Towards Optimal Channel Allocation in Cellular Reuse Partitioning Networks

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Abstract

We consider the problem of channel allocation for simple cellular network models with reuse partitioning. A number of call requests are present at the different zones of each cell and interference constraints prevent the use of the same channel in certain combination of zones at different cells. For linear networks, we obtain a channel allocation algorithm that maximizes system capacity without violating the interference constraints. Unlike the single reuse factor case, the problem can not be reduced to coloring problem for interval graphs. For the circular array case, we derive an asymptotically optimal channel allocation scheme; while for the hexagonal arrays, we derive a bound that improves the adapted bounds from single reuse factor case.

1 Introduction

Reuse partitioning is a technique for increasing wireless cellular system capacity by using multiple channel reuse factors in the same cellular system. The available channels are split among several reuse patterns with different reuse factors. Channels from the group with smallest reuse factor are assigned to mobile units with the best received signal quality. Typically these mobiles are close to the base station and can tolerate the increased interference. On the other hand, channnels with the highest reuse factor are assigned to the mobiles close to the periphery of the cell, so as to maintain an acceptable signal to interference ratio (SIR). The reuse partitioning technique reduces the SIR for the mobiles which are well above the required threshold, increasing system capacity at the same time.

The benefits of reuse partitioning has been adequately addressed in the literature. It was suggested in [6] that by using two reuse factors of $N_1 = 3$ and $N_2 = 9$, it is possible to achieve an increase in channel capacity of about 30 percent over that achieved by a single reuse factor of $N = 7$, for a system with objective SIR of 17 db. The idea of reuse partitioning, combined with a traffic adaptive channel assignment for highway microcellular radio systems was introduced in [2] and the results indicate the capacity may be doubled compared to that of fixed channel assignment. In [11] it has been shown that by using more than one reuse patterns and considering the assignment failure rate as performance measure, it is possible to achieve significant capacity improvements. Performance bounds for this model were presented in [10]. Finally, in [9], the authors show that for a two-zone cellular system, a substantial increase of carried traffic can be accomplished by allowing calls to overflow to outer zones, at the expense of very unbalanced blocking for the traffic of the two zones.

In this paper we attempt to precisely characterize the maximum achievable system capacity for simple reuse partitioning cellular networks. A number of call requests are present at the different zones of each cell and interference constraints prevent the use of the same channel in certain combination of zones at different cells. We wish to determine the minimum number of channels required to satisfy the call requirements. For linear arrays, we show that unlike the single reuse factor case, this problem can not be reduced to coloring problem for interval graphs. The presence of additional interference constraints imposed by reuse partitioning increases the complexity of the the channel allocation problem. The main result of this paper is an algorithm that allocates required channels to the calls so that a minimum number of channels is required and the the complex interference constraints imposed by reuse partitioning are not violated. A simple formula for the minimum number of required channels is also derived. For more complex networks, the single reuse factor case is difficult and we expect the reuse partitioning case to be no easier. For the circular array case, we derive an asymptotically optimal channel allocation scheme; while for hexagonal arrays, we derive a bound that improves the adapted bounds from single reuse factor case.

2 Linear array

Consider a linear cellular network with cells that are numbered sequentially from 1 to N. Each cell is partitioned in K concentric zones as shown in Figure 1; zone K is the outermost zone while zone 1 the innermost.

Calls in different cells may reuse the same channel subject to constraints on the spatial proximity of the calls. Let (i,k) denote the k^{th} zone of the i^{th} cell. Two calls in zones (i,k) and (j,l) may use the same channel if and only if $|i-j| \geq \max\{k,l\}$. This implies for example, that two calls in innermost zones of any two cells may reuse the same channel; also, two calls in zone 2 of two cells can reuse the same frequency only if the cells are not adjacent etc. As an example, Figure 2 shows the collection of zones which cannot use the same channels as those used in a specific zone.

