CHAPTER EIGHT

Optically Pumped Solid-State Lasers

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Optically Pumped Solid-State Lasers

8.1 Introduction

In this chapter we shall discuss in some detail the operating principles, characteristics, and design features of solid-state lasers in which the laser medium is an insulating or glassy solid. In many of these lasers the active particles are impurity ions doped into a host matrix. These lasers are pumped optically, generally with a pulsed or continuous lamp, although they can also be pumped by another laser. Our discussion will build on the brief introduction to one of this class of laser, the ruby laser, given in Chapter 3. The chapter will conclude with a discussion of the characteristics of the radiation emitted by such lasers and how this radiation can be modified and controlled in time.

8.2 Optical Pumping in Three- and Four-Level Lasers

The optical pumping process in an insulating solid-state laser can be illustrated schematically with reference to Fig. (8.1). Light from the pumping lamp(s) excites ground state particles into an absorption band, labelled 3 in the figure. Ideally, particles that reach this state should transfer rapidly into the upper laser level, level 2. If this transfer process is preferentially to level 2 rather than to level 1 a population inversion will result between levels 2 and 1 and laser action can be obtained. The drain transition from level 1 back to the ground state should be fast,

Fig. 8.1.

to keep level 1 from becoming a "bottleneck." The performance of the laser will be influenced by several factors.

8.2.1 Effective Lifetime of the Levels Involved

The length of time a particle can remain in an excited level is governed by its *effective* lifetime. This lifetime is influenced by both radiative and nonradiative processes. The transfer transition from level 3 to level 2 in Fig. (8.1) is generally nonradiative. Particles make the transition by dumping their excess energy into the lattice – they literally heat up the medium. Such a nonradiative process from level i to level j can be described by a rate coefficient X_{ij}, similar to a spontaneous emission coefficient A_{ij}. The overall rate at which particles leave level i is

$$\frac{dN_i}{dt} = -\sum_j N_i A_{ij} - \sum_j N_i X_{ij}, \tag{8.1}$$

where the summation in j runs over all levels below level i in energy. The effective lifetime of level i is

$$\tau_i = 1 / \sum_j (A_{ij} + X_{ij}). \tag{8.2}$$

The nonradiative decay of an excited state can be thought of as a collisional process, in a similar way to stimulated emission, in which a quantized packet of acoustic energy within the solid, called a *phonon*, collides with the excited particle. These quantized acoustic energy packets always exist in a solid, even at absolute zero: they correspond closely to the waves we considered in our discussion of black-body radiation, except that the modes being counted are now vibrational motions of the particles constituting the solid. The perturbation of excited states by these phonons leads to line broadening, which can be substantial. Broad absorption bands in a solid result from the line-broadened "smearing" together of levels that would be sharp and distinct in the vapor phase. The vastly increased numbers of neighboring particles in a solid (or liquid) compared to a gas causes large amounts of line broadening, although the extent of the broadening can vary greatly from level to level.

8.2.2 Threshold Inversion in Three- and Four-Level Lasers

If the lower laser level is very close to ($E_1 \ll kT$), or is, the ground state, the system is a three-level laser. In such a laser the threshold inversion is substantially higher than in a four-level system ($E_1 \gg kT$). If the total number of particles per unit volume participating in the pumping and lasing process is N we can write

$$N = N_3 + N_2 + N_1 + N_{gs}, \tag{8.3}$$

where the subscripts indicate the level involved, and N_{gs} is the population density in the ground state. Eq. (8.3) assumes that other levels of the system are not actively involved and that their populations are negligible.

In a three-level laser level 1 is essentially the ground state so we can write

$$N = N_3 + N_2 + N_1. \tag{8.4}$$

In a good solid-state laser, level 3 transfers its excitation very rapidly to level 2 so its population can be neglected. In this case

$$N = N_2 + N_1. \tag{8.5}$$

The threshold inversion is

$$N_t = N_2 - \frac{g_2}{g_1} N_1. \tag{8.6}$$

From Eqs. (8.5) and (8.6)

$$(N_2)_{3-level} = \frac{(g_2/g_1)N + N_t}{[(g_2/g_1 + 1)]}, \tag{8.7}$$

which if $g_2 = g_1$ gives $N_2 \simeq N/2$.

In a good four-level laser in which the lower laser level remains relatively depopulated we can write

$$(N_2)_{4-level} \simeq N_t.$$

Consequently, the relative rate at which level 2 must be excited to produce an inversion (all other factors such as decay rates being comparable) is

$$\frac{(N_2)_{3-level}}{(N_2)_{4-level}} \simeq \frac{(g_2/g_1)N + N_t}{[(g_2/g_1) + 1]\, N_t}. \tag{8.8}$$

Usually, $N \gg N_t$, and if we take the simple case $g_2 = g_1$, the ratio of pumping rates becomes $N/2N_t$. This number can easily be 10^4 or more, demonstrating once again how surprising it was that a three-level laser was the first ever to be operated.

8.2.3 Quantum Efficiency

If the average energy of photons from the pumping lamp is $h\overline{\nu}_{pump}$ and the laser photon is $h\nu$ the intrinsic quantum efficiency of the pumping process is

$$\eta_i = \frac{h\nu}{h\overline{\nu}_{pump}}. \tag{8.9}$$

8.2.4 Pumping Power

The rate at which particles must be excited to the upper laser level to sustain a population inversion is

$$R_2 = N_2/\tau_2 (\text{particles m}^{-3}\text{s}^{-1}). \tag{8.10}$$

If the average probability factor for a particle in level 3 transferring to level 2 rather than elsewhere is $\overline{\eta}$ (called the branching factor) then the rate at which level 3 must be pumped is

$$R_3 = \frac{N_2}{\overline{\eta}\tau_2}. \tag{8.11}$$

The corresponding absorbed pump power is

$$P_A = \frac{N_2 h\overline{\nu}_{pump}}{\overline{\eta}\tau_2}. \tag{8.12}$$

8.2.5 Threshold Lamp Power

To determine the power and spectral characteristics of the lamp(s) needed to create an inversion, we must relate the threshold pumping rate R_2 to the absorption coefficient in the pump band, and electrical and geometrical factors that determine how efficiently the lamp generates pump light and couples this into the laser medium.

