TABLE II
20-USER WEIGHT VECTORS OF LENGTH 15

$$\vec{w}^{(1)} = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,1),\ \vec{w}^{(2)} = (0,0,0,0,0,0,0,0,0,0,0,0,0,1,0),$$
$$\vec{w}^{(3)} = (0,0,0,0,0,0,0,0,0,0,0,0,-1,1,1),\ \vec{w}^{(4)} = (0,0,0,0,0,0,0,0,0,0,0,1,0,0,0),$$
$$\vec{w}^{(5)} = (0,0,0,0,0,0,0,0,0,0,-1,1,0,0,1),\ \vec{w}^{(6)} = (0,0,0,0,0,0,0,0,0,-1,0,1,0,1,0),$$
$$\vec{w}^{(7)} = (0,0,0,0,0,0,0,0,1,-1,-1,1,-1,1,1),\ \vec{w}^{(8)} = (0,0,0,0,0,0,0,1,0,0,0,0,0,0,0),$$
$$\vec{w}^{(9)} = (0,0,0,0,0,0,-1,1,0,0,0,0,0,0,1),\ \vec{w}^{(10)} = (0,0,0,0,0,-1,0,1,0,0,0,0,0,1,0),$$
$$\vec{w}^{(11)} = (0,0,0,0,1,-1,-1,1,0,0,0,0,-1,1,1),\ \vec{w}^{(12)} = (0,0,0,-1,0,0,0,1,0,0,0,1,0,0,0),$$
$$\vec{w}^{(13)} = (0,0,1,-1,0,0,-1,1,0,0,-1,1,0,0,1),\ \vec{w}^{(14)} = (0,1,0,-1,0,-1,0,1,0,-1,0,1,0,1,0),$$
$$\vec{w}^{(15)} = (-1,1,1,-1,1,-1,-1,1,1,-1,-1,1,-1,1,1).$$

$$\vec{z} \leftarrow \vec{z} - \vec{v}_5 = (1,\ 1,\ 2,\ 1,\ 0,\ 0,\ 1,\ 0,\ 1,\ 1,\ 2,\ 1,\ 0,\ 0,\ 1).$$
$$q = 4$$
$$\vec{w}^{(4)}\vec{z}^T = 1 \Rightarrow e_4 = e_1^{(4)} = 1$$
$$\vec{z} \leftarrow \vec{z} - \vec{v}_4 = (0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1).$$
$$q = 3$$
$$\vec{w}^{(3)}\vec{z}^T = 1 \Rightarrow e_3 = e_1^{(3)} = 1$$
$$\vec{z} \leftarrow \vec{z} - \vec{v}_3 = (0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0)$$
$$\Rightarrow e_2 = e_1 = 0.$$

So $e_3 = e_4 = e_5 = e_6 = e_8 = e_9 = e_{10} = e_{12} = e_{13} = e_{15} = e_{16} = e_{17} = e_{18} = 1$, $e_{11} = e_{20} = 2$, and $e_1 = e_2 = e_7 = e_{14} = e_{19} = 0$. With $s_i = e_i - 1$, that implies $s_1 = s_2 = s_7 = s_{14} = s_{19} = -1$, $s_{11} = s_{20} = 1$, and $s_3 = s_4 = s_5 = s_6 = s_8 = s_9 = s_{10} = s_{12} = s_{13} = s_{15} = s_{16} = s_{17} = s_{18} = 0$. Hence, the receiver uniquely identifies that $-\vec{c}_1$, $-\vec{c}_2$, $-\vec{c}_7$, $-\vec{c}_{14}$, $-\vec{c}_{19}$, $\vec{c}_{11}$, and $\vec{c}_{20}$ are sent from the active users and the rest of users are idle.

IV. SUMMARY

In this correspondence we use Lindstrom combinatory detecting set for the construction of a set of spreading sequences for S-CDMA systems. These spreading sequences can support the number of users far exceeding the current S-CDMA approaches, and hence improves the sum rate for S-CDMA systems.

ACKNOWLEDGMENT

The authors wish to thank the anonymous referees for their helpful comments and suggestions.

REFERENCES

[1] M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt, *Spread Spectrum Communications Handbook.* New York: McGraw-Hill, 1994.

[2] J. L. Massey and T. Mittelholzer, "Welch's bound and sequence sets for code-division multiple-access systems," in *Sequences II* (Positano, Italy, 1991), R. Capocelli, A. De Santis, and U. Vaccaro, Eds. New York: Springer, 1993, pp. 63–78.

[3] E. A. Lee and D. G. Messerschmitt, *Digital Communication.* Boston, MA: Kluwer, 1994.

[4] Y. W. Wu and S. C. Chang, "Coding scheme for multiple-access adder channel with and without idle users," *Electron. Lett.*, vol. 31, pp. 523–524, Mar. 1995.

[5] G. H. Khachatrian and S. S. Martirossian, "A new approach to the design of codes for synchronous-CDMA systems," *IEEE Trans. Inform. Theory,* vol. 41, pp. 1503–1506, Sept. 1995.

[6] B. Lindstrom, "On a combinatorial problem in number theory," *Can. Math. Bull.*, vol. 8, pp. 477–490, 1965.

Scheduling and Performance Limits of Networks with Constantly Changing Topology

Leandros Tassiulas, *Member, IEEE*

***Abstract*— A communication network with time-varying topology is considered. The network consists of M receivers and N transmitters that, in principle, may access every receiver. An underlying network state process with Markovian statistics is considered that reflects the physical characteristics of the network affecting the link service capacity. The transmissions are scheduled dynamically, based on information about the link capacities and the backlog in the network. The region of achievable throughputs is characterized. A transmission scheduling policy is proposed that utilizes current topology state information and achieves all throughput vectors achievable by any anticipative policy. The changing topology model applies to networks of Low-Earth Orbit (LEO) satellites, meteor-burst communication networks, and networks with mobile users.**

***Index Terms*—Low-Earth Orbit (LEO) satellite networks, throughput analysis, time-varying networks, wireless networks.**

I. INTRODUCTION

In this correspondence we consider a changing topology network model that captures some essential features of wireless networks with changing topology, including Low-Earth Orbit (LEO) satellite net-

Manuscript received December 26, 1993; revised February 25, 1996. This research was supported in part by NSF under Grant NCR-9406415, and by the AFOSR under Grant 95-1-061.

