

CHAPTER NINETEEN

The Electrooptic and Acoustooptic Effects and Modulation of Light Beams

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The Electrooptic and Acoustooptic Effects and Modulation of Light Beams

In this chapter we shall explain how the distortion produced in a crystal lattice by the application of an electric field or by the passage of a sound wave affects the propagation of light through the crystal. These effects – the electrooptic and acoustooptic effects are of considerable practical importance as they can be used to amplitude and phase modulate light beams, shift their frequencies and alter the direction in which they travel.

19.1 Introduction to the Electrooptic Effect

When an electric field is applied to a crystal, the ionic constituents move to new locations determined by the field strength, the charge on the ions and the restoring force. As we saw in Chapter 18 unequal restoring forces along three mutually perpendicular axes in the crystal lead to anisotropy in the optical properties of the medium. When an electric field is applied to such a crystal, in general, it causes a change in the anisotropy. These changes can be described in terms of the modification of the indicatrix by the field – both in terms of the principle refractive indices of the medium and in the orientation of the indicatrix. If these effects can be described, to first order, as being linearly proportional to the applied field then the crystal exhibits the *linear* electrooptic effect. We shall see

that this results only if the crystal lattice lacks a center of symmetry. * So, some cubic crystals can exhibit the linear electrooptic effect. If the crystal possess a center of symmetry (or is even an isotropic material such as a gas or liquid) a change in optical properties can result that depends, to first order, on the square of the applied field. This is the *quadratic* electrooptic effect. Both the linear and quadratic electrooptic effects can be used effectively in various optical devices.

19.2 The Linear Electrooptic Effect

For any cartesian coordinate system the equation of the indicatrix has the general form

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1 \tag{19.1}$$

If x, y, z are chosen to be principal axes the cross terms in yz, xz and xy disappear and, in general, the equation of a biaxial indicatrix (Eq. (18.52)) results. For the linear electrooptic effect, an applied electric field changes each of the coefficients $\left(\frac{1}{n^2}\right)_i$ above in a way that is linearly dependent on the E_x, E_y and E_z components of the field. This change in these coefficients is described by the electrooptic tensor \bar{r} through the relationship

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_{j=1}^3 r_{ij} E_j \tag{19.2}$$

where $E_1 = E_x, E_2 = E_y, E_3 = E_z$. So for example:

$$\Delta\left(\frac{1}{n^2}\right)_1 = r_{11} E_x + r_{12} E_y + r_{13} E_z \tag{19.3}$$

and

$$\begin{pmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \Delta\left(\frac{1}{n^2}\right)_3 \\ \Delta\left(\frac{1}{n^2}\right)_4 \\ \Delta\left(\frac{1}{n^2}\right)_5 \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{19.4}$$

Typical magnitudes of the r_{ij} coefficients are $10^{-12} m/V$. The form of the matrix describing \bar{r} depends on the symmetry of the crystal and is

* A crystal posses a center of symmetry if there are identical particles in the lattice at vectors \mathbf{r} and $-\mathbf{r}$ where \mathbf{r} is a position vector measured from an arbitrary origin.

closely related to the symmetry of the piezoelectric* tensor \bar{d} , which relates polarization produced in a medium to stress σ^{**} . The matrix describing \bar{r} is the transpose of the matrix describing \bar{d} . Nye has tabulated in a convenient way the form of \bar{d} . Table (19.1) is a slight variant on the format given by Nye. ***

With an electric field applied the equation of the indicatrix changes from Eq. (19.1) to

$$\begin{aligned} & \left[\frac{1}{n_1^2} + \Delta\left(\frac{1}{n^2}\right)_1 \right] x^2 + \left[\frac{1}{n_2^2} + \Delta\left(\frac{1}{n^2}\right)_2 \right] y^2 + \left[\frac{1}{n_3^2} + \Delta\left(\frac{1}{n^2}\right)_3 \right] z^2 \\ & + 2 \left[\frac{1}{n_4^2} + \Delta\left(\frac{1}{n^2}\right)_4 \right] yz + 2 \left[\frac{1}{n_5^2} + \Delta\left(\frac{1}{n^2}\right)_5 \right] xz \\ & + 2 \left[\frac{1}{n_6^2} + \Delta\left(\frac{1}{n^2}\right)_6 \right] xy = 1 \end{aligned} \quad (19.5)$$