The absorption band has a lineshape function $g_3(\nu)$. If the energy density of the pump radiation in the laser medium is $\rho_P(\nu)$ then the rate at which level 3 is excited is

$$R_3 = \int N_0 B_{03} g_3(\nu) \rho_P(\nu) d\nu. \tag{8.13}$$

If we assume plane wave illumination of the laser medium then we can write $\rho(\nu) = I(\nu)/c$ and Eq. (8.13) becomes

$$R_3 = \int N_0 \frac{c^2 A_{30}}{8\pi\nu^2} \frac{I(\nu)}{h\nu} g_3(\nu) d\nu. \tag{8.14}$$

From Eq.(8.14) we can recognize $N_0 c^2 A_{30} g_3(\nu)/8\pi\nu^2$ as the absorption (negative gain) coefficient of the laser medium $\alpha(\nu)$. So

$$R_3 = \int \frac{I(\nu)\alpha(\nu)d\nu}{h\nu}. \tag{8.15}$$

The integration covers the range of frequencies encompassed by the absorption band.

If we allow for the possibility of a frequency-dependent branching factor, the rate of excitation of level 2 is

$$R_2 = \int \frac{I(\nu)\alpha(\nu)\eta(\nu)}{h\nu} d\nu. \tag{8.16}$$

If the absorption band over which the integration is carried out is narrow, of width $\Delta\nu_3$, we can replace the quantities in (8.16) by averages and write

$$R_2 = \overline{I}(\nu)\overline{\alpha}(\nu)\overline{\eta(\nu)}\Delta\nu_3/h\overline{\nu}_{pump}. \tag{8.17}$$

8.3 Pulsed Versus CW Operation

If the pumping lamp is a flashlamp of duration $t_p \ll \tau_2$, then at the end of the flash, all the excited particles will still be in level 2. In this case

$$N_2 = t_p\overline{I(\nu)} \;\; \overline{\alpha(\nu)} \;\; \overline{\eta(\nu)}\Delta\nu_3/h\overline{\nu}_{pump}. \tag{8.18}$$

The flashlamp energy needed to achieve $N_2 \geq N_t$ depends additionally on the following factors: The electrical efficiency e of the lamp defined as

$$e = \frac{\text{joules of light energy out}}{\text{capacitor joules in}}. \tag{8.19}$$

(For xenon flashlamps this efficiency factor can be as high as 80%).[8.1] The fraction f of this light in the spectral region that will pump the absorption band is

$$f = \frac{\text{light energy within absorption band}}{\text{total light energy}}, \tag{8.20}$$

and the geometrical efficiency g with which the light is coupled to the laser medium, which usually takes the form of a cylindrical rod or flat slab

$$g = \frac{\text{light energy within absorption band reaching laser medium}}{\text{total light energy within absorption band}}. \tag{8.21}$$

The total energy that must reach the laser crystal, in the right spectral region, to reach threshold can be written as

$$U_t = t_p\overline{I(\nu)} = \frac{N_t h\overline{\nu}_{pump}}{\overline{\alpha}(\nu)\overline{\eta(\nu)}}. \tag{8.22}$$

8.3.1 Threshold for Pulsed Operation of a Ruby Laser

In the ruby laser the typical Cr^{3+} ion concentration is $\sim$ 0.05% by weight, equivalent to about 10^{25} Cr^{3+} m^{-3}. Since this is a three-level laser, roughly half these ions must be excited to the upper laser level to reach threshold. Therefore, we can take $N_t = 5 \times 10^{24}$ m^{-3}. Fig.(3.3) shows that the approximate average absorption coefficient of ruby in the 350–600 nm region is about 100 m^{-1}. Assuming efficient transfer from the pump bands we take $\overline{\eta}(\sim) = 1$. The average pump wavelength is $\simeq$ 475 nm. Therefore,

$$U_t \simeq \frac{5 \times 10^{24} \times 6.66 \times 10^{-34} \times 3 \times 10^8}{100 \times 475 \times 10^{-9}} = 21 \text{ kJ m}^{-2}.$$

For a cylindrical ruby crystal 20 mm long $\times$ 10 mm diameter, if the lamp energy reaches the crystal uniformly over its curved surface the threshold input energy reaching the crystal in the appropriate spectral band is about 13 J. For a modern flashlamp the electrical efficiency e can be 50% or more. The spectral output of the lamp will approximate a black body with superimposed spectral features, as shown in Fig. (8.2). It is reasonable to assume that 25% of the emitted light is in the ruby pump bands. If the optical arrangement of lamp(s) and crystal is efficient the geometrical factor g should be 50% or better. The electrical input to reach threshold will therefore be about 200 J. Small ruby crystals in an efficient pumping arrangement such the axisymmetric ellipsoidal cavity shown in Fig. (8.3) can have thresholds as low as 34 J.

To achieve CW operation in a solid-state insulating laser the pumping rate of the upper laser level must be sufficient to maintain $N_2 = N_t$ in the face of all the spontaneous relaxation processes. The absorbed power per unit volume must be

$$U_t = \frac{N_t h\overline{\nu}_{pump}}{\tau_2\overline{\eta(\nu)}}. \tag{8.23}$$

8.3.2 Threshold for CW Operation of a Ruby Laser

Although the ruby laser is not commonly operated in a CW mode it is interesting to calculate its threshold pump rate. We use the same values for the parameters in Eq. (8.23) as before, with the addition of the value

Fig. (8.2).

for τ_2, 3 ms, to get

$$U_t = \frac{5 \times 10^{24} \times 6.626 \times 10^{-34} \times 3 \times 10^{8}}{3 \times 10^{-3} \times 475 \times 10^{-9}} \simeq 7 \times 10^{8} \text{ W m}^{-3}.$$

In the first CW ruby laser experiments, Nelson and Boyle[8.3] obtained threshold pump rates of this order. They had to use a 850 W mercury xenon lamp focused onto a ruby crystal 11.5 mm long by 0.61 mm in diameter kept at liquid nitrogen temperature inside a dewar to achieve 300 mW of power actually absorbed in the crystal.

8.4 Threshold Population Inversion and Stimulated Emission Cross-Section

The small-signal gain at the center of the line can be written in the form

$$\gamma_0 = \left(N_2 - \frac{g_2}{g_1} N_1\right) \sigma_0, \tag{8.24}$$

where σ_0 is called the stimulated emission cross-section. It serves as a useful parameter for comparing different laser media. From Eq. (2.30) it is clear that for a homogeneously broadened line, for which

$$g(\nu_0, \nu_0) = \frac{2}{\pi \Delta\nu}, \tag{8.25}$$

$$\sigma_0 = \frac{c^2 A_{21}}{4\pi^2 \nu^2 \Delta\nu}. \tag{8.26}$$

For the ruby laser $\sigma_0 \simeq 1.2 \times 10^{-24}$ m^2.

