ENEE 691 OPTICAL COMMUNICATION SYSTEMS
First Examination, Thursday, March 29, 2001, 2:00 - 3:15pm
CLOSED BOOK, NO NOTES, CALCULATORS PERMITTED

ANSWER THREE (3) QUESTIONS - IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT

(1) Explain the concept of numerical aperture (NA) of an optical fiber and calculate its value for a fiber with $n_1=1.46$, $\Delta=0.002$.

A point source of light with a total power output of 10mW is placed on axis and 20mm away from the perpendicular cleaved end face of a fiber with $n_1=1.46$, $\Delta=0.002$, and a fiber core radius of 25$\mu$m. Use the ray model to calculate how much power will be guided in the fiber.

Repeat this calculation with the point source placed against the cleaved face of the fiber.

(2) If the light in a single mode fiber crosses a perpendicular cleaved end face there is a back-reflection because of the index change. Calculate the reflectance in this case if $n_1=1.455$.

What coating (thickness and refractive index) could you place on the end face to reduce the reflectance to zero?

A common way to reduce back-reflection is to the cleave the fiber at an angle of $8^\circ$ away from perpendicular. If a quarter wavelength thick layer is placed on this cleaved face what is the back-reflectance for a P-wave if the layer has $d=\lambda/4$, refractive index $n=1.3$?

Hint: the effective impedance for a P-wave is $Z \cos \theta$. The transformed impedance formula is:

$$Z''_3 = Z'_2 \left( \frac{Z'_3 \cos k_2 d' + j Z'_3 \sin k_2 d'}{Z'_2 \cos k_2 d' + j Z'_2 \sin k_2 d'} \right).$$

In the following arrangement estimate how much power emerges at port 5.

![Diagram of optical fiber system with 3dB couplers and matched facets.]
(3) Choose any modern semiconductor laser design that you are familiar with and explain in as much detail as you can how it works. Explain some of the features of the output radiation and how factors such as the bandgap, laser length and lateral dimensions, and refractive index behavior affect the properties of the output radiation.

(4) The V-number of an optical fiber can be written as

\[ V = \frac{2\pi n_1}{\lambda_0} a \sqrt{2 \Delta}. \]

Explain the significance of this quantity and what it tells you about the performance of an optical fiber.

An optical fiber has core radius \( a = 12 \mu m \), \( n_1 = 1.46 \), \( \Delta = 0.005 \), and is being used at \( \lambda_0 = 1.55 \mu m \). Calculate, in the weakly guiding approximation, how many LP modes will propagate, and how many modes (TE, TM, EH, and HE) all together. Compare your answer with the number of modes that can be estimated from the V-number. A table of zeros of various Bessel functions is supplied below.

Hint: for an EH mode, \( m = \nu + 1 \).

Bessel function zeros:

\[ J_0: \quad 2.4048, \quad 5.52008, \quad 8.65373 \]
\[ J_1: \quad 3.83171, \quad 7.01559, \quad 10.17347 \]
\[ J_2: \quad 5.13562, \quad 8.41724, \quad 11.61984 \]
\[ J_3: \quad 6.38016, \quad 9.76102, \quad 13.0152 \]
\[ J_4: \quad 7.58834, \quad 11.06471, \quad 14.37254. \]
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First Examination, Tuesday, April 2, 1996, 3:30 - 4:45pm

**ANSWER THREE (3) QUESTIONS – IF MORE THAN THREE ARE ANSWERED, BEST THREE WILL COUNT**

(1) The contributions to noise in a p-i-n photodiode are

**Shot Noise:** \( \langle i^2_{N_1}(f) \rangle = 2e \bar{f} \Delta f \).

**Johnson Noise:** \( \langle i^2_{N_2}(f) \rangle = 4kT \Delta f / R \)

A particular p-i-n photodiode has responsivity 0.6A/W at 0.8μm. What is its NEP (WHz^{1/2}) when \( T=300K, R=5 \) ohm? (5pts.)