The use of the index i on $\Delta\left(\frac{1}{n^2}\right)_i$ is an example of what is called *contracted* notation: i takes the values 1,2,3,4,5, and 6 where $1 \equiv xx$, $2 \equiv yy$, $3 \equiv zz$, $4 \equiv yz$, $5 \equiv xz$, $6 \equiv xy$. Therefore the index i represents the

* *Piezoelectricity* is the phenomenon in which application of a force to a crystal causes a voltage to develop between the faces of the crystal. The voltage appears because the force has caused polarization – separation of positive and negative charges within the medium.

** *Stress* is force per unit area and can take the form of compressive or shear stress. For a more detailed discussion see J.F. Nye, *Physical Properties of Crystals*, Oxford University Press, Oxford, 1957.

*** His piezoelectric coefficients d_{ij} describe polarization produced by stress in the form

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix}$$

so, for example

$$P_x = d_{11}\sigma_{11} + d_{12}\sigma_{22} + d_{13}\sigma_{33} + d_{14}\sigma_{23} + d_{15}\sigma_{13} + d_{16}\sigma_{12}$$

σ_{12} is a shear stress – it is the stress produced in the y direction by a force acting on the face of a volume perpendicular to x . To be consistent with our notation for \bar{r} , where factors of 2 appear in the cross terms in Eq. (19.1) we need to write

$$P_x = d_{11}\sigma_{11} + d_{12}\sigma_{22} + d_{13}\sigma_{33} + \left(\frac{1}{2}d_{14}\right)2\sigma_{23} + \left(\frac{1}{2}d_{15}\right)2\sigma_{13} + \left(\frac{1}{2}d_{16}\right)2\sigma_{12}$$

So all Nye's d_{ij} coefficients where $i \neq j$ are double the value they have in our definition, where in symmetry terms $d_{ij} \equiv r_{ji}$.

Table 19.1 Symmetry of the linear electrooptic tensor for the 32 crystal classes.

zero element	non-zero element
equal non-zero element	
equal non-zero elements of opposite sign	
Centrosymmetric classes	
All elements zero	

specific quadratic term with which it is associated in the equation of the indicatrix. We shall have cause to use this notation further. So, if our original axes were principal axes the indicatrix becomes

The application of the electric field has introduced cross terms so x, y, z are no longer appropriate principal axes – the indicatrix has been changed in shape and rotated in space.

In a centrosymmetric crystal the change in shape of the indicatrix produced by a field \mathbf{E} must be the same as for field $-\mathbf{E}$, as these 2 fields are identical as far as the lattice is concerned. Therefore

$$\Delta \left(\frac{1}{n^2} \right)_i = \sum_{j=1}^3 r_{ij} E_j = - \sum_{j=1}^3 r_{ij} E_j \tag{19.7}$$

which can only be true if $r_{ij} = 0$. Consequently, there is no *linear* electrooptic effect (or any piezoelectric effect) in a centrosymmetric crystal.

The linear electrooptic effect is closely related to the inverse piezoelectric effect – where application of an electric field to a non-centrosymmetric crystal leads to an actual change in crystal shape (Ref. Nye). The change in crystal shape leads to strain and the shape and orientation of the indicatrix are altered. With this additional contribution to the change in index coefficients we can write

$$\Delta \left(\frac{1}{n^2} \right)_i = \sum_{k=1}^6 p_{ik} S_k + \sum_{j=1}^3 r_{ij} E_j \tag{19.8}$$

j takes the values 1, 2, 3; i and k takes the values 1 → 6 (contracted

notation). S_j is a component of the strain;* p_{ij} is the *elasto-optic* tensor. At high frequencies the inertia of the crystal prevents it straining macroscopically, so the first term on the right of Eq. (19.8) is zero. At high frequencies the residual change in indices is called the *Pockel's* effect. At low frequencies the elasto-optic effects cannot be ignored. However, since the deformation leading to strain is generally caused by the inverse piezoelectric effect** – and is therefore also related to applied field – it is possible to incorporate all changes in index into a single low-frequency linear electrooptic tensor \bar{r}^{dc} and write

$$\Delta \left(\frac{1}{n^2} \right)_i = \sum_{j=1}^3 r_{ij}^{dc} E_j \quad (19.9)$$

