Meeting QOS Requirements in a Cellular Network with Reuse Partitioning*

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ABSTRACT

Reuse partitioning is a technique for providing more efficient spectrum reuse in cellular radio systems. A cell in such a system is divided into concentric zones, each associated with an overlaid cell plan. Calls that arise in the periphery of the cell have fewer channels in their availability than those arising close to the base station and therefore they experience higher blocking rates. In this paper we consider the problem of balancing uniformly the blocking probability throughout the cell offering a fair treatment to the whole area within the cell, by controlling the allocation to the different channel layers. A policy that minimizes the maximum blocking probability experienced at any location of the cell is identified and is shown to be of threshold type. The policy satisfies any achievable constraint on the blocking rate uniformly throughout the cell. An adaptive scheme that adjusts the threshold based on estimates of the blocking probabilities in the different zones of the cell is proposed. This scheme tracks the optimal threshold effectively without any knowledge of the traffic parameters. Simulation study shows that substantial capacity improvements are achieved by the application of the optimal channel assignment policy, over the uncontrolled system.

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1 Introduction

Recently the variety of mobile radio communications services has been progressing and the number of subscribers has rapidly been growing. Moreover new mobile radio communication services are being introduced one after another. As a result, the demand for radio frequencies is growing rapidly making the efficient use of the spectrum one of the most significant problems. The infrastructure needed to support such communications often includes the use of many fixed radio transceiver sites that serve as gateways for communication with wireless units. The fixed transceivers are networked through fixed links. This arrangement allows high spectral reuse in the service area and fosters improved bandwidth utilization with limited power. In this paper we analyze the concept of reuse partitioning as a technique to increase frequency utilization.

In cell planning one of the most important parameters is the reuse distance which is defined as the minimum distance between two cells that may use the same channels (carriers in FDM or slots in TDM) without violating pre-specified signal-to-interference ratio (SIR) constraints. Reuse factor is a cell plan parameter equivalent to the reuse distance. It is the minimum number of channels needed to establish one call connection at each cell without reusing a channel in cells located closer than the reuse distance.

Reuse partitioning is the technique of using multiple reuse factors in the same cellular system. The main objective of reuse partitioning is to provide an increase in system capacity over that which can be achieved with a single factor, without relaxing SIR performance requirements. The underlying principle behind reuse partitioning is to degrade SIR performance for these mobiles that already have more than adequate transmission quality while offering greater protection to those mobiles that require it. The goal is to produce an overall SIR distribution that satisfies reception quality constraints while bringing about a general increase in system capacity. The available channels are split among several reuse patterns with different reuse factors N_i. Channel allocation within the i-th group is then determined by the reuse factor N_i. Mobile units with the best received signal quality will be preferentially assigned to the group of channels having the smallest reuse factor value, while those with the poorest received signal quality will be only assigned to the group of channels having the largest reuse factor value. Typically mobile units close to a cell-site are served by channels from a group having a small value for N_i since they receive high signal level and therefore are more resistant to interference than a mobile close to the perimeter of the cell. Mobile units that are far from the cell-site are served by channels from a group having a large value for N_i. One may visualize this structure as if each cell is divided into concentric zones, where a mobile in each zone-i achieves a sufficient SIR level by being assigned a channel from reuse pattern-i or some pattern associated with a zone closer to the cell perimeter.

The concept of using different reuse patterns in cellular systems has been addressed in the literature. It was suggested in [2] that by using two reuse factors of $N_1 = 3$ and $N_2 = 9$ we may achieve an increase in channel capacity of 30 percent over that which is achieved by a single reuse factor of $N = 7$, for a system with objective SIR of 17 db. The idea of reuse partitioning, combined with a traffic adaptive channel assignment for highway microcellular radio systems is introduced in [1] and the results indicate the capacity may be doubled compared to that of fixed channel assignment. Also in [11] has been shown that by using more than one reuse patterns and considering the assignment failure rate as performance measure we may achieve significant capacity improvements. Furthermore Zander in [10] has proposed upper and lower performance bounds for the class of channel assignment algorithms that use some arbitrary predictor of signal quality, considering as performance measure the failure rate which involves the calls that either are not assigned a channel at all, or have been assigned a channel that fails to meet the SIR requirement. Moreover for a two-zone system it was shown in [3] that a substantial increase of carried traffic can be accomplished by allowing calls to overflow to outer zones, at the expense of very unbalanced blocking for the traffic of the two zones.

The difference in the blocking for the traffic streams of the different zones is a drawback of the reuse partitioning approach. Mobiles located comparatively far away from the site will in general have fewer channels at their disposal. Therefore the area around the base station has some preferential treatment compared to the outer area of the cell. This will give to mobiles in different parts of the system different grade of service in terms of the blocking probability experienced. The goal of this work is to alleviate this drawback in a reuse partitioned system and to offer a fair treatment of the whole area within the cell by minimizing uniformly the blocking probability within the whole cell. We formulate the problem of fair channel assignment as a multiobjective optimization problem of minimizing the maximum blocking probability experienced at any location of the cell. We identify a threshold type class of policies that always contains a policy that solves the multiobjective optimization problem. Based on the above result we propose a policy that adjusts the threshold adaptively based on the blocking history of the system. This policy does not rely on the knowledge of the traffic conditions in the system. With a simulation study we confirm the good performance of that policy.

The paper is organized as follows: In the next section we provide a general description of the concept of reuse partitioning. In section 3 we formulate and solve the optimization problem for a two-zone cell system. In section 4 we point out some important factors for the design of a system with reuse partitioning, while in section 5 we present some simulation results that indicate the performance improvement achieved by the optimal control policy. Finally section 6 presents an adaptive algorithm for estimating the parameters involved in the optimal control policy.

2 The concept of Reuse Partitioning

In cellular networks there is a trade-off between the density of frequency reuse and the area that is covered by a base station that has been allocated that frequency. If a frequency is reused very densely, let say by every base station, then only a very restricted area around the base station can be covered since the SIR exceeds the required threshold only very close to the base station. As the reuse becomes sparser and sparser the area around the base station that is covered is increased. Normal cellular practice is to specify signal to interference ratio of 18 dB or higher. A seven-cell reuse pattern (N=7) is needed in order to provide coverage to the whole plane when the SIR is constrained to be over 18 dB. If we use a reuse factor of 1, which means that each frequency is used by every base station, then this frequency can be used only within an area around the base station, hence defining a subcell within the cell and providing coverage of only a portion of the whole plane (fig.1). For a reuse factor of 3, that is each frequency is used by 1/3 of the base stations, the area of the corresponding subcell increases but there is still some area that is not covered by any base station (fig.2). Reuse partitioning is based on the idea of combining together more than one reuse patterns in frequency planning. A fraction of the frequencies is allocated with reuse factor 7, while the rest is allocated by a denser reuse pattern.

In order to implement the idea of reuse partitioning we need to estimate the part of a cell which is covered by channels belonging to a specific reuse pattern. We assume that the received carrier power G_{ij} is inversely proportional to D_{ij}^4, where D_{ij} is the distance measured from the transmitter located at point-i to the receiver located at point-j (i.e. $G_{ij} = bD_{ij}^{-4}$, where b is a constant). Considering a reuse pattern with factor N_i we find the radius L_i of the subcell whose points satisfy the minimum pre-specified SIR. Since the radio signal attenuates with the $4th$ power of the distance the interference by the co-channel cells in the second or larger tier is highly dominated by that caused by the interfering cells in the first tier and therefore can be considered negligible ([4]). If V is the set of the interfering cells in the first tier then the SIR at point j is given by:

$$(SIR)_j = \frac{G_{ij}}{\sum_{k \in V} G_{kj}}. \tag{1}$$

Denoting by R the radius of the original cell (i.e. the cell that its size provides full coverage of the plane), we can express the radius L_i of the subcell corresponding to the reuse factor N_i as $L_i = a_i R$. In order to estimate a_i we have to satisfy the SIR requirement for the worst possible case, which is for a point on a vertix of the subcell of radius L_i. Solving the equation (1) for the worst case and for SIR of 18 dB we find that for a reuse factor of $N_1 = 1$ we have $a_1 = 0.22$ and for $N_2 = 3$ we have $a_2 = 0.36$. The coverage structure resembles a pyramidal overlay of cells. The base of the structure is formed by contiguous hexagonal groups of cells that are served by channels from the highest reuse factor group. Overlaid layers are formed by noncontiguous hexagonal groups of cells, of decreasing radius, that are served by channels from lower reuse factor groups. The idealized coverage structure is illustrated in figure 1 for a system using two reuse factors $N_A = 1$ and $N_B = 7$ and in figure 2 for a system using $N_A = 3$ and $N_B = 7$.

Reuse partitioning is implemented by dividing the available spectrum into two or more groups of mutually exclusive channels. Channel allocation within the i-th group is then determined by the reuse factor N_i. Considering a system that is given a frequency allocation of S channels, in a canonical cellular system the number C of channels that is assigned to each cell site is determined by the reuse factor N and is given by $C = S/N$. If we consider that we have divided the cell into K zones, where each zone-i corresponds to a reuse factor N_i, then the channels which have been allocated according to the factor N_i can be used up to zone-i. Assuming that a fraction p_i of the S channels has been allocated with reuse factor N_i, where $\sum_{i=1}^{K} p_i = 1$, then the number of channels assigned to a particular cell according to the reuse factor N_i is: $K_i = \frac{p_i S}{N_i}$. Clearly: $S = \sum_{i=1}^{K} N_i K_i$. In the following we are going to consider that each cell is divided into two zones (i.e. K=2).

