Joint Optimal Channel Base Station and Power Assignment for Wireless Access

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Abstract— **The fundamental problem underlying any phase (hand-off, new connection, etc.) of a dynamic resource allocation algorithm in a wireless network is to assign transmission powers, forward (downstream) and reverse (upstream) channels, and base stations such that every mobile of the system can establish a connection. Each one of these problems separately has been studied extensively. In this work we consider the joint problem in a system with two base stations. An algorithm that achieves the optimal assignment is provided. It involves the computation of a maximum matching in a graph that captures the topological characteristics of the mobile locations. The traffic capacities, in terms of expected number of connections per channel, of the forward and reverse channel are obtained and compared, for both cases of power control and nonpower control. It turns out that when the transmission power is fixed, the capacities of the forward and reverse channel are different, while when power control is allowed they are the same. For systems with two mobiles the capacities of the forward and reverse channels are studied analytically. Finally, several versions of the two-way channel assignment problem are studied.**

I. Introduction

THE PROVISION of personal communication services (PCS's) is the goal of the evolution of integrated communication systems. The basic PCS philosophy is that the underlying telecommunication infrastructure will provide user-to-user, location independent, communication services. The services supported by PCS's generate a large volume of communication traffic with diverse burstiness characteristics and quality of service requirements. The wireless network should be able to support this traffic and to meet the quality of service requirements. Efficient utilization of the limited radio spectrum will be vital for meeting the anticipated traffic demands of PCS's.

The saturation of the advanced mobile phone service (AMPS) cellular network that provides mobile phone services today, is a strong indication of the problems that will arise by the scarcity of the spectrum in a network with considerably increased traffic. Advances both in the physical layer as well as the access and networking layer will be necessary. In the physical layer, sophisticated modulation, coding and signal processing techniques will be needed to achieve high bit-rate transmission in the digital mobile radio channel. In the access and networking layer, channel allocation schemes that achieve maximum spatial channel reuse will be needed to achieve high spectrum utilization.

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In the existing analog system, the frequency channels of each cell are fixed and preallocated off-line during the frequency planning phase. A call can be served only if the cell where it arises has a free channel, otherwise it is lost. In the digital system of the near future, there will be great flexibility in spectrum management and control. All the channels will be potentially available to all base stations, to be allocated in a dynamic fashion. Also the transmission powers both of the base stations and the users will be controllable. This great flexibility will increase considerably the traffic capacity of the system if it is managed properly.

In a network with dynamic power and channel control, the channels the powers and the base stations can be reassigned at any time, even in the middle of a call. As the mobiles change positions, the powers of the received signals change and a reassignment (hand-off) might be required to retain the connection. When a new call request arises, a reassignment of the existing calls might be required to accommodate it. The fundamental problem underlying any phase (hand-off, new connection, etc.) of a dynamic resource allocation algorithm in a wireless network is the following.

Problem (P): Given a number of channels, a number of mobiles, a number of base stations and the path losses among them, assign transmission powers, channels and base stations such that all mobiles have a connection.

In this paper, we study the above problem and obtain a solution for the case of two base stations, while in [13] we consider the resource allocation problem in a general network with arbitrary number of base stations and mobiles. It is shown here, that the optimal assignment is obtained by the solution of the maximum matching problem in an appropriate graph. The graph has as nodes the mobiles and links connect any two mobiles which can share the same channel by appropriate base station assignment and power selection. There is one-to-one correspondence between matchings and frequency assignments where the pairs of the mobiles that correspond to the links of the matching share the same channel. A maximum matching corresponds to maximal channel reuse or equivalently to provision of connections using the minimal number of channels.

Based on the optimal assignment algorithm the traffic capacity, appropriately defined, is studied then. Let $C_N^f(C_N^r)$ be the minimum number of channels needed to establish a forward (reverse) connection to N mobiles, randomly and uniformly distributed, when the transmission power is fixed. The traffic capacity T_N^f in the forward direction without power control is defined as

$$T_N^f = \frac{N}{E[C_N^f]} \tag{1}$$

and similarly for the reverse direction. For both the forward and reverse channels T_2^f and T_2^r are computed, while in general, T_N^f and T_N^r are obtained by simulation. For the forward channel the limit of the forward traffic capacity T_N^f, as N increases, is obtained analytically. It turns out that for small N, $T_N^f > T_N^r$ while for large N the inequality is reversed. In general the capacity is different for the forward and reverse channel. The case of power control is considered next. For every configuration of N mobiles, the minimum number of channels necessary to establish a connection in the forward and reverse direction for every mobile, $\tilde{C}_N^f$ and $\tilde{C}_N^r$, respectively, are equal. Hence, the corresponding capacities $\tilde{T}_N^f$ and $\tilde{T}_N^r$, defined as in (1), are equal, $\tilde{T}_N^f = \tilde{T}_N^r = \tilde{T}_N$. Note, that by $\tilde{C}$ and $\tilde{T}$ we denote the minimum number of channels needed in the system and the corresponding traffic capacity respectively, when the powers are controllable parameters. We obtain $\tilde{T}_N$ by simulation and compare it with T_N^f and T_N^r. As it is expected, a considerable improvement on the capacity is observed. The problem of two-way channel assignment is also studied and similar results are obtained.

The paper is organized as follows. In Section II, the general allocation problem is formulated rigorously. In Section III, the optimal assignment through the matching problem is given. Several versions of the two-way channel assignment are also discussed. In Section IV-A, the traffic capacity in the nonpower control case is studied. The power control case is considered in Section IV-B.

II. Problem Formulation

In this section, we introduce some notation and define the allocation problem rigorously. Even though we will study exclusively the case of two base stations in this paper, we formulate the general problem. There are L communication channels available in the system. The channels may be either frequency bands in an frequency division multiple access (FDMA) system or a carrier and a time slot in a time division multiple access (TDMA) system or different codes in a code-division multiple-access (CDMA) system. There are M base stations and N mobiles at arbitrary locations with respect to the base stations. The path loss coefficients G_{ij} between any base station i and mobile j are provided. They characterize completely the propagation properties of the system in the sense that when i transmits power P_i, j receives power $G_{ij}P_i$. We denote by P_{il}^f the transmitted power from base station i in the forward channel l. Similarly, P_{jl}^r denotes the transmitted power from mobile j in the reverse channel l. Cochannel interference is the prevailing interference type; this is equal to $\sum_{k\neq i} G_{kj}P_{kl}^f$ in mobile j receiving from base station i at channel l. The carrier to interference ratio $(C/I)_{jl}^f$ at mobile j in channel l is equal to

$$\left(\frac{C}{I}\right)_{jl}^f = \frac{G_{ij}P_{il}^f}{\sum\limits_{k\neq i} G_{kj}P_{kl}^f}. \tag{2}$$

The interference constraint at mobile j that receives on channel l is satisfied if

$$\left(\frac{C}{I}\right)_{jl}^f \geq T \tag{3}$$

where T is a threshold imposed by physical layer's constraints. The constraint (3) is on the area mean carrier power and the interference power. T is selected such that (3) guarantees that the effect of fast and slow fading will not be detrimental on the link quality. Similarly, the carrier to interference ratio $(C/I)_{il}^r$ at base station i in channel l is equal to

$$\left(\frac{C}{I}\right)_{il}^r = \frac{G_{ij}P_{jl}^r}{\sum\limits_{n\neq j} G_{in}P_{nl}^r}. \tag{4}$$

The reverse radio link from mobile j to base station i at channel l satisfies the interference constraints if

$$\left(\frac{C}{I}\right)_{il}^r \geq T. \tag{5}$$

The problem of joint channel power and base station allocation is illustrated in Figs. 1 and 2. Clearly the three problems are interrelated. For certain channel allocations and base station assignments there may be power vectors that satisfy the interference constraints while for others may be not. Therefore, these problems need to be considered jointly.

A. Forward Channel Assignment Problem

A forward channel assignment is specified by a function $C^f(\,)\colon \{1, \cdots, N\} \rightarrow \{1, \cdots, L\}$ with the interpretation that $C^f(j)$ is the channel at which mobile j is receiving. A base station assignment for the forward channel is specified by a function $B^f(\,)\colon \{1, \cdots, N\} \rightarrow \{1, \cdots, M\}$ where $B^f(j)$ is the base station from which j is receiving. A forward channel assignment $C^f(\,)$ and base station assignment $B^f(\,)$ are *jointly admissible* if at most one mobile is assigned to each base station in each channel. In other words if

$$C^f(i) = C^f(j) \Rightarrow B^f(i) \neq B^f(j).$$

A *forward channel-base station assignment* is a pair (C^f, B^f) of jointly admissible forward channel and base station assignments respectively. A forward channel-base station assignment is feasible if there exists a transmission power assignment such that at each mobile the carrier to interference ratio from its assigned base station in the assigned channel, exceeds the required threshold T. That is

$$\max_{P^f\geq 0}\left\{\min_{j=1,\cdots,N}\left\{\frac{G_{B^f(j)j}P^f_{B^f(j)C^f(j)}}{\sum\limits_{k\neq B^f(j)} G_{kj}P^f_{kC^f(j)}}\right\}\right\} \geq T. \tag{6}$$