We shall not deal explicitly with this point further, but it should be noted that whenever a coefficient r_{ij} is written in what follows, this coefficient may be significantly different in magnitude between low and high frequencies. Table (19.2) gives some examples for important linear electrooptic materials.

One additional effect is worth noting – the photoelastic effect. This is the change in index coefficients produced directly by applied stress (no applied electric field is involved). For this effect

$$\Delta \left(\frac{1}{n^2} \right)_i = \sum_{k=1}^6 \pi_{ik} \sigma_k \quad (19.10)$$

k runs from 1 \rightarrow 6 (contracted notation). The π_{ik} are piezo-optical coefficients whose typical magnitude is $10^{-12} m^2/\text{newton}$; the σ_k are the components of the stress (for further discussion see Nye).

19.3 Longitudinal Electrooptic Modulation

To illustrate how an applied electric field affects an electrooptic crystal it is instructive to consider the specific important material KDP (potassium dihydrogen phosphate, KH_2PO_4). The related material

* Each strain component is a measure of the distortion of the crystal shape in a particular axial direction or of the rotation of the structure about a particular axial direction. For further details see Nye.

** If strain results solely from the inverse piezoelectric effect then

$$r_{ij}^{dc} = r_{ij} + \sum_{k=1}^6 p_{ik} d_{jk}.$$

Fig. 19.1.

KD*P(KD_2PO_4), ADP (ammonium dihydrogen phosphate, $(NH_4)H_2PO_4$) and AD*P, $(NH_4)D_2PO_4$) behave similarly. For these materials, symmetry $\bar{4}2m$, the electrooptic tensor has the form

$$\bar{r} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \quad (19.11)$$

In the presence of an applied field $\mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{y}} + E_z \hat{\mathbf{k}}$ the indicatrix, written in its original principal axes coordinates is

$$\frac{x^2 + y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y xz + 2r_{63}E_z xy = 1 \quad (19.12)$$

For a field applied in the z direction only Eq. (19.12) becomes

$$\frac{x^2 + y^2}{n_0^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \quad (19.13)$$

To find the new principal axes we note that Eq. (19.13) is symmetric in x and y , so the new x and y axes must be rotated by 45° from the original axes, as shown in Fig. (19.1). The z axis is unchanged.

The old axes are related to the new according to

$$\begin{aligned} x &= x' \cos 45^\circ + y' \sin 45^\circ \\ y &= -x' \sin 45^\circ + y' \cos 45^\circ \end{aligned} \quad (19.14)$$

Substitution in Eq. (19.13) gives

$$\left(\frac{1}{n_0^2} - r_{63}E_z\right)x'^2 + \left(\frac{1}{n_0^2} + r_{63}E_z\right)y'^2 + \frac{z^2}{n_e^2} = 1 \quad (19.15)$$

The indicatrix now no longer has any cross terms so our choice of new

principal axes was correct. The crystal has now become biaxial with

$$n_{x'}^2 = \frac{n_0^2}{1 - n_0^2 r_{63} E_z}; \quad n_{y'}^2 = \frac{n_0^2}{1 + n_0^2 r_{63} E_z}; \quad n_{z'}^2 = n_e^2 \quad (19.16)$$

If $n_0^2 r_{63} E_z \ll 1$, which is true for any reasonable electric field, these equations become

$$n_{x'} = n_0(1 + \frac{1}{2} n_0^2 r_{63} E_z); \quad n_{y'} = n_0(1 - \frac{1}{2} n_0^2 r_{63} E_z); \quad n_{z'} = n_e \quad (19.17)$$