Two aspects of channel management arise in such systems. The allocation of nominal channels to each reuse pattern (i.e to each zone) which takes place in the cell planning process and the actual assignment of channels to serve the calls in each zone, which is performed in real time. Here we study explicitly the second issue and we gain some insight for the first only through a numerical study.

3 Policies for channel assignment in a two zone cell system

We consider a two-zone cell system where a set S_1 of K_1 channels of the total number of the available channels is assigned to each cell according to a reuse factor N_1 and a set S_2 of K_2 channels are assigned according to a reuse factor N_2. In accordance with the constraints enforced by the SIR requirements mentioned in the previous sections, calls arriving in zone-1 can access all the $(K_1 + K_2)$ channels, while calls arriving in zone-2 can use only channels from set S_2. Therefore, in the absence of any control on the channel assignment the blocking probability experienced by a mobile in zone-2 will be higher than that experienced by a mobile in zone-1. Calls are generated in a cell with rate λ calls/time unit. The traffic is splitted in the two zones according to a factor f, $0 < f < 1$, which is determined by the traffic distribution in the cell and by the reuse factors. Therefore the arrival rates of calls in zone-1 and zone-2 are: $\lambda_1 = f\lambda$ and $\lambda_2 = (1-f)\lambda$, respectively.

Let i(t) and j(t) be the total number of busy channels in S_1 and S_2 respectively at time t, where $i(t) \leq K_1$ and $j(t) \leq K_2$. A call originated in zone-2 may be blocked because there is no available channel in S_2, while a call originated in zone-1 may be blocked for one of the following reasons: either there is no available channel in both sets S_1 and S_2 or there is no available channel in S_1 but although there is available channel in S_2 it is worthwhile reserving it for a call of zone-2. The channel assignment control policy determines whether a new call request will be granted a channel and which is the actual channel to be allocated. We wish to control the channel assignment in a reuse partitioned system with two zones such that we offer a fair treatment to the whole area within the cell by minimizing uniformly the blocking probability for both zones.

Let $P_{1,g}$, $P_{2,g}$ be the blocking probabilities experienced by calls arising in the inner and outer zones respectively when the channel assignment is controlled by policy g. The problem of fair channel assignment is formulated as follows:

(P): $\min\{\max(P_{1,g}, P_{2,g})\}$,
over all possible channel assignment policies g.

We show that the problem (P) has always a solution within a class of threshold policies. Those policies are as follows. When a call in zone-1 is generated if at least one zone-1 channel is free then it is forwarded there. If no channels of zone-1 are available then we accept the call if the number of busy zone-2 channels is less than a threshold T, we reject it if the busy zone-2 channels are more than T, while if exactly T zone-2 channels are occupied then we accept the call with some probability q and reject it with probability 1-q. Problem (P) is formulated as a Markov Decision Process (MDP) optimization problem (subsection 3.1). It is shown that this problem is equivalent to a constrained MDP optimization problem with linear cost (subsection 3.1). The analysis of the latter problem shows the existence of threshold type optimal policies (subsection 3.2).

3.1 Markov Decision Process Formulation

We assume that calls arrive in the cell in a Poisson stream with rate λ calls/time unit. Therefore the arrival of calls in each zone-i can be described by a Poisson process with rate λ_i calls/time unit. The corresponding service times of the calls are i.i.d exponential random variables with mean $\frac{1}{\mu}$ time units. The system corresponds to the blocking queueing system shown in figure 3. The state at time t is the two-dimensional vector $\mathbf{X}(t) = (i(t), j(t))$, with i(t) and j(t) as defined before. We use the notation $\{\mathbf{X}(t), t \geq 0\}$ for the stochastic process that describes the evolution of the system. The state space of that process is $\mathcal{X} = \mathcal{X}^1 \times \mathcal{X}^2$ where $\mathcal{X}^1 = \{0, 1, \ldots, K_1\}$ and $\mathcal{X}^2 = \{0, 1, \ldots, K_2\}$. Transitions among the states in $\mathcal{X}$ are described in terms of the operators: $A_l : \mathcal{X} \to \mathcal{X},\ D_l : \mathcal{X} \to \mathcal{X},\ l = 1, 2,$ defined by

$A_1(i,j) = (i+1, j)$ if $i < K_1$

$A_1(i,j) = (i, (j+1)^*)$ if $i = K_1$

$A_2(i,j) = (i, (j+1)^*)$

$D_1(i,j) = ((i-1)^+, j)$

$D_2(i,j) = (i, (j-1)^+)$

where,

$n^+ = max\{n, 0\}$

$m^* = min\{m, K_2\}$

The transition A_l represents an arrival of a call in zone-l, while D_l stands for the departure of a call served by a channel belonging to the set S_l of channels. Let us denote by z_t^i the probability of blocking a customer of zone-i at time t. Since a customer of zone-i can be blocked only at the moment of its arrival it is obvious that $z_t^i = 0$, if the transition (denoted by ω) is such that $\omega \neq A_{1(2)}$ and $z_t^i \in [0,1]$ if $\omega = A_{1(2)}$. Under all the assumptions mentioned above $\{\mathbf{X}(t)\}$ is a two-dimensional continuous time Markov process.

We first argue that the original continuous-time system can be converted into an equivalent discrete-time system by sampling it at certain random instants of time. If a server (channel) is idle assume that it is serving a "dummy" (or "imaginary") customer. Now consider the original continuous-time system sampled at the sequence of random times when either 1) an arrival occurs, 2) a"real" customer departs (a call is completed), or 3) a "dummy" customer departs. This sampled system is a discrete-time system which is a faithful replica of the original continuous-time system. This procedure of converting a continuous-time problem into a discrete-time problem is well recorded in the literature and rigorous validations are readily available. In Lippman [5] or Rosberg, Varaiya and Warland [9] it is shown that the discrete and continuous time problems are equivalent in that for infinite horizon cost criteria the optimal policies for the two formulations coincide. In the following we consider the discrete time problem.

Let $0 < t_1 < t_2 < \ldots < t_k < \ldots$ be the transition epochs (due to arrivals or departures) of the process $\{\mathbf{X}(t)\}$. Consider the discrete time Markov chain $\{\mathbf{X_k}\}$ with $\mathbf{X_k} = \mathbf{X}(t_k)$, $z_k^1 = z_{t_k}^1$, $z_k^2 = z_{t_k}^2$ with $0 \leq z_k^i \leq 1$ for $i = 1,2$ and the following transition probabilities, where $\lambda_1, \lambda_2, \mu$ are normalized such that while they are still in the same proportion to one another, the following holds: $\lambda_1 + \lambda_2 + (K_1 + K_2)\mu = 1$:

$$P(X_{k+1}/X_k = (i,j), z_k) = \begin{cases} \lambda_1 & if \quad X_{k+1} = A_1 X_k, \quad z_k^1 = 0 \\ \lambda_1 & if \quad X_{k+1} = X_k, \quad z_k^1 = 1 \\ \lambda_2 & if \quad X_{k+1} = A_2 X_k, \quad z_k^2 = 0 \\ \lambda_2 & if \quad X_{k+1} = X_k, \quad z_k^2 = 1 \\ i\mu & if \quad X_{k+1} = D_1 X_k \\ j\mu & if \quad X_{k+1} = D_2 X_k \\ (K_1 + K_2)\mu - i\mu - j\mu & if \quad X_{k+1} = X_k. \end{cases}$$

A control policy is any process $z = (z_1, z_2, \ldots)$, where $z_k = (z_k^1, z_k^2)$, such that $z_k^i = 0$ if $\omega \neq A_{1(2)}$ and $z_k^i \in [0,1]$ if $\omega = A_{1(2)}$. By G we denote the space of all policies of that form.

The long run average blocking cost for zone-2 calls associated with policy $g \in G$ is:

$$P_{2,g}(x) = \limsup_{n\to\infty} \frac{1}{n} E_x^g[\sum_{k=0}^{n-1} z_k^2], \quad x \in \mathcal{X},$$

where $E_y^g[\ ]$ denotes the expectation with respect to the probability measure induced by policy g, on the state process starting in state-y. Similarly for the blocking cost regarding zone-1 calls we have:

$$P_{1,g}(x) = \limsup_{n\to\infty} \frac{1}{n} E_x^g[\sum_{k=0}^{n-1} z_k^1], \quad x \in \mathcal{X}.$$

Therefore problem (P) can be stated rigorously as follows:

(P'): $\min\{\max(P_{1,g}(x), P_{2,g}(x))\}$,
over all channel assignment policies $g \in G$ and for any initial state $x \in \mathcal{X}$.

Notice that since $\{\mathbf{X}(t)\}$ is a finite state space Markov chain, the minimum in problem (P') is always achievable by some policy. To avoid unnecessary complexity we use min in the formulation of (P') instead of inf.
Problem (P') can be shown to be equivalent to the following constrained optimization problem:

(P''): $\min\{P_{2,g}(x)\}$
under the constraint: $P_{1,g}(x) \leq P_0$,
over all channel assignment policies $g \in G$ and for any initial state $x \in \mathcal{X}$.