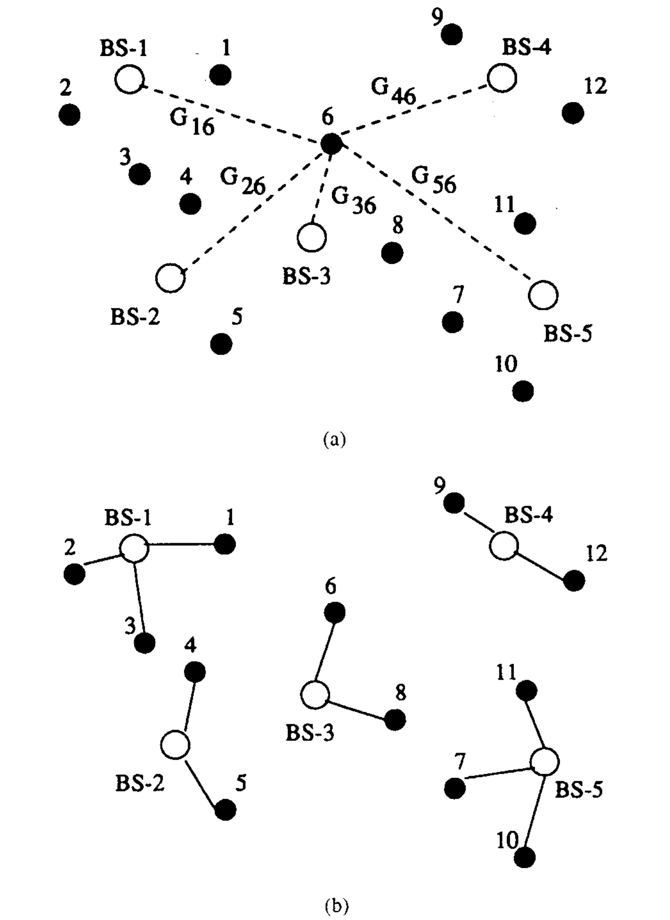

Fig. 1. (a) A number of mobiles need to establish a forward and reverse connection with some base stations. The transmission gains G_{ij} are given between any two locations i and j. (b) A base station is selected by each mobile for its forward and its reverse links. The two base stations need not be the same.

The resource allocation problem can be stated as follows:

$$\max_{(C^f,B^f)}\left\{\max_{P^f\geq 0}\left\{\min_{j=1,\cdots,N}\left\{\frac{G_{B^f(j)j}P^f_{B^f(j)C^f(j)}}{\sum\limits_{k\neq B^f(j)} G_{kj}P^f_{kC^f(j)}}\right\}\right\}\right\} \quad (7)$$

where (C^f, B^f) is a jointly admissible forward channel base station assignment. If the optimal value of the objective function is larger than the threshold T then the channel and base assignment (C^f, B^f) and the power assignment P^f that achieve the maximum provide a feasible allocation.

B. Reverse Channel Assignment Problem

A reverse channel assignment is specified by a function $C^r(): \{1, \cdots, N\} \rightarrow \{1, \cdots, L\}$ with the interpretation that $C^r(j)$ is the channel at which mobile j is transmitting. A base station assignment for the reverse channel is specified by a function $B^r(): \{1, \cdots, N\} \rightarrow \{1, \cdots, M\}$ where $B^r(j)$ is the base station to which j is transmitting. The reverse channel assignment problem can be formulated similarly to the forward channel assignment problem.

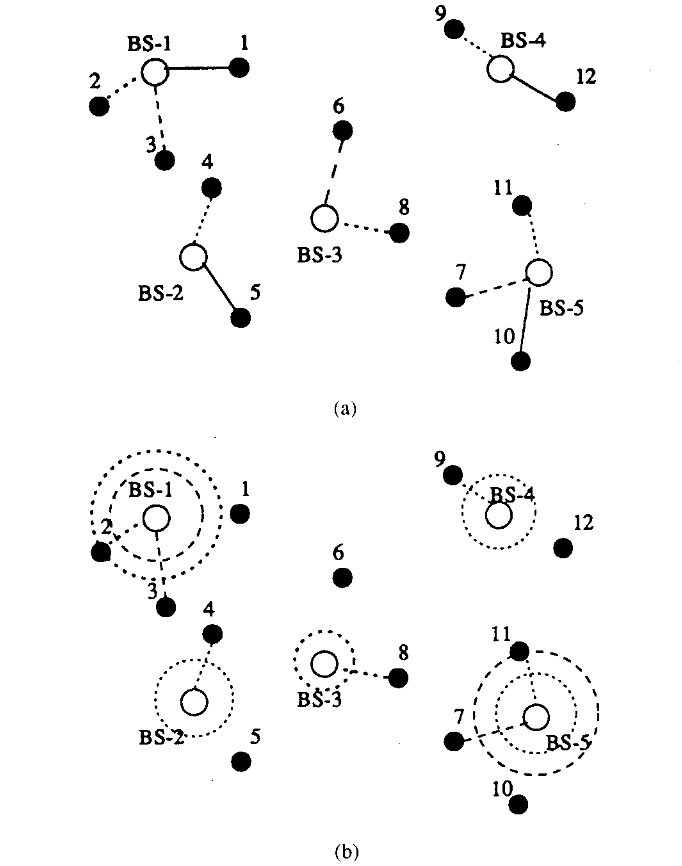

Fig. 2. (a) A channel is selected by each link. The cochannel links are represented by the same type of line. (b) The transmission powers of the cochannel links should be selected such that the interference constraints at each mobile are satisfied.

C. Two-Way Channel Assignment Problem

There are several versions of the two-way channel assignment problem depending on the constraints we pose. In the existing analog system, a channel is prespecified as forward or reverse and it can be used only in one direction. This is certainly not a physical constraint since in principal the same channel can be used in both directions simultaneously in sufficiently spatially separated locations. In the future digital systems, this constraint can be eliminated and a channel may be used for communication in both directions. Moreover, another constraint, usually imposed, that affects the system capacity is that a mobile should communicate with the same base station in both directions. If we ignore the practical constraints imposed in specific systems, each mobile may use any channel and base station for each one of the reverse and forward connections. An *admissible two-way channel base station assignment* in this case is a quatraple $\mathbf{CB}() = [C^f(), C^r(), B^f(), B^r()]$ for which the following are satisfied

$C^f(i) \neq C^r(i), i = 1, \cdots, N$ A mobile cannot talk and listen in the same channel.

$C^f(i) = C^f(j) \Rightarrow B^f(i) \neq B^f(j), i, j = 1, \cdots, N, i \neq j$. A base station can talk at most to one mobile per channel.

$C^r(i) = C^r(j) \Rightarrow B^r(i) \neq B^r(j), i, j =$

$1, \cdots, N, i \neq j$. A base station can listen at most to one mobile per channel.

$C^f(i) = C^r(j) \Rightarrow B^f(i) \neq B^r(j), i, j = 1, \cdots, N, i \neq j$. A base station cannot talk and listen at the same channel.

The problem can be stated as follows:

$$\max_{\mathbf{CB}} \left\{ \max_{(P^f, P^r)\geq 0} \left\{ \min_{j=1,\cdots,N} \left\{ \frac{G_{B^f(j)j} P^f_{B^f(j)C^f(j)}}{\sum_{k\neq B^f(j)} G_{kj} P^f_{kC^f(j)}} \right\} \min_{j=1,\cdots,N} \left\{ \frac{G_{B^r(j)j} P^r_{jC^r(j)}}{\sum_{n\neq j} G_{B^r(j)n} P^r_{nC^r(j)}} \right\} \right\} \right\} \tag{8}$$

where $\mathbf{CB} = (C^f, C^r, B^f, B^r)$ is an admissible two way *channel* base station assignment.

If the optimal value of the objective function is larger than the threshold T then the channel assignment **CB** and the power assignment P^f and P^r that achieves the maximum provide a feasible allocation. The above optimization problems arise constantly in the management and control of a wireless network. In the generality that it is stated, these problems are clearly hard optimization problems.

The channel assignment problem in cellular networks (when the powers and base stations are preassigned and fixed) has been shown to be equivalent to a generalized graph coloring problem [9], [14], which is known to be NP-hard [7], [9]. If we have only cochannel interference then the graph is obtained by representing each cell by a vertex with an edge connecting two vertices if the involved cells are forbidden from using common channels. The problem is to assign channels (or colors) to the vertices of this graph such that adjacent vertices are assigned disjoint sets of color. The assignment should use as few channels (colors) as possible. Since no efficient algorithm that solves this problem exists, many heuristic channel assignment algorithms have been suggested and evaluated in the literature (i.e., [3], [5], [6], and [18]).

In [16], the optimum transmitter power control problem for the case of preassigned base stations and considering only one channel is studied. A power control scheme that is optimum in the sense that it maximizes the number of simultaneous connections in the system is presented. The algorithm removes combinations of cells and computes the eigenvalues of each reduced system until the (C/I) requirement is fulfilled. Such a submatrix problem is computationally hard and at the present it is not known if an algorithm exists to solve the problem in polynomial time. However, the satisfiability problem is solvable ([1] and [16]). That is, for a given set of cochannel links, whether there exist feasible power vector that guarantees (C/I) ratios at all mobiles above a prespecified threshold. If not then the previous step by step removal algorithm needs to be applied. Foschini [4] and Zander [15] proposed synchronous distributed power control algorithms. In [12], Mitra extended Foschini's synchronous algorithm in allowing asynchrony between the various mobiles and base stations reducing the need for coordination. In [2], several algorithms for joint power control and channel assignment with varying complexities are proposed and compared, while in [10] different adaptive methods for efficiently allocating the available radio resources have been combined and evaluated.