If a wave propagates in the z direction through such a crystal the retardation is

$$\Delta\phi = \frac{2\pi L}{\lambda_0} (n_{x'} - n_{y'}) = \frac{2\pi L n_0^3 r_{63} E_z}{\lambda_0} \quad (19.18)$$

The crystal has become a waveplate whose retardation is linearly proportional to the field. It is common practice to characterize such a crystal by the voltage, V_π , necessary to produce a retardation of π . Since this applied voltage acts across a length L of crystal, $E_z = V_\pi/L$, so from (Eq. (19.18))

$$V_\pi = \frac{\lambda_0}{2n_0^3 r_{63}} \quad (19.19)$$

For a crystal of KDP which has $r_{63} = -10.5 \times 10^{-12} \text{ m/V}$, $n_0 = 1.51$ and for an operating wavelength of 632.8 nm, Eq. (19.19) gives $V_\pi = 8752V$. A crystal of KDP with this longitudinally applied voltage has been transformed into a half-wave plate and will, therefore, transform a linearly polarized input wave into a linearly polarized output beam whose polarization direction has been rotated from its original direction. If the input wave is polarized at 45° to the x' and y' directions, as shown in Fig. (19.2), that is polarized parallel to the original x or y directions of the undisturbed indicatrix, then the half-wave plate rotates the plane of linear polarization by 90° . When the crystal is operated in this way, as the applied voltage increases from 0 to V_π the output makes a transition from linear polarization \rightarrow elliptical polarization with the major axis aligned along the original polarization direction \rightarrow circular polarization \rightarrow elliptical polarization with the minor axis aligned along the original polarization direction \rightarrow 90° rotated linear polarization as shown in Fig. (19.3). We shall see shortly how this modulation of the polarization state can be used to make an electrooptic amplitude modulator.

Longitudinal electrooptic modulators of this sort, in which the field is applied along the light propagation direction, are somewhat inconvenient insofar as the light must pass through the electrodes used to apply the field: To minimize loss the electrodes are usually deposited as a fine metallic grid on the crystal surfaces shown in Fig. (19.2).

Fig. 19.3.

Fig. 19.4.

19.4 Transverse Electrooptic Modulation

It is more convenient to apply the electric field transversely between electrodes placed on side faces of the crystal as shown in Fig. (19.4). As an example of how this is done we can consider the important electrooptic material lithium niobate $LiNbO_3$.*

Its crystal symmetry is 3m so the form of its linear electrooptic tensor is

$$\underline{r} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix} \quad (19.20)$$

* This material is also strongly piezoelectric and is used to fabricate acoustooptic modulators.

Fig. 19.5.

Without an applied electric field the crystal has uniaxial symmetry with an indicatrix of the form

$$\frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_e^2} = 1 \quad (19.21)$$

with a field applied in the y principal axis direction the indicatrix becomes

$$\left(\frac{1}{n_0^2} - r_{22}E_y\right)x^2 + \left(\frac{1}{n_0^2} + r_{22}E_y\right)y^2 + \frac{z^2}{n_e^2} + 2r_{51}E_yyz = 1 \quad (19.22)$$

Clearly, because of the introduction of the cross term in yz the original principal axes are no longer appropriate. The effect of the field has been to render the crystal *biaxial* and rotate the indicatrix in the yz plane. Let us assume that the indicatrix has been rotated so that its new principal axes y', z' are at an angle θ to the original principal axes, as shown in Fig. (19.5). The x direction remains a principal axis with the field E_y applied, as no cross term involving x has been introduced into the equation of the indicatrix.

We can write the old coordinates in terms of the new as

$$\begin{aligned} x &= x' \\ y &= y' \cos \theta - z' \sin \theta \\ z &= z' \cos \theta + y' \sin \theta. \end{aligned} \quad (19.23)$$

If we substitute from Eq. (19.23) into Eq. (19.22) the condition that x', y', z' are the new principal axes is that the cross term in z', y' must vanish, that is

$$\left(\frac{1}{n_e^2} - \frac{1}{n_0^2} - r_{22}E_y\right) \sin \theta \cos \theta + r_{51}E_y(2 \cos^2 \theta - 1) = 0 \quad (19.24)$$