Denote by P_a the optimal value resulting from the optimization problem (P'). Clearly the following proposition holds.

Proposition 1 *An optimal policy g for the problem (P'') for P_0 equal to P_a is optimal for problem (P') as well.*

From proposition 1 it is clear that if problem (P'') is solved by threshold type policies then problem (P') is solved by threshold type policies as well.

3.2 Optimal policy for the constrained problem

In this section we show that the optimal control policy g for the constrained optimization problem (P'') described above is of threshold type. It can be readily shown by using samplepath arguments that accepting a type-1 call (a call originated in zone-1) whenever there are free channels in S_1 (i.e. when at state $x = (i,j), i < K_1$ and there is a transition $\omega_k = A_1$ then $z_k^1 = 0$) and placing it to one of these free channels is the action of the optimal policy at such states. Hence in the following we will consider only policies that act as specified above.

The constrained optimization problem can be reduced to one without constraint through the introduction of Lagrange multipliers. In the following we define and solve the corresponding Lagrangian problem. Then using the results from the solution of the Lagrangian problem we deduce the structure of the optimal policy for the constrained optimization problem.
Define: $B^{\lambda}(\mathbf{X}(t_k), z_k) = z_k^2 + \lambda z_k^1$.
For each fixed multiplier $\lambda \geq 0$ define the Lagrangian:

$$J_g(x) = \limsup_{n\to\infty} \frac{1}{n} E_x^g[\sum_{k=0}^{n-1} B^{\lambda}(\mathbf{X}(t_k), z_k)], \quad x \in \mathcal{X}. \tag{2}$$

An admission policy g_d^{λ} is said to be average cost optimal policy, if it minimizes (2),
i.e. if $J_{g_d}^{\lambda} \leq J_g^{\lambda}(x), \quad x \in \mathcal{X}$, for any policy g $\in G$.
In order to study the optimization problem associated with the long run average cost in (2) we need to consider the optimization problem associated with the β-discounted cost of horizon n, defined next.

$$V_{g,n}^{\lambda,\beta}(x) = E_x^g \sum_{k=0}^{n-1} \beta^k (z_k^2 + \lambda z_k^1) \tag{3}$$

where $0 < \beta < 1$.
By $V_n^{\lambda,\beta}(x)$ we denote the minimum β-discounted cost of horizon n, over all policies $g \in G$. It is known that since the Markov decision process under consideration has finite state space, the β-optimal cost is achieved for some policy and this satisfies the dynamic programming (DP) equation:

$$V_{k+1}^{\lambda,\beta}(x = (i,j)) = \quad \min_{z_k \in [0,1]^2} \quad \{z_k^2 + \lambda z_k^1 + \beta\lambda_1(1 - z_k^1)V_k^{\lambda,\beta}(A_1 x)$$

$$
\begin{aligned}
&+\beta\lambda_1 z_k^1 V_k^{\lambda,\beta}(x)\\
&+\beta\lambda_2(1-z_k^2)V_k^{\lambda,\beta}(A_2x)\\
&+\beta\lambda_2 z_k^2 V_k^{\lambda,\beta}(x)\\
&+\beta\mu i V_k^{\lambda,\beta}(D_1x)\\
&+\beta\mu j V_k^{\lambda,\beta}(D_2x)\\
&+\beta[(K_1+K_2)\mu - i\mu - j\mu]V_k^{\lambda,\beta}(x)\}. \qquad (4)
\end{aligned}
$$

Rewriting the DP equation as

$$
\begin{aligned}
V_{k+1}^{\lambda,\beta}(x) &= \min_{z_k\in[0,1]^2} && \{z_k^1(\lambda - \beta\lambda_1[V_k^{\lambda.\beta}(A_1x) - V_k^{\lambda,\beta}(x)])\\
&&& +z_k^2(1-\beta\lambda_2[V_k^{\lambda,\beta}(A_2x) - V_k^{\lambda,\beta}(x)])\}\\
&+ && \beta\lambda_1 V_k^{\lambda,\beta}(A_1x) + \beta\lambda_2 V_k^{\lambda,\beta}(A_2x)\\
&+ && \beta\mu i V_k^{\lambda,\beta}(D_1x) + \beta\mu j V_k^{\lambda,\beta}(D_2x)\\
&+ && \beta[(K_1+K_2)\mu - i\mu - j\mu]V_k^{\lambda,\beta}(x), \qquad (5)
\end{aligned}
$$

we get the following criterion for blocking a customer of zone-1:

$$
\begin{array}{lcl}
V_k^{\lambda,\beta}(A_1x) - V_k^{\lambda,\beta}(x) > \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 1\\
V_k^{\lambda,\beta}(A_1x) - V_k^{\lambda,\beta}(x) < \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 0\\
V_k^{\lambda,\beta}(A_1x) - V_k^{\lambda,\beta}(x) = \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 \in \{0,1\}.
\end{array}
$$

Since we have limited ourselves to policies that take $z_k^1 = 0$ when at state $x_{k-1} = (i,j)$ and $i < K_1$ the above relation can be rewritten as:

$$
\begin{array}{lcl}
V_k^{\lambda,\beta}(K_1,j+1) - V_k^{\lambda,\beta}(K_1,j) > \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 1\\
V_k^{\lambda,\beta}(K_1,j+1) - V_k^{\lambda,\beta}(K_1,j) < \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 0\\
V_k^{\lambda,\beta}(K_1,j+1) - V_k^{\lambda,\beta}(K_1,j) = \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 \in \{0,1\}.
\end{array}
$$

Similarly for blocking a customer of zone-2 we get the following criterion:

$$
\begin{array}{lcl}
V_k^{\lambda,\beta}(i,j+1) - V_k^{\lambda,\beta}(i,j) > \frac{\lambda}{\beta\lambda_2} & \Longrightarrow & z_k^2 = 1\\
V_k^{\lambda,\beta}(i,j+1) - V_k^{\lambda,\beta}(i,j) < \frac{\lambda}{\beta\lambda_2} & \Longrightarrow & z_k^2 = 0\\
V_k^{\lambda,\beta}(i,j+1) - V_k^{\lambda,\beta}(i,j) = \frac{\lambda}{\beta\lambda_2} & \Longrightarrow & z_k^2 \in \{0,1\}.
\end{array}
$$

From the above relations we notice that if the function $V_n^{\lambda,\beta}(x)$ is convex then the threshold structure of the optimal policy follows readily. In the following subsection we prove the convexity of $V_n^{\lambda,\beta}(x)$.

3.2.1 Convexity of $V_n^{\lambda,\beta}(x)$

We have been unable to show that $V_n^{\lambda,\beta}(x)$ is convex by using the DP equation directly. Instead we deduced convexity of $V_n^{\lambda,\beta}(x)$ by formulating a linear programming problem equivalent to the Markov decision problem and using duality theorems ([9]). In the following we present the theorem that states the convexity of $V_n^{\lambda,\beta}(x)$ omitting the proof which is quite lengthy and can be found in [8].

Theorem 1 *$V_n^{\lambda,\beta}(x)$ is convex for $x \in N^2$.*

From the monotonicity of $V_n^{\lambda,\beta}$ and the exponential decay of the cost the following lemma follows easily.

Lemma 1 *For any $\beta < 1$, $\lim_{n\to\infty} V_n^{\lambda,\beta}(x)$ exists and $V_\infty^{\lambda,\beta}(x) = \lim_{n\to\infty} V_n^{\lambda,\beta}(x) < \infty$*

The minimum cost for the infinite horizon is:

$$V^{\lambda,\beta}(x) = \min_{g\in G} E_x^g \sum_{k=0}^{\infty} \beta^k (z_k^2 + \lambda z_k^1)$$

Moreover $V^{\lambda,\beta}(x)$ is the unique solution of optimality condition (4) (for the infinite horizon). From the uniqueness of $V^{\lambda,\beta}(x)$ and from Lemma 1 it follows that: $V^{\lambda,\beta}(x) = V_\infty^{\lambda,\beta}(x)$.
From theorem 1 and from Lemma 1 we have the following corollary.

Corollary 1 *For $\beta < 1$ $\;V^{\lambda,\beta}(x) = V_\infty^{\lambda,\beta} = \lim_{n\to\infty} V_n^{\lambda,\beta}(x)$ is convex.*

3.2.2 Structure of the optimal policy

In this section we will describe the structure of the optimal policy for the discounted cost problem.