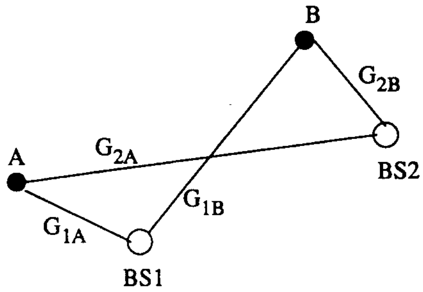

Fig. 3. System geometry and link gains.

III. Optimal Assignment

In this section, we propose an algorithm that achieves the optimal channel, base station and power assignment, in the sense that minimizes the number of channels needed to establish communication for all mobiles in the forward and reverse directions. Note, that minimizing the number of channels is equivalent to the satisfiability problem (P). The procedure we propose is based on the solution of a maximum matching problem in an appropriate graph and is applicable for both the cases of power control (i.e., the powers are controllable parameters) and nonpower control (i.e., fixed powers). Before proceeding with the description of the algorithm we present some preliminary results on the conditions that should be satisfied such that two transmitter-receiver pairs can make use of the same channel. Since problem (P) is the same in the forward and reverse channel assignment we focus on the forward channel in the following.

Lets denote by A and B two transceivers that may communicate with the transceivers 1 and 2, respectively (Fig. 3). We consider as forward channel a channel used for communication from transmitter 1(2) to receiver $A(B)$ and as reverse a channel that is used for communication in the opposite direction. The next theorem states the conditions for having channel reuse. Although the results of the following theorem could follow from the more general solution for M base stations and M mobiles presented in [8] and [16] we outline here the proofs for the sake of completeness.

Theorem 1:

a) Both receivers A and B can receive on the same forward channel if the powers P^f_{1l} and P^f_{2l} transmitted by transmitters 1 and 2 satisfy the following relation:

$$\frac{T}{\left(\frac{G_{1A}}{G_{2A}}\right)} \leq \frac{P^f_{1l}}{P^f_{2l}} \leq \frac{\left(\frac{G_{2B}}{G_{1B}}\right)}{T}. \tag{9}$$

b) When the power is controlled the necessary and sufficient condition for transceivers A and B to receive on the same forward channel and/or transmit on the same reverse channel is the following:

$$\sqrt{\frac{G_{2B}G_{1A}}{G_{1B}G_{2A}}} \geq T \tag{10}$$

and the transmission powers P_{1l}^f, P_{2l}^f that achieve it, if possible, are such that

$$\frac{P_{1l}^f}{P_{2l}^f} = \sqrt{\frac{G_{2B}G_{2A}}{G_{1B}G_{1A}}}. \tag{11}$$

Proof: The expressions for the carrier to interference ratios at receivers A and B can be written as follows:

$$\left(\frac{C}{I}\right)_{Al}^f = \frac{G_{1A}P_{1l}^f}{G_{2A}P_{2l}^f} \geq T$$
$$\left(\frac{C}{I}\right)_{Bl}^f = \frac{G_{2B}P_{2l}^f}{G_{1B}P_{1l}^f} \geq T. \tag{12}$$

Similar expressions hold for the carrier to interference ratios at receivers 1 and 2. Part a) and relation (10) in part b) can be easily concluded by the rearrangement of those expressions. Now we proceed to prove relation (11) of part b) of the theorem. The transmission powers P_{1l}^f and P_{2l}^f that do not violate the interference constraints at receivers A and B, if this is possible, are those that maximize the minimum of the carrier to interference ratios at the receivers A and B. Therefore we must select powers P_{1l}^f and P_{2l}^f in order to optimize the following objective function

$$\max_{P^f}\left\{\min\left\{\frac{G_{1A}P_{1l}^f}{G_{2A}P_{2l}^f}, \frac{G_{2B}P_{2l}^f}{G_{1B}P_{1l}^f}\right\}\right\}. \tag{13}$$

Note that

$$\min\left\{\frac{G_{1A}P_{1l}^f}{G_{2A}P_{2l}^f}, \frac{G_{2B}P_{2l}^f}{G_{1B}P_{1l}^f}\right\} = \begin{cases} \dfrac{G_{2B}}{G_{1B}}\dfrac{P_{2l}^f}{P_{1l}^f} & \text{if } \dfrac{P_{1l}^f}{P_{2l}^f} \geq \sqrt{\dfrac{G_{2B}G_{2A}}{G_{1B}G_{1A}}} \\ \dfrac{G_{1A}}{G_{2A}}\dfrac{P_{1l}^f}{P_{2l}^f} & \text{if } \dfrac{P_{1l}^f}{P_{2l}^f} < \sqrt{\dfrac{G_{2B}G_{2A}}{G_{1B}G_{1A}}}. \end{cases}$$

Therefore, the optimal value of the objective function is achieved if

$$\frac{P_{1l}^f}{P_{2l}^f} = \sqrt{\frac{G_{2B}G_{2A}}{G_{1B}G_{1A}}} \tag{14}$$

and is equal to $\sqrt{G_{1A}G_{2B}/G_{2A}G_{1B}}$. □

From part b) of the above theorem we can easily conclude that when the powers are controlled the forward and reverse channel assignment problems are equivalent, in the sense that it is stated in the following corollary. In [17], it was also shown that in the case of power control the achievable carrier to interference ratios in the forward and reverse directions are identical at every instant.

Corollary 1: When the powers are adjustable parameters, two transceivers that communicate with another pair of transceivers can share one forward channel if they can share one reverse channel.

It should be noted that the results of Theorem 1 and Corollary 1 are valid under the assumption of no background noise. In this case if relation (10) is satisfied then we can always adjust the transmitted powers according to (11) such that both the constraints of relation (12) are satisfied, even if the maximum allowable transmitted power is finite and limited. If we take into consideration the existence of background noise then in the case that the maximum allowable transmitted power is limited Corollary 1 may not hold and therefore the traffic capacities of the forward and reverse links may not be equal.

Note, that if there are only two base stations at most two mobiles can share the same channel. Hence, at any feasible assignment there will be a number of channels used by only one mobile and the rest of the channels used by two mobiles. An assignment that requires minimum number of channels minimizes the number of mobiles that use a channel by themselves, or equivalently maximizes the number of mobiles that can share a channel. An optimal assignment corresponds to a maximum matching in an appropriate graph.

Associate with each configuration of mobiles a compatibility graph $G = (V, E)$ created as follows. The nodes of the graph are in one-to-one correspondence with the mobiles. An edge connects two mobiles if they can share the same channel with appropriate base station assignment and power selection. A matching $\mathcal{M}$ of the compatibility graph G is a subset of the edges with the property that no two edges of $\mathcal{M}$ share the same node. Every edge in $\mathcal{M}$ is called matched edge. Maximum matching is a matching that has the maximum possible number of edges. Clearly the set of possible assignments are in one-to-one correspondence with the set of matchings. The next theorem follows readily.

Theorem 2: The minimum number of channels C_N^f is equal to the number of edges in a maximum matching of the corresponding compatibility graph G (cardinality of the maximum matching) plus the number of nodes that are not incident upon any matched edge. The same result holds for the power control case, too.

The theorem suggests the following way of computing optimal assignments for any configuration of mobiles.

1) Create the compatibility graph G by identifying all possible mobile pairs that can use the same channel (using Theorem 1).
2) Find a *maximum matching* of the compatibility graph G.
3) Allocate the same channel to the mobile pairs that correspond to the edges of the maximum matching. For each such pair make the base station and power assignment.

In accordance with the results, we presented in Theorem 1 the identification of the pairs of mobiles that can share a channel in the forward direction can be based on relation (9) for the nonpower control case and on relation (10) for the power control case. In the latter case the optimal power assignment is achieved if the transmitted powers P_{1l}^f, P_{2l}^f for

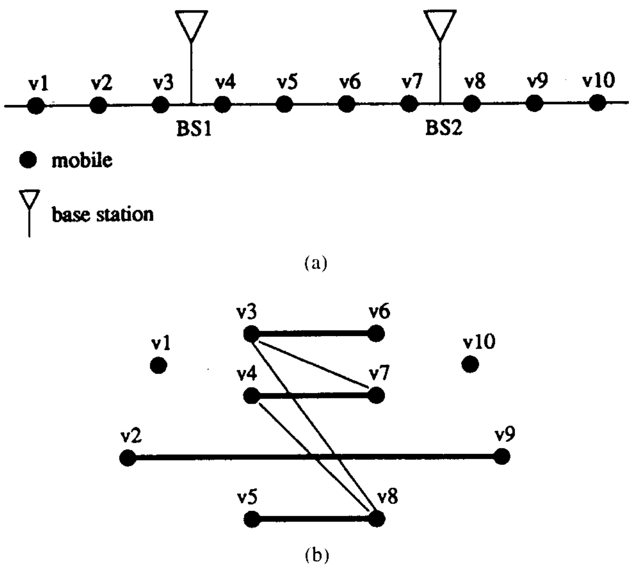

Fig. 4. (a) Two base stations and $N = 10$ mobiles distributed on a line and (b) compatibility graph G. The heavy lines represent a maximum matching.

a pair of mobiles that share a channel are chosen to satisfy relation (11). As we mentioned before similar expressions hold for the reverse channel, too. Hence, connectivity of the compatibility graph is different in the cases of power control, nonpower control, forward and reverse channel assignment. Note that the computationally intensive part of the algorithm is the identification of the maximum matching of the compatibility graph. The fastest algorithm known that solves the maximum matching problem for a general graph is described in [11] and proposes an implementation with running time $O(\sqrt{|V|}\,|E|)$.