We can assume that $r_{51}E_y$ is small enough that θ is a small angle and

write $\sin \theta \simeq \theta$, $\cos \theta \simeq 1$. Eq. (19.24) then gives

$$\theta = \frac{-r_{51}E_y}{\left(\frac{1}{n_e^2} - \frac{1}{n_0^2} - r_{22}E_y\right)} \quad (19.25)$$

with $r_{51} = 28 \times 10^{-12} \text{m/V}$, $r_{22} = 3.4 \times 10^{-12} \text{m/V}$, $n_e = 2.21$, $n_0 = 2.3$ and a voltage of 1kV applied across a 1mm thick crystal.

$$\theta = \frac{-28 \times 10^{-12} \times 10^6}{\left(\frac{1}{2.21}\right)^2 - \left(\frac{1}{2.3}\right)^2 - 3.4 \times 10^{-12} \times 10^6} = 1.78 \text{ mrad} = 0.1^\circ \quad (19.26)$$

Indeed, θ is a small angle. Since in LiNbO_3 $n_0 > n_e$, θ is a negative angle so in reality the rotation of the axes is clockwise, not counter clockwise as shown in Fig. (19.5). With respect to the new principal axes the equation of the indicatrix is now

$$\begin{aligned} \left(\frac{1}{n_0^2} - r_{22}E_y\right)x'^2 + \left[\left(\frac{1}{n_0^2} + r_{22}E_y\right)\cos^2\theta + \frac{\sin^2\theta}{n_e^2} + r_{51}E_y \sin 2\theta\right]y'^2 \\ + \left[\left(\frac{1}{n_0^2} + r_{22}E_y\right)\sin^2\theta + \frac{\cos^2\theta}{n_e^2} - r_{51}E_y \sin 2\theta\right]z'^2 = 1 \end{aligned} \quad (19.27)$$

and with $\cos \theta \simeq 1$, $\sin \theta \simeq \theta$

$$\begin{aligned} \left(\frac{1}{n_0^2} - r_{22}E_y\right)x'^2 + \left(\frac{1}{n_0^2} + r_{22}E_y + 2r_{51}E_y\theta + \frac{\theta^2}{n_e^2}\right)y'^2 \\ + \left(\left(\frac{1}{n_0^2} + r_{22}E_y\right)\theta^2 - 2r_{51}E_y\theta + \frac{1}{n_e^2}\right)z'^2 = 1 \end{aligned} \quad (19.28)$$

the new principal refractive indices satisfy the equations

$$\begin{aligned} \frac{1}{n_{x'}^2} &= \frac{1}{n_0^2} - r_{22}E_y \\ \frac{1}{n_{y'}^2} &= \frac{1}{n_0^2} + r_{22}E_y + 2r_{51}E_y\theta + \frac{\theta^2}{n_e^2} \\ \frac{1}{n_{z'}^2} &= \left(\frac{1}{n_0^2} + r_{22}E_y\right)\theta^2 - 2r_{51}E_y\theta + \frac{1}{n_e^2} \end{aligned} \quad (19.29)$$

Since θ is very small we can actually neglect the rotation of the indicatrix and write its equation as

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad (19.30)$$

where

$$\begin{aligned} n_x &= \frac{n_0}{(1 - n_0^2 r_{22} E_y)^{1/2}} = n_0 \left(1 + \frac{1}{2} n_0^2 r_{22} E_y\right) \\ n_y &= \frac{n_0}{(1 + n_0^2 r_{22} E_y)^{1/2}} = n_0 \left(1 - \frac{1}{2} n_0^2 r_{22} E_y\right) \\ n_z &= n_e \end{aligned} \quad (19.31)$$

For a wave travelling in the z direction the indices of refraction experienced by wave polarized along x and y are n_x and n_y respectively. For the transversely operated electrooptic modulator shown in Fig. (19.4) the retardation is

$$\Delta\phi = \frac{2\pi L}{\lambda_0} (n_x - n_y) = \frac{4\pi L n_0^3 r_{22} V}{\lambda_0 d} \quad (19.32)$$

The half-wave voltage V_π of the modulator is the voltage required to make it act as a half-wave plate.