Proposition 2 *If state (K_1, j) is a blocking state for zone-1 calls then all states (K_1, j'), with $j' > j$ are also blocking states for zone-1 calls.*

Proof. If $j' = K_2$ then obviously is a blocking state. Consider now the general case that $j' < K_2$. Assuming that (K_1, j) is a blocking state, in order to prove that $(K_1, j+1)$ is also a blocking state we must prove that: $V^{\lambda,\beta}(K_1, j+2) - V^{\lambda,\beta}(K_1, j+1) > \frac{\lambda}{\beta\lambda_1}$. Since (K_1, j) is a blocking state for zone-1 calls we have: $V^{\lambda,\beta}(K_1, j+1) - V^{\lambda,\beta}(K_1, j) > \frac{\lambda}{\beta\lambda_1}$. By convexity of $V^{\lambda,\beta}(\ )$ we get:

$$V^{\lambda,\beta}(K_1, j+2) - V^{\lambda,\beta}(K_1, j+1) \geq V^{\lambda,\beta}(K_1, j+1) - V^{\lambda,\beta}(K_1, j) > \frac{\lambda}{\beta\lambda_1}$$

and our claim is true. □

Define: $\nabla_j V^{\lambda,\beta}(i,j) = V^{\lambda,\beta}(i, j+1) - V^{\lambda,\beta}(i,j)$.

Up to this point the optimal policy for the Lagrangian problem has one of the two forms $g_1^\lambda = \{(z_k^1, z_k^2)\}$ and $g_2^\lambda = \{(z_k^1, z_k^2)\}$ which regarding the zone-1 calls can be described by the following expressions, if at state (i,j).

For policy g_1^λ

$$\begin{array}{lcl}
i < K_1 & \Longrightarrow & z_k^1 = 0 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) < \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 0 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) > \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 1 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) = \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 0.
\end{array}$$

For policy g_2^λ

$$\begin{array}{lcl}
i < K_1 & \Longrightarrow & z_k^1 = 0 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) < \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 0 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) > \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 1 \\
i = K_1 \ \ and \ \ \nabla_j V^{\lambda,\beta}(K_1, j) = \frac{\lambda}{\beta\lambda_1} & \Longrightarrow & z_k^1 = 1.
\end{array}$$

The convexity of $V^{\lambda,\beta}(x)$ implies that $\nabla_j V^{\lambda,\beta}(x)$ is monotone increasing. Therefore the quantity $\nabla_j V^{\lambda,\beta}(x)$ changes the relation of its value compared to the value of $\frac{\lambda}{\beta\lambda_1}$ at most once. Therefore there exist a $j^*_{\lambda,\beta}$ such that the above mentioned two policies can be transformed to the following forms:

For policy g_1^λ

$$z_k^1 = \begin{cases} 0 & if \quad j < j^*_{\lambda,\beta} \\ 1 & if \quad j > j^*_{\lambda,\beta} \\ 0 & if \quad j = j^*_{\lambda,\beta} \end{cases}$$

For policy g_2^λ

$$z_k^1 = \begin{cases} 0 & if \quad j < j^*_{\lambda,\beta} \\ 1 & if \quad j > j^*_{\lambda,\beta} \\ 1 & if \quad j = j^*_{\lambda,\beta} \end{cases}$$

We have already defined that:

$$V_g^{\lambda,\beta} = \lim_{n\to\infty} E^g \sum_{k=0}^{n-1} \beta^k (z_k^2 + \lambda z_k^1) = P_{2,g}^{\lambda,\beta} + \lambda P_{1,g}^{\lambda,\beta},$$

where $P_{2,g}^{\lambda,\beta} = \lim_{n\to\infty} E^g \sum_{k=0}^{n-1} \beta^k z_k^2$ and $P_{1,g}^{\lambda,\beta} = \lim_{n\to\infty} E^g \sum_{k=0}^{n-1} \beta^k z_k^1$.

Denote by g^λ the policy which satisfies the DPE for parameter λ for the $V^{\lambda,\beta}$ and use the notation $V^{\lambda,\beta} = V_{g^\lambda}^{\lambda,\beta}$, $P_2^{\lambda,\beta} = P_{2,g^\lambda}^{\lambda,\beta}$ and $P_1^{\lambda,\beta} = P_{1,g^\lambda}^{\lambda,\beta}$.

Lemma 2 *$V^{\lambda,\beta}$ is monotone non-decreasing in λ and $P_1^{\lambda,\beta}$ is monotone non-increasing in λ.*

Proof. These assertions are all a consequence of the fundamental inequality that reads:

$$\begin{aligned} V_{g^\lambda}^{\lambda+k,\beta} - V_{g^\lambda}^{\lambda,\beta} &\geq V_{g^{\lambda+k}}^{\lambda+k,\beta} - V_{g^\lambda}^{\lambda,\beta} \\ &\geq V_{g^{\lambda+k}}^{\lambda+k,\beta} - V_{g^{\lambda+k}}^{\lambda,\beta} > 0. \end{aligned}$$

for any $\lambda \geq 0$ and $k > 0$. Then

$$kP_1^{\lambda,\beta} \geq V^{\lambda+k,\beta} - V^{\lambda,\beta} \geq kP_1^{\lambda+k,\beta} \tag{6}$$

These prove both our claims. $\square$

Now we turn to the constrained optimization problem. Notice that as $P_{1,g}^{\lambda,\beta}$ and $P_{2,g}^{\lambda,\beta}$ have been defined above represent the corresponding β-discounted blocking costs for the constrained optimization problem. We introduce : $\gamma = inf\{\lambda : \ P_1^{\lambda,\beta} \leq P_0\}$. By the monotonicity of $P_1^{\lambda,\beta}$ this γ is well defined. Because of the form of the two policies g_1^λ and g_2^λ which satisfy the Lagrangian problem for λ we notice that: $P_{1,g_1^\lambda}^{\lambda,\beta} < P_{1,g_2^\lambda}^{\lambda,\beta}$. Now according to [6], if for some $\lambda \geq 0$ it holds $P_{1,g_i^\lambda}^{\lambda,\beta} = P_0$ then g_i^λ is the optimal policy for the constrained problem, too. If such λ does not exist then according to [6] for the above defined γ we have: $P_{1,g_1^\gamma}^{\gamma,\beta} < P_0$ and $P_{1,g_2^\gamma}^{\gamma,\beta} > P_0$

and the policy $f_q = qg_1^\gamma + (1-q)g_2^\gamma,\ \ q \in [0,1]$ is the optimal policy for the constrained problem and furthermore $P_{1,f_q}^{\gamma,\beta} = P_0$.

Therefore the optimal control policy, denoted by π_c, when at state (i,j) at time k, has the following threshold form for zone-1 calls:

$$z_k^1 = \begin{cases} 0 & i < K_1 \\ 0 & i = K_1, \quad j < j^* \\ 1-q & i = K_1, \quad j = j^* \\ 1 & i = K_1, \quad j > j^*. \end{cases}$$

Following similar arguments, we conclude that the optimal control policy for calls of zone-2 has a similar threshold form with the one that holds for zone-1 calls, but possibly with a different threshold. Of course in the case of a two-zone system one of the two thresholds should be the maximum number of allocated channels in zone-2 (i.e. K_2).

3.2.3 Average cost optimal policy

Using standard techniques from the theory of Markov Decision Processes and Dynamic Programming we can characterize the optimal policy for the average cost case. Based on the results we obtained for the β-discounted problem and some results that relate the β-discounted and the average cost problem in [7] we get the following structure for the optimal control policy π_c regarding calls of zone-1, when at state (i,j):

$$z_k^1 = \begin{cases} 0 & i < K_1 \\ 0 & i = K_1, \quad j < j^* \\ 1-q & i = K_1, \quad j = j^* \\ 1 & i = K_1, \quad j > j^*. \end{cases}$$

In the following we outline the necessary steps to prove the above argument. As we saw in section 3.2.2 the threshold j^* for the β-discounted problem depends on β and the difference: $V^{\lambda,\beta}(K_1, j+1) - V^{\lambda,\beta}(K_1,j)$. In the average cost case the optimal threshold j^* depends on the difference: $h^\lambda(K_1, j+1) - h^\lambda(K_1,j)$, where $h^\lambda = \lim_{n\to\infty}(V^{\lambda,\beta_n}(x) - V^{\lambda,\beta_n}(0,0))$ for $x \in \mathcal{X}$, for some sequence $\beta_n \to 1$. First we note that the above limit exists and therefore $h(\ )$ is well defined. Then according to [7] $h^\lambda(x)$ inherits the structure form of $V^{\lambda,\beta}(x)$ and therefore is convex. The proof is concluded following similar arguments with those used in section 3.2.2 for the β-discounted problem.

In order to calculate the parameters involved in the optimal policy we may use the standard computational approach of policy iteration. But usually this method turns out to be impractical since it is time consuming requiring many computations and is also based on the exact knowledge of the traffic statistics. In the section 6 we propose an adaptive method which tracks the optimal threshold effectively without any knowledge of the traffic parameters.

4 Design issues: Selection of reuse factors/Channel allocation to the different reuse factors

There are several relevant design issues in a system with reuse partitioning in addition to the selection of the channel assignment policy. Two of them are the selection of reuse factors for the different cell layers and the allocation of the spectrum to the different layers. Both of these may affect the performance of the system since they affect, either directly or indirectly, the number of channels allocated to each different zone as well as the total number of channels allocated to a whole cell. The selection of these parameters depends on the performance criteria that the system should meet and on the traffic statistics of the different zones of a cell.