To achieve threshold the minimum inversion, N_t is

$$N_t = \left(N_2 - \frac{g_2}{g_1} N_1\right)_t = \frac{1}{\sigma_0}\left(\alpha - \frac{1}{\ell} \ell\text{n} r_1 r_2\right). \tag{8.27}$$

Therefore, the stimulated emission cross-section together with parameters of the laser cavity allow quick estimation of the threshold inversion. We shall meet some examples of this later.

8.5 Paramagnetic Ion Solid-State Lasers

A large number of paramagnetic ions from the iron, rare-earth (lanthanide) and actinide groups of the periodic table exhibit laser action when doped into a large number of host crystals or glasses. In a separate class of solid-state lasers – the so-called *stoichiometric* lasers the active ion is not an impurity dopant but is an intrinsic part of the lattice. These lasers have not to date achieved significant importance and we will not discuss them further. The large diversity of doped crystalline lasers is demonstrated by Table (8.1), which lists some of the most important such lasers, together with some of their relevant operating parameters. These are all optically pumped lasers; most are four-level systems. By far the most important is the Nd^{3+} laser, which is operated successfully in a variety of host materials, for example in yttrium aluminum garnet ($Y_3Al_5O_{12}$,YAG), calcium tungstate ($CaWO_4$), lithium yttrium fluoride ($LiYF_4$,YLF), $Ca_5(PO_4)_3F$(FAP), $YAlO_3$(YALO), gadolinium gallium garnet (GGG), scandium-substituted GGG (GSGG), and in various glasses. Another important laser of this type is the holmium (Ho^{3+}) laser in $LiYF_4$ (YLF), frequently with erbium ions (Er^{3+}) or thulium ions (Tm^{3+}) added to the host to assist in the absorption of flashlamp energy and transfer into the Ho^{3+} upper laser level.

The chromium (Cr^{3+}) in $BeAl_2O_4$ (alexandrite) laser, is interesting as its laser wavelength can be tuned continuously from 700 to 800 nm.

The Ti^{3+} in Al_2O_3, the titanium–sapphire laser, is a very attractive source for the generation of tunable near-infrared radiation and has largely replaced dye lasers in this application. Fig. (8.4) shows the absorption and emission spectroscopic properties of the Ti^{3+}:Al_2O_3 material. When pumped by a high power argon ion laser, a power output in excess of 1 W can be obtained[8.4].

Although specific details of their operation differ from one doped insulating crystal or glass laser to other, and many of the general features of their construction and operation are similar. The Nd:YAG laser serves as a benchmark for this discussion.

Fig. (8.3).

8.6 The Nd:YAG Laser

YAG has a combination of desirable properties as a host medium for Nd^{3+} ions: it has relatively high thermal conductivity, which allows it to disperse the waste heat from the optical pumping process; it has high mechanical strength, and can be grown as crystals of large size with good optical quality. The Nd^{3+} ions substitute within the YAG lattice in a single site so the emission and absorption lines are homogeneously broadened. Typical Nd^{3+} doping densities range up to 1%.

An energy level diagram for the Nd^{3+} ions in YAG is shown in Fig. (8.5). The primary pumping process is absorption of lamp energy from the ground state ${}^4I_{9/2}$ into the ${}^4F_{5/2}$ level followed by transfer to the upper laser level ${}^4F_{3/2}$. This consists of a number of closely spaced levels. Two of these, labelled R_2 and R_1 in Fig. (8.5), serve as the upper levels of the closely spaced transitions ℓ_2 and ℓ_1. The stronger of these, ℓ_2, has $\lambda_2 = 1.06415$ μm, the other, ℓ_1, has $\lambda_1 = 1.0646$ μm. Each of these transitions is substantially homogeneously broadened, with $\Delta\nu \sim 2 \times 10^{11}$ Hz, and because they are so close in frequency, $\sim 10^{11}$ Hz apart, they form a single asymmetric lineshape. This lineshape, shown in Fig. (8.6) contributes to the effective overall gain of the laser, which sees its peak gain near 1.06415 μm. It is interesting in a case like this to compute the effective spontaneous emission coefficient describing the transition.

8.6.1 Effective Spontaneous Emission Coefficient

At ambient temperature the populations of the two levels R_2 and R_1 are in thermal equilibrium. The ratio of their populations is the ratio of

Fig. (8.4).

Fig. (8.5).

Fig. (8.6).

their degeneracy factors.

$$\frac{N_{R_2}}{N_{R_1}} = \frac{g_{R_2}}{g_{R_1}} = \frac{2}{3}.$$

The effective spontaneous emission coefficient A_{21} is related to the coefficients for ℓ_2 and ℓ_1 separately according to

$$A_{21} = A_{\ell_1}\frac{N_{R_1}}{N_{R_2}} + A_{\ell_2}. \tag{8.28}$$

For values $A_{\ell_2} = 1440$ s^{-1}, $A_{\ell_1} \simeq 250$ s^{-1}; $A_{21} = 1815$ s^{-1}. With these parameters the stimulated emission cross-section for the Nd:YAG laser is $\simeq 9 \times 10^{-23}$ m^2.

8.6.2 Example – Threshold Pump Energy of a Pulsed Nd:YAG Laser

We take for an example a crystal 50 mm long by 5 mm in diameter. One end of the crystal has a reflectance of 100%, the other a reflectance of 92%. For a good quality crystal $\alpha \simeq 0$. The threshold gain is

$$\gamma_t = \alpha - \frac{1}{\ell}\ell n r_1 r_2 = 0.833 \text{ m}^{-1}.$$

Therefore, from Eq. (8.27) the threshold inversion is $N_t \simeq 9.3 \times 10^{21}$ m^{-3} –

more than 500 times smaller than for the ruby laser. The energy that must be absorbed to reach threshold is

$$U_t = \frac{N_t h\overline{\nu}_{pump}}{\overline{\eta(\nu)}}, \tag{8.29}$$

which with $\overline{\eta(\nu)} \simeq 1$ and a pump wavelength of 810 nm gives $U_t = 2280$ J m^{-3}. For the crystal specified this gives a threshold absorbed energy of 2.24 mJ.

Overall conversion efficiencies from electrical energy to radiant emission in the Nd^{3+} absorption bands may range as high as 15% so the necessary electrical input to reach threshold may be as small as 15 mJ. With these small thresholds it is not surprising that high energy pulsed operation of Nd:YAG lasers is routine. Single transverse mode outputs up to 50 J are readily available. Although 1.06 μm is the commonest wavelength produced by these lasers, a laser transition at 1.3188 μm is also easy to obtain that has the $^4I_{13/2}$ level in Fig. (8.5) as its lower level.