From Eq. (19.32), if $\phi = \pi$

$$V_\pi = \frac{\lambda_0 d}{4L n_0^3 r_{22}} \quad (19.33)$$

For example: with $d = 5\text{mm}$, $L = 10\text{mm}$, $n_0 = 2.3$, $r_{22} = 3.4 \times 10^{-12}\text{m/V}$ at an operating wavelength of 530nm - $V_\pi = 1600\text{V}$.

To illustrate further how a change in the orientation of the applied field and the light propagation direction affects the way in which an electrooptic crystal behaves, let us now consider what happens when light propagates through a LiNbO_3 crystal in the y direction, but with the field applied along the z direction.

In this case the indicatrix becomes

$$\left(\frac{1}{n_0^2} + r_{13} E_z\right)x^2 + \left(\frac{1}{n_0^2} + r_{13} E_z\right)y^2 + \left(\frac{1}{n_e^2} + r_{33} E_z\right)z^2 = 1 \quad (19.34)$$

The application of the field has left the crystal uniaxial, but the new ordinary and extraordinary refractive indices satisfy

$$\begin{aligned} \frac{1}{n_0'^2} &= \left(\frac{1}{n_0^2} + r_{13} E_z\right) \\ \frac{1}{n_e'^2} &= \left(\frac{1}{n_e^2} + r_{33} E_z\right) \end{aligned} \quad (19.35)$$

Since $r_{13} E_z \ll \frac{1}{n_0^2}$ and $r_{33} E_z \ll \frac{1}{n_e^2}$ for any reasonable applied field strengths,

$$\begin{aligned} n_0' &= \frac{n_0}{(1 + r_{13} n_0^2 E_z)^{1/2}} = n_0 \left(1 - \frac{1}{2} r_{13} n_0^2 E_z\right) \\ n_e' &= \frac{n_e}{(1 + r_{33} n_e^2 E_z)^{1/2}} = n_e \left(1 - \frac{1}{2} r_{33} n_e^2 E_z\right) \end{aligned} \quad (19.36)$$

If a wave passes through such a crystal in the y direction it will, in

Fig. 19.6.

general, split into an O-wave polarized in the x direction, and an E-wave polarized along the z direction. For the geometry shown in Fig. (19.6) the retardation is

$$\Delta\phi = \frac{2\pi L}{\lambda_0}(n'_e - n_0) \quad (19.37)$$

which gives

$$\Delta\phi = \frac{2\pi L}{\lambda_0}(n_e - n_0 + \frac{1}{2}(r_{33}n_e^3 - r_{13}n_0^3)E_z) \quad (19.38)$$

The half wave voltage of the crystal used in this way is

$$V_\pi = \frac{\lambda_0 d}{L(r_{33}n_e^3 - r_{13}n_0^3)} \quad (19.39)$$

With $d = 5\text{mm}$, $L = 10\text{mm}$, $r_{33}n_e^3 - r_{13}n_0^3 = 224 \times 10^{-12}\text{m/V}$, and our operating wavelength of 530nm ; $V_\pi = 1183\text{V}$. Although this is a slightly lower half wave voltage than in the equation configuration discussed previously, there is a potential drawback to using the crystal in this way. Eq. (19.38) shows that the crystal operated in this orientation acts as a waveplate – because the wave is not propagating along the optic axis. The field-independent birefringent retardation $2\pi L(n_e - n_0)/\lambda_0$ will be temperature sensitive and can lead to practical stability problems. However, in practice this difficulty can be overcome by building an electrooptic device using pairs of crystals in series as shown in Fig. (19.7). The orientation of the x and z axes in the 2 crystals are arranged to be orthogonal so that the constant term in Eq. (19.38) cancels out. In order that the field dependent retardation in Eq. (19.38) does not also cancel out the field polarity in the second crystal must be reversed.

Fig. 19.7.