Clearly, in a two zone cell system, the ratio of blocking for zone-1 traffic to blocking for zone-2 traffic can be changed by changing the numbers of channels in the two sets. Generally we assume that the total radio bandwidth is constant, but this does not mean that the total number of channels is constant irrespective of the division of channels between the two zones. This property stems from the fact that a channel can be used once in each reuse pattern, and when it is moved from a N_2-cell reuse pattern to the N_1-cell reuse pattern, the number of cells where the channel is used is multiplied by a factor of $\frac{N_2}{N_1}$. Therefore the different channel allocation patterns result to different total number of channels associated with a cell. Specifically as the number of channels assigned according to the reuse factor N_1 increases, so does the total nunber of channels allocated to the whole cell, since zone-1 channels can be reused more densely than zone-2 channels (i.e. for $N_1 = 1$ we can use a channel in zone-1 in every base station).

The allocation of the channels to the different reuse patterns is closely related with the statistics of the traffic in each zone of a cell and depends on the purpose of the division. In our case the purpose is to accomplish a certain blocking level within the whole cell. As we have already mentioned the factor f denotes the portion of the traffic in zone-1, while (1-f) denotes the portion of traffic which belongs to zone-2. Therefore f is directly related to the density of the traffic in the zones of the cell. As f increases more channels should be allocated to zone-1.

For any given channel allocation pattern, assuming that the ratio of the offered traffic rate in zone-1 to the offered traffic in zone-2 is fixed, there will be a maximum traffic load denoted by λ^* that can be handled by the system such that the the blocking probability in both zones is less than or equal to some prespecified value P_0. The control policy that results from the optimization problem (P''), since it achieves to balance the blocking probabilities in the two zones if this is possible, will be the one which achieves the maximum offered load λ^* that satisfies the blocking requirements. Among all the different possible channel allocation patterns we expect that there will be a channel allocation pattern that corresponds to the maximum achievable traffic that can be handled by the system under prespecified blocking probability requirements in the two zones. Any further increase in the number of channels in the inner zone may result in an overall increase of the total number of channels allocated to the whole cell, but the number of channels that can be accessed by mobiles in zone-2 decreases resulting in an increase of the zone-2 blocking probability. Therefore after some point in order to meet the blocking requirements the traffic in zone-2 has to decrease, and since zone-2 traffic usually corresponds to the larger portion of the traffic in the cell the overall offered traffic has to decrease.

The selection of the most appropriate reuse factors is another important design issue which affects the performance of a system. The reuse factor associated with the outer zone should be selected in order to provide coverage to the whole plane, satisfying the SIR requirements to every point of the system (i.e. for SIR of 18 dB a seven-cell reuse pattern is needed). The use of small reuse factors for the inner zones increases the total number of channels assigned to each cell, since small reuse factors correspond to dense reuse of the channels in the system. On the other hand by using a larger reuse factor for the inner zone (i.e. $N_1 = 3$, that is each frequency can be used by 1/3 of the base stations) we increase the area covered by zone-1 and therefore we increase the factor f, but the reuse of the channels becomes sparser. Therefore the total number of the channels available in each cell of the system may be smaller than the corresponding number in the case of using smaller reuse factors, but the distribution of the traffic among the two zones may be such that, overall, the total maximum traffic achieved by the system is larger compared to the traffic achieved by a system which uses smaller reuse factors for the inner zone, in order to meet specific blocking probability requirements.

However design practice recommends the placement of the base stations to the more heavily loaded areas, since under such a design channels can be reused according to denser reuse patterns in the areas with high traffic density. In the following section, through a numerical study, we are going to gain some insight of the influence of these factors on the performance of the system.

5 Performance comparisons

In the following we will compare the performance of three different channel assignment policies for a fixed channel allocation pattern. We will consider the performance of the conventional system, where all the channels are allocated following the same reuse pattern, the performance of a system using reuse partitioning with the optimal policy and the performance of a system with reuse partitioning but without channel assignment control which means that a call can access any free channel which satisfies the SIR requirement (we will refer to that policy as the non-threshold policy). We also present some results that indicate how do the design issues mentioned in section 4 affect the performance of the system.

The SIR requirement of our sample system corresponds to reuse factor of $N = 7$. The total number of channels available in the system is $S = 350$. Therefore a conventional system would allocate 50 channels per cell. In the system with reuse partitioning we also consider that each cell is divided into two concentric zones corresponding to reuse factors of either 1 and 7 or 3 and 7. In the following graphs that we compare the performance of the three different channel assignment policies we are going to consider two different cases for the distribution of the calls between the two zones of the cell. In the first case we consider the situations where we have larger concentration of mobiles in the zone around the base station than that in the outer zone. In the following, to represent such traffic patterns we consider that the factor f is either proportional to the radius of the inner zone (i.e. f is equal to 0.22 for reuse factor of 1 and SIR of 18 dB or higher, as explained in section 2), or equal to 0.5 for traffic distributions with even higher density in zone-1. This traffic consideration is justified by the design principle that the base stations are placed to the areas where more heavy loads are expected. We will also consider the case where the traffic is uniformly distributed over the cell and therefore the portion of the traffic assigned to the inner zone is proportional to the area of the inner zone.

In figure 4 we present the performance results for factor f equal to 0.22 and an allocation of $K_1 = 14$ channels in the inner zone and $K_2 = 48$ channels in the outer zone. The horizontal axis represents the offered load to the system in Erlangs, while in the vertical axis we place the blocking probabilities. The graph contains several curves which correspond to the blocking probabilities of the conventional system and the reuse partitioned system employing the optimal policy as well as the non-threshold policy. We notice that both policies for the system with reuse partitioning result in a substantial increase in the capacity of the system, of about 20 percent, over the simple $N = 7$ conventional system. This happens because by using the reuse partitioning concept, in some sense, we increase the total number of channels available in each cell. It is obvious that for the non-threshold policy the substantial increase of the traffic has been accomplished at the expense of very unbalanced blocking for the calls in the two different zones. This is due to the fact that calls from zone-1 are allowed to access zone-2 channels when there are no available channels in zone-1, hence seizing a number of the channels allocated to zone-2 and increasing the blocking probability experienced by a call originated in zone-2. The proposed optimal threshold policy achieves to balance these blocking probabilities, resulting simultaneously in an extra decrease up to 15 percent of the maximum blocking probability (of zone-2 in our case), which actually corresponds to an extra increase of the maximum traffic that can be handled by a system with specific objectives for the blocking probability. This happens because in this case the optimal threshold policy allows zone-1 calls to access only a portion of the channels of zone-2, determined by the threshold, hence allowing some channels allocated in zone-2 to be accessed only by calls originated in that zone and offering some protection to the blocking of zone-2 calls. Therefore we increase zone-1 blocking which was too low under the non-threshold policy, and decrease the much higher blocking of zone-2, resulting at a point where both blocking probabilities become equal to some value P_0. This point corresponds to the maximum traffic load accepted by the system, under the constraint that all mobiles in the system experience a blocking probability less than or equal to that value P_0. In figure 5 we notice that the optimal control policy attains more significant improvements on the system capacity for the cases where we have high traffic in the inner zone. In this graph we observe that under the optimal threshold policy the maximum blocking probability in the system is decreased significantly, especially under heavy traffic where the decrease is about 40%. Obviously such a decrease corresponds to an extra increase of the maximum traffic that can be handled by a system with specific objectives for the blocking probability. Moreover the improvements on the system capacity will be more impressive if we relax the SIR requirement (i.e. SIR=14 dB).

In figure 6 we plot the same performance results as in graph 4 but considering the factor f equal to 0.05 and an allocation pattern of $K_1 = 7$ channels to zone-1 and $K_2 = 49$ channels to zone-2. Again we confirm the performance improvement of the reuse partitioned system over the conventional system. In this case, for a given traffic load the decrease in the maximum blocking probability under the optimal threshold policy compared to that in the conventional system is about 10 percent. The improvement is smaller than the cases we considered in figures 4 and 5 due to the fact that only a very small portion of the overall cell traffic corresponds to zone-1, while the remaining traffic should be handled by the 49 channels allocated to zone-2 which follow the 7-cell reuse pattern. In low and moderate traffic the optimal threshold policy and the non-threshold policy produce similar values for the blocking probability of zone-2, since even in the non-threshold policy the overflow traffic from zone-1 towards channels of zone-2 is very small compared to the actual traffic of zone-2 and therefore does not affect substantially the blocking of zone-2 calls. Under heavy traffic the optimal threshold policy, that in our case results to a threshold 0 not allowing calls from zone-1 to access any channel of zone-2, presents an improvement on the maximum traffic load achieved in the system, over the non-threshold policy, but does not achieve to balance the blocking probabilities in the two different zones. This happens because due to the initial channel allocation pattern, the 7 channels allocated to zone-1 are enough to keep the blocking of zone-1 in much lower levels than that of zone-2, even if no calls from zone-1 are allowed to overflow to zone-2 channels.