8.7 CW Operation of the Nd:YAG Laser

CW laser operation of Nd:YAG lasers is possible because of their low threshold pump requirements. Outputs of tens of watts are readily obtained in the TEM_{00} mode. To operate the Nd:YAG crystal described

Fig. (8.7).

above in a CW mode requires a steady pumping power (W m^{-3}) of

$$P_t = \frac{N_t h\overline{\nu}_{pump}}{\tau_2 \overline{\eta(\nu)}}, \tag{8.30}$$

where if spontaneous emission is the dominant loss process from the upper laser level $\tau_2 = 1/A_{21}$. With the parameter used above

$$P_t = 4.15 \times 10^6 \text{ J m}^{-3}$$

and for the 50 mm $\times$ 5 mm diameter crystal $P_t = 4.07$ W.

Because CW arc lamps are not as efficient at converting electrical energy to light as are flashlamps, the factors e, f, and g can be taken to be on the order of 10%, 10%, and 50% respectively. The lamp power required to reach threshold would be $\simeq$ 800 W. The overall efficiency for producing laser power is on the order of 1–3%, as can be seen from Fig. (8.7), which shows the actual performance characteristics of some CW Nd:YAG lasers pumped by krypton arc lamps.

8.8 The Nd^{3+} Glass Laser

The construction of doped crystalline lasers of high power or energy output, particularly in configurations where the output of a laser oscillator is amplified by successive crystals, requires the growth of crystals of substantial size. This must be accomplished without the introduction of optical inhomogeneities into the crystal. In practice, most such crystals are grown by using a small seed crystal and pulling a larger crystal from the melt – the so-called Czochralski method, which is illustrated in Fig. (8.8). Such methods become difficult to use if very large crystals are needed. However, the use of glasses to serve as host media, particularly

Table (8.2). *Spectroscopic properties of the ${}^4F_{3/2} \to {}^4I_{11/2}$ transition of Nd^{3+} ions in different glasses at 295 K*[8.8].

Glass	Refractive index n	Cross-section (10^{24} m^2)	Wavelength λ_p (nm)	Line–width Δ_{eff}(nm)	Lifetime τ_R (μs)
Oxides					
Silicates	1.46 to 1.75	0.9 to 3.6	1057 to 1088	34 to 55	170 to 1090
Phosphates	1.49 to 1.63	2.0 to 4.8	1052 to 1057	22 to 35	280 to 530
Borates	1.51 to 1.69	2.1 to 3.2	1054 to 1063	34 to 38	270 to 450
Germanates	1.61 to 1.71	1.7 to 2.5	1060 to 1063	36 to 43	300 to 460
Tellurites	2.0 to 2.1	3.0 to 5.1	1056 to 1063	26 to 31	140 to 240
Halides					
Fluoroberyllates	1.28 to 1.38	1.6 to 4.0	1046 to 1050	19 to 29	460 to 1030
Fluoroaluminates	1.41 to 1.48	2.2 to 2.9	1049 to 1051	30 to 33	420 to 570
Fluorozirconates	1.52 to 1.56	2.9 to 3.0	1049	26 to 27	430 to 450
Chlorides	1.67 to 1.91	6.0 to 6.3	1062 to 1064	19 to 20	180 to 220

for Nd^{3+} ions, circumvents this difficulty as extremely large pieces of doped glass are easily fabricated.

In a glass the environments of the Nd^{3+} ions vary much more than in a crystalline material because of the random structural character of the glass matrix. Therefore, not only is the laser substantially inhomogeneously broadened, but it has a much broader linewidth than, for example, Nd:YAG. The laser transition linewidth is typically on the order of 300 cm^{-1}, $\sim 10^{13}$ Hz, and its actual shape varies from one glass to another, as can be seen from Fig. (8.9) and Table (8.2). The lifetime of the upper laser level also depends on the type of glass and the Nd^{3+} concentration as can be seen from Fig. (8.10). Fig. (8.11) shows a typical energy level diagram for Nd^{3+} ions in glass together with the absorption spectrum of the material.

The world's most powerful lasers are based on this material. Because their linewidths are much larger than in YAG, their inversion threshold is higher; however, once they are pumped sufficiently above threshold they offer comparable (energy-out)/(energy-in) performance to Nd:crystalline lasers. The relatively long upper laser level lifetime $\sim$100 μs, allows them to store larger quantities of energy – up to 0.5 MJ m^3. The best example of their capability is in their use in inertial confinement fusion (ICF) research, which will be discussed in more detail in Chapter 24. In such high energy applications the output of a well controlled Nd:YAG crys-

Fig. (8.8).

Fig. (8.9).

Fig. (8.10).

tal laser oscillator will be amplified up to a million times by a series of laser amplifiers of progressively larger aperture, as shown schematically in Fig. (8.12). These amplifiers utilize large slabs of Nd:glass material placed at Brewster's angle and pumped through their flat faces with

Fig. (8.11).

arrays of linear flashlamps. A large oscillator/amplifier(s) chain of this kind will contain many additional components over and above the oscillator, its mirror feedback system, and the successively larger amplifiers. There will be Faraday isolators (see Chapter 14), which allow light to pass in one direction but not the other, beam expanders, spatial filters for smoothing the transverse spatial profile of the laser beam (see Chapter 16), electro-optic switches (see Chapter 19), and nonlinear crystals for harmonic generation (see Chapter 21).

The large Nova laser at the Lawrence Livermore National Laboratory delivers a final amplified pulse energy of approximately 100 kJ at 1.06 μm in 1 ns – corresponding to a peak power of 100 TW (terawatts). By the use of nonlinear techniques a substantial part (25 kJ) of this laser energy can be converted to its third harmonic (0.35 μm) for irradiation of nuclear fusion targets. In such multi-megajoule laser systems the advantages of Nd:glass becomes apparent: it has a high damage threshold so can amplify high intensity pulses; and because of its large linewidth its gain coefficient is smaller than for Nd:YAG, which helps to eliminate the occurrence of spontaneous parasitic oscillations in amplifier sections.

Amplification of very short optical pulses can be achieved with a glass laser amplifier, because of the broad linewidth $\sim 10^{13}$ Hz, the amplification of pulses $\sim$ 1 ps in length is feasible. In mode-locked operation the broad linewidth of the Nd:glass laser allows it to generate pulses in the 2–20 ps range.

