

Parseval's theorem:

$$\int_{-\infty}^{\infty} g^{*}(t)\, h(t)\, dt = \int_{-\infty}^{\infty} G^{*}(j\omega)\, H(j\omega) \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} G^{*}(j2\pi f)\, H(j2\pi f)\, df$$

if put a^{*} = complex conjugate into

$\langle x, y\rangle = \int_{-\infty}^{\infty} x^{T*}(t)\, y(t)\, dt$ Then

$$\langle v^{i}, v^{i}\rangle_{t} = \langle V^{i}, V^{i}\rangle_{f}, \quad \langle v^{n}, v^{n}\rangle_{t} = \langle V^{n}, V^{n}\rangle_{f}$$

$$E(\infty) - \|v^{i}\|_{t}^{2} - \|v^{n}\|_{t}^{2} = \|V^{i}\|_{f}^{2} - \|V^{n}\|_{f}^{2}$$

$$= \|V^{i}\|^{2} - \|S \cdot V^{i}\|^{2} = \|V^{i}\|^{2} - \|S\|^{2} \cdot \|$$

$\|V^{i}$