In the case of the uniformly distributed traffic in the cell we may attain some improvement in the system performance if we increase the portion of traffic that corresponds to inner zone which is actually equivalent to increasing the area covered by the inner zone. This can be achieved by using larger reuse factors for the reuse partitioned system. An indication of such improvements can be seen in figure 7. This figure represents the performance of a system using two reuse factors of $N_1 = 3$ and $N_2 = 7$. The radius of the inner zone resulting from the reuse factor 3, is as calculated in section 2, equal to 0.36. Therefore in this case the split traffic factor f for uniformly distributed traffic in the cell is about 0.13. We consider a channel allocation pattern of $K_1 = 7$ channels in the inner zone and $K_2 = 47$ channels in the outer zone. As we expected, we can see from figure 7 that the performance results under the optimal threshold policy are better than those under the two other policies (conventional system and reuse partitioned system with non threshold policy). Moreover comparing figures 6 and 7 we note that for given blocking probability requirements, the system using the reuse factors 3 and 7 under the optimal threshold policy shows an increase of about 5% on the maximum achievable traffic load, compared to that which is achieved by a system using the reuse factors of 1 and 7.

In figure 8 we plot the achievable maximum offered traffic to the system which meets the performance requirement that the blocking probability in each zone is less than 4%, against different sets of channel allocation to the two zones for different values of factor f. As we expected for every f, which actually corresponds to some traffic distribution over the cell, there is a channel allocation pattern which achieves the maximum acceptable offered load to the system, under the use of the optimal threshold policy. Each of the curves shows a behavior where initially the maximum achievable offered traffic increases with the increase of the number of channels in the inner zone, since by such a change we increase the number of channels available in the cell and after reaching a maximum point each curve starts to descend because any further increase in the number of channels in the inner zone may result in a large increase of the blocking probability in zone-2, as explained in section 4.

Moreover we see that as the factor f increases so does the maximum achievable offered load, as well as the channel allocation pattern at which this maximum occurs. This happens because large f corresponds to traffic distributions with high density traffic in zone-1 and therefore in order to satisfy the blocking probability requirement in zone-1 and zone-2 simultaneously we have to allocate more channels in zone-1. Therefore we actually move channels from a reuse pattern of 7 to a reuse pattern of 1, resulting in an increase to the total number of channels in each cell, since these channels can be used at 7 times as many locations. These observations reinforces the design principle of placing the base stations at the more heavily loaded areas.

6 Adaptive estimation of the threshold

In order to implement the optimal policy we need to calculate the threshold, denote it by T, as well as the randomized factor q which indicates the decision of accepting or blocking a call of zone-1 when being at state (K_1, T). In this section we present an algorithm for updating these parameters on line, which actually approaches the optimal values of T and q fast enough and starting from any initial values. The algorithm relies on measurements of the blocking probabilities and does not need the statistical parameters of the system. The intuition behind the algorithm is explained below. We must note that, since calls from zone-1 and zone-2 under the threshold policy share the resources allocated to zone-2 in a way indicated by the threshold, the blocking probabilities in the two zones P_1 and P_2 change in a reverse way with a change of the threshold. That is, for a given traffic load, an increase to the threshold applied to zone-2 channels results to a decrease of P_1 and an increase of P_2, since calls originated in zone-1 may access more channels of zone-2. Similarly a decrease in the threshold results to a decrease of P_2 and an increase of P_1.

The adaptive algorithm updates the estimates of the threshold T and the randomized factor q at certain time instances based on the current estimates of the blocking probabilities. Let by T_n and q_n denote the control parameter iterates. By P_i^n we denote the estimate of the blocking probability in zone-i at the n-th iteration. It should be clear that $T_n \in \{0, 1, \ldots, K_2\}$ and $q_n \in [0, 1)$. The threshold and the randomized factor are represented both by the variable $h_n = T_n + q_n$, with $h_n \in [0, K_2]$. The threshold is the integer part of this variable while the randomized factor is the fractional part.

A recursive algorithm should have the form:

$$\begin{aligned} h_{n+1} &= I[h_n + b_n] \\ T_{n+1} &= \lfloor h_{n+1} \rfloor \\ q_{n+1} &= h_{n+1} - \lfloor h_{n+1} \rfloor \end{aligned} \tag{7}$$

where b_n is an estimate of the necessary adjustments on the variable $(T_n + q_n)$ based on the estimates of the blocking probabilities in the two zones and $I[\,]$ is defined by:

$$I[x] = \begin{cases} x & if \;\; 0 \leq x \leq K_2, \\ 0 & if \;\; x < 0, \\ K_2 & if \;\; x > K_2. \end{cases}$$

Therefore it remains to describe how we can estimate the b_n at each iteration as well as how the estimates of the blocking probabilities are updated. Since our target is to balance the blocking probabilities in the two zones when this is possible, a good choice for our estimate b_n which indicates the necessary adjustments should depend on the difference: $P_1^n - P_2^n$. We tried different algorithms where we considered b_n to be of the form: $b_n = a(P_1^n - P_2^n)$, where a was a constant. The sign of this difference indicates to which direction, increase or decrease, the current values of T and q should change. The convergence of such algorithms was often depended on the initial values of T and q, as well as on the order of the blocking probabilities in the two zones and on the amount that the difference between these two blocking probabilities changes by a change on the control parameters. Hence different values of the constant a were necessary for different systems and different traffic characteristics in order the algorithms to converge fast enough. These observations make the use of such algorithms impractical.

Therefore we turned our attention to algorithms where b_n is, in some sense, an estimate of the derivative of the difference of the blocking probabilities in the two zones. An algorithm which somehow captures this idea can be of the form:

$$b_n = \frac{P_1^n - P_2^n}{\mid (P_1^n - P_2^n) - (P_1^{n-1} - P_2^{n-1}) \mid} \Delta_{T,q}^n \tag{8}$$

where

$$\Delta_{T,q}^n = \mid (T_n + q_n) - (T_{n-1} + q_{n-1}) \mid \tag{9}$$

Actually based on the fact that a change of $\Delta_{T,q}^n$ on the factors T and q produces a change of: $\mid (P_1^n - P_2^n) - (P_1^{n-1} - P_2^{n-1}) \mid$ on the difference between P_1 and P_2, b_n indicates the proposed change on T and q in order to balance the blocking probabilities in the two zones in the next step. Therefore relations (7), (8) and (9) describe an adaptive algorithm for estimating the optimal values of T and q. It remains now to investigate how often should the algorithm iterate. In order to answer this question we need to clarify how the blocking probabilities P_1^n and P_2^n are calculated.

Since we want the system to be able to keep track of the sudden changes on the traffic characteristics in the two zones we have to decrease as much as possible the effect of the past of the system on the estimation of P_1^n and P_2^n. In the following we consider that the estimates of the blocking probabilities P_1^n and P_2^n are based on the last w calls only. We will iterate every w calls in the system. In order the algorithm to converge to the optimal values of the control parameters fast enough starting from any initial values for the T and q, and to be able to follow the changes in the traffic characteristics this w should be small. On the other hand a faithful estimate of the blocking probabilities in the two zones, which eliminates the random factors and errors involved in the simulation methods, should consider a w large enough. A numerical investigation for systems similar to those considered in the previous sections, indicated that a w in the order of thousands of calls is sufficient.

In figure 9 we present some numerical results on the convergence of the algorithm implemented on a system with allocation of 14 channels in the inner zone and 48 channels in the outer zone. We consider offered load of $\lambda = 517$ calls/time unit and a split traffic factor f equal to 0.2. In the vertical axis we present the values of the control parameters T and q. For representation reasons we use only one number to express these parameters. The integer part of this number corresponds to the threshold and the decimal part corresponds to the factor q. In the horizontal axis we present the iterations of the algorithm which actually correspond to the evolution of the system expressed in multiples of w incoming calls. This graph contains two curves. Both of them show how the algorithm proceeds in estimating the optimal threshold and factor q as the system evolves starting from different initial values of T=10, q=0 and T=33, q=0, respectively. As we see independently of the initial values of the control parameters the algorithm converges to the actual optimal values which in our case are: T=39 and q=0.8. As seen by figure 9 this adaptive scheme favors the large changes on the values of the control parameters, if these are necessary, therefore approaching the optimal values quite fast. Thereafter the changes are very smooth in the neighborhood of the optimal values of the control variables.

As we see in figure 10, this scheme captures the idea of being able to keep track of the changes on the traffic characteristics of the system. Specifically in this graph we present the results of estimating the optimal control parameters in the same manner with graph 9 but at some random points we change suddenly the call arrival rate λ. Therefore we start the system with low load of $\lambda = 300$ (at point A) and initial T=10 and q=0, we suddenly change λ at point B to 517 and finally we increase λ to 700 at point C. The actual optimal values for these different traffic characteristics are as follows: for $\lambda = 300$ T=48 and q=0; for $\lambda = 517$ T=39 and q=0.8; for $\lambda = 700$ T=0 and q=0. As we see the algorithm is able of making the necessary adjustments on the values of the threshold and factor q in order to reach the actual values fast enough.

7 Conclusions-Further research

In this paper we have seen that for the same SIR performance constraints, reuse partitioning has the potential for obtaining a significant increase in system capacity when compared to a system that uses only a single reuse factor. We considered the problem of characterizing the channel assignment policy in a system using two different reuse patterns, which provides the same grade of service to every part in the system by minimizing uniformly the blocking probability within the whole cell. By such a policy we achieve to alleviate the unfairness introduced by the approach of the reuse partitioning. We proved that for such a system the optimal channel assignment policy π_c is of threshold type. Specifically when a call in zone-1 is generated and no channels of zone-1 are available then we accept the call if the number of busy zone-2 channels is less than a threshold T, we reject it if the busy zone-2 channels are more than T, while if exactly T zone-2 channels are occupied then we accept the call with some probability q and reject it with probability 1-q.