We consider the following channel allocation problem. A number of call requests C_{ik} are present at each zone (i,k). Channels have to be assigned to calls so that each call gets one

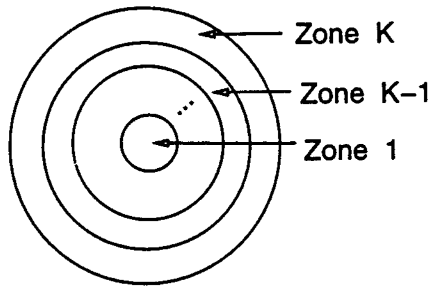

Figure 1: A cell with several zones

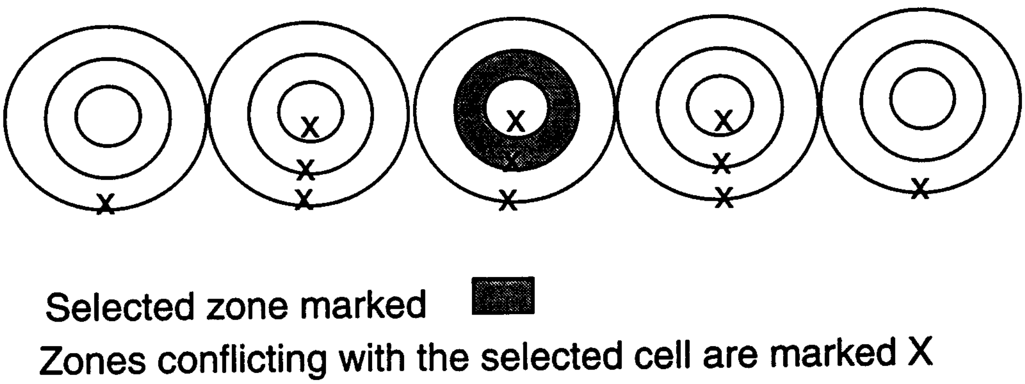

Figure 2: Illustration of channel reuse: $K = 3$

channel, there are no conflicts and the minimum number of channels is used.

The above problem can be formulated as a node coloring problem for an appropriately defined *interference graph*. This graph contains one node for each zone and two nodes in the graph are adjacent if a channel can not be simultaneously assigned to two calls in the corresponding zones. With each node in the interference graph associate a weight equal to the number of call requests in the corresponding zone. Then the above problem is identical to finding the chromatic number of the weighted interference graph. The relationship between channel assignment and graph coloring has been recognized early on and there is a lot of work where static and dynamic channel assignment schemes rely on coloring algorithms of appropriate interference graphs [3, 5, 8].

Node coloring problems are NP-hard for general graphs but efficient algorithms exist for some special classes of graphs, e.g. perfect graphs. A graph $G = (V, E)$ is a perfect graph [4] if its independent set polytope can be completely characterized by the clique constraints. The independent set polytope of the graph G is the convex hull of vectors of the form $1(S)$, $S \subset V$,

S an independent set; where $1_i(S)$, the i^{th} component of vector $1(S)$, equals 1 if node $i \in S$ and 0 otherwise. A simple example of a perfect graph is an interval graph, a graph whose nodes can be represented by intervals on the real line such that two nodes are adjacent if and only if the corresponding intervals do not overlap.

If each cell has only two zones, then the associated interference graph is an interval graph and hence, a perfect graph for which efficient coloring methods exist. To show that the interference graph is an interval graph, associate interval $(2i, 2i+1)$ on the real line to node $(i, 1)$, and interval $(2(i-1)+3/4, 2(i+1)+1/2)$ to node $(i, 2)$, $i = 0, 1, 2, \cdots$.

Interestingly however, when there are more than 2 zones per cell then the interference graph may not be an interval graph (see Appendix). In what follows, we show that the interference graph in the general case is still a perfect graph and hence the number of channels can be completely described by clique constraints. These constraints can be easily enumerated and hence the minimum number of channels can be computed as well. Our proof is constructive and instead of relying on general coloring methods for perfect graphs [4], we develop a simple coloring/channel allocation scheme for establishing the perfectness of the graph.

We first describe an optimal channel allocation scheme. The channels are allocated to the cells sequentially starting from cell 1 to cell N. In each cell the channels are allocated to the zones sequentially as well, starting from the calls in zone K and moving up to the calls in zone 1. All the calls in a certain zone are allocated before the algorithm moves on with the allocation of the calls in the next zone.

Assume that channels have been allocated to all calls in cells 1 through $i-1$ as well as to all calls in zones K through $k-1$ of cell i. The calls in zone (i, k) are then allocated by the following steps:

1. Consider all the channels that have been used already in one of the cells 1 through i. Reuse first those channels for the calls in zone (i, k) as follows.