8.9 Geometrical Arrangements for Optical Pumping

In Chapter 3 we have already briefly discussed the optical pumping arrangement in which the laser crystal was pumped by a helical lamp wrapped around the crystal, and the elliptical reflector scheme in which flashlamp and crystal lie along the two focal lines of an elliptical cross-section cylindrical reflector. Other optical pumping arrangements are also commonly used.

In the elliptical reflector arrangement, all the light from the lamp (along one focal line) passes through the crystal (along the other focal line). In this arrangement linear lamp and laser rod are quite separate, which makes access and replacement of either a simple matter. High purity, deionized water coolant can be circulated directly inside the reflector to keep both lamp and laser rod cool. This arrangement is very efficient and convenient but the illumination it provides is not axisymmetric. The side of the laser crystal nearest to the flashlamp receives its illumination directly, and the greater intensity of this direct illumination produces a population inversion in the system which is greater on the side of the crystal nearest to the lamp. The resultant laser beam has an asymmetric intensity distribution that is skewed towards the side of the laser rod facing the lamp.

The close-coupling optical pumping arrangements shown in Fig. (8.13) are simple and convenient to manufacture, and provide comparable efficiency to elliptical cavities, although again without axial symmetry (unless very many close coupled lamps are used). However, the close proximity of flashlamp and laser can lead to thermal dissipation problems. If the laser crystal is heated by the pumping lamp its oscillation threshold will rise. This can occur for several reasons: because of an increase in the linewidth of the laser transition, thermal population of the lower laser level, or particularly, the production of refractive index inhomogeneities in the laser crystal. In very high power solid-state laser systems, especially Nd^{3+}/glass, the laser medium is frequently fabricated as a series of flat slabs placed at Brewster's angle and optically pumped through their flat faces by arrays of linear flashlamps. Special reflector arrangements have been developed for efficiently coupling light from these flashlamp arrays into the laser slabs, as shown in Fig.(8.14).

8.9.1 Axisymmetric Optical Pumping of a Cylindrical Rod

An ingenious method for obtaining axisymmetric irradiation of the laser crystal is to use an ellipsoidal mirror (an ellipsoid of revolution) as shown in Fig. (8.3). Practically all the light from the flashlamp passes through the laser crystal. However, even, in this case where the illumination of the laser crystal is axisymmetric, the volume illumination of the crystal

Fig. (8.12).

Fig. (8.13).

is not uniform. This can be illustrated by considering a simple two-dimensional model where a cylindrical transparent rod is uniformly illuminated around its circumference as shown in Fig. (8.15). Note that groups of rays that do not strike the surface of the rod normally will be focused through the curved surface by refraction. We assume that the illumination consists of many, incoherent plane waves of equal intensity which have their electric vectors parallel to the axis of the cylinder[8.12] (it can be shown that the final result is the same if the electric vectors of the incident waves are perpendicular to the cylinder axis). If we assume that interference effects between these waves are "smeared out," then the energy density in the rod due to all the illuminating waves can be taken as the sum of their individual energy densities.

We consider the group of waves whose propagation vectors lie within a small range of angles $d\alpha$. Their contribution to the total average energy

Fig. (8.14).

density ρ_0, at a point outside the rod, is

$$\Delta\rho_0 = \frac{\rho_0}{2\pi} d\alpha, \tag{8.31}$$

since the total energy density comes from waves covering the total circular angle of 2π. Now, if there are m waves altogether (where in the limit we will let m become infinitely large) the energy density associated with each wave is ρ_0/m. If the electric field of each wave is of the form $E\cos(\omega t + \phi)$ (where E becomes infinitely small as $m \to 0$) then the time-averaged energy density outside the rod due to an individual wave is

$$d\rho_0 = \overline{\epsilon_0 E^2 \cos^2(\omega t + \phi)}, \tag{8.32}$$

where the bar indicates time averaging, which gives

$$d\rho_0 = \frac{1}{2}\epsilon_0 E^2. \tag{8.33}$$

The total energy density due to the m waves is

$$\rho_0 = \sum_m d\rho_0 = \frac{1}{2}\epsilon_0 \sum_m E^2 = \frac{1}{2}\epsilon_0 E_0^2, \tag{8.34}$$

where E_0 is the root mean square value of the total electric field due to all the waves. Thus, from (8.31) and (8.34)

$$\Delta\rho_0 = \frac{\epsilon_0 E_0^2}{4\pi} d\alpha. \tag{8.35}$$

The group of waves which make this contribution to the energy density, strike the outer wall of the cylindrical rod at angle α, as shown in Fig. (8.16). Their incident intensity is

$$I = c_0 \Delta\rho \tag{8.36}$$

and the intensity transmitted through the surface, from Eq. (4.30) with

$n' = 1$, is

$$I_t = nt^2 I, \tag{8.37}$$

where t is the transmission coefficient, which in this case is given by †

$$t = \frac{2\cos\alpha}{\cos\alpha + n\cos\beta}. \tag{8.38}$$

The angle of refraction β is related to the angle of incidence α by Snell's law

$$\frac{\sin\alpha}{\sin\beta} = n, \tag{8.39}$$

where n is the refractive index of the cylinder. The total internal flux transmitted through a small area of surface dA of unit length along the cylinder axis is, from (8.37)

$$F^0_{int} = nt^2 dAI\cos\beta, \tag{8.40}$$

since $dA\cos\beta$ is the effective area of the element of surface normal to the direction of propagation of the wave.

Once this group of waves has entered the cylinder they reflect around inside, maintaining internal angles of incidence and reflection of β, as shown in Fig. (8.16). Each time they strike the wall their intensity is diminished by a factor r^2, where r is the reflection coefficient given by†

$$r = \frac{n\cos\beta - \cos\alpha}{n\cos\beta + \cos\alpha}. \tag{8.41}$$

We consider the contribution that these waves make to the energy density in a small annulus of the cylinder of radius R and thickness dR. On any given pass across the cylinder, between internal reflections, the waves strike this region at an angle θ where

$$\theta = \text{arc } \sin(R_0 \sin\beta/R), \tag{8.42}$$

where, of course, if $R < R_0\sin\beta$, the particular group of waves does not pass through the part of the cylinder of radius R. If the flux that strikes this annular region is F^j_{int}, where F^j_{int} is the internal flux after j internal reflections, then the stored energy within the annulus due to this passage of the internally reflected wave is

$$du^j(r) = \frac{2F^j_{int} n dR}{c_0\cos\theta}, \tag{8.43}$$

which gives

$$du^j(r) = \frac{2r^{2j} F^0_{int} nRdR}{c_0\sqrt{R^2 - R_0^2\sin^2\beta}}. \tag{8.44}$$

† See Appendix 4.