19.5 High Frequency Waveguide Electrooptic Modulators

In a high frequency electrooptic modulator the electrical drive to the modulator must be applied using some form of stripline configuration so that the induced phase or polarization modulation propagates at, or close to, the phase velocity of the light signal travelling through the modulator. If this is not done, the light propagating through the crystal sees an electric field of varying phase as it propagates and the effects of the field on the phase or polarization state of the wave can cancel out.

Fig. (19.) shows such a travelling wave electrooptic modulator (Suzanne Hiser, Fig. (1.2)). The electric field acts on the waveguide region of the electrooptic crystal as shown in Fig. (19.) (Suzanne Hiser, Fig. (2.4)). In LiNbO₃, for example, the waveguiding region is made by diffusing titanium into the crystal. The buffer layer is usually Al₂O₃ or SiO₂, and prevents the propagating wave, in this geometry a TM wave, from interacting with the metal electrodes. The undiffused hammer is typically 7μ wide, the buffer layer 0.26μm thick, with a 3μm thick Cr/Au electrode on the top. The electrode thickness is chosen to be larger than the skin depth at the design frequency. The mode difference between guiding region and substrate is typically 0.01-0.02.

In the Z-cut, TM mode of operation, the electric field of the mode has E_z , E_x components. The E_z component of the wave sees the change in index $\Delta(\frac{1}{n^2})_3 = r_{33}E_z$, this is the only change in the radiation.

For LiNbO₃

$${}^=r = \begin{pmatrix} 0 & -3.4 & 8.6 \\ 0 & 3.4 & 8.6 \\ 0 & 0 & 30.8 \\ 0 & 28 & 0 \\ 28 & 0 & 0 \\ -3.4 & 0 & 0 \end{pmatrix} \times 10^{-12} m/v$$

typical optical losses are $\sim |dB/cm$ at 633 nm and $\sim 0.3dB/cm$ at $\lambda = 1.32\mu m$

$$\Delta(1/n^2) = {}^=r \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

For the Z - cut the applied field is in the E_z direction.

$$\Delta\left(\frac{1}{n^2}\right)_3 = r_{33}E_z$$

For the X cut, the field is also applied in the z direction, and the wave propagates in the y direction E_z is the only electric field component of the mode, which is a TE mode—it has $|E|_x$ and H_y magnetic field components. With the X cut $\Delta\left(\frac{1}{n^2}\right)_3 = r_{33}E_z$ is the only change in the . Various kinds of waveguide structure can be fabricated in LiNbO₃, as shown in Fig. (19.) (Suzanne Hiser, Figs. 2.1 and 2.2). In the Mach-Zehnder interferometer configuration the waves propagating in the two arms can experience different phase shifts so can emerge in a cut of phase. The device can be used as an amplitude modulator or switch. If the phase shift between the 2 arms is ϕ the output intensity I_{out} will be added to the output intensity I_{in} according to

$$\frac{I_{out}}{I_{in}} = \frac{1}{2} (1 + \cos \phi) = \cos^2 \phi/2$$

Variation of ϕ in such a device will not produce a linear modulator. However, if the device is biased by $\pi/2$, for example by having one interferometer arm slightly longer than the other (as shown in Fig. (19.)) (Suzanne Hiser, Fig. (2.2)) then we can write $\phi = \pi/2 + \Delta\phi$. In this case

$$\begin{aligned} \frac{I_{out}}{I_{in}} &= \frac{1}{2}(1 + \cos(\pi/2 + \Delta\phi)) \\ &= \frac{1}{2}(1 + \cos \pi/2 \cos \Delta\phi - \sin \pi/2 \sin \Delta\phi) \\ &= \frac{1}{2} - \frac{1}{2}\Delta\phi \end{aligned}$$

for small phase modulation $\Delta\phi$.

This provides linear modulation of the intensity about a near value $I_{\text{out}} = \frac{1}{2}I_{\text{in}}$. To achieve passive biasing of LiNbO₃ the path difference is $\Delta L = 152$ nm, this provides $\phi = \pi/2$. Typical devices are 10-20 mm long with the guiding region 20 μm apart.