   Start from cell $i-1$ and go up to cell 1 doing the following for each cell j. Consider all the channels used in zones of cell j which are not conflicting with zone (i, k), if any. From these channels, identify those that are not already allocated to calls in zone (i, k) and allocate as many as necessary. The order of selection of zones of cell j is not important. If there are remaining calls without a channel allocated in zone (i, k) then proceed to cell $j-1$ and do the same.

2. If all the channels that are already used are exhausted and there are calls in zone (i, k) without an allocated channel, then use new channels arbitrarily.

We claim that the number of channels required by the above algorithm is equal to the size of the maximum weighted clique of the interference graph. Note that the number of required channels must be no less than the size of the maximum weighted clique of the interference graph. Hence it suffices to show that the algorithm requires a number of channels equal to the size of some clique in the interference graph. In fact, the following stronger property holds for every zone (i, k).

P1. For any cell m, $m \leq i$, consider all the calls in the cells m through $i-1$ as well as the calls in the zones (i, K) through (i, k). Consider the weighted interference subgraph that corresponds to just those calls. The total number of distinct channels used for the allocation of calls in this group of cells is equal to the size of some clique in the interference subgraph.

Theorem 1. *Property P1 holds for any zone* (i, k), $i = 1, \cdots, N$, $k = 1, \cdots, K$.

Proof. The proof is by induction. $P1$ clearly holds for zone $(1, K)$. We show that it will hold for zone (i, k) when it holds for zone $(i, k+1)$, $k = K-1, \cdots, 1$; and that it will hold for zone (i, K) when it holds for zone $(i-1, 1)$.

Let m be an arbitrary cell, $m \leq i$. We consider two cases in verifying P1.

Case A: All calls in zone (i, k) are allocated channels that are used already either by calls in cells m through $i-1$ or by calls in zones (i, K) through $(i, k+1)$.

In this case the total number of distinct channels, say C, used in cells m through $i-1$ and zones (i, K) through $(i, k+1)$ is equal to the total number of distinct channels used in cells m through $i-1$ and zones (i, K) through (i, k). The induction hypothesis ensures that there is a clique in the interference graph corresponding to the cells m through $i-1$ and zones (i, K) through $(i, k+1)$, with size equal to C. That clique is the one for which property P1 is satisfied for zone (i, k) as well.

Case B: The calls in zone (i, k) need additional channels than those used by calls in cells m through $i-1$ and in zones (i, K) through $(i, k+1)$.

Consider the weighted interference graph corresponding to calls in cells $m, \cdots, i-1$ and zones $(i, K), (i, K-1), \cdots, (i, k)$. We specify the maximal weighted clique U in this graph. The set U consists of three types of nodes.

(a) All the nodes corresponding to zones $(i, K), (i, K-1), \cdots, (i, k)$.

(b) All the nodes corresponding to any zone of cells $m, \cdots, i-k$ that are conflicting with zone (i, k).

(c) From the induction hypothesis, there exists a clique Q in the weighted interference subgraph consisting of nodes corresponding to calls in the set of cells $i-k+1, \cdots, i-1$ and the size of this clique is equal to the total number of distinct channels needed by the algorithm to allocate channels to calls to this set of cells. Consider the set of nodes in Q.

We claim first that U is a clique. This follows from the following arguments.

- All the nodes of type (a) are conflicting among themselves; they are also conflicting with nodes of type (b) by definition and finally they are conflicting with nodes of type (c) since they are conflicting with any zone in the cells $i-k+1, \cdots, i-1$.
- A zone (j,l) of type (b) has the property that $i-j < \max\{k,l\}$, since it conflicts with zone (i,k). Furthermore, $\max\{k,l\} = l$, otherwise $i-j < k$, a contradiction. Hence a zone (j,l) of type (b) conflicts with a zone (j',l') of type (c) since $j'-j < i-j < l \leq \max\{l,l'\}$. Furthermore, any two zones of type (b) are conflicting among themselves since they each conflict with (i,k). This can be seen as follows. Take two nodes (j,l) and (j',l') of type (b). Suppose $j < j'$. Since they both conflict with (i,k) arguing as before, we have $\min\{l,l'\} > k$. Therefore $j'-j < i-j < \max\{k,l\} = l \leq \max\{l,l'\}$ which shows that (j,l) and (j',l') do indeed conflict.
- All nodes of type (c) are conflicting among themselves by definition.