If the area of the detector is 1mm^2 what is its detectivity? (1pt.)

The detector is being used in a 100Mb/s communication link with a source power of 10dBm with a number \( N_T \) of 20dB taps on the channel. (defined by \( 10 \log \frac{P_{channel}}{P_{tar}} = 20 \)) Estimate how many taps can be tolerated between source and detector if a received S/N ratio of 21.5dB is required. (4pts.)

(2) Explain qualitatively the various kinds of dispersion in an optical fiber that can affect the performance of a fiber optic communication channel. (5pts.)

For a step-index fiber with \( n_{core}=1.6, n_{cladding}=1.5 \), with a core radius of 25μm and a wavelength \( \lambda_0 = 1.55μm \) estimate the maximum dispersion (\( \Delta t/L \)) that might result (2pts.). Derive a simple expression that provides an estimate of the maximum core radius that will ensure single mode operation. (3pts.) (Hint: use a simple 2-D ray model.)

(3) Explain the concepts:

(i) A ray invariant in an optical fiber (1pt.)

(ii) Caustic surfaces in an optical fiber (1pt.)

(iii) Numerical aperture of an optical fiber (1pt.)

The ray function \( g(r) \) can be written as

\[
g(r) = n^2(r) - \bar{\beta}^2 - l^2 \frac{\rho^2}{r^2}.
\]

Explain how this equation can be used to describe the path of a ray in a fiber. (3pts.)

A graded index fiber has a core radius of 30μm, with \( n_0=3, n_{cladding}=1.5 \) and \( n(r) = n_0 - \frac{1}{2}n_2r^2 \).

A particular ray has \( \bar{\beta} = 2, \bar{l} = 0 \). What can you say about its path in the fiber? (2pts.) What if \( l = 0.2 \)? (2pts.)

(4) Discuss three of the following:

(a) Photomultiplier detectors

(b) Photon counting

(c) Solitons

(d) Noise figure of an optical receiver

Useful information: Boltzmann’s constant: \( 1.38 \times 10^{-23}J/K \), Planck’s constant \( 6.626 \times 10^{-34}Js \), velocity of light \( c_0=2.997 \times 10^8 m/s \), magnitude of electronic charge \( 1.6 \times 10^{-19}C \).
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Take Home Examination, Thursday May 2 - Thursday May 9, 1996

ANSWER THREE (3) QUESTIONS ONLY

(1) Prove that if a single-mode optical fiber is operated at its minimum dispersion wavelength \( \lambda_D \) (the wavelength where \( \lambda_D^2 \frac{d^2n}{d\lambda^2}\lambda_D = 0 \)) then the pulse from a laser of spectral width \( \Delta \lambda \) (FWHM) will spread through material dispersion according to

\[
\frac{\tau}{L} = (-)\gamma^2 \frac{\lambda_D^3}{8c_0} \frac{d^3n}{d\lambda^3}\lambda_D,
\]

where \( \gamma = \Delta \lambda / \lambda_D \).

Compute this value for a silica fiber. Consult appropriate literature to find appropriate values for the constants.

(2) A slab waveguide has \( n_1 = 4, n_2 = 2, 2d = \mu m \). Calculate and plot the waveguide dispersion curve for the lowest order TE mode.

(3) Design a free-space optical fiber link to operate between two satellites in a direct detection mode at 100MB/s (NRZ) with a BER < \( 10^{-9} \) over a range of 10km. Assume that the transmitter produces a Gaussian laser beam at a wavelength of 810nm. Consider the following issues:

(a) The size of the transmitter telescope, as this will affect the spot size and beam divergence of the transmitted beam. Keep this as small as possible.

(b) The size of the receiver telescope aperture to collect sufficient signal. Keep this as small as possible.

(c) The type of detector chosen, its bandwidth, dark current, NEP, and electronics.

(d) The power of the transmitter. Keep this as small as possible.

Remember there are trade-offs in the design.