The results have shown that by applying the optimal channel assignment policy π_c to the system we succeed not only to balance the blocking probabilities in the two zones of the system alleviating the unfairness in the quality of service in the different parts of the system, but we can also achieve the maximum offered load that can be handled by the system under the performance requirements that the blocking probabilities in both zones should be less than or equal to some prespecified value.

We have also presented an adaptive way of estimating the factors involved in the optimal channel assignment policy based on the blocking history in the two zones, which approaches the optimal values of the control parameters fast enough starting from any initial values and is able to keep track of the changes on the traffic requirements in the cell, although this scheme does not rely on the knowledge of the traffic conditions in the system.

Throughout our study we pointed out some design issues which play important role on the efficiency of the resource management functions and affect the performance of the system. One such important issue for the cell planning process is how do we choose the different reuse patterns. Another design decision is the allocation of the nominal channels to each reuse pattern (i.e. to each zone). In this paper we gave some numerical results that indicate how these factors affect the performance of the system but these problems still remain open for further investigation. It is also possible that the allocation of channels to the different reuse patterns will be done dynamically depending on the variations of the traffic requirements to the different zones of the cell. This is another open problem for further investigation.

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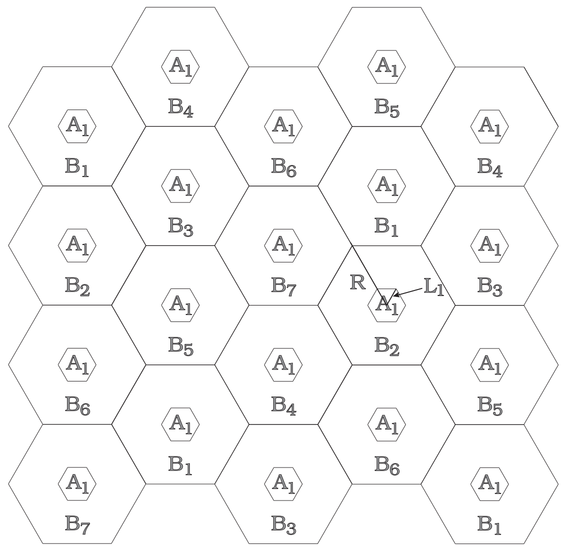

Figure 1: Cellular grid partitioned into two reuse groups with $N_A = 1$ and $N_B = 7$. The frequency A_1 that is used by every base station can be used only within the small cell.

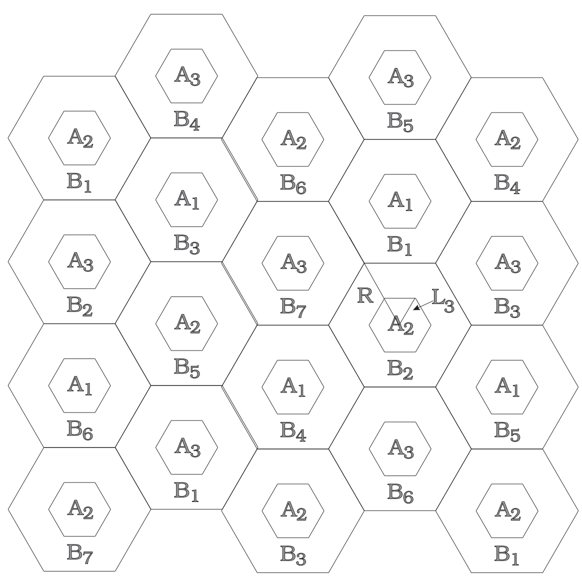

Figure 2: Cellular grid partitioned into two reuse groups with $N_A = 3$ and $N_B = 7$. Frequencies A_1, A_2, A_3 which are reused with reuse pattern 3 can be used only within a restricted area around the base station.

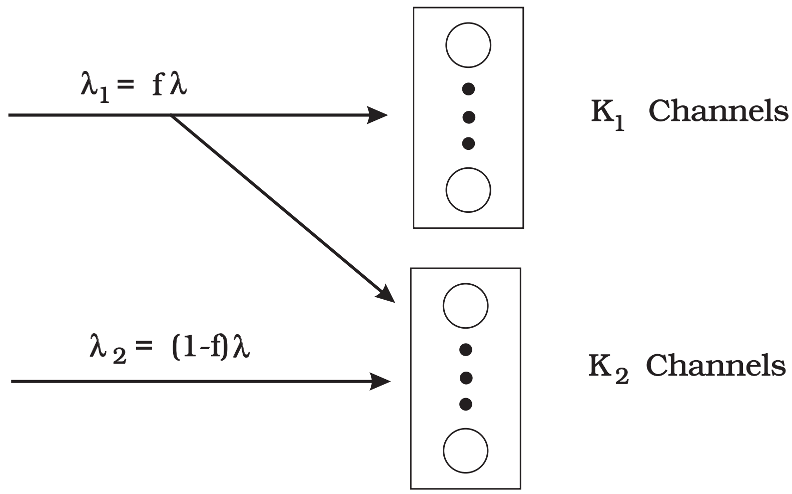

Figure 3: Blocking queueing model for a cell divided into two zones.

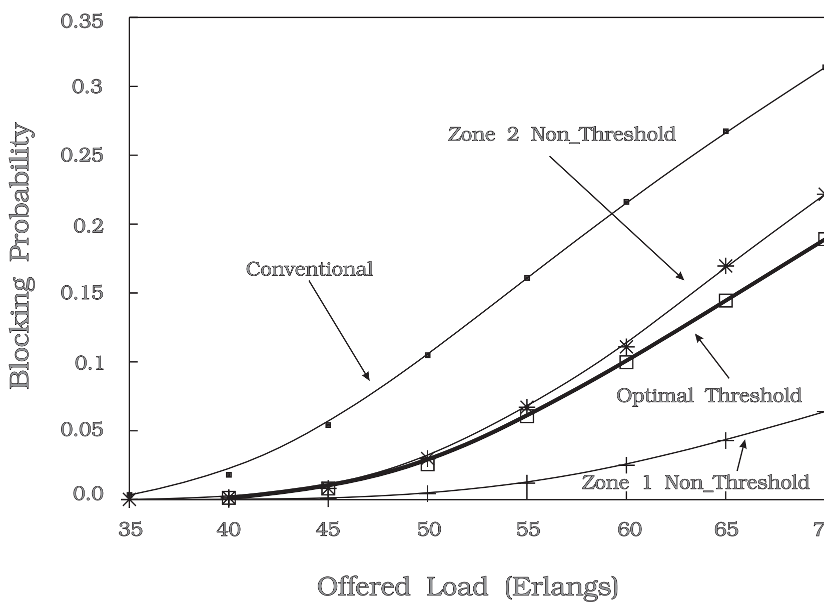

Figure 4: Blocking probability as a function of offered traffic. For the reuse partitioned system $N_1 = 1,\ N_2 = 7,\ K_1 = 14,\ K_2 = 48,\ f = 0.22$.

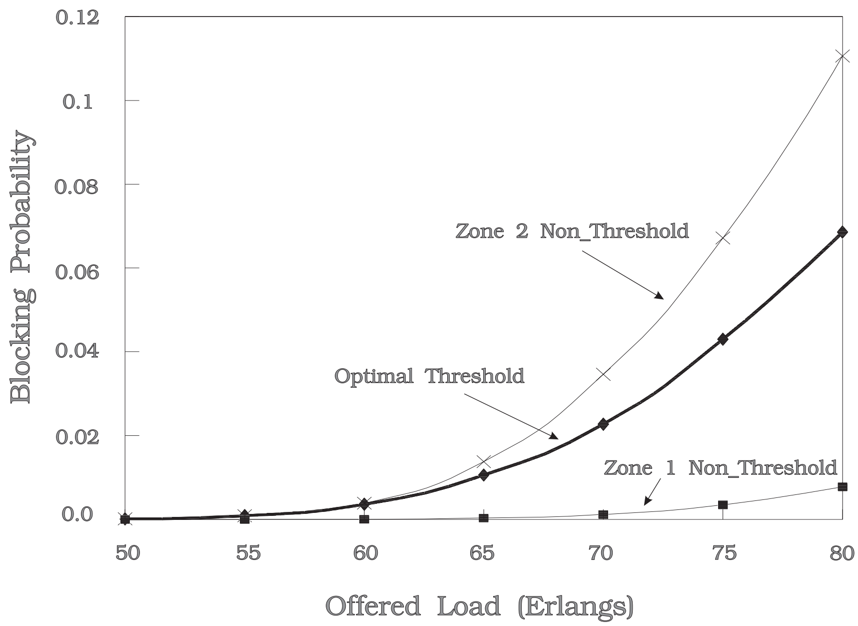

Figure 5: Blocking probability as a function of offered traffic. For the reuse partitioned system $N_1 = 1,\ N_2 = 7,\ K_1 = 42,\ K_2 = 44,\ f = 0.5$.