This establishes that U is a clique.

We now show that the size of U equals the total number of distinct channels used by the algorithm to allocate channels to calls in cells $m, \cdots, i-1$ and zones $(i,K), (i,K-1), \cdots, (i,k)$. The channels allocated to calls in zones $(i,K), (i,K-1), \cdots, (i,k)$ are certainly included in U by definition. Also, the channels allocated to calls in cells $i-k+1, \cdots, i-1$ is equal to the size of Q and $Q \subset U$ by construction. Finally, consider channels allocated by the algorithm to cells $m, \cdots, i-k$. If a zone in this set of cells is conflicting with (i,k) then the channels allocated to that zone are included in U as well by construction. If on the other hand, a zone in this set of cells does not conflict with (i,k), then all these channels are allocated by the algorithm to (i,k), and hence these channels are also included in U. This completes the proof of the theorem. □

In summary, we have provided an algorithm which provides a conflict free allocation of channels to calls in a linear array. The number of channels used by this algorithm is equal to the maximal weighted clique of the associated interference graph and is the minimum possible. This also establishes the fact that the interference graph is a perfect graph since its chromatic number is only determined by clique constraints.

One way to determine the miminum number of required channels is of course to run the algorithm. But it is also of interest to see if there is a way to characterize all the cliques which would then lead to an expression for the minimum number of required channels. This is done next.

We first show that there is a smaller set of cliques which determine the minimum number of channels. Consider cliques U of the following type: for each level k, $i \leq k \leq K$, U contains k consecutive zones, with the property that the k cells for level k consitute a subset of the $k+1$ cells of level $k+1$. As an example, for $K=3$, two cliques of the above form are given by

(a) $(i,3),(i+1,3),(i+2,3),(i,2),(i+1,2),(i,1)$ and (b) $(i,3),(i+1,3),(i+2,3),(i+1,2),$ $(i+2,2)$, $(i+2,1)$. We claim that all the maximal cliques that need to be considered for the evaluation of the bound are of this type. Consider an arbitrary clique Q. Let m_k, M_k be the minimum and maximum cells respectivelly where a zone of level less than or equal to k that belongs to Q may arise. Consider the set of zones S that includes for each level k all the zones $(m_k,k),(m_k+1,k),..,(M_k,k)$. It can be easily verified that this set of zones is a clique and clearly it is a superset of Q. Furthermore it can be easily verified that S is a subset of a maximal clique of the type specified earlier.

One can derive an recursive expression to determine the number of required channels. Recall that C_{ik} denotes the number of required channels in zone (i,k). Let $\{N_i^{(k)}\}_{1\leq i\leq N, 1\leq k\leq K}$ denote some intermediate quantities that are related by the recursion

$$N_i^{(k+1)} = \sum_{l=i}^{i+k} C_{l,k+1} + \max\left\{N_i^{(k)}, N_{i+1}^{(k)}\right\}.$$

The initial condition in the recursion is given by

$$N_i^{(2)} = C_{i,2} + C_{i+1,2} + \max\{C_{i,1}, C_{i+1,1}\}$$

and the total number of required channels is given by

$$\max_{0\leq i\leq N-K} N_i^{(K)}.$$

The proof follows from the above discussion on the special class of determining cliques and is omitted for brevity.

3 Other cellular topologies

3.1 Circular array

Consider N cells are arranged in a circular array, that is, cell $N-1$ is adjacent to cell 0. Each cell is partitioned into K concentric zones. Two cells (i,k) and (j,l), $i<j$ can reuse the same frequency if $i+\max\{k,l\} \leq j$ holds and either $j+\max\{k,l\} < N$ holds or $(j+\max\{k,l\})$ (mod N) $\leq i$ holds. Let C_{ik} denote the number of call requests in zone (i,k). We wish to determine a channel allocation that assigns one channel to a call without conflicts and uses the minimum number of channels.