The author is with the Electrical Engineering Department and the Institute for Systems Research, University of Maryland, College Park, MD 20742 USA.

Publisher Item Identifier S 0018-9448(97)02319-5.

works, terrestrial mobile, and meteor-burst communication systems. The model is general enough to include networks with multiple nodes and arbitrary configurations. The transmission capacity of the links may vary with time in several different fashions. In changing connectivity models, the link is either available to transmit in its full capacity or unavailable, hence the capacity is a binary process. In general, the capacity may take one among multiple values. This is the case when adaptive channel coding is used and the coding rate decreases as the channel quality deteriorates. The temporal variation of the capacity is modeled by a hidden Markov chain. More specifically, the link capacity is a function of the state of the underlying Markov chain that models the temporal variation of the network.

The focus of the correspondence is on the scheduling and the throughput of the time-varying topology network. Initially, the effect of the changing topology is isolated by considering the system with no exogenous arrivals, and with an infinite supply of packets in each transmitter. The region of the traffic rates achievable by any policy is obtained. The case of exogenous arrivals is considered next. The region of achievable throughputs is characterized. It contains the throughput vectors achievable by a large class of policies, including anticipative policies which can rely on the statistics of the topology process and which may utilize the topology state for scheduling. A policy is proposed which achieves any achievable throughput. It schedules the transmissions at slot t based on the backlog at slot t as well as the transmission quality of the different links. This policy needs no statistics of the topology or the arrival processes for scheduling. Furthermore, it is nonanticipative and does not need full knowledge of the state of the topology.

A queueing system that captures the phenomenon of changing connectivity in the single-node case was considered by Tassiulas and Ephremides in [5]. The changing connectivity was represented by Bernoulli processes. The system considered here generalizes the one considered in [5] in the following aspects. First, a network with arbitrary topology is considered. Second, the topology is represented by a hidden Markov model instead of an independent and identically distributed (i.i.d.) process. The cases of periodic connectivity process (LEO satellite networks) and certain dependent topology processes arising in meteor-burst networks are included. Third, anticipative scheduling policies are considered which may base the transmission scheduling decisions on the knowledge of the future connectivity pattern. Fourth, link capacities that may take values in a set are considered. That is, a link is not necessarily in either connected or disconnected state but it may be in a variety of states that represent different transmission qualities. Systems where the effect of changing connectivity arises in a somewhat different context were studied by Carr and Hajek [1] and Tassiulas and Papavassiliou [6].

The queue length process in the changing connectivity model is a hidden Markov chain, influenced by the underlying topology process. Its stability analysis is done by considering the drift of a quadratic Liapunov function, averaged by the stationary distribution of the topology process. The drift between consecutive time instances may be nonnegative for some states of the topology process. If we consider though the drift between time instances appropriately spaced such that the topology process reaches its steady state, then only the steady-state statistics of the topology process matters. Hence it is enough for stability to study the drift averaged by the stationary distribution of the topology process. The same approach may turn out to be appropriate for the stability analysis of systems with Markov modulated arrivals.

The correspondence is organized as follows. The model is specified in Section II. In Section III the region of achievable throughputs for a system with infinite packet supplies in the nodes is obtained. In Section IV the system is considered with exogenous arrivals, and the maximum throughput scheduling policy is obtained.

II. The Changing Topology Network

The network consists of N transmitters and M receivers. Each transmitter may attempt transmission to one of the receivers at each time instant. The success of the transmission depends on the other transmission attempts at the same time as well as the *topology state* of the network. The topology state includes all the characteristics of the network that affect the transmission and may change with time. These may be the connectivity status of the links in networks with changing connectivity, the transmission rate for links with changing quality, etc. Other characteristics, not directly related to the transmission properties, may be included in the topology state. For example, in networks where the variation of the transmission characteristics has some periodic structure, the topology state will include the current state of the transmission characteristics as well as the phase. We assume that there is a finite number L of possible topology states; the set of all possible topology states is denoted by $\mathcal{S}$. We consider slotted time, with slot lengths equal to a packet transmission time. The topology state may change at the slot boundaries and its value at slot t is denoted by the variable $\boldsymbol{S}(t)$. The topology process $\{\boldsymbol{S}(t)\}_{t=1}^{\infty}$ is assumed to be an irreducible (possibly periodic) finite state-space Markov chain.

The transmission of transmitter i to receiver j at slot t is successful with probability $Q_{ij}(t)$. The time-varying topology is represented by the variation with time of the probabilities of successful transmission $Q_{ij}(t)$. These probabilities depend on the topology state as well as on which transmitters attempt transmission toward which receivers at each slot. Throughout the correspondence we assume that given the topology state at time t and the transmission attempts at that slot, the outcome of the transmission at t is independent of the past topology states or transmission attempts. The transmission attempts at slot t are denoted by the binary *transmission vector*

$$\boldsymbol{R}(t) = (R_{ij}(t) : i = 1, \cdots, N j = 1, \cdots, M)$$

where $R_{ij}(t)$ is equal to 1 if transmitter i attempts transmission to receiver j at that slot and 0 otherwise. One possibility is that a receiver may be able to listen to only one transmitter at a time, in which case at most one transmitter should attempt to access a particular receiver. If the receivers have multireception capabilities, then it is possible that more than one transmitter may attempt transmission to the same receiver simultaneously. The set of all possible transmission vectors is denoted by $\mathcal{R}$. Let $Q_{ij}: \mathcal{S} \times \mathcal{R} \rightarrow [0, 1]$ be the function that determines the probability of success in the transmission from i to j at t based on $\boldsymbol{R}(t)$, $\boldsymbol{S}(t)$; that is,

$$Q_{ij}(t) = Q_{ij}(\boldsymbol{S}(t), \boldsymbol{R}(t)).$$

The probability of success function captures all the changing topology features which are relevant for the transmission control. Furthermore, this formulation has within its scope arbitrary configurations of the network. If there is no communication link from some transmitter i to receiver j then $Q_{ij}(\boldsymbol{s}, \boldsymbol{r})$ is equal to zero for every pair $\boldsymbol{r}, \boldsymbol{s}$.