Next we apply the algorithm in a simple example. In this example, and throughout the rest of the paper, we assume that the signal strength decreases inversely proportional to d_{ij}^4, where by d_{ij} we denote the distance between transmitter i and receiver j. Then, relation (12) can be simplified as

$$\left(\frac{C}{I}\right)_{Al}^{f} = \left(\frac{d_{2A}}{d_{1A}}\right)^4 \geq T$$
$$\left(\frac{C}{I}\right)_{Bl}^{f} = \left(\frac{d_{1B}}{d_{2B}}\right)^4 \geq T. \tag{15}$$

Throughout our numerical studies the threshold T is taken equal to 18 dB, unless otherwise indicated.

Example 1: Consider a system with two base stations and $N = 10$ mobiles distributed on a line, as shown in Fig. 4. The optimum reverse channel assignment will be identified.

By v_i, $i = \{1, 2, \cdots, 10\}$ we denote mobile-i while by BS_j, $j = \{1, 2\}$ we denote base station j. For the system that is depicted in Fig. 4(a) we identify first all possible pairs of mobiles that can share a common channel in reverse direction. We create the corresponding compatibility graph represented by its adjacency matrix $C{:}\, N \times N$, called the compatibility matrix, where:

$$C[i, j] = \begin{cases} 1, & \text{if } v_i \text{ and } v_j, \text{ can share a channel,} \\ 0, & \text{otherwise.} \end{cases}$$

In our case, this matrix has the following form:

$$C = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&1&1&1&0&0 \\ 0&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0&1&1&0&0&0&0&0&0 \\ 0&0&1&1&1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix}.$$

The corresponding compatibility graph G is shown in Fig. 4(b). In order to compute the minimum number of channels C_{10}^r needed to accommodate the mobiles of graph G we must first identify the maximum matching of graph G. For the graph of Fig. 4(b) a maximum matching is

$$\mathcal{M} = \{[v_2, v_9], [v_3, v_6], [v_4, v_7], [v_5, v_8]\}$$

shown as heavy lines in Fig. 4(b). We let the pairs of mobiles matched by $\mathcal{M}$ share the same channel. For example v_2 transmits to base station 1 on the same channel that v_9 transmits to base station 2. The cardinality of $\mathcal{M}$ is 4. Therefore, the total number C_{10}^r of channels is equal to four plus the number of nodes that are not incident upon any matched edge (v_1 and v_{10}), that is $C_{10}^r = 4 + 2 = 6$. □

Most of the cases of the two-way channel assignment problem can be reduced to a maximum matching problem in an appropriate compatibility graph. First we consider the nonpower control case. Initially we assume that each channel can be used in one direction only (either as forward or as reverse channel) and that each mobile may use any base station for the forward and reverse connection. Then the two-way channel assignment problem is reduced to the solution of the forward and reverse channel assignment problems and the total number of channels needed to accommodate all the mobiles in the system is equal to $(C_N^f + C_N^r)$.

Now consider the case that a channel can be used in both directions, if this is preferable, and each mobile may use different base stations for the forward and reverse connection. The corresponding compatibility graph is as follows. Each mobile-i is represented by two nodes in graph G that correspond to the forward and reverse connection of the mobile, respectively. An edge connects two nodes if and only if the corresponding mobiles can share the same channel for their communication in the directions indicated by the two nodes. Then, the total number of channels needed to satisfy all the interference constraints in the system is equal to the cardinality of the maximum matching of graph G plus the number of nodes that are not incident upon any matched edge.

Now we turn our attention to the cases that the powers are adjustable parameters. First assume that each channel can be used only in one direction. Then the total number of channels needed to accommodate all the mobiles is equal to $2\tilde{C}_N^f$. This happens because as we have seen in Corollary 1, if two mobiles can share one forward channel then they can share one reverse channel, too.

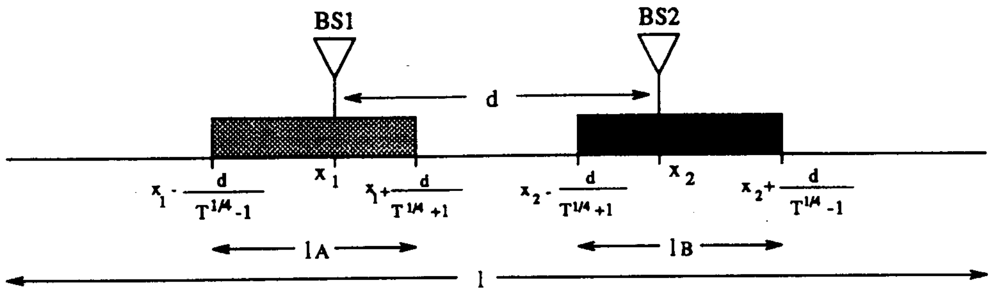

Fig. 5. Forward channel assignment. Lightly (heavily) shaded is the segment of the line that mobile $A(B)$ should fall in order to have simultaneous communication of both mobiles on the same forward channel.

Lets now consider the most general case that a channel can be used in both directions. First we assume that a mobile should use the same base station for its communication in both directions. As we conclude from Theorem 1 if two mobiles can share a forward channel l_1 then they can also share a reverse channel l_2 and therefore they can form a pair of mobiles that can make use of two channels to communicate in both directions. It can also be shown that if a mobile A uses channel l_1 in forward direction and mobile B uses the same channel in reverse direction then these two mobiles can also make use of one channel l_2 in the opposite directions, i.e., mobile A can transmit to base 1 on l_2 and mobile B can receive from base 2 on l_2. However, if any of the above conditions are satisfied then those two mobiles can form a pair that can share common channels for their communication. Then we can create a graph G where as nodes we consider all the mobiles and an edge connects two nodes if and only if the corresponding mobiles can share the same channel, in some way. Following similar reasoning as before, we can easily see that the problem of finding the optimal joint channel base assignment is reduced to the identification of the maximum matching of the above created graph G. Actually single nodes in the graph G represent those mobiles for which there is no way to share any channel with any other mobile and they should make use of their own channels in both directions. Denote by C_h the cardinality of the maximum matching of G plus the number of the nodes that are not incident upon any matched edge. Then, the total number of channels needed to accommodate all the mobiles of graph G is equal to C_h multiplied by two in order to count for the communication in both directions.

If we eliminate the constraint that a mobile should communicate with the same base station in both directions then a procedure similar to the corresponding case under fixed powers can be followed.

IV. Evaluation of Traffic Capacities

In this section, the traffic capacities of the forward and reverse channel, as defined in the introduction, are obtained and compared for the cases of power control and nonpower control. The mobiles are considered to be randomly and uniformly distributed either on a line or on the plane.

A. Nonpower Control

Assume that all base stations and mobiles use the same transmission powers, equal to P. As we have already seen it is possible two mobiles to be able to reuse the same channel in the forward direction, while they cannot reuse a reverse channel. The opposite is also possible. As we will see next the traffic capacities of the channels in each direction are different as well. In general, the traffic capacities T_N^f and T_N^r in the forward and reverse direction are obtained by simulation.

1) Forward Channel: For the forward channel there is a region around each base station 1 and 2 such that two mobiles A and B may reuse the same forward channel if and only if one mobile is in the region of base station 1 and the other belongs to the region of base station 2. In the following the boundaries of those regions are computed both in the linear and the planar cases.

Initially consider a line of length l units and denote by d the distance between the two base stations (Fig. 5). Denote by x_i, $i \in \{1, 2\}$ and x_j, $j \in \{A, B\}$ the coordinates that identify the positions of base stations 1 and 2 and of mobiles A and B, respectively, on the line. Because the desired signal and the interference at a mobile come from the base stations whose position on the line is fixed, we can consider each mobile separately. The seeking regions can be easily identified by satisfying the inequalities of relation (15). It turns out that the region around base station 1 is a segment (we refer to as segment A) with length l_A given by

$$l_A = \frac{d}{T^{1/4}+1} + \frac{d}{T^{1/4}-1}.$$

Segment A is represented by the lightly shaded portion of the line in Fig. 5. Similarly, the corresponding region around base station 2 is a segment (segment B) with length $l_B = l_A$ and is depicted by the heavily shaded part of line in Fig. 5.

In the following, we are going to obtain the boundaries of the regions around base stations 1 and 2 for the planar case. The positions of the two base stations are fixed and identified by the pairs (x_1, y_1) and (x_2, y_2) where by x_i and y_i we denote, respectively, the horizontal and vertical coordinates of base station i in a Cartesian coordinate system (Fig. 6). Denote by (x_A, y_A) and (x_B, y_B) the corresponding pairs of coordinates of mobiles A and B, respectively, and by d the distance between the two base stations.