It is well known [1] that the special case of the above problem with 1 zone per cell is a hard problem because of the presence of holes, antiholes etc. in the associated interference graph. Therefore, we concentrate on suboptimal allocations.

Consider opening up the circular array after cell $N-1$. Then it becomes a linear array for which we know the number of required channels to meet the call requirements. Let N^*

denote this quantity. Since a circular array presents more constraints, we know that at least N^* channels would be required in this case as well. We adapt an argument presented in [1] to construct an algorithm that needs $\frac{N}{N-K}N^*$ channels. This algorithm becomes optimal for large arrays as $N/K \to \infty$.

The algorithm works in N cycles. In cycle n, the set of cells $n, (n+1) \pmod N, \cdots, n+N-K \pmod N$ are considered for allocation. The channel requirements in each cycle are reduced to $C_{ik}/(N-K)$. We make the following observations.

- The cells considered in each cycle form a linear array since the additional constraints from the endpoints are absent. Hence the algorithm of the previous section can be used to assign, in each cycle, $C_{ik}/(N-K)$ frequencies to zone (i,k) using a total of $N^*/(N-K)$ channels.

- Each zone (i,k) is allocated channels in $N-K$ cycles out of the N cycles. Hence after N cycles, zone (i,k) gets its required C_{ik} channels.

- After N cycles, a total of $\frac{N}{N-K}N^*$ channels are needed

The fractional frequencies can be handled naturally. If zone (i,k) requires $x.y$ frequencies in a cycle, it is assigned x frequencies in all but the last cycle and $x+1$ frequencies in the last cycle.

3.2 Planar hexagonal array

Consider a hexagonal array with (1,3) cell reuse pattern. Each cell has two zones. A frequency can be reused in the inner zone of every cell. A frequency can be reused in the outer zones of two cells if those two cells do not touch each other along any face.

We consider the co-ordinate system shown in Figure 3. Let C^0_{xy} and C^1_{xy} denote the channel requirements in the inner and outer zones of cell (x,y). We want to obtain an upper bound to the total number of required channels. Many upper bounds are possible and none seem to dominate others.

If we do not treat the inner zones in any special manner, then one can think of $C^0_{xy} + C^1_{xy}$ to be the number of required frequencies in cell (x,y). Then following the arguments in [1], atmost

$$\max_{x,y} \max_{(i,j)\in\mathcal{A}} 1.5 \left[C^0_{xy} + C^1_{xy} + C^0_{x+i,y+j} + C^1_{x+i,y+j}\right] \tag{1}$$

frequencies are needed, where $\mathcal{A}$ is the set $\{(0,1),(1,-1),(-1,0)\}$. This bound may not be good when $\{C^0_{x,y}\}$ are large, since we do not use the fact that frequencies can be reused in inner zones of every cell.

We now develop another upper bound by providing an channel allocation scheme. First some notation is needed. Let the outer zones of the cells be colored RED, YELLOW and