The transmission control policy is the collection of transmission vectors $\{\boldsymbol{R}(t)\}_{t=1}^{\infty}$. In general, the $\boldsymbol{R}(t)$' s might be random vectors that take values in $\mathcal{R}$. In the transmission policies of interest, the transmission vector depends on the system state. The vector $\boldsymbol{R}(t)$ may be a function of the backlog and the topology state at t, and in some cases of the future topology states $\boldsymbol{S}(t+1), \boldsymbol{S}(t+2), \cdots$ as well, since those are available to the controller in certain applications. In Section III, the throughput region is characterized for a saturated system with infinite backlogs. Ergodic policies for which the long-run average throughput exists are considered. In the study of the system with exogenous arrivals, only stationary policies are considered. The restriction to stationary policies does not affect, in any significant manner, the region of achievable throughput as it will be shown later.

In the throughput analysis we will see that nothing is gained by considering policies that make the scheduling decisions based on information about the future topology states since the throughput region achieved by nonanticipative policies coincides essentially with the throughput region achieved by anticipative policies. Before we proceed to the analysis, we discuss in more detail how the above model applies to some practical systems.

A. Networks of LEO Satellites

Each satellite is equipped with several transmitters and receivers. Each transmitter corresponds to one of the N transmitters in the model. Similarly, for the receivers. Each transmitter may transmit only to one receiver at a time. Also each receiver may listen to one transmitter at a time. In this case, the topology will be more easily represented by the introduction of the *connectivity variables.* Receiver j may listen to transmitter i at slot t if and only if the binary random variable $C_{ij}(t)$, the *connectivity variable* between i and j is equal to 1. At every time slot the *connectivity state* of the network is represented by the random vector

$$\boldsymbol{C}(t) = (C_{ij}(t),\ i = 1, \cdots, N, j = 1, \cdots, M).$$

Because of the periodic orbital movement, the variation of the distance between any two satellites is periodic. Therefore, the connectivity $C_{ij}(t)$ between any transmitter–receiver pair i–j is periodic as well. The connectivity vector is also periodic with period the least common multiple of the periods of $C_{ij}(t)$ which will be denoted by T. The connectivity vector incorporates all the relevant topology information for scheduling. Nevertheless, it does not qualify as a topology state because it is possible that the same connectivity states occur several times during a period; therefore, the *phase* of the connectivity is needed as well. Let $p(t) = t \bmod T$ and $\boldsymbol{S}(t) = (\boldsymbol{C}(t), p(t))$. The connectivity $\boldsymbol{C}(t)$ and the *phase* $p(t)$ constitute a state for the topology process. The state space is

$$\mathcal{S} = \{0,1\}^{NM} \times \{0, \cdots, T-1\}.$$

The transmission from transmitter i to receiver j at t is successful if $C_{ij}(t) = 1$ and i is the only transmitter that attempts transmission to j. Therefore,

$$Q_{ij}(t) = C_{ij}(t) R_{ij}(t) \prod_{l \neq i} (1 - R_{lj}(t)) \prod_{l \neq j} (1 - R_{il}(t)).$$

A generalization of the satellite network specified above is the case where the receivers have multireception capabilities. In a CDMA system, for instance, each receiver may receive successfully more than one packet in the same slot coming from several transmitters. The probability of successful reception $Q_{ij}(t)$, depends on the physical positions of the transmitters and receivers, on the transmitters which attempt transmission to receiver j, as well as on the spread-spectrum signaling scheme that is used for access and it can take more than two values in general.

B. Meteor-Burst Communication Networks

Ionized layers from showers of meteorites act as a reflector to the electromagnetic radiation and make feasible a *meteor-burst communication channel.* This channel is available for transmission only during the period of the meteor bursts which occur at random time instances and have a random duration. In a packet communication link built using a meteor-burst channel, a packet transmission is successful only if it initiates and ends during a meteor-burst period. Several models have been considered for the distribution of the burst period and the intermittent periods, including exponential [3] as well as more general distributions, those of the hitting times of appropriate Markov chains [8].

A network built on meteor-burst channels fits in our changing topology model as follows. The topology state is a vector with one state variable for each link. This variable represents the evolution of the availability of the link. Given the state of link i at slot t, the probability of successful transmission is equal to zero if the slot starts in an intermittent period, or equal to the probability that the meteor-burst period will end before the end of the slot if the slot starts during a meteor-burst period. In the case of exponentially distributed bursts, the state of the channel is binary, indicating only whether the slot starts during a meteor-burst period or not. For more general distributions, the topology state will be multivalued.

III. System Throughput Region

As a first step toward the investigation of the control problem in the changing topology network, the region of throughputs achievable by all ergodic policies is characterized in this section. A saturated system is considered, where an infinite supply of packets is present in the network at the beginning of time. The transmissions that take place at slot t as they are determined by the scheduling policy, are represented by the indicator vector $\boldsymbol{R}(t)$. Due to access constraints and depending on the topology state, some of these transmissions are successful and others are not. Let $D_{ij}(t)$ be a binary random variable that is equal to 1 if there is a successful transmission from transmitter i to receiver j at slot t and to 0 otherwise.

$$\boldsymbol{D}(t) = (D_{ij}(t),\ i = 1, \cdots, N, j = 1, \cdots, M)$$

is the *departure vector* at time t, that is, the indicator vector representing the successful transmissions. The time average throughput up to slot t is

$$\boldsymbol{H}(t) = \frac{1}{t} \sum_{\tau=1}^{t} \boldsymbol{D}(\tau). \tag{1}$$

We consider the class of policies $\mathcal{G}$ for which the long-run average throughput exists; that is,

$$\lim_{t \to \infty} \boldsymbol{H}(t) = \boldsymbol{h} \quad \text{a.s.} \tag{2}$$

where $\boldsymbol{h}$ is a constant vector. Define the throughput region $\overline{\mathcal{H}}$ of the system to be the set of all throughput vectors achievable by policies in $\mathcal{G}$. The characterization of $\overline{\mathcal{H}}$ involves some probability distribution associated with the steady-state behavior of the topology process.

In general, the topology process will be a periodic Markov chain and its period will be denoted by T. The state space is partitioned in the sets $\mathcal{S}^v, v = 0, 1, \cdots, T-1$ such that

$$\boldsymbol{S}(tT + v) \in \mathcal{S}^v, \quad v = 0, 1, \cdots, T-1.$$

It is assumed that all the states in $\mathcal{S}^v$ communicate, therefore, each one of the processes $\{\boldsymbol{S}^v(t)\}_{t=1}^{\infty}$

$$\boldsymbol{S}^v(t) = \boldsymbol{S}(tT + v), \quad v = 0, 1, \cdots, T-1$$

is an aperiodic irreducible Markov chain with state space $\mathcal{S}^v$ that has a stationary distribution denoted by p^v. Let p be the distribution on $\mathcal{S}$ defined as

$$p(s) = \frac{1}{T} \sum_{v=0}^{T-1} p^v(s), \quad s \in \mathcal{S}.$$

This is the stationary distribution of the topology state process. For notational convenience in the following we denote $\boldsymbol{S}(tT + v)$ by $\boldsymbol{S}^v(t)$. The necessary and sufficient condition for a vector $\boldsymbol{h}$ to belong to $\overline{\mathcal{H}}$ is the following.

