

file: c:\math\mcd80\mcd_rwn\singval.mcd RWN 10/24/01
 example of singular value decomposition $A=UDV^T$, U and V orthogonal, D diagonal ≥ 0

$$A := \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -2 & 1 & 2 \\ 3 & 4 & -3 & -4 \\ -3 & -4 & 3 & 4 \end{bmatrix}$$

Call to get singular value decomposition

$$Msvd := svd(A)$$

$$svalues := svds(A)$$

$$svalues = \begin{bmatrix} 10.93 \\ 0.732 \\ 0 \\ 0 \end{bmatrix}$$

$$Msvd = \begin{bmatrix} -0.286 & -0.647 & 0.707 & 0 \\ 0.286 & 0.647 & 0.707 & 0 \\ -0.647 & 0.286 & 0 & 0.707 \\ 0.647 & -0.286 & 0 & 0.707 \\ -0.407 & 0.578 & -0.707 & 0 \\ -0.578 & -0.407 & 0 & -0.707 \\ 0.407 & -0.578 & -0.707 & 0 \\ 0.578 & 0.407 & 0 & -0.707 \end{bmatrix}$$

first rows of $M=U$, last $=V$

submatrix call: $submatrix(A,r1,r2,c1,c2)$ returns for A
 where $r1$ =start row, $r2$ =endrow, $c1$ =start column, $c2$ =end column

$$U := submatrix(Msvd, 0, 3, 0, 3)$$

$$U = \begin{bmatrix} -0.286 & -0.647 & 0.707 & 0 \\ 0.286 & 0.647 & 0.707 & 0 \\ -0.647 & 0.286 & 0 & 0.707 \\ 0.647 & -0.286 & 0 & 0.707 \end{bmatrix}$$

$$V := submatrix(Msvd, 4, 7, 0, 3)$$

$$V = \begin{bmatrix} -0.407 & 0.578 & -0.707 & 0 \\ -0.578 & -0.407 & 0 & -0.707 \\ 0.407 & -0.578 & -0.707 & 0 \\ 0.578 & 0.407 & 0 & -0.707 \end{bmatrix}$$

transpose is ctrl-1

$$U \cdot U^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$V \cdot V^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$diag(svalues) = \begin{bmatrix} 10.93 & 0 & 0 & 0 \\ 0 & 0.732 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

As a check:

$$U \cdot diag(svalues) \cdot V^T = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -2 & 1 & 2 \\ 3 & 4 & -3 & -4 \\ -3 & -4 & 3 & 4 \end{bmatrix}$$