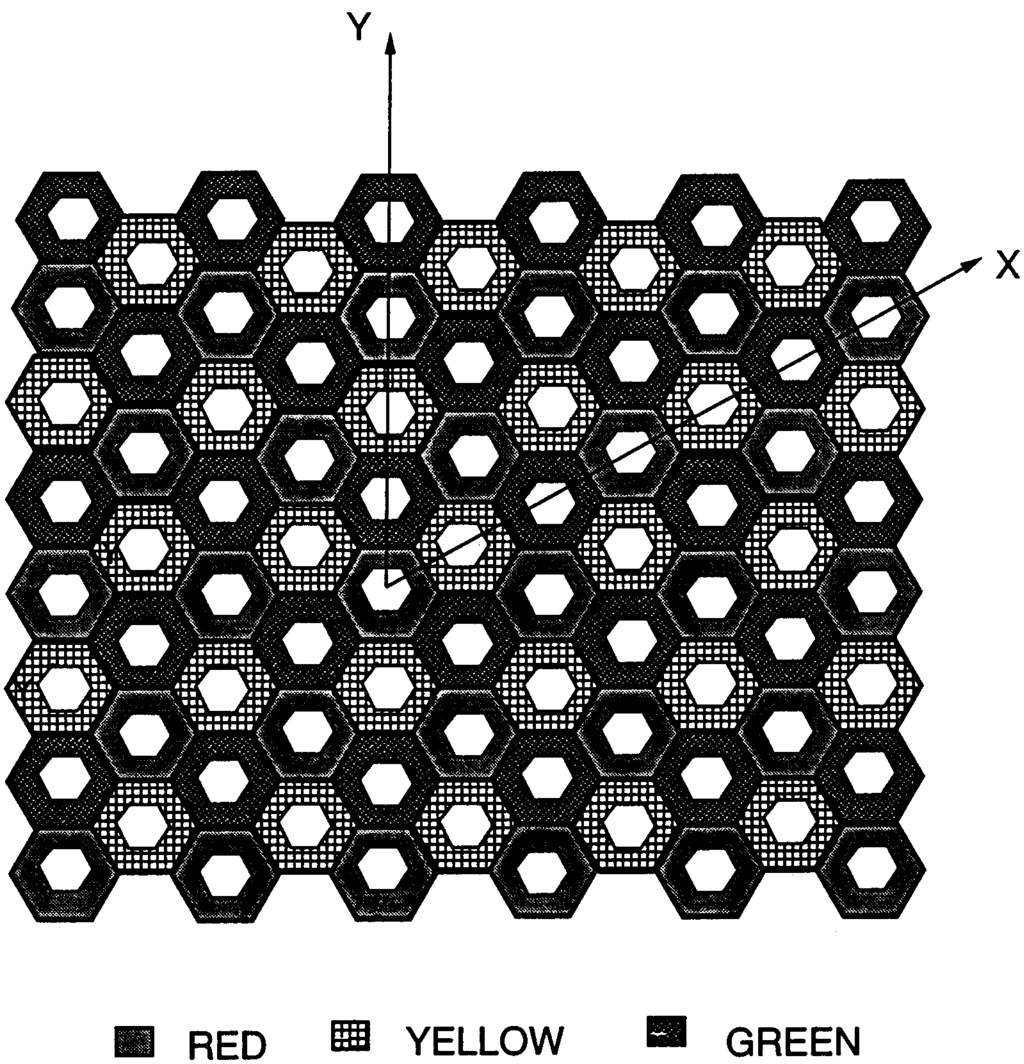

Figure 3: Coloring and co-ordinate system for hexagonal array with (1,3) spatial reuse pattern

GREEN. Two cells are neighbors if they touch each other along a face. Note that the set of GREEN (resp. YELLOW,RED) neighbors of a RED (resp. GREEN, YELLOW) cell (x, y) is given by $(x+i, y+j) : (i, j) \in \mathcal{A}$, where $\mathcal{A} = \{(0,1), (1,-1), (-1,0)\}$ as defined in the previous paragraph. The channel allocation scheme follows. *Allocate channels to the inner zones of every cell. Then allocate channels to outer zones in the order RED, YELLOW and GREEN. Repeat the (RED, YELLOW, GREEN) cycle twice, each time allocating half the required frequencies to each outer zone. Note that channels can be allocated to a RED colored outer zone only when all the inner zones in the cells around that zone has been allocated.*

We derive a bound on the number of channels used by the above algorithm. Let $\mathcal{A}_1$ denote the set $(0,0),(\pm 1,0),(0,\pm 1),(\pm 1,\mp 1)$. Denote the quantities

$$N_1^* = \max_{x,y}\left\{C_{xy}^1 + \max_{(i,j)\in\mathcal{A}} C_{x+i,y+j}^1 + \max_{(i_1,j_1)\in\mathcal{A}_1} C_{x+i_1,y+j_1}^0\right\}$$

and

$$N_2^* = \max_{x,y}\max_{(i,j)\in\mathcal{A}}\left\{C_{xy}^1 + C_{x+i,y+j}^1\right\}.$$

Theorem 2. *The number of channels used by the above channel allocation scheme is less than or equal to*

$$N_1^* + N_2^*. \tag{2}$$

As a remark, observe that the bound (2) can be tighter than (1) in some circumstances; consider for example the case where channel requirements in every zone of every cell is the same.