(4) A heterodyne receiver for a digital link has a responsivity of 0.6A/W and 3nA of dark current. The temperature is 25°C, the load resistance is 50ohm, and the bandwidth is 500MHz. The received optical power is 10nW when a binary “one” is received.

(a) How much local oscillator power is needed to make the S/N just 1dB less than the shot noise limit?

(b) If this were a direct detection system with a bandwidth of 250MHz, what signal power is required to give the same S/N as part (a)?
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Final Examination, Wednesday, May 22, 1996, 10:30am - 12:30pm

ANSWER FOUR (4) QUESTIONS - IF MORE THAN FOUR ARE ANSWERED, BEST FOUR WILL COUNT

(1) Prove that the noise spectrum of shot noise is (5pts.)

\[ \langle i_N^2(f) \rangle = 2e_i \Delta f. \]

What is the minimum detectable signal in a direct detection shot-noise-limited system with responsivity \( R \)? (3pts.)

If a photomultiplier is being used to detect a weak signal what likely sources of noise are likely to prevent shot-noise-limited performance being achieved? (2pts.)

(2) Explain how an avalanche photodiode operates. (4pts.)

The contributions to noise in an avalanche photodiode are

Shot Noise: \( \langle i_N^2(f) \rangle = 2e_i \Delta f. \)

Johnson Noise: \( \langle i_N^2(f) \rangle = 4kT \Delta f/R. \)

A particular APD has quantum efficiency of 0.8 at 1.3\( \mu \)m and a noise factor of 10. What is its NEP (WHz\(^{-1/2}\)) when \( T=300K \), \( R=50 \) ohm? (3pts.)

The detector is being used in a 1GHz communication link with a source power of 10dBm. What is the power margin (if any) for a BER of \( 10^{-9} \) if there is 30dB of link loss. (3pts.)

(3) Explain the concepts refractive index and group refractive index. (1pt.) Show that the group refractive index is (2pts.)

\[ n_g = n - \lambda dn/d\lambda, \]

and thereby prove that the pulse spreading caused by material dispersion in a step-index fiber not being operated at its minimum dispersion wavelength is (5pts.)

\[ \Delta T/L = (-) \frac{\lambda^2}{c_0} \frac{d^2n}{d\lambda^2} \frac{\Delta \lambda}{\lambda}. \]

For conventional step-index silica fiber operated at 1.55\( \mu \)m, \( \lambda^2 d^2n/d\lambda^2 = -0.01 \). What is the pulse spreading in ps/km/nm? (2pts.)

(4) Discuss three of the following:

(a) retardation in an uniaxial crystal
(b) the operation of an electro-optic phase modulator
(c) photon counting with photomultiplier tubes
(d) the spectral absorption of silica fibers

(5) A free-space optical link uses a laser of power 50mW at a wavelength of 1.06\( \mu \)m. The distance between source and receiver is 10km. The transmitter provides a Gaussian beam with spotsize \( w_0=2 \)mm, the receiver has an aperture of diameter 100mm. The laser is modulated in a NRZ format at 100Mb/s. The detector receives all the light that enters the receiver aperture. The detector is a p-i-n diode with dark current of 10nA, responsivity 0.8A/W, and is coupled to an amplifier with an input resistance of 50ohm and noise figure of 6dB. Calculate:

(a) the various source of noise (2pts.)
(b) the received signal power (3pts.)
(c) the signal to noise ratio at the output of the amplifier (2pts.)

What l.o. power would be needed to ensure shot-noise-limited operation? (3pts.)

Hint: information in question (2) should be useful. The beam divergence angle of a Gaussian beam is

\[ \theta_{\text{beam}} = \frac{\Delta}{w_0}. \]

(6) Explain how a modern semiconductor of the kind that would be used in a high data rate link is constructed and operates. Explain carefully features of the design that you discuss that ensure efficient operation and describe what the output spectrum from the device is like. If the laser is modulated at a high data rate what additional effects occur?