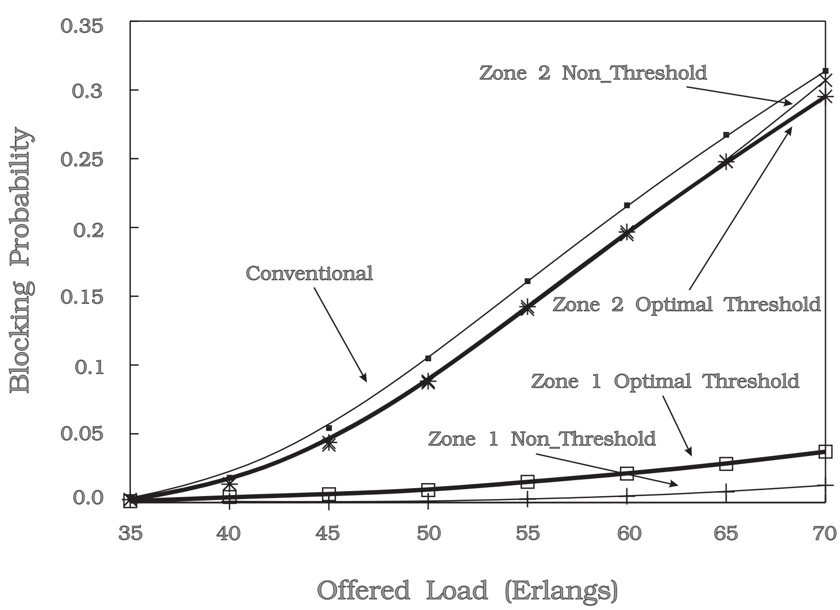

Figure 6: Blocking probability as a function of offered traffic. For the reuse partitioned system $N_1 = 1,\ N_2 = 7,\ K_1 = 7,\ K_2 = 49,\ f = 0.05$.

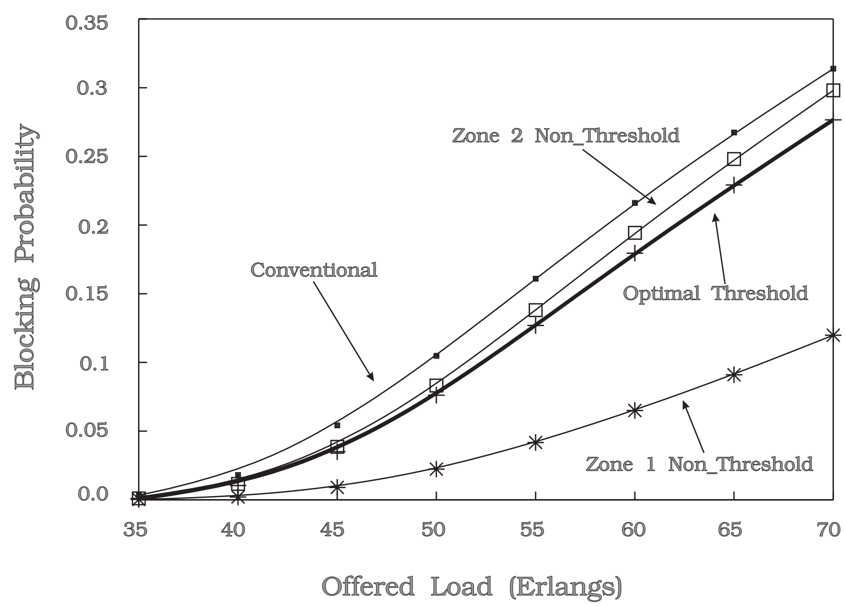

Figure 7: Blocking probability as a function of offered traffic. For the reuse partitioned system $N_1 = 3,\ N_2 = 7,\ K_1 = 7,\ K_2 = 47,\ f = 0.13$.

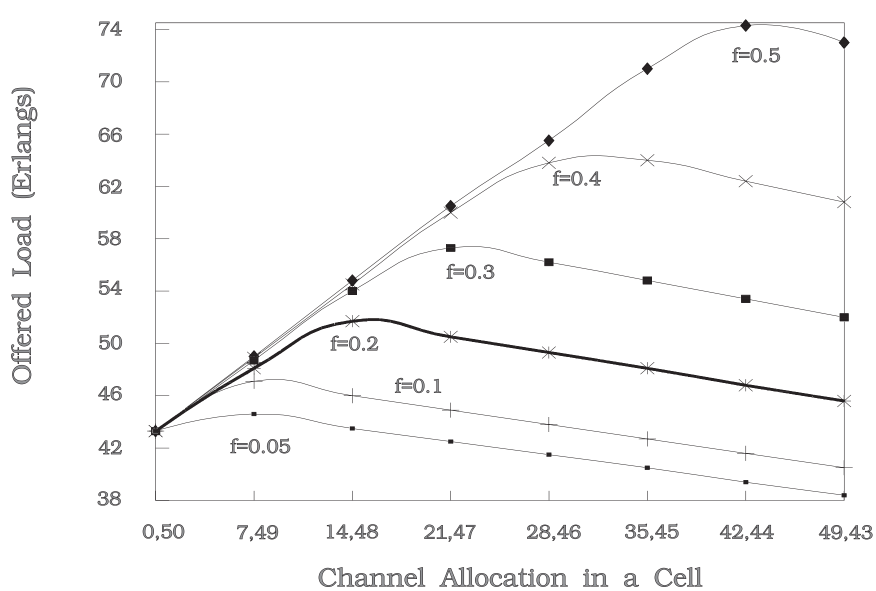

Figure 8: Maximum achievable offered load to the system meeting the requirement of blocking probability less than 4% in every zone under the optimal threshold policy, as a function of the channel allocation pattern in the two zones of the cell.

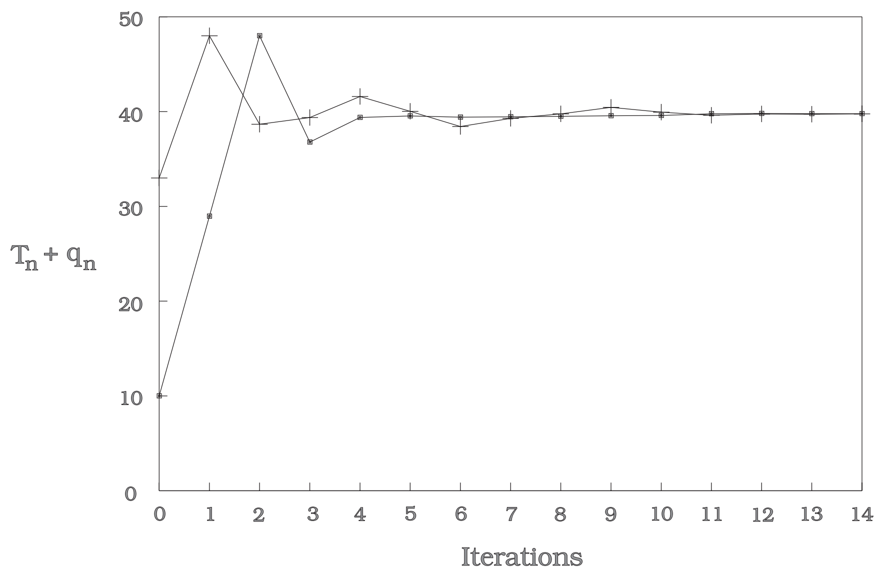

Figure 9: Evolution of the estimate $T_n + q_n$ under the adaptive algorithm for different initial values.

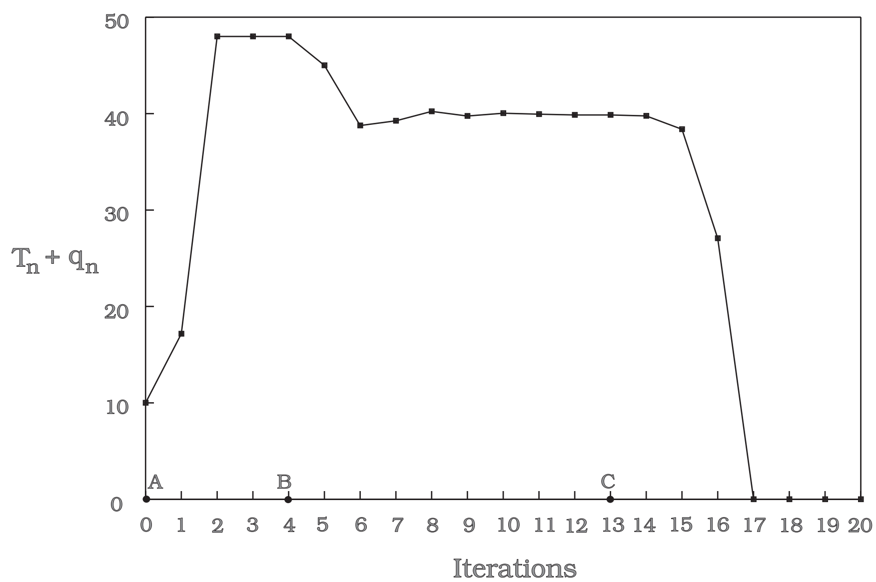

Figure 10: Evolution of the estimate $T_n + q_n$ under the adaptive algorithm for time varying traffic. At points A, B and C there are changes in the arrival rate ($\lambda_A = 300,\ \lambda_B = 517,\ \lambda_C = 700$).