The total stored energy in the annulus is

$$du(r) = \sum_{j=0}^{\infty} \frac{2r^{2j} F^0_{int} nRdR}{c_0\sqrt{R^2 - R_0^2 \sin^2\beta}}, \tag{8.45}$$

which substituting from (8.40) and summing the geometric series gives

$$du(r) = \frac{2n^2t^2 I \cos\beta dARdR}{c_0\sqrt{R^2 - R_0^2 \sin^2\beta}(1-r^2)}. \tag{8.46}$$

The energy density in the annulus, since its volume is $2\pi RdR$, is

$$d\rho(r) = \frac{n^2t^2 I \cos\beta dA}{\pi c_0\sqrt{R^2 - R_0^2 \sin^2\beta}(1-r^2)}. \tag{8.47}$$

The total energy density at radius r due to all the waves that strike an area $2\pi R_0$ of the outer cylinder in a small angular range $d\alpha$ at α is

$$\Delta\rho(r) = \frac{2n^2t^2 I R_0 \cos\beta}{c_0\sqrt{R^2 - R_0^2 \sin^2\beta}(1-r^2)}. \tag{8.48}$$

Now, from (8.38) and (8.41)

$$\frac{t^2}{1-r^2} = \frac{\cos\alpha}{n\cos\beta}, \tag{8.49}$$

and since $I = c_0\Delta\rho_0$ and $\Delta\rho_0 = (\rho_0/2\pi)\, d\alpha$,

$$\Delta\rho(r) = \frac{n\rho_0 R_0}{\pi} \frac{\cos\alpha d\alpha}{\sqrt{R^2 - R_0^2 \sin^2\alpha/n}}. \tag{8.50}$$

The total energy density at radius R, $\rho(R)$, is obtained by integrating (8.50) over the range of angles of incidence α that allow waves to reach radius R

$$\rho(R) = \frac{n\rho_0 R_0}{\pi} \int_{-\alpha_0}^{\alpha_0} \frac{\cos\alpha d\alpha}{\sqrt{R^2 - R_0 \sin^2\alpha/n}}, \tag{8.51}$$

where $\sin\alpha_0$ is the smaller of 1 and nR/R_0. The solution of (8.51) is

$$\frac{\rho(R)}{\rho_0} = n^2 \qquad \text{for } 0 \le R \le R_0/n$$

$$= \left(\frac{2n^2}{\pi}\right) \text{arc } \sin\left(\frac{R_0}{nR}\right) \qquad \text{for } R_0/n \le R \le R_0, \tag{8.52}$$

which is shown in Fig. (8.17) for the case $n = 1.76$.

The energy density within the cylinder is a maximum, and constant, for all radii $\le R_0/n$. Fig. (8.17) also shows the energy density as a function of radius inside a cylindrical rod when the incident radiation consists of all possible polarizations in three dimensions. If the cylinder absorbs the incident radiation then these energy density functions are

Fig. (8.15).

Fig. (8.16).

modified as illustrated in Fig. (8.18), which shows both the two- and three-dimensional calculated energy density functions for different values of d where $d = 2R_0\times$ absorption coefficient.

It is clear from Figs. (8.17) and (8.18) that if a cylindrical laser crystal is made up as a composite rod with an inner absorbing core, for example Cr^{3+} doped α-Al_2O_3 (pink ruby) of radius R_0/n, and an outer sheath of transparent material of the same refractive index, for example α-Al_2O_3 (sapphire) as shown in Fig. (8.19), then very close to uniform illumination of the amplifying medium will be achieved. Such composite laser rods do, in fact, have lower oscillation thresholds than comparable uniform rods although manufacture of these composite rods is more difficult.

8.10 High Power Pulsed Solid-State Lasers

The input energies to the flashlamps used in pumping high energy solid

Fig. (8.17).

state lasers can be very large, ranging up to several thousand joules for each lamp. Typical flash durations in these systems are in the 100 μs–1 ms range. These large energy inputs can cause severe heating of the laser rod and consequently restricts operation to pulsed operation, unless special cooling arrangements are incorporated.

For this reason, high power solid state lasers cannot be built just by increasing the diameter and length of a cylindrical laser rod. Large diameter rods cannot eliminate waste heat effectively, and are also difficult to fabricate.

The highest available energy outputs are obtained from Nd^{3+} glass lasers, which at high power levels are efficiently pumped in a face-pumped slab configuration, one example of such a system, which contains some interesting features, is shown in Fig. (8.20). In this arrangement the thermal and optical pumping inhomogeneity problems associated with large diameter cylindrical laser crystals are considerably reduced. Irradiation of the crystal is efficient over a large surface/volume ratio and large size glass slabs, placed at Brewster's angle, can be used.

Development of high power Nd^{3+} glass systems of this kind is continuing in many laboratories as part of a program to obtain laser-induced fusion. Current attention is focused on the development of lasers with output energies up to 1 MJ to be delivered to pellets of nuclear fuel on time scales from 100 ps–1 ns. This corresponds to peak powers of 10^{14}–10^{15} W.

8.11 Diode-Pumped Solid-State Lasers

The development of efficient, high power semiconductor lasers using GaAlAs operating near 800 nm has created an important new class of

Fig. (8.18).

optically pumped lasers in which a crystalline laser is optically pumped by a semiconductor laser. We shall discuss the characteristics of the latter in detail in Chapter 13. The most common laser of this kind is the Nd:YAG operating at 1.06 μm or 1.32 μm, although other hosts for the Nd^{3+} ions, such a YLF, $La_2Be_2O_5$ (BEL) and YVO_4, have been used. A fortuitous concidence between available high power GaAlAs lasers at $\sim$809 nm and the absorption of Nd^{3+} ions into the $4F_{5/2}$ pump band, see Fig. (8.4) provides for very efficient conversion from 809 nm to laser oscillation at 1.06 μm, over 10% conversion is easily obtained with pump powers of $\sim$200 mW.

Table (8.3) summarizes the diode-pumped solid-state laser transitions that have been obtained to date[8.15]. One laser in this host that is worthy of note is the 2.1 μm holmium YAG laser, which has potential optical radar (LIDAR) applications because its wavelength makes it far less of an eye hazard than lasers operating at 1.06 μm and shorter wavelengths.