C1 There exist nonnegative numbers $c_{sr}, s \in \mathcal{S}, r \in \mathcal{R}$ such that

$$\sum_{r \in \mathcal{R}} c_{sr} \le 1, \quad s \in \mathcal{S} \tag{3}$$

for which we can express the vector h as

$$h = \sum_{s \in \mathcal{S}} p(s) \sum_{r \in \mathcal{R}} c_{sr} Q(s, r) \tag{4}$$

where $\mathcal{S}$ is the state space of the topology process, $\mathcal{R}$ the set of possible transmission vectors, and

$$Q(s, r) = (Q_{ij}(s, r): i = 1, \cdots, N, j = 1, \cdots, M).$$

Note that for every initial distribution of the topology state process, the distribution of $S^v(t)$ will converge to its stationary distribution. Hence only the stationary distribution of the topology process affects the throughput. The coefficients c_{sr} can be interpreted as the fraction of time in which transmission vector r is selected by the scheduler, when the system is in state s. Note also that condition **C1** involves the stationary distribution of $S(t)$ and not the transition probabilities of the process. The proof of the following proposition is omitted for brevity.

Proposition 1: A throughput vector h is achievable by a policy in $\mathcal{G}$, if and only if condition **C1** holds.

IV. Scheduling with Limited State and Statistics Information

For any achievable throughput vector h that is a vector satisfying **C1**, a simple randomized transmission policy that achieves h can be easily obtained. Specifically, the transmission vector R is selected with probability c_{sr} when in state S ,assuming $r \neq 0$. The transmission vector $r = 0$ is selected with probability $c_{0s} + 1 - \Sigma_{r \in \mathcal{R}} c_{sr}$ when in state S. In order to compute c_{sr}, the stationary distribution of the topology process is required. Furthermore, the topology states $S(t)$ should be available to the controller at each slot t, since the transmission probabilities are conditional on the state. In this section, a policy is presented that achieves any achievable throughput without knowledge of the topology process statistics. The policy is dynamic and relies on the queue lengths for scheduling. The topology state is not needed by the controller; only the probabilities of successful transmission $Q_{ij}(t)$ at each slot are utilized for scheduling. For instance, in the case of periodic connectivity processes, the controller needs to know only the current connectivities and not the phase of the connectivity process. This is an important aspect of the policy since in certain cases the topology state of the system is not easily measurable while the probabilities of success are. Finally, the policy needs no information on the arrival rates.

The system is considered with exogenous arrivals. At every time slot t a number of packets $A_{ij}(t)$ arrive at transmitter i to be transmitted to receiver j. We will assume that the arrival process

$$\{(A_{ij}(t): i = 1, \cdots, N, j = 1, \cdots, M)\}_{t=1}^{\infty}$$

is i.i.d. and also

$$E[A_{ij}^2(t)] = \tilde{a}_{ij} < \infty, \quad \text{for } i = 1, \cdots, N, j = 1, \cdots, M.$$

The arrival rate of traffic at transmitter i for receiver j is

$$a_{ij} = E[A_{ij}(t)]$$

and the arrival rate vector is

$$a = (a_{ij}: i = 1, \cdots, N, j = 1, \cdots, M).$$

It is assumed that if there is traffic from transmitter i to receiver j, that is $a_{ij} > 0$, then there is a state s and a transmission vector r such that $Q_{ij}(s, r) > 0$.

A throughput vector h is considered achievable if there is a transmission scheduling policy under which the system is stable when the arrival rate vector a is equal to h. Let $X_{ij}(t)$ be the number of packets waiting at transmitter i to be transmitted to receiver j by the beginning of slot t; let

$$X(t) = (X_{ij}(t): i = 1, \cdots, N, j = 1, \cdots, M)$$

be the corresponding queue length vector and $\mathcal{X} = Z_+^{NM}$ the state space. If $a_{ij} = 0$, then $X_{ij}(t) = 0$ for all t. Define $X^v(t) = X(tT + v)$. The system is defined to be stable if the following holds.

S1 The queue length process $\{X^v(t)\}_{t=0}^{\infty}$ converges in distribution to some vector $\tilde{X}^v$ that is independent of the initial condition and such that $E[\tilde{X}^v] < \infty$ for all $v = 0, 1, \cdots, T-1$.

Stationary anticipative policies with finite scheduling horizon are considered, which constitute the class of policies $\mathcal{G}^0$ defined as follows. A stationary policy g belongs to $\mathcal{G}^0$ if there exists a constant k^g (which may vary from policy to policy) such that the transmission vector $R(t)$ is independent of the arrival and topology processes conditionally on $X(t), S(t), S(t+1), \cdots, S(t+k^g)$; that is,

$$\begin{aligned} P[A \cap \{R(t) \in B\} | X(t), S(t), \cdots, S(t+k^g)] \\ = P[A | X(t), S(t), \cdots, S(t+k^g)] \\ \cdot P[R(t) \in B | X(t), S(t), \cdots, S(t+k^g)] \end{aligned}$$

where B is any subset of $\mathcal{R}$ and A any event on the probability space where the arrival and topology processes are defined. Hence $\mathcal{G}^0$ contains all policies g at which the transmission vector at time t may be a function of the backlog at t as well as the topology states up to k^g slots in the future. The following additional condition is imposed on the policies in $\mathcal{G}^0$.

C2 If at time t there are nonempty queues in the system $(X(t) \neq 0)$ and for the current state $S(t)$ there is a transmission vector r such that $Q_{ij}(S(t), r) > 0$ for some i, j such that $X_{ij}(t) > 0$, then the transmission vector $R(t)$ selected by a policy $g \in \mathcal{G}^0$ is such that $Q_{lm}(S(t), R(t)) > 0$, for some l, m such that $X_{lm}(t) > 0$.