Proof. Let $T_{x,y}(k)$ denote the total number of channels after allocating to cell (x,y) in the k^{th} cycle; $k=1,2$. We now obtain some expressions from the operation of the algorithm. The policy first allocates channels to the inner zones and then to RED cells; therefore,

$$T_{x,y}(1) = \max_{(i,j)\in\mathcal{A}_1} C_{x+i,y+j}^0 + 0.5\, C_{x,y}^1; \quad (x,y):\ \text{RED}. \tag{3}$$

Observe that the $\{(x+i,y+j):(i,j)\in\mathcal{A}_1\}$ represents the 6 cells touching (x,y). Similarly, in both cycles, a YELLOW (respectively GREEN) colored outer cell is allocated after all the RED (respectively YELLOW) colored neighboring cells. Hence,

$$T_{x,y}(k) = \max_{(i,j)\in\mathcal{A}} T_{x+i,y+j}(k) + 0.5\, C_{x,y}^1; \quad (x,y):\ \text{YELLOW},\ \ \text{GREEN},\quad k=1,2. \tag{4}$$

Finally, a RED colored outer cell is allocated in the second cycle after allocating to all its GREEN neighbors; hence,

$$T_{x,y}(2) = \max_{(i,j)\in\mathcal{A}} T_{x+i,y+j}(1) + 0.5\, C_{x,y}^1; \quad (x,y):\ \text{RED}. \tag{5}$$

Since the total number of required channels is simply $T_{x,y}(2)$ for a GREEN colored zone (x,y), we are only interested in deriving $T_{x,y}(2)$ for (x,y) : GREEN, in terms of $\{C_{xy}\}$ using the above equations (3-5). First use (4) recursively in (4) to yield for (x,y): GREEN:

$$T_{x,y}(2) = \max_{(i_m,j_m)\in\mathcal{A},1\le m\le 2} T_{x+i_1+i_2,y+j_1+j_2}(2) + \max_{(i_1,j_1)\in\mathcal{A}} 0.5C_{x+i_1,y+j_1}^1 + 0.5C_{x,y}^1. \tag{6}$$

This can be done since for (x,y): GREEN, $(x+i,y+j)$ is YELLOW for $(i,j)\in\mathcal{A}$. Now observe that $(x+i_1+i_2,y+j_1+j_2)$ in (6) is RED so that (5) can be used to yield

$$\begin{aligned} T_{x,y}(2) = {} & \max_{(i_m,j_m)\in\mathcal{A},1\le m\le 3} T_{x+i_1+i_2+i_3,y+j_1+j_2+j_3}(1) + \max_{(i_m,j_m)\in\mathcal{A},1\le m\le 2} 0.5C_{x+i_1+i_2,y+j_1+j_2}^1 \\ & + \max_{(i_1,j_1)\in\mathcal{A}} 0.5C_{x+i_1,y+j_1}^1 + 0.5C_{x,y}^1. \end{aligned} \tag{7}$$

Since $(x + i_1 + i_2 + i_3, y + j_1 + j_2 + j_3)$ in (7) is GREEN, the process repeats for the first cycle ($k = 1$). It is not difficult to derive the final expression for (x, y) : GREEN (an empty summation: $\sum_x^y, y < x$ is taken to be 0):

$$T_{x,y}(2) = \max_{\substack{(i_m, j_m) \in \mathcal{A}, 1 \le m \le 5 \\ (i_6, j_6) \in \mathcal{A}_1}} \left\{ 0.5 \sum_{m=0}^{5} C^1_{x + \sum_{l=1}^{m} i_l, y + \sum_{l=1}^{m} j_l} + C^0_{x + \sum_{l=1}^{6} i_l, y + \sum_{l=1}^{6} j_l} \right\}. \quad (8)$$

By pairing terms inside $\{\cdots\}$ on the right hand side in the above equation, it is easy to see that $T_{x,y}(2) \le N_1^* + N_2^*$. For instance, the sum of the first two terms is less than $N_2^*/2$ since the corresponding cells touch each other. Similarly, the sum of the third and fourth terms is also less than $N_2^*/2$. Finally, the sum of the three remaining terms is N_1^* by definition. This establishes the upper bound (2). □

4 Discussion

Certain coloring problems in graphs capturing the interference constraints arising in cellular networks with reuse partitioning were considered in this paper. An optimal algorithm for a linear network was derived, while suboptimal algorithms and bounds for circular and planar networks were obtained as well. Optimal and suboptimal packing algorithms as those proposed here are useful in designing dynamic channel assignment with good performance.