Diode-pumped lasers in which the semiconductor lasers are arranged to inject their pump radiation through the cylindrical faces of a laser rod are in many ways akin to lamp-pumped lasers of the same kind. However, because of the high electrical efficiency of GaAlAs laser diodes ($> 10\%$) the overall electrical to optical conversion efficiency of these lasers is very high, approaching 10%.

If the diode laser pump radiation is injected along the axis and matched to the transverse mode geometry of the solid state laser then very stable, narrow linewidth laser oscillation can be obtained: Fig. (8.21) shows the clever monolithic Nd:YAG laser design of Kane and Byer[8.16]. In this laser the magneto-optical properties of YAG are used to obtain unidirectional amplification of a travelling wave in the laser cavity. This prevents the spatial hole-burning that can occur in a stand-

Table (8.3). *Diode-Pumped Solid-State Lasers.*

Ion	Transition	Wavelength (μm)	Operating temperature (K)
Nd^{3+}	${}^4F_{3/2} \rightarrow {}^4I_{11/2}$	1.06	300
	${}^4F_{3/2} \rightarrow {}^4I_{13/2}$	1.32	300
	${}^4F_{3/2} \rightarrow {}^4I_{9/2}$	0.95	
Ho^{3+}	${}^5I_7 \rightarrow {}^5I_8$	2.1	300
Er^{3+}	${}^4I_{13/2} \rightarrow {}^4I_{13/2}$	2.8	300
	${}^4I_{13/2} \rightarrow {}^4I_{9/2}$	1.6	300
Tm^{3+}	${}^3F_4 \rightarrow {}^3H_5$	2.3	300
U^{3+}	${}^4I_{11/2} \rightarrow {}^4I_{9/2}$	2.61	4.2
Dy^{2+}	${}^5I_7 \rightarrow {}^5I_8$	2.36	1.9
Yb^{3+}	${}^2F_{7/2} \rightarrow {}^2F_{5/2}$	1.03	77

Fig. (8.19).

ing wave homogeneously broadened laser, which can allow multiple longitudinal modes to oscillate.

8.12 Relaxation Oscillations (Spiking)

When a laser system is pushed into a state of population inversion, as soon as its gain passes the threshold value the oscillation grows in amplitude and, by reducing the gain through stimulated emission, stabilizes

Fig. (8.20).

the gain at the loss line. The approach to a stable oscillation may occur smoothly, as if the approach to the equilibrium situation were damped. However, oscillation about the equilibrium position is frequently observed. This oscillation appears as a time-varying output laser intensity following the attainment of a population inversion sufficient to start the oscillation. It is called "spiking." It results from the attempt of the intracavity field and the population inversion to become balanced and can be visualized as an oscillatory energy exchange process between the intracavity intensity and the population inversion. As the intracavity intensity increases, the upper level population is reduced so the gain falls – this reduces the intracavity intensity. This reduction of the intracavity intensity causes the gain to increase again. The intracavity intensity then increases again, and so on.

If we observe the output intensity from, for example, a pulsed ruby laser, we generally see the sort of behavior illustrated in Fig. (8.22). The laser emission in successive spikes tends to shift between different longitudinal and transverse modes. Suppose the emission in one spike is in a longitudinal mode of order m so that the oscillation frequency is

$$\nu = \frac{mc}{2\ell}\left(\lambda = \frac{2\ell}{m}\right),$$

then the standing wave field in the laser cavity has nodes and antinodes of intensity in a certain pattern, as shown in Fig. (8.23). Following laser emission in the spike the inverted population is depleted near the antinodes of the field relative to the nodes; this causes a longitudinal mode of different order to oscillate in the next spike. This mode will have higher gain because its field antinodes will be near the field nodes of the previously oscillating mode as shown schematically in Fig. (8.23).

One phenomenon which can suppress this longitudinal mode jumping

Fig. (8.21).

is called *spatial cross-relaxation*, where an excited particle at position A in Fig. (8.23) can rapidly move to a position B. This smears out any spatial sinusoidal variation of gain within the laser medium and sustains oscillation on a particular high gain longitudinal mode. In a crystal lattice, because the excited particles are fixed, it is relatively difficult for them to transfer excitation from one position to another. However, the transfer can still occur radiatively, the relative efficiency of this process depending on the Einstein coefficient A_{21} for the laser transition, and the absorption coefficient of the laser crystal on this transition (which, of course, depends on the atom concentration and the linewidth of the transition).

Jumping from one transverse mode to another in successive spikes is understandable in much the same way. Oscillation in one TEM mode depletes a certain region of the crystal preferentially. This favors subsequent oscillation on a different TEM mode, which has its maxima of intracavity intensity in the least depleted spatial region of the laser medium.

Frequency and mode jumping of the above kinds also tend to be enhanced by nonuniformity of optical pumping, and heating of the crystal during the flash, which produces spatial refractive index variations. Simultaneous oscillation on different longitudinal and transverse modes may occur even though the laser transition is homogeneously broadened. The simultaneously oscillating modes arrange to use different spatial regions of the crystal, so although in theory they can compete for the same group of atoms, in fact they do not. A simple illustration of how this can happen is given in Fig. (8.24). In gaseous lasers spatial cross-relaxation is fast and in *homogeneously* broadened gas lasers the above

Fig. (8.22).

type of process, which would lead to multi-mode oscillation, is generally suppressed.

8.13 Rate Equations for Relaxation Oscillation

In describing relaxation oscillation in a laser we follow the approach first given by Statz and deMars[8.17] . We assume that laser oscillation builds up in a single mode of the optical resonator.