Together with the fact that for every i, j for which there is nonzero traffic from i to j there are s and r such that $Q_{ij}(s, r) > 0$, the condition **C2** guarantees that from any initial state the network will hit the empty state with probability one under any policy in $\mathcal{G}^0$, if the arrivals are frozen. This property is needed to claim irreducibility of certain processes in the proof of Proposition 2. Note that the above condition is a type of nonidling condition imposing that if there are nonempty queues in the system, the transmission vector should be selected such that at least one will receive some service. Condition **C2** is satisfied by all policies of interest. Let $\mathcal{H}$ be the region of throughput vectors achievable by any policy in $\mathcal{G}^0$. We will show that the necessary and sufficient condition for a vector h to belong to $\mathcal{H}$ is the following.

C3 There exist nonnegative numbers $c_{sr}, s \in \mathcal{S}, r \in \mathcal{R}$, such that

$$\sum_{r \in \mathcal{R}} c_{sr} < 1, \quad s \in \mathcal{S} \tag{5}$$

for which we can express the vector h as

$$h = \sum_{s \in \mathcal{S}} p(s) \sum_{r \in \mathcal{R}} c_{sr} Q(s, r). \tag{6}$$

Proposition 2: If the system is stable under some policy in $\mathcal{G}^0$ then the arrival rate vector satisfies condition **C3**.

Proof: The process

$$\{(X(t), S(t), \cdots, S(t+k^g))\}_{t=1}^{\infty}$$

is an irreducible Markov chain for any policy $g \in \mathcal{G}^0$. Condition **S1**, that holds if the system is stable, together with the finiteness of the state space of the topology process implies that the process

$$\{(\boldsymbol{X}(t), \boldsymbol{S}(t), \cdots, \boldsymbol{S}(t+k^g))\}_{t=1}^{\infty}$$

has an honest stationary distribution. Assume that the system starts with the initial state distributed according to the stationary distribution. Then the process $\{\boldsymbol{X}(t), \boldsymbol{S}(t)\}_{t=0}^{\infty}$ is stationary. With $\boldsymbol{H}(t)$ and $\boldsymbol{D}(t)$ as defined in Section III, we have that the achievable throughput vector $\boldsymbol{h}$ is

$$\boldsymbol{h} = E[\boldsymbol{H}(t)] = E[\boldsymbol{D}(t)].$$

Consider a modification $\hat{g} = \{\hat{\boldsymbol{R}}(t)\}_{t=0}^{\infty}$ of policy $g = \{\boldsymbol{R}(t)\}_{t=0}^{\infty}$, where $\hat{R}_{ij}(t) = R_{ij}(t)$ if $X_{ij}(t) \neq 0$ and $\hat{R}_{ij}(t) = 0$ if $X_{ij}(t) = 0$. Clearly, the evolution of the system is identical under the two policies. Using this fact and conditioning on $\boldsymbol{S}(t), \boldsymbol{R}(t)$ and the event $\{\boldsymbol{X}(t) \neq 0\}$ successively we get

$$\begin{aligned} \boldsymbol{h} &= E[\boldsymbol{D}(t)] = E[E[\boldsymbol{D}(t)|\boldsymbol{S}(t), \boldsymbol{R}(t)]] \\ &= E[E[\boldsymbol{D}(t)|\boldsymbol{S}(t), \hat{\boldsymbol{R}}(t)]] \\ &= E[E[\boldsymbol{D}(t)|\boldsymbol{S}(t), \hat{\boldsymbol{R}}(t), \boldsymbol{X}(t) \neq 0]]P[\boldsymbol{X}(t) \neq 0] \\ &= E[Q(\boldsymbol{S}(t), \hat{\boldsymbol{R}}(t))]P[\boldsymbol{X}(t) \neq 0] \\ &= \sum_{\boldsymbol{s} \in \mathcal{S}} P[\boldsymbol{S}(t) = \boldsymbol{s}]E[Q(\boldsymbol{S}(t), \hat{\boldsymbol{R}}(t))|\boldsymbol{S}(t) = \boldsymbol{s}]P[\boldsymbol{X}(t) \neq 0] \\ &= \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \sum_{\boldsymbol{r} \in \mathcal{R}} P[\hat{\boldsymbol{R}}(t) = \boldsymbol{r}|\boldsymbol{S}(t) = \boldsymbol{s}]P[\boldsymbol{X}(t) \neq 0]Q(\boldsymbol{s}, \boldsymbol{r}). \end{aligned} \quad (7)$$

Define

$$c_{\boldsymbol{sr}} = P[\hat{\boldsymbol{R}}(t) = \boldsymbol{r}|\boldsymbol{S}(t) = \boldsymbol{s}]P[\boldsymbol{X}(t) \neq 0]. \quad (8)$$

Note that under the stationary distribution and because of the irreducibility, $P[\boldsymbol{X}(t) = 0] > 0$, or equivalently $P[\boldsymbol{X}(t) \neq 0] < 1$. Hence for $c_{\boldsymbol{sr}}$ as defined in (8), condition **C3** holds. □

Consider the policy π_0 that schedules at slot t the vector

$$\boldsymbol{R}(t) = \arg \max_{\boldsymbol{r} \in \mathcal{R}} \sum_{i=1}^{N} \sum_{j=1}^{M} Q_{ij}(\boldsymbol{S}(t), \boldsymbol{r}) X_{ij}(t). \quad (9)$$

$\boldsymbol{R}(t)$ is selected based on the backlog and the transmission success probabilities at slot t. Besides the success probabilities, no further knowledge about the topology state is required. Furthermore, neither the statistics of the state process, nor of the arrivals need to be known for the selection of $\boldsymbol{R}(t)$. In the following it is shown that π_0 stabilizes the network under condition **C3**, therefore, it has a stability region that coincides with $\mathcal{H}$, the region of achievable throughputs by policies in $\mathcal{G}^0$. Define $\boldsymbol{Y}(t) = (\boldsymbol{X}(t), \boldsymbol{S}(t))$ and $\boldsymbol{Y}^v(t) = (\boldsymbol{X}^v(t), \boldsymbol{S}^v(t))$.

Proposition 3: Under policy π_0 and when condition **C3** holds, the process $\{\boldsymbol{Y}^v(t)\}_{t=0}^{\infty}$ converges weakly to a random vector $\hat{\boldsymbol{Y}}^v = (\hat{\boldsymbol{X}}^v, \hat{\boldsymbol{S}}^v)$, such that

$$E\hat{\boldsymbol{X}}^v < \infty \quad (10)$$

for all $v = 0, 1, \cdots, T-1$.