Because it is not possible to match the velocity of the excitation field and optical field over a long distance at high modulation fields, a phase-reversed scheme can be used to extend the bandwidth. After each length of the device over which the optical and drive fields get out of phase by π the polarity of the drive field is reversed as shown in Fig. (19.) (SH, Fig. (2.7)). Such a device will in theory have a pass band response. Broad frequency response can be obtained by aperiodically varying the length between phase reversals of the drive field, as shown in Fig. (19.) (SH, Fig. (2.11)).

19.6 Straight Electrode Modulator

In a electrooptic modulator of the straight type high frequency response is limited by traveling wave mismatch effects. We can represent the drive voltage at position z along this structure as

$$V(z, t) = V_0 \sin(\omega_m t - \frac{\omega_m}{c_0} n_m z)$$

where c_0 is the velocity of light in vacuo, ω_m is the drive frequency, and n_m is the effective refractive index for the drive field (such that the phase velocity of the drive field is c_0/n_m).

The optical field propagates with phase velocity,

$$c = \frac{c_0}{n}$$

where n is the effective refractive index for the waveguide mode.

If the optical field enters the crystal at $t = 0$ at time t_0 then the voltage that is seen as it propagates can be formed from Eq. (19.) by writing $t = z_n/c_0$ which is the time it takes the optical wave to reach location z

$$V(z) = V_0 \sin(\frac{\omega_m z}{c_0} (n - n_m))$$

The electrooptically induced change in the index n can be written as

$$\Delta n(z) = aV(z)$$

where a is a constant that depends on the specific geometry and crystal being used.

The total phase shift experienced in propagating a length ℓ through the

crystal is

$$\begin{aligned}\Delta\phi &= \int_0^\ell \frac{\omega \Delta n(z)}{c_0} dz \\ &= \frac{aV_0\omega}{c_0} \int_0^\ell \sin\left(\frac{\omega_m z}{c_0}(n - nm)\right) dz\end{aligned}$$

which gives

$$\begin{aligned}\Delta\phi &= -\frac{aV_0\omega}{c_0} \cos\left[\frac{\omega_m z}{c_0}(n - nm)\right] \frac{c_0}{\omega_m(n - nm)} \Big|_0^\ell \\ &= -\frac{aV_0}{(n - nm)} \left[\cos \frac{\omega_m \ell}{c_0}(n - nm) - 1 \right]\end{aligned}$$

By making the substitution

$$\theta = \pi\omega_m/\omega_c$$

where

$$\omega_c = \frac{\pi c_0}{(n_m - n)\ell}$$

the total phase shift can be written as

$$\begin{aligned}\Delta\phi &= \frac{aV_0\ell}{\pi c_0} \omega_c [\cos \theta - 1] \\ &= \frac{aV_0\ell\omega_m}{2c_0(\theta/2)} [\cos^2 \theta/2 - \sin^2 \theta/2 - 1] \\ &= \frac{aV_0\ell\omega_m}{2c_0(\theta/2)} [-2 \sin^2 \theta/2] \\ &= -\frac{aV_0\omega_m\ell}{c_0} \left[\frac{\sin(\theta/2)}{\theta/2} \right] \sin \theta/2 \\ \Delta\phi &= -\frac{aV_0\omega_m^2\pi\ell}{2\omega_c c_0 (\pi\omega_m/2\omega_c)} \frac{\sin^2 \theta/2}{\theta/2} \\ &= -\frac{a\pi V_0\omega_m^2\ell}{2c_0\omega_c} \sin^2 c(\theta/2)\end{aligned}$$

where*

$$\Delta\phi = -\frac{a\pi V_0\omega_m^2\ell^2}{2c_0\pi c_0} (n_m - n) \sin^2 c(\theta/2)$$

The optimum length for the modulator satisfies

$$\frac{\omega_m \ell (n - n_m)}{c_0} = \pi, \text{ or } \omega_m = \omega_c, \theta = \pi$$

For LiNbO_3 $n_m = 4.2$, $n = 2.146$, so for example, with $\ell = 10$ mm $n - n_m = 2.05$, the cutoff frequency is 7.36 Hz.

* $\sin c(\theta/2) = \sin(\theta/2)/\sin \theta/2$.