The two mobiles can share the same forward channel if both inequalities of (15) are satisfied simultaneously. Rearranging

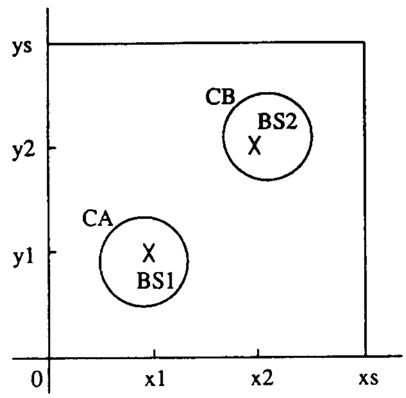

Fig. 6. Two base stations placed on the plane. Forward channel assignment. Mobiles A and B should fall into circles C_A and C_B, respectively, in order to share a forward channel.

the terms of the first inequality of relation (15) and after some algebra we get

$$\left(x_A - \frac{T^{1/2}x_1 - x_2}{T^{1/2}-1}\right)^2 + \left(y_A - \frac{T^{1/2}y_1 - y_2}{T^{1/2}-1}\right)^2 \leq \frac{T^{1/2}d^2}{(T^{1/2}-1)^2}. \quad (16)$$

From the latter expression we see that the carrier to interference ratio at mobile A is larger than the imposed threshold T if mobile A belongs to the interior of a circle (we refer to as circle C_A) with center at (x_0^A, y_0^A) and radius r_A, where

$$(x_0^A, y_0^A) = \left(\frac{T^{1/2}x_1 - x_2}{T^{1/2}-1}, \frac{T^{1/2}y_1 - y_2}{T^{1/2}-1}\right)$$
$$r_A = \frac{T^{1/4}d}{T^{1/2}-1}.$$

Similarly the carrier to interference ratio at mobile B satisfies the imposed constraint if mobile B belongs to the interior of a circle C_B with center at (x_0^B, y_0^B) and radius r_B, where

$$(x_0^B, y_0^B) = \left(\frac{T^{1/2}x_2 - x_1}{T^{1/2}-1}, \frac{T^{1/2}y_2 - y_1}{T^{1/2}-1}\right)$$
$$r_B = \frac{T^{1/4}d}{T^{1/2}-1}.$$

The two circles C_A and C_B are depicted in Fig. 6. Simultaneous communication of both mobiles on the same forward channel exist if one mobile belongs to the interior of circle C_A and the other belongs to the interior of circle C_B.

For the case of forward channel assignment without power control the computation of the optimal assignment can be done using a simpler method rather than the general algorithm developed in Section III. Assume that there are N mobiles randomly distributed on the plane and denote by $S_A(S_B)$ the set that consists of those mobiles that belong to the region around base station 1(2). Denote by N_A and N_B the cardinalities of sets S_A and S_B, respectively. Every pair of mobiles such that one belongs to set S_A and the other belongs to set S_B can share the same forward channel. Obviously we can create at most $\min(N_A, N_B)$ such pairs. Every mobile that does not belong to one of those pairs must use a separate channel. Therefore, the minimum number of channels C_N^f needed to establish communication for all the mobiles in the forward direction is given by

$$C_N^f = N - \min(N_A, N_B). \quad (17)$$

Lets denote by P_A and P_B the probabilities that a mobile belongs to the sets S_A and S_B, respectively. For the linear case, those probabilities are as follows:

$$P_A = \frac{l_A}{l}$$

and

$$P_B = \frac{l_B}{l}.$$

Now simultaneous communication of the two mobiles on the same forward channel is possible if one mobile belongs to segment A and the other to segment B. This may happen with probability P^f where

$$P^f = 2P_AP_B = \frac{8d^2T^{1/2}}{l^2(T^{1/2}-1)^2}. \quad (18)$$

Similarly for the planar case the corresponding probabilities are given by

$$P_A = \frac{\pi r_A^2}{x_s y_s}$$

and

$$P_B = \frac{\pi r_B^2}{x_s y_s}.$$

Therefore, both mobiles can share the same forward channel with probability P^f equal to

$$P^f = \frac{2\pi^2 r_A^2 r_B^2}{x_s^2 y_s^2}. \quad (19)$$

In our case, since the regions around the two base stations are equal to each other, probabilities P_A and P_B are also equal and in the following we denote them by P_I. In general, the number of channels C_N^f in the forward direction cannot be obtained analytically and the maximum matching problem need to be solved. Consequently the traffic capacity T_N^f is obtained by simulation. The limiting behavior of T_N^f is specified next.

Theorem 3: The forward traffic capacity T_N^f satisfies the following:

$$\lim_{N\to\infty} T_N^f = \frac{1}{1-P_I}. \quad (20)$$

Proof: Denote by N_0 the number of mobiles that do not belong to sets S_A and S_B. Then, the minimum number of channels C_N^f needed in the forward direction can be written as follows:

$$C_N^f = N_0 + \max(N_A, N_B). \quad (21)$$

Obviously: $N_0 + N_A + N_B = N$. The expected number of channels $E[C_N^f]$ is given by

$$E[C_N^f] = E[N_0 + \max(N_A, N_B)] = E\{N_0 + E[\max(N_A, N_B)|N_0]\} \quad (22)$$

N_0 is a random variable (r.v.) with binomial distribution of order N and $p = 1 - P_A - P_B = 1 - 2P_I$. Therefore

$$E[N_0] = N(1 - 2P_I). \tag{23}$$

Now we proceed to calculate the $E[\max(N_A, N_B)/N_0]$. Notice that, conditioned on N_0, N_A is a r.v. with binomial distribution of order $(N - N_0)$ and $p = 1/2$. Therefore, we can write

$$\begin{aligned} E&[\max(N_A, N_B)|N_0] \\ &= E[\max(N_A, N - N_0 - N_A)|N_0] \\ &= \sum_{\zeta=\lceil (N-N_0)/2\rceil}^{N-N_0} \zeta \binom{N-N_0}{\zeta} (\tfrac{1}{2})^{N-N_0} \\ &\quad + \sum_{\zeta=\lceil (N-N_0+1)/2\rceil}^{N-N_0} \zeta \binom{N-N_0}{\zeta} (\tfrac{1}{2})^{N-N_0} \\ &= \frac{1}{2}(N-N_0) + \left\lceil \frac{N-N_0}{2} \right\rceil \binom{N-N_0}{\left\lceil \frac{N-N_0}{2}\right\rceil} \left(\frac{1}{2}\right)^{N-N_0}. \end{aligned} \tag{24}$$

Substituting relation (24) in (22) we get

$$E[C_N^f] = E\left[\frac{1}{2}N + \frac{1}{2}N_0 + \left\lceil \frac{N-N_0}{2} \right\rceil \binom{N-N_0}{\left\lceil \frac{N-N_0}{2}\right\rceil} \left(\frac{1}{2}\right)^{N-N_0}\right]. \tag{25}$$

Therefore, we have (26), shown at the bottom of the page. As N increases, from relations (23) and (26) we conclude that

$$T_N^f \longrightarrow \frac{1}{1 - P_I}. \tag{27}$$

2) Reverse Channel: In the reverse channel assignment, since the receiver is the base station, the desired signal as well as the interference come from the mobiles whose position is not fixed. Therefore, the exact position of a mobile affects the set of positions of the other mobile for which the simultaneous communication of both mobiles on the same channel is feasible. Lets fix the position of mobile A and consider the region that the other mobile should fall into in order to be able to use the same reverse channel with A. The limits of this region depends on the position of mobile A, unlike to the case of the forward channel. In the linear case the boundaries of those regions are closed form functions of the position of mobile A. Those closed forms change in different regions of the line and they are depicted in Fig. 7. In this figure, heavily shaded is the part of the line that contains all the possible positions of mobile B, for a given position of mobile A, such that both mobiles can make use of the same reverse channel. Lightly shaded is the portion of the line where mobile A should be in order for the limits of the heavily shaded regions to be given by the specific formulas. The analytic formulas change in different regions of the line as it is illustrated in Fig. 7(a)–(c). For a uniform distribution the length of the heavily shaded segment over l, for fixed x_A is the conditional probability that two mobiles can reuse the same channel when one is at x_A. By integration we conclude that the probability P^r that both mobiles communicate on the same reverse channel is given by

$$P^r = \frac{8d^2T^{1/2}}{l^2(T-1)}. \tag{28}$$

Similarly, for the planar case, given the position (x_A, y_A) of mobile A, the two mobiles can share the same reverse channel if mobile B belongs to the intersection of the exterior of a circle with center at (x_1, y_1) and radius R_1 such that

$$R_1^2 = T^{1/2}[(x_1 - x_A)^2 + (y_1 - y_A)^2]$$

and the interior of the circle with center at (x_2, y_2) and radius R_2 such that

$$R_2^2 = \frac{(x_2 - x_A)^2 + (y_2 - y_A)^2}{T^{1/2}}.$$

Two mobiles may use either one or two channels for their communication in one direction. Therefore, the expected number of forward channels $E[C_2^f]$ and reverse channels $E[C_2^r]$ needed in order to satisfy the interference constraints in forward and reverse direction, respectively, is given by

$$\begin{aligned} E[C_2^f] &= 1P^f + 2(1 - P^f) \\ &= 2 - P^f. \end{aligned} \tag{29}$$

$$\begin{aligned} E[C_2^r] &= 1P^r + 2(1 - P^r) \\ &= 2 - P^r. \end{aligned} \tag{30}$$

For the linear case P^f and P^r are given by expressions (18) and (28), respectively. Comparing relations (18) and (28), we see that

$$E[C_2^f] < E[C_r^2]. \tag{31}$$

In Fig. 10, we plot the probability of using only one channel in every direction for the communication of two mobiles that are randomly and uniformly distributed on a line with two base stations, versus the line length. The probability of using one forward channel is larger than the probability of using one channel in the reverse direction. In Fig. 11, we can see the corresponding expected number of channels in each direction (forward, reverse, and total), in order to satisfy the interference constraints for the mobiles and the base stations.

