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ENEE 610 To Consider #7

1. Read chapter 4, pp. 128 - 179, on analyzing linear time-invariant networks.

2. A 3n-port circulator is a device with the scattering matrix

$$S_{circ} = \begin{bmatrix} 0_n & 0_n & 1_n \\ 1_n & 0_n & 0_n \\ 0_n & 1_n & 0_n \end{bmatrix}$$

Show that the scattering matrix at the first set of n ports is a product when terminated in the second and third sets of n ports by n-ports of respective scattering matrices S_2 and S_3. What is the physical meaning and how does this result change if S_{circ} is replaced by its transpose?

3. Find the state variable realization, R={A, B, C, D}, for the cascade of two sub-systems in terms of the state variable realizations R_1 and R_2 of the sub-systems.

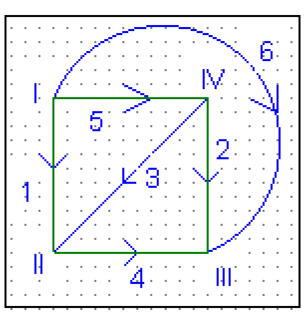
4. Since any polynomial with real coefficients can be factored into the product of degree two and degree one polynomials with real coefficients, use the results of 3. above to show that a real realization is equivalent to one with A a direct sum of 2x2 and 1x1 real companion matrices. What will the resulting form of B, C, and D be? Show that there need be at most one 1x1 term in the direct sum for A.

5. Using the results of 4. give a design of a real (linear, time-invariant) single input single output system as a cascade of degree two and degree one sub-systems. Carry this design out on

$$\frac{Vout}{Vin}(s) = T(s) = \frac{8(s^2+3s+4)(s+6)}{(s+3)(2s^2+3s+4)(s^2+5s+7)}$$

Note the differences in different realizations obtained by associating numerator factors with different denominator factors.

6. For the following graph where nodes are numbered by Roman numerals:



a) By exhibiting all possible trees determine how many possible trees there are; check by the result of problem 3.12, p. 124.

b) For the tree of branches 1,2,3 set up cut-set and the tie-set matrices.

c) For the tree of branches 1,2,3 set up the cut-set matrix and for the different tree of branches 1,2,5 set up the tie-set matrix; repeat by using branches 4,5,6 for the tie-set tree. Check what relations may be between the cut- and tie-set matrices and how they compare with results when using the same tree for both the cut-set and the tie-set matrix.

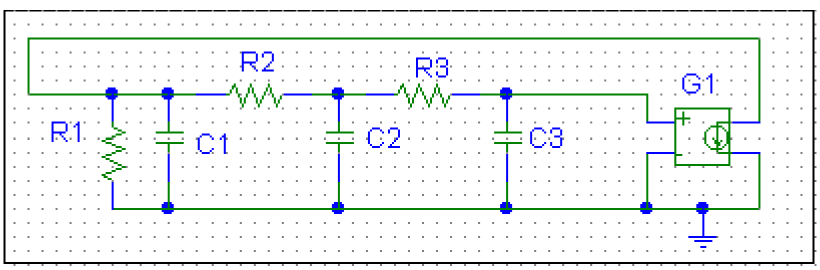
d) Assume that branch 1 is a current source, 2 a capacitor, and 3,4,5 resistors, set up the node equations and the loop equations and compare.

7. If the graph of 6. above has branches 1 & 2 as resistors R_1 & R_2, branch 6 as Vin, branch 2 as R_{load} in series with V_{dd}, branch 4 as resistor R_s, branches 3 and 5 being for an NMOS transistor (source connected to bulk = substrate) with branch 3 as Drain, D, to Source, S, and branch 5 as Gate, G, to Source

a) Draw the circuit

b) Set up using the graph a set of describing equations assuming the transistor described by $I_G=0$, $I_D=f(V_{DS},V_{GS})=-I_S$ for an appropriate function f(.,.). Give what would be an appropriate function f(.).

8. For the following circuit set up a graph with the capacitors in the tree and from that obtain on an admittance basis a set of equations describing the circuit. Put these into state variable form with the output vector comprising the voltages on the resistors, $y=v_{out}(t)$. Assuming non-zero initial conditions on all capacitors find Vout(s), the Laplace transform of $v_{out}(t)$.



9. The equations

$$\frac{dx}{dt} = yz$$
$$\frac{dy}{dt} = -zx$$
$$\frac{dz}{dt} = -k^2 xy$$

yield the Jacobi elliptic functions x(t)=sn(t,k), y(t)=cn(t,k), z(t)=dn(t,k) for 0<k<1 and initial conditions

x(0)=0=sn(0,k), y(0)=1=cn(0,k), z(0)=1=dn(0,k).

Using the PSpice Gvalue, or the Spice polynomial VCCS, set up a circuit in Pspice or Spice to realize the Jacobi elliptic functions. Plot these functions versus time for various k and make a 3-D plot of z versus x,y.

Reference: K. R. Meyer, "Jacobi Elliptic Functions from a Dynamical Systems Point of View," The American Mathematical Monthly, Vol. 108, No. 8, October 2001, pp. 729-737