Proof Outline: Under policy π_0, the process $\{\boldsymbol{Y}^v(t)\}_{t=0}^{\infty}$ is a Markov chain. The proof of the proposition is based on the study of the drift of a Liapunov function, that is, the sum of the squares of the backlogs in the system. The drift depends on both the topology state and the queue length vector. For some topology states, the drift might be positive for arbitrarily large values of the queue length vector. In Lemma 1 it is shown that for large enough queue length vector, if the drift is over the topology states with the stationary distribution of the topology process, it becomes negative. In Lemma 2 it is shown that the k-step drift becomes negative for sufficiently large k. The proof of the lemma is based on the following idea. The distribution of the topology process will converge to its stationary distribution for any initial state at time t. When the statistics of the topology process approaches stationarity, then the one-step drift of the Liapunov function will be negative. If k is large enough such that the distribution of the topology process is close to its stationary for a large time period then the cumulative drift from time t to $t+k$ will become negative. The proof of the proposition is concluded based on the negative drift of the Liapunov function. □

The proof of Proposition 3 will follow after the next two lemmas. In the remainder of the correspondence we use the following notational convention; in the inner product of two vectors, the transposition superscript of the second vector is omitted whenever no confusion arises. We will see later in the proof of Lemma 2 that

$$\boldsymbol{a}\boldsymbol{X}^v(t) - \max_{\boldsymbol{r} \in \mathcal{R}} (Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t)) \quad (11)$$

is the dominant term in the drift

$$E[V(\boldsymbol{Y}^v(t+1)) - V(\boldsymbol{Y}^v(t))|\boldsymbol{X}^v(t), \boldsymbol{S}^v(t)] \quad (12)$$

for large $\boldsymbol{Y}^v(t)$, where

$$V(\boldsymbol{Y}(t)) = \sum_{i=1}^{N} \sum_{j=1}^{M} X_{ij}^2(t).$$

In view of this fact, the next lemma shows that for large $\boldsymbol{Y}^v(t)$, if the drift in (12) is weighted by the stationary distribution of the topology state, then it becomes negative. In the rest of the correspondence, the definition of the norm $||\cdot||$ is

$$||\boldsymbol{x}|| = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{M} x_{ij}^2}, \qquad \text{for } \boldsymbol{x} \in \mathcal{X}.$$

Lemma 1: If condition **C3** is satisfied then

$$\boldsymbol{a}\boldsymbol{X}^v(t) - \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \max_{\boldsymbol{r} \in \mathcal{R}} (Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t)) \leq -\epsilon||\boldsymbol{X}^v(t)||. \quad (13)$$

Proof: By using the fact that $\boldsymbol{a}$ satisfies condition **C3** and the representation that is implied for $\boldsymbol{a}$, we get after some calculations

$$\begin{aligned} &\boldsymbol{a}\boldsymbol{X}^v(t) - \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \max_{\boldsymbol{r} \in \mathcal{R}} (Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t)) \\ &= \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \Bigg(\sum_{\boldsymbol{r} \in \mathcal{R}} c_{\boldsymbol{sr}} (Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t) \\ &\quad - \max_{\boldsymbol{r} \in \mathcal{R}} Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t)) \Bigg) \\ &\quad - \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \left(1 - \sum_{R \in \mathcal{R}} c_{\boldsymbol{sr}} \right) \max_{R \in \mathcal{R}} Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t) \\ &\leq - \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \left(1 - \sum_{\boldsymbol{r} \in \mathcal{R}} c_{\boldsymbol{sr}} \right) \max_{\boldsymbol{r} \in \mathcal{R}} Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t) \\ &\leq -m \sum_{\boldsymbol{s} \in \mathcal{S}} p(\boldsymbol{s}) \max_{\boldsymbol{r} \in \mathcal{R}} Q(\boldsymbol{s}, \boldsymbol{r})\boldsymbol{X}^v(t) \end{aligned} \quad (14)$$

where

$$m = \min_{\boldsymbol{s} \in \mathcal{S}} \left\{ 1 - \sum_{\boldsymbol{r} \in \mathcal{R}} c_{\boldsymbol{sr}} \right\} > 0.$$

A direct implication of condition **C3** is that for any l, j for which $a_{lj} > 0$, there exists a state $\boldsymbol{s}$ and a transmission vector $\boldsymbol{r}$ such that

$$p(\boldsymbol{s})Q_{lj}(\boldsymbol{s}, \boldsymbol{r}) > 0.$$

Therefore,

$$\min_{l,j} \max_{\boldsymbol{s} \in \mathcal{S}, \boldsymbol{r} \in \mathcal{R}} \{p(\boldsymbol{s})Q_{lj}(\boldsymbol{s}, \boldsymbol{r})\} = m' > 0$$

from which we get

$$-m\sum_{\boldsymbol{s}\in S}(p(\boldsymbol{s})\max_{\boldsymbol{r}\in\mathcal{R}}\{Q(\boldsymbol{s},\boldsymbol{r})\boldsymbol{X}^v(t)\})$$
$$\leq -mm'\max_{ij}X_{ij}^v(t)\leq -mm'\frac{1}{\sqrt{MN}}\|\boldsymbol{X}^v(t)\|. \quad (15)$$

From (14) and (15) the lemma follows with $\epsilon = mm'/\sqrt{MN}$. □

The following lemma shows that if the squared backlog in the network is large enough at time t then it is reduced at time $t+k$ if k is large enough.

Lemma 2: If condition **C3** is satisfied and the scheduling policy π_0 is adopted, then there exist $B_1, \epsilon_0, k > 0$ such that

$$E[V(\boldsymbol{Y}^v(t+k)) - V(\boldsymbol{Y}^v(t))|\boldsymbol{Y}^v(t), \boldsymbol{S}(t)]$$
$$< -\epsilon_0\|\boldsymbol{X}^v(t)\| \quad \text{if } V(\boldsymbol{Y}^v(t)) > B_1. \quad (16)$$

Proof: See the Appendix. □

To proceed we need the following proposition due to Tweedie [7], which is presented here in a form appropriate for the problem under consideration.