$$T_N^f = \frac{N}{E[C_N^f]} = \frac{N}{E\left[\frac{1}{2}N + \frac{1}{2}N_0 + \left(\frac{1}{2}\right)^{N-N_0} \left\lceil \frac{N-N_0}{2} \right\rceil \binom{N-N_0}{\left\lceil \frac{N-N_0}{2}\right\rceil}\right]}. \tag{26}$$

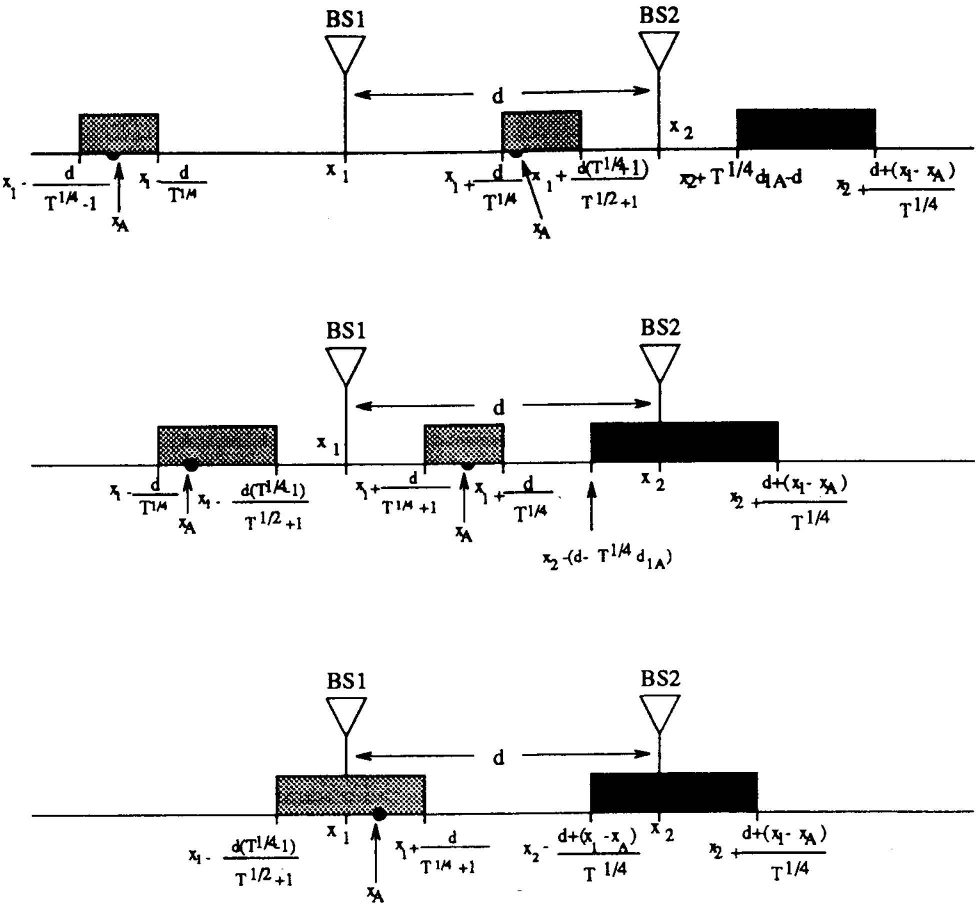

Fig. 7. Reverse channel assignment. Heavily shaded are all the possible positions of mobile B, for a fixed position x_A of mobile A, such that both mobiles can share a reverse channel. d_{1A} denotes the distance of mobile A from base station 1.

From relation (31) and the definitions of the forward and reverse traffic capacities, it follows readily that

$$T_2^f > T_2^r. \tag{32}$$

For an arbitrary number of mobiles we evaluate the expected optimal number of channels by simulation, using the algorithm developed in Section III to compute the minimum number of channels for every instance. In Fig. 12, we present the capacities (number of mobiles per channel) of the forward and reverse channel, versus the number of mobiles that are randomly and uniformly distributed on a line with length $l = 2.20$. The distance between the two base stations is taken equal to one unit. It turns out from that figure that for small $N (N < 6)$ $T_N^f > T_N^r$, while for large N the opposite is true. Therefore, in general, the forward and reverse channels are not equivalent. In Fig. 13, the same quantities are depicted for threshold $T = 14$ dB. In that case the difference between T_N^f and T_N^r is higher and $T_N^r > T_N^f$ for $N > 2$. Similar qualitative results hold for the case that we have N mobiles distributed on the plane.

3) Two-Way Channel Assignment: Now we consider the joint forward and reverse channel assignment problem. There are several versions of the two-way channel assignment problem depending on the constraints we pose. First assume that one channel can be used only in one direction (i.e., either as forward or as reverse channel). In the following, we consider that a mobile communicates with the same base station in both directions. For two mobiles in order to satisfy the interference constraints for both forward and reverse communication we need at least two channels—one for forward and one for reverse direction—and at most four channels in the case that each connection is established on a different channel in every direction.

Fig. 8 depicts all the possible positions of mobile B (heavily shaded part of line) for a fixed position of mobile A such that both mobiles can share one forward and one reverse channel. Throughout our analysis, we observed that in some cases whenever the use of the same reverse channel is possible for both mobiles then the communication on the forward direction can also be done in one channel (i.e., when mobile A is on left of base 1 and mobile B is on right of base 2), while in some other cases (i.e., when both mobiles between the two base stations) the opposite is true. Therefore, we see that some positions of the mobiles on the line favor the establishment of both connections on the same forward channel, while others favor the use of a common reverse channel for both mobiles. Similarly to the reverse channel assignment, by integrating over all possible values of the position x_A of mobile A, we conclude that the probability that two channels are enough to satisfy the interference constraints in our system is equal to

$$\frac{2d^2}{l^2 T^{1/4}(T^{1/4}-1)} + \frac{6d^2}{l^2(T^{1/4}+1)^2} + \frac{2d^2}{l^2 T^{1/4}(T^{1/4}+1)^2}.$$

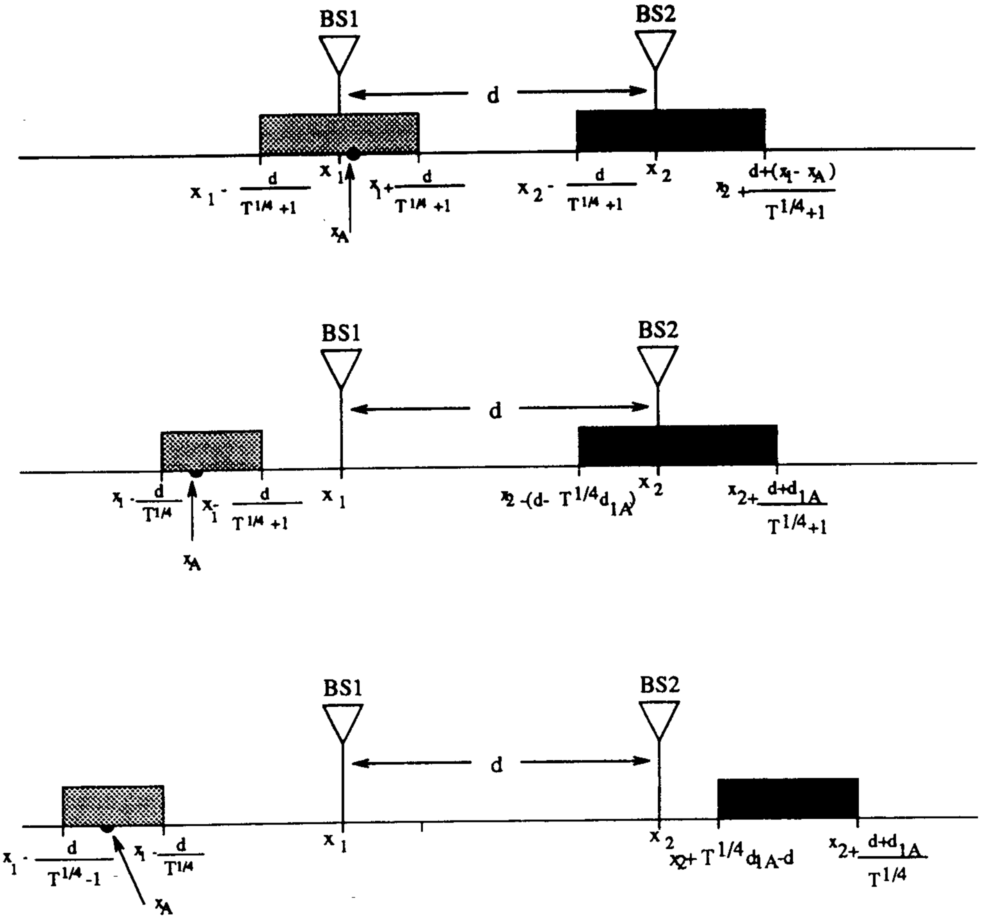

Fig. 8. Joint unidirectional channel assignment. Heavily shaded are all the possible positions of mobile B, for a fixed position x_A of mobile A, such that both mobiles can share one forward and one reverse channel, assuming that each channel is used in one direction only. d_{1A} denotes the distance of mobile A from base station 1.