5 Appendix

In this appendix, we show that the interference graph corresponding to the linear array with 3 zones per cell is not an interval graph.

Suppose the interference graph is indeed an interval graph. Therefore, there exists an interval graph representation. Let I_{ik} denote the interval on the real line associated with the node of the interference graph corresponding to zone (i, k). Also let $L(I_{ij})$ (respectively $R(I_{ij})$) denote the left (respectively right) end-point of the interval I_{ij}.

We first consider the case where interval I_{11} is on the left of interval I_{21}, that is $R(I_{11}) < L(I_{21})$. Since I_{12} intersects both I_{11} and I_{21}, we have $L(I_{12}) \le R(I_{11}) \le L(I_{21}) \le R(I_{12})$ as shown in Figure 4(a). Since I_{31} does not intersect any one of I_{11}, I_{21} and I_{12}, it must be that interval I_{31} is either on the right or left of I_{11}, I_{21} and I_{31}. In other words, either $L(I_{31}) > \max\{R(I_{21}), R(I_{12})\}$ as shown in Figure 4(a) or $R(I_{31}) < \min\{L(I_{12}), L(I_{11})\}$ as shown in Figure 4(b). Accordingly consider the following two subcases.

Case 1: I_{31} is on the right of I_{11}, I_{21} and I_{31}. We will show that I_{21} and I_{43} cannot intersect which is a contradiction. Since I_{31} is on the right of I_{21}, $R(I_{21}) < L(I_{31})$. Since I_{13} and I_{31} intersect, $L(I_{31}) \le R(I_{13})$. Also, since I_{13} and I_{43} do not intersect and I_{31} and I_{43} intersect, we have $R(I_{13}) \le L(I_{43})$. Combining the above three inequalities, we derive $R(I_{21}) < L(I_{43})$

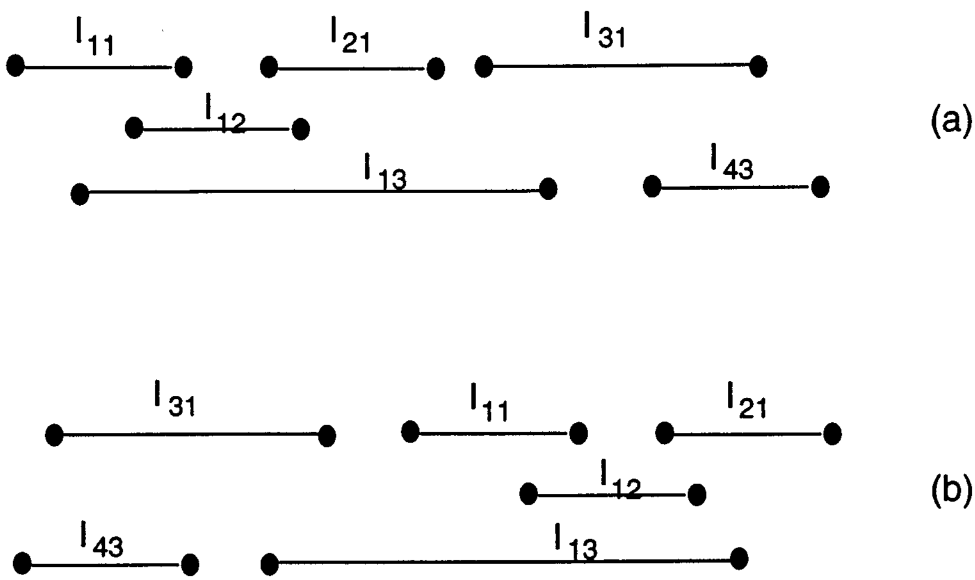

Figure 4: Interference graphs of reuse partitioning networks are not interval graphs: Two cases

which implies that I_{21} and I_{43} do not intersect, a contradiction. This case is illustrated in Figure 4(a).
Case 2: I_{31} is on the left of I_{11}, I_{21} and I_{31}. Arguments similar to that given in Case 1 can be given to show that I_{21} and I_{43} cannot intersect, a contradiction.

The discussion of the case where interval I_{11} is on the right of interval I_{21} is very similar and is skipped for brevity.

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