Proposition 4 (Tweedie): Suppose that $\{Y_n\}_{n=1}^{\infty}$ is an aperiodic and irreducible Markov chain with countable state space S. Let $f(y), g(y)$ be real nonnegative functions such that $g(y)\geq f(y)$, $y\in A^c$ where A is a finite subset of S. If

$$E(g(Y_2)|Y_1 = y) < \infty, \qquad y\in A \quad (17)$$

and

$$E(g(Y_2)|Y_1 = y) < g(y) - f(y), \qquad y\in A^c \quad (18)$$

then the Markov chain is ergodic and

$$Ef(\hat{Y}) < \infty$$

where the random variable $\hat{Y}$ has the steady-state distribution of the Markov chain $\{Y_n\}_{n=1}^{\infty}$.

Proof of Proposition 3: The proof follows easily from Lemma 2 and Proposition 4. Let $f, g: \mathcal{X}\times S\to R^+$ be such that

$$g(\boldsymbol{y}) = \sum_{i=1}^{N}\sum_{j=1}^{M}x_{ij}^2 \qquad f(\boldsymbol{y}) = \epsilon_0\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{M}x_{ij}^2}$$

where $\boldsymbol{y} = (\boldsymbol{x},\boldsymbol{s})$. Let $A^c = \{\boldsymbol{y}: V(\boldsymbol{y}) > B_1\}$, for B_1 as it is specified in Lemma 2. We can easily see that (17) holds in this case. From Lemma 2 we have that if **C3** holds, then there is a k such that for the subsequence $\boldsymbol{Z}^v(t) = \boldsymbol{Y}^v(kt), t = 1,\cdots$, which is a Markov chain as well, condition (18) holds. From these two conditions, Proposition 3 follows from Proposition 4. □

Appendix
Proof of Lemma 2

For any $m\leq 0$ we have

$$E[V(\boldsymbol{Y}(tT+v+m+1)) - V(\boldsymbol{Y}(tT+v+m))|\boldsymbol{Y}^v(t)]$$
$$= E[(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))$$
$$\cdot(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))^T|\boldsymbol{Y}^v(t)]$$
$$+ 2E[(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))$$
$$\cdot\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]. \quad (19)$$

Since the second moments of the arrivals are finite we have

$$E[(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))$$
$$\cdot(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))^T|\boldsymbol{Y}^v(t)]$$
$$\leq\sum_{i=1}^{N}\sum_{j=1}^{M}E[(A_{ij}(t)+1)^2] = \sum_{i=1}^{N}\sum_{j=1}^{M}(\tilde{a}_{ij}+2a_{ij}+1). \quad (20)$$

Notice that we can write

$$E[V(\boldsymbol{Y}^v(t+k)) - V(\boldsymbol{Y}^v(t))|\boldsymbol{Y}^v(t)] = \sum_{m=0}^{Tk-1}E[V(\boldsymbol{Y}(tT+m+v+1))$$
$$- V(\boldsymbol{Y}(tT+m+v))|\boldsymbol{Y}^v(t)]$$

and from (19) and (20) we get

$$E[V(\boldsymbol{Y}^v(t+k)) - V(\boldsymbol{Y}^v(t))|\boldsymbol{Y}^v(t)]$$
$$\leq kc_1 + 2\sum_{m=0}^{Tk-1}E[(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))$$
$$\cdot\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)] \quad (21)$$

where we define

$$c_1 = T\sum_{i=1}^{N}\sum_{j=1}^{M}(\tilde{a}_{ij}+2a_{ij}+1).$$

In the following we focus on the second term of the sum in the right side of (21) and we bound it appropriately from above. Let $\boldsymbol{D}(t)$ be the departure vector as it has been defined in the beginning of Section III. Then we have

$$E[(\boldsymbol{X}(tT+m+v+1)) - \boldsymbol{X}(tT+m+v))$$
$$\cdot\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]$$
$$= E[\boldsymbol{A}(tT+m+v)\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]$$
$$- E[\boldsymbol{D}(tT+m+v)\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]. \quad (22)$$

In the following, we upper-bound each one of the terms in the difference in (22). From the fact that

$$\boldsymbol{X}(tT+m+v)\leq\boldsymbol{X}^v(t) + \sum_{l=0}^{m-1}\boldsymbol{A}(tT+l+v)$$

we get

$$E[\boldsymbol{A}(tT+m+v)\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]\leq\boldsymbol{a}\boldsymbol{X}^v(t)+m\boldsymbol{a}\boldsymbol{a}^T. \quad (23)$$

For the second term in the difference in the right side of (22) we get

$$E[\boldsymbol{D}(tT+m+v)\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]$$
$$= E[E[\boldsymbol{D}(tT+m+v)\boldsymbol{X}^T(tT+m+v)|\boldsymbol{X}(tT+m+v),$$
$$\boldsymbol{S}(tT+m+v)]|\boldsymbol{Y}^v(t)]$$
$$= E[\boldsymbol{X}(tT+m+v)E[\boldsymbol{D}^T(tT+m+v)|\boldsymbol{X}(tT+m+v),$$
$$\boldsymbol{S}(tT+m+v)]|\boldsymbol{Y}^v(t)]$$
$$= E[\max_{\boldsymbol{r}\in\mathcal{R}}Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\boldsymbol{X}^T(tT+m+v)|\boldsymbol{Y}^v(t)]. \quad (24)$$

The following holds with probability one.

$$\max_{\boldsymbol{r}\in\mathcal{R}}Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\boldsymbol{X}^T(tT+m+v)$$
$$= \max_{\boldsymbol{r}\in\mathcal{R}}Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\cdot\left(\boldsymbol{X}^v(t)+\sum_{l=0}^{m-1}\boldsymbol{A}(tT+l+v) - \sum_{l=0}^{m-1}\boldsymbol{D}(tT+l+v)\right)^T$$
$$\geq\max_{\boldsymbol{r}\in\mathcal{R}}(Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\boldsymbol{X}^v(t))$$
$$- \max_{\boldsymbol{r}\in\mathcal{R}}\left(Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\cdot\sum_{l=0}^{m-1}\boldsymbol{D}(tT+l+v))^T\right)$$
$$\geq\max_{\boldsymbol{r}\in\mathcal{R}}(Q(\boldsymbol{S}(tT+m+v),\boldsymbol{r})\boldsymbol{X}^v(t)) - mNM. \quad (25)$$