The corresponding probability of using three channels for the two-way channel assignment is equal to

$$\frac{2d^2}{l^2T^{1/4}(T^{1/4}-1)^2}+\frac{2d^2}{l^2T^{1/4}(T^{1/4}+1)^2}+\frac{2d^2}{l^2}\cdot\left[\frac{2}{T^{1/2}-1}-\frac{1}{(T^{1/4}+1)^2}-\frac{1}{T^{1/4}(T^{1/4}+1)}\right]+\frac{2d^2}{l^2}\left[\frac{2}{T^{1/2}+1}-\frac{2}{(T^{1/4}+1)^2}-\frac{1}{T^{1/4}(T^{1/4}+1)^2}\right].$$

Now we eliminate the constraint that a channel can be used for communication in one direction only and allow each channel to be used in both directions, if possible. If we assume that a mobile or a base station can not talk and listen on the same channel then we can easily see that we need at least two channels in order to satisfy all the interference constraints. If two channels are enough to satisfy all the interference constraints for the communication of both mobiles, this means that either the two mobiles can share one forward channel l_1 and one reverse channel l_2 or the forward link of one mobile and the reverse link of the other share the same channel. In Fig. 9, we present all the possible positions of mobile B, for a given position of mobile A, such that two channels are enough to satisfy all the interference constraints for the communication in both directions if a channel can be used either in one direction or in both directions. Fig. 9 can be interpreted in the same way with Figs. 7 and 8. The corresponding probability is equal to

$$\frac{2d^2}{l^2T^{1/4}(T^{1/4}-1)}+\frac{2d^2}{l^2(T^{1/4}+1)^2}+\frac{2d^2}{l^2}\left[\frac{1}{T^{1/2}}+\frac{1}{T^{1/4}(T^{1/4}+1)}\right].$$

B. Power Control

Assume now that the transmission power is controllable. This means that the transmitter can adjust its power every time that is involved in a new connection. Because of the equivalence of the forward and reverse channel, as it is stated in Corollary 1, it suffices to limit ourselves only in the forward direction. In the following, we study analytically the linear case. Since the transmitted powers from the base stations to the mobiles are adjustable parameters we conclude that, although the desired signal and the interference at a mobile come from the base stations that have fixed positions, the exact position of one mobile affects the set of positions of the other mobile for which the two mobiles can share a channel. Lets fix the position of mobile A and consider the region that the other mobile should fall into in order to be able to use the same channel with mobile A. This region depends on the position of mobile A and can be specified using Theorem 1. Let $d_{1A}(d_{2B})$

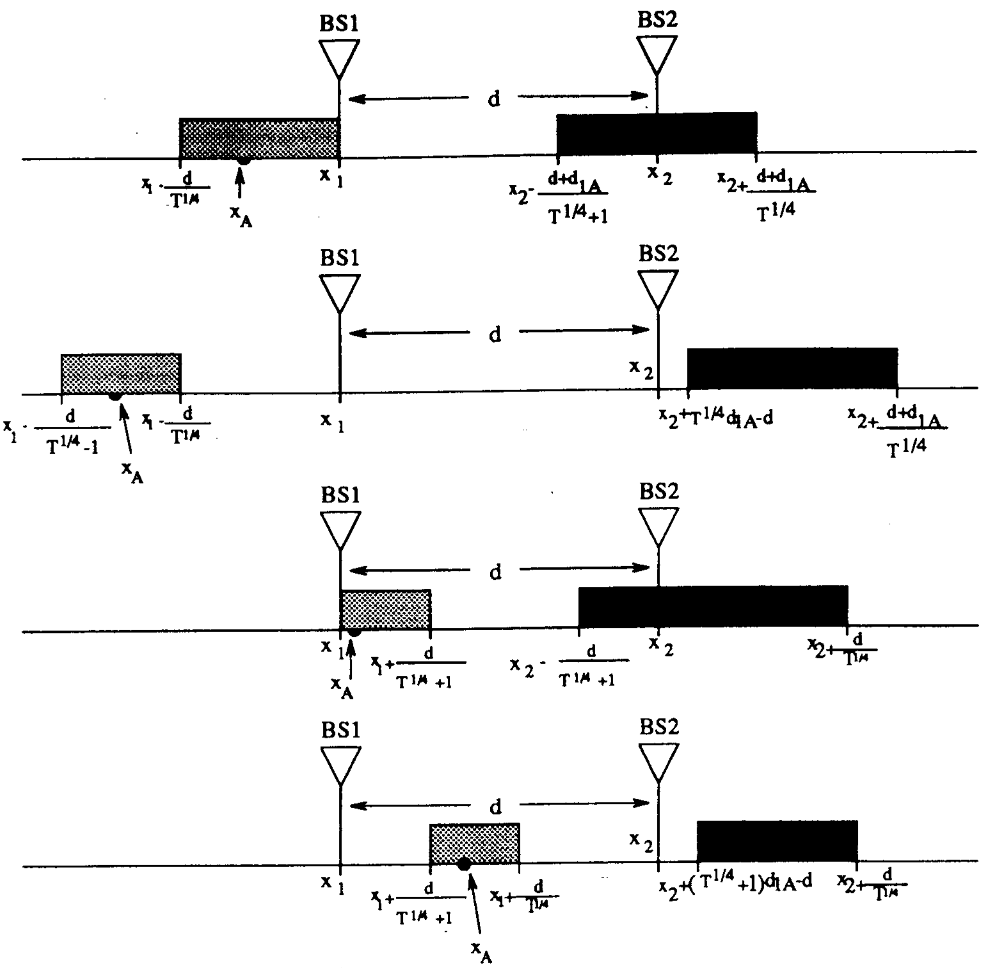

Fig. 9. Joint bidirectional channel assignment. Heavily shaded are all the possible positions of mobile B for a given position x_A of mobile A, such that two channels are enough to satisfy all the interference constraints in forward and reverse directions, assuming that a channel can be used in both directions. d_{1A} the distance of mobile A from base station 1.

be the distance between mobile $A(B)$ and base station 1(2). Depending on the positions of mobiles A and B on the line with respect to the fixed positions of the base stations we distinguish the following cases.

Case I: Mobile A on the left of base 1 and mobile B on the right of base 2:
if

$$0 \le d_{1A} \le \frac{d}{T^{1/2}-1}$$

then

$$d_{2B} \ge 0$$

if

$$d_{1A} > \frac{d}{T^{1/2}-1}$$

then

$$0 \le d_{2B} < \frac{d^2 + dd_{1A}}{(T^{1/2}-1)d_{1A} - d}.$$

Case II: Mobile A on the left of base 1 and mobile B between the two base stations:
if

$$d_{1A} \ge 0$$

then

$$0 \le d_{2B} \le \frac{d^2 + dd_{1A}}{(T^{1/2}+1)d_{1A} + d}.$$

Case III: Mobile A between the two base stations and mobile B on the right of base 2:
if

$$0 \le d_{1A} \le \frac{d}{T^{1/2}+1}$$

then

$$d_{2B} \ge 0$$

if

$$\frac{d}{T^{1/2}+1} < d_{1A} \le d$$

then

$$0 \le d_{2B} \le \frac{d^2 - dd_{1A}}{(T^{1/2}+1)d_{1A} - d}.$$

Case IV: Both mobiles between the two base stations:
if

$$0 \leq d_{1A} \leq d$$

then

$$0 \leq d_{2B} \leq \frac{d^2 - dd_{1A}}{(T^{1/2} - 1)d_{1A} + d}.$$

Case V: Both mobiles on the left of base station 1:
if

$$0 \leq d_{1A} < \frac{d}{T^{1/2} - 1}$$

then

$$d_{2B} \geq \frac{d^2 + dd_{1A}}{d - (T^{1/2} - 1)d_{1A}}$$

if

$$d_{1A} \geq \frac{d}{T^{1/2} - 1}$$

then no solution exists.

Case VI: Both mobiles on the right of base station 2:
if

$$d_{1A} \geq d$$

then

$$0 \leq d_{2B} \leq \frac{dd_{1A} - d^2}{(T^{1/2} - 1)d_{1A} + d}.$$

The probability $\tilde{P}^f$ that both mobiles can communicate on the same forward channel can be computed by integrating d_{2B} for all possible values of d_{1A} for all the above cases. In general, the expected minimum number of channels $E[\tilde{C}_N]$ needed to establish a connection to N mobiles, when the powers are adjustable parameters, cannot be obtained analytically and the maximum matching problem need to be solved. Therefore, the traffic capacity $\tilde{T}_N$ is obtained by simulation.

Now we turn our attention to the case that each channel can be used in both directions. The positions of two mobiles that allow communication in both directions are given by the union of the corresponding positions of the two mobiles for the cases where a channel is used exclusively in one direction and the cases where each mobile talks on one channel and listens on the other channel. In the following, we provide the corresponding relations between d_{1A} and d_{2B} for each of the cases we considered before.

Case I: Mobile A on the left of base 1 and mobile B on the right of base 2:
if

$$0 \leq d_{1A} \leq \frac{d}{T^{1/2} - 1}$$

then

$$d_{2B} \geq 0$$

if

$$d_{1A} > \frac{d}{T^{1/2} - 1}$$

then

$$0 \leq d_{2B} < \frac{d^2 + dd_{1A}}{(T^{1/2} - 1)d_{1A} - d}.$$

Case II: Mobile A on the left of base 1 and mobile B between the two base stations:
if

$$d_{1A} \geq 0$$

then

$$0 \leq d_{2B} \leq \frac{d^2 + dd_{1A}}{T^{1/2}d_{1A} + d}.$$

Case III: Mobile A between the two base stations and mobile B on the right of base 2:
if

$$0 \leq d_{1A} \leq \frac{d}{T^{1/2}}$$

then

$$d_{2B} \geq 0$$

if

$$\frac{d}{T^{1/2}} < d_{1A} \leq d$$

then

$$0 \leq d_{2B} \leq \frac{d^2 - dd_{1A}}{T^{1/2}d_{1A} - d}.$$

Case IV: Both mobiles between the two base stations:
if

$$0 \leq d_{1A} \leq d$$

then

$$0 \leq d_{2B} \leq \frac{d^2 - dd_{1A}}{(T^{1/2} - 1)d_{1A} + d}.$$

Case V: Both mobiles on the left of base station 1:
if

$$0 \leq d_{1A} < \frac{d}{T^{1/2} - 1}$$

then

$$d_{2B} \geq \frac{d^2 + dd_{1A}}{d - (T^{1/2} - 1)d_{1A}}$$

if

$$d_{1A} \geq \frac{d}{T^{1/2} - 1}$$

then no solution exists.

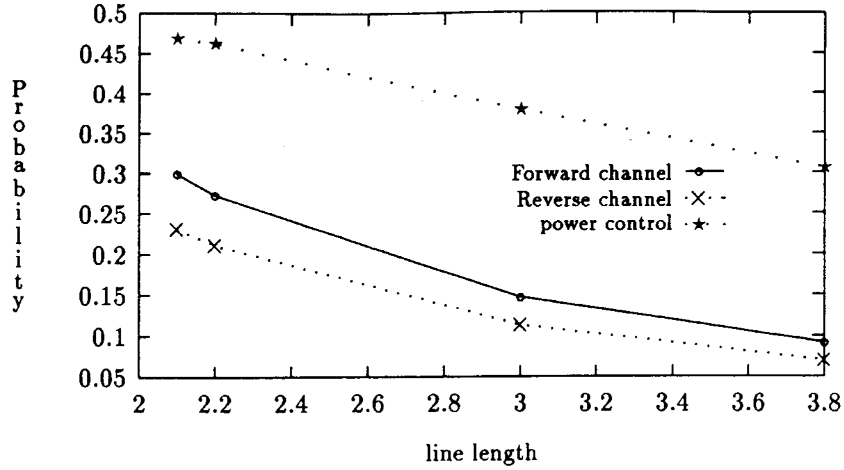

Fig. 10. Probability of using only channel for the communication of two mobiles in one direction (linear case, $d = 1$).

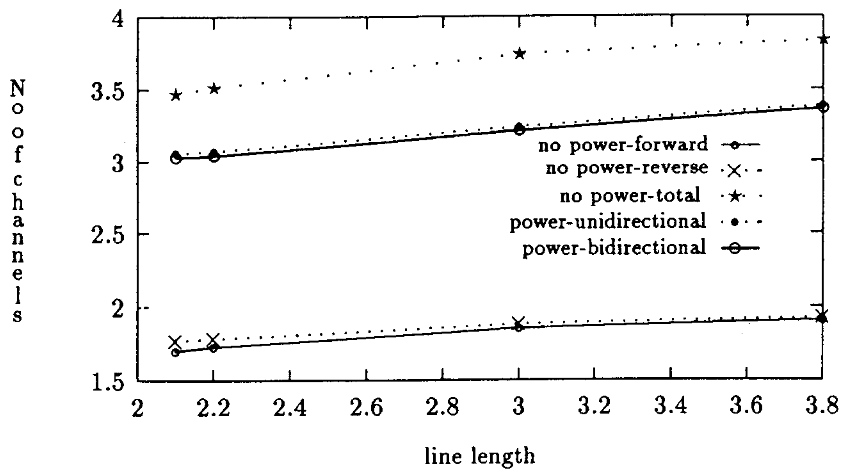

Fig. 11. Expected number of channels for the communication of two mobiles (linear case, $d = 1$).

Case VI: Both mobiles on the right of base station 2: if

$$d_{1A} \geq d$$

then

$$0 \leq d_{2B} \leq \frac{dd_{1A} - d^2}{(T^{1/2} - 1)d_{1A} + d}.$$

Notice that in Cases II and III the regions for which two channels are enough to satisfy all the interference constraints are larger than the corresponding regions that we found when each channel can be used in one direction only, while in the other cases they remain the same.

As we have already explained the probabilities $\tilde{P}^f$ and $\tilde{P}^r$ of using one forward and one reverse channel respectively, for the power control case are equal. In Fig. 10 these probabilities are represented by the same curve and as we notice they are much higher than those for the nonpower control case. Thus, the expected number of channels when the powers are controllable parameters is much lower than the corresponding value for the case of fixed powers. This result is clearly depicted in Fig. 11. By allowing each channel to be used in both directions, if this is desirable, we achieve a further reduction in the expected number of channels.

The traffic capacities for the power control case for an arbitrary number of mobiles are evaluated by simulation. The corresponding curves are depicted in Fig. 12. The first curve (identified by the label power control-uni) corresponds to the case that each channel can be used only in one direction, while the second curve (identified by the label power control-bi) corresponds to the case that a channel can be used in both directions. Fig. 13 presents the same quantities but using a threshold of 14 dB. Comparing them to those for the nonpower control case we verify the considerable improvement on the traffic capacity. Moreover, the use of bidirectional channels results in a small extra increase in the corresponding capacity.

V. Conclusion

The fundamental problem of the dynamic resource allocation in a wireless network has been considered here. An algorithm that achieves the optimal channel, base station and power assignment, in the sense that minimizes the number of used channels, for any system with two base stations and arbitrary number of mobiles is provided. The traffic capacities of the forward and reverse channel assignment are obtained

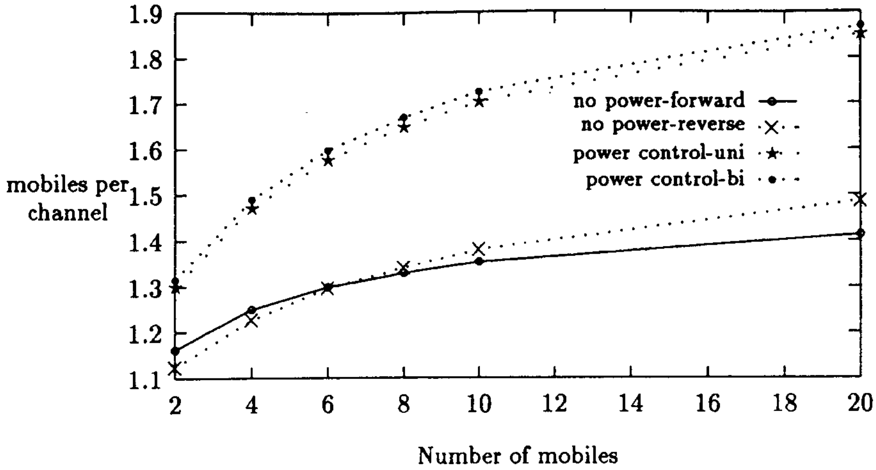

Fig. 12. Expected number of mobiles per channel (linear case, $d = 1$)—threshold of 18 dB.

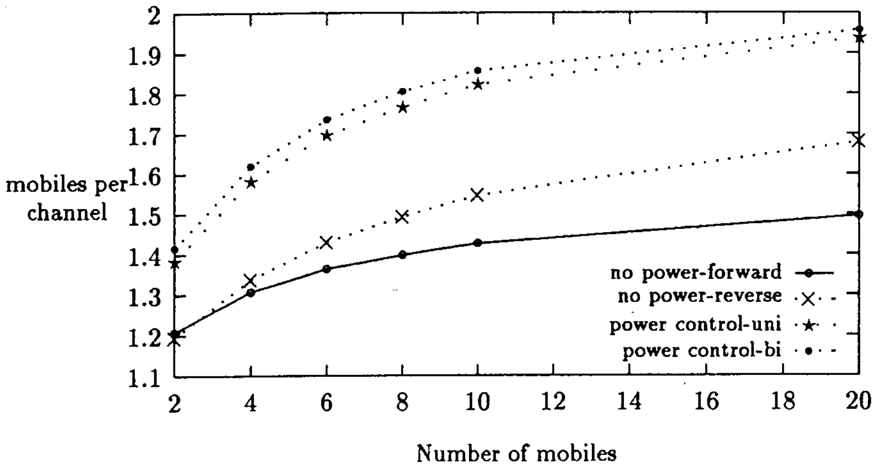

Fig. 13 Expected number of mobiles per channel (linear case, $d = 1$)—threshold of 14 dB.

and it turns out that they are different in the nonpower control case while they are the same when the powers are controllable parameters. Several versions of the two-way channel assignment problem, depending on the constraints we pose, are also discussed. For the case of two base stations and two mobiles the conditions under which the mobiles can share one channel in every direction, as well as the corresponding probabilities are obtained and compared under the power control and nonpower control cases.

A similar approach to the one we developed in order to identify the joint optimal channel base station and power assignment could be also applied in the general case that we consider an arbitrary number of base stations and mobiles. The same steps could be followed, but instead of identifying pairs of mobiles that can share a channel and therefore reducing the problem to the solution of a maximum matching on the appropriate compatibility graph, we must identify all the possible subsets of mobiles that can make use of the same channel. Then, we have to select those subsets that correspond to the use of the minimum number of channels in the system, as well as to assign base stations and select powers. As the number of base stations and mobiles increases such a procedure becomes intractable. Therefore, some heuristic algorithms that approximate the optimal assignment should be devised based on the steps followed by the optimal algorithm for two base stations. Such algorithms will have great practical value. In [13], we provide a heuristic algorithm that actually does the joint resource allocation in a general network with M base stations and N mobiles and we verify the large capacity improvements that can be achieved through the integration of the channel base station and power assignment.

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