By taking expectations in (25) conditioning on Y^v we get

$$\begin{aligned}E[\max_{r\in\mathcal{R}}(Q(S(tT+m+v),r)X^T(tT+m+v))|Y^v]\\ \geq E[\max_{r\in\mathcal{R}}(Q(S(tT+m+v),r)X^v(t))|Y^v(t),\\ S^v(t)]-mNM\\ \geq \sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))P[S(tT+m+v)\\ =s|S^v(t)]-mNM.\end{aligned}\quad(26)$$

By replacing in (22) from the relations (23), (24), and (26), we get

$$\begin{aligned}E[(X(tT+m+v+1))-X(tT+m+v))\\ \cdot X^T(tT+m+v)|Y^v(t)]\\ \leq aX^v(t)+maa^T+mNM\\ -\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\\ \cdot P[S(tT+m+v)=s|S^v(t)].\end{aligned}\quad(27)$$

By summing (27) over m we get

$$\begin{aligned}\sum_{m=0}^{kT-1}E[(X(tT+m+v+1))\\ -X(tT+m+v))X^T(tT+m+v)|Y^v(t)]\\ \leq (kT)^2(aa^T+NM)+kTaX^v(t)\\ -\sum_{m=0}^{kT-1}\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))P[S(tT+m+v)=s|S^v(t)]\\ \leq k^2c_2+\sum_{m=0}^{kT-1}(aX^v(t)-\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))p^{m\oplus v}(s))\\ +\sum_{m=0}^{kT-1}\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\\ \cdot(p^{m\oplus v}(s)-P[S(tT+m+v)=s|S^v(t)]\end{aligned}\quad(28)$$

where we define $c_2 = T^2(aa^T + NM)$ and $\oplus$ denotes addition $\bmod T$. After some calculations and from Lemma 1 we get

$$\begin{aligned}\sum_{m=0}^{kT-1}(aX^v(t)-\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))p^{m\oplus v}(s))\\ =\sum_{s\in\mathcal{S}}((aX^v(t)-\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\sum_{m=0}^{kT-1}p^{m\oplus v}(s))\\ =kT\sum_{s\in\mathcal{S}}((aX^v(t)-\max_{r\in\mathcal{R}}(Q(s,r)X^v(t)))p(s))\\ \leq -\epsilon kT||X^v(t)||\end{aligned}\quad(29)$$

where ϵ is as it has been determined in the proof of Lemma 1. Also we have

$$\begin{aligned}\sum_{m=0}^{kT-1}\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\\ \cdot(p^{m\oplus v}(s)-P[S(tT+m+v)=s|S^v(t)])\\ \leq\sum_{m=0}^{kT-1}\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\epsilon(m)\\ \leq\sum_{m=0}^{kT-1}LeX^v(t)\epsilon(m)\leq\sum_{m=0}^{kT-1}LNM||X^v(t)||\epsilon(m)\end{aligned}\quad(30)$$

where

$$\epsilon(m)=\max_{s'\in\mathcal{S}}\max_{v=0,\cdots,T-1}\max_{s\in\mathcal{S}}|p^{m\oplus v}(s)-P[S(tT+m+v)=s|S^v(t)=s']|$$

and e is the unit vector. Clearly, we have

$$\epsilon(m)\leq 1.$$

From the fact that the processes $S^v(t)$ are irreducible aperiodic finite state-space Markov chains we get that

$$\lim_{m\to\infty}\epsilon(m)=0.$$

Let m_0 be such that

$$\epsilon(m)\leq\frac{\epsilon}{2LNM},\qquad \text{for } m\geq m_0.\quad(31)$$

Assuming that k is selected such that $kT > m_0$, we can write from (30) and (31)

$$\begin{aligned}\sum_{m=0}^{kT-1}\sum_{s\in\mathcal{S}}\max_{r\in\mathcal{R}}(Q(s,r)X^v(t))\\ \cdot(p^{m\oplus v}(s)-P[S(tT+m+v)=s|S^v(t)])\\ \leq\sum_{m=0}^{m_0-1}LNM||X^v(t)||+\sum_{m=m_0}^{kT-1}\frac{\epsilon}{2}||X^v(t)||.\end{aligned}\quad(32)$$

By replacing in (28) from (29) and (32) and from the resulting inequality back to (21) we get

$$\begin{aligned}E[V(Y^v(t+k))-V(Y^v(t))|Y^v(t)]\\ \leq kc_1+2k^2c_2-2\epsilon kT||X^v(t)||\\ +2\sum_{m=0}^{m_0-1}LNM||X^v(t)||+2\sum_{m=m_0}^{kT-1}\frac{\epsilon}{2}||X^v(t)||\\ \leq kc_1+2k^2c_2+m_0 2LNM||X^v(t)||-\epsilon kT||X^v(t)||\\ =kc_1+2k^2c_2+m_0\left(2LNM-\frac{\epsilon}{2}kT\right)||X^v(t)||\\ -\frac{\epsilon}{2}kT||X^v(t)||.\end{aligned}\quad(33)$$

If we have

$$k>\frac{4LNM}{\epsilon T}\quad\text{and}\quad B_1>\left((kc_1+2k^2c_2)\frac{4}{\epsilon kT}\right)^2$$

then the lemma holds with $\epsilon_0 = (\epsilon kT)/4$. □

REFERENCES

[1] M. Carr and B. Hajek, "Scheduling with asynchronous service opportunities with applications to multiple satellite systems," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 1820–1833, Dec. 1993.

[2] *IEEE Commun. Mag.* (Special Issue on Satellite and Terrestrial Systems and Serives for Travelers), vol. 29, Nov. 1991.

[3] J. D. Oetting, "An analysis of meteor-burst communications for military applications," *IEEE Trans. Commun.*, vol. COM-28, pp. 1591–1601, Sept. 1980.

[4] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 1936–1948, Dec. 1992.

[5] ——, "Dynamic server allocation to parallel queues with randomoly varying connectivity," *IEEE Trans. Inform. Theory*, vol. 39, pp. 466–478, Mar. 1993.

[6] L. Tassiulas and S. Papavassiliou, "Optimal anticipative scheduling with asynchronous transmission opportunities," *IEEE Trans. Automat. Contr.*, to be published.

[7] R. L. Tweedie, "The existence of moments of stationary Markov chains," *J. Appl. Prob.*, vol. 20, pp. 191–196, 1983.

[8] Y. Chandramouli, M. F. Neuts, and V. Ramaswami, "A queueing model for meteor burst packet communication systems," *IEEE Trans. Commun.*, vol. 37, Oct. 1989.