Let q be the number of photons in the mode per unit volume. The mode corresponds to a monochromatic radiation field at frequency ν' whose energy density is a δ-function

$$\rho(\nu) = qh\nu'\delta(\nu - \nu'). \tag{8.53}$$

The upper and lower laser levels have population densities N, N_1, and effective lifetimes τ, τ_1, respectively. We assume that the laser is a *good* one such that $\tau_2 \gg \tau_1$, and $N \gg N_1$. We neglect N_1. The population inversion in this case is the same as N and obeys the differential equation

$$\frac{dN}{dt} = R - NW - \frac{N}{\tau}, \tag{8.54}$$

where R is the rate per unit volume at which particles are fed into the upper laser level, and W is the stimulated emission rate. In this case

$$W = \int qh\nu'\delta(\nu - \nu')B_{21}g(\nu_0, \nu)d\nu, \tag{8.55}$$

which gives

$$W = qh\nu' B_{21}g(\nu_0, \nu') = qB. \tag{8.56}$$

The constant B is

$$B = h\nu' B_{21}g(\nu_0, \nu') = \frac{c^3}{8\pi\nu'^2}A_{21}g(\nu_0, \nu'), \tag{8.57}$$

which is the ratio of the spontaneous emission rate per frequency interval to the mode density. Eq. (8.54) therefore becomes

$$\frac{dN}{dt} = R - qBN - \frac{N}{\tau}. \tag{8.58}$$

The equation describing the rate of change of the number of photons in the mode is

$$\frac{dq}{dt} = qBN - \frac{q}{\tau_0} + N\epsilon A_{21}, \tag{8.59}$$

where τ_0 is the time constant of the laser cavity, for example given by Eq. (4.11), and ϵ is the probability that a photon will be emitted spontaneously into the particular mode we are considering. Given the very large number of available modes for spontaneous emission we neglect the term containing ϵ in Eq. (8.59). We cannot solve Eqs. (8.58) and (8.59), which are called the *Statz–deMars* equations, exactly, although it is easy to examine their solution numerically using a computer. To find an analytic solution we consider small departures from the equilibrium situation in which $dN/dt = 0$, and $dq/dt = 0$.

In equilibrium, $N = N_0$, where from Eq. (8.59)

$$N_0 = \frac{1}{B\tau_0} \tag{8.60}$$

and from Eqs. (8.58), and (8.60)

$$q_0 = R\tau_0 - \frac{1}{B\tau}. \tag{8.61}$$

There are no photons in the mode when $q_0 = 0$. This occurs when the pumping rate is at its threshold value, R_t, where

$$R_t = \frac{1}{B\tau\tau_0}. \tag{8.62}$$

We define a pumping factor r, where

$$r = \frac{R}{T_t}, \tag{8.63}$$

so Eq. (8.61) can be written

$$q_0 = \frac{r-1}{B\tau}. \tag{8.64}$$

We solve Eqs. (8.58) and (8.59) approximately by considering small oscillations about the equilibrium position. We write

$$\begin{aligned} N(t) &= N_0 + N'(t), \qquad N' \ll N_0, \\ q(t) &= q_0 + q'(t), \qquad q' \ll q_0. \end{aligned} \tag{8.65}$$

Substitution in Eqs. (8.58) and (8.59) gives

$$\frac{dN'}{dt} = R - B(q_0 + q')(N_0 + N') - \left(\frac{N_0 + N'}{\tau}\right), \tag{8.66}$$

$$\frac{dq'}{dt} = B(q_0 + q')(N_0 + N') - \left(\frac{q_0 + q'}{\tau_0}\right). \tag{8.67}$$

Neglecting the product of small quantities gives

$$\frac{dN'}{dt} = \left(R - Bq_0N_0 - \frac{N_0}{\tau}\right) - N'\left(Bq_0 + \frac{1}{\tau}\right) - Bq'N_0, \tag{8.68}$$

$$\frac{dq'}{dt} = \left(Bq_0N_0 - \frac{q_0}{\tau_0}\right) + Bq_0N' + q'\left(BN_0 - \frac{1}{\tau_0}\right). \tag{8.69}$$

By virtue of Eqs. (8.58) and (8.59) the first term in Eq. (8.68) and the first and third terms in Eq. (8.69) are zero. That is

$$\frac{dN'}{dt} = -N\left(Bq_0 + \frac{1}{\tau}\right) - q'BN_0 \tag{8.70}$$

and

$$\frac{dq'}{dt} = NBq_0. \tag{8.71}$$

Differentiation of Eq. (8.70) followed by substitution from Eq. (8.71) gives

$$\frac{d^2N'}{dt^2} + \frac{dN'}{dt}\left(Bq_0 + \frac{1}{\tau}\right) + \frac{N'q_0B}{\tau_0} = 0. \tag{8.72}$$

Use of Eq. (8.61) gives us

$$\frac{d^2N'}{dt^2} + \frac{dN'}{dt}(RB\tau_0) + N'\left(RB - \frac{1}{\tau\tau_0}\right) = 0. \tag{8.73}$$

Similarly,

$$\frac{d^2q'}{dt^2} - Bq_0\frac{dN'}{dt} = 0. \tag{8.74}$$

Substitution from (8.70) gives

$$\frac{d^2q'}{dt^2} + Bq_0\left(Bq_0 + \frac{1}{\tau}\right)N' + q'Bq_0BN_0 = 0, \tag{8.75}$$

which becomes

$$\frac{d^2q'}{dt^2} + \frac{dq'}{dt}\left(Bq_0 + \frac{1}{\tau}\right) + q'\left(\frac{q_0B}{\tau_0}\right) = 0. \tag{8.76}$$

Finally, from eq. (8.61)

$$\frac{d^2q'}{dt^2} + \frac{dq'}{dt}\left(RB\tau_0\right) + q'\left(RB - \frac{1}{\tau\tau_0}\right) = 0. \tag{8.77}$$

So, q' and N' both obey the same differential equation, which is logical since the fluctuations in photon density are closely coupled to fluctuations in population inversion.

For a trial solution of Eq. (8.77) we try $q' = q_0e^{-(\eta - i\xi)t}$. Remembering that

$$q_0 = \left(\frac{RB\tau_0 - 1/\tau}{B}\right)$$

we find

$$\eta = \frac{1}{2}\left(q_0 B + \frac{1}{\tau}\right) = \frac{RB\tau_0}{2} = \frac{r}{2\tau} \tag{8.78}$$

and

$$\xi = \sqrt{\frac{q_0 B}{\tau_0} - \frac{q_0^2 B^2}{4} - \frac{q_0 B}{2\tau} - \frac{1}{4\tau^2}} = \sqrt{\frac{(r-1)}{\tau\tau_0} - \left(\frac{q_0 B}{2} + \frac{1}{2\tau}\right)^2}. \tag{8.79}$$

Eq. (8.79) can be simplied by using Eqs. (8.61)–(8.63) to give

$$\xi = \sqrt{\frac{(r-1)}{\tau\tau_0} - \left(\frac{r}{2\tau}\right)^2}. \tag{8.80}$$

The photon density is a damped oscillatory function of angular frequency ξ, provided

$$\left(\frac{r-1}{\tau\tau_0}\right) > \left(\frac{r}{2\tau}\right)^2,$$

of the form $q' = q_0 e^{-\eta t} e^{i\xi t}$.

If we take the specific example of a ruby laser with

$$\tau = 3 \times 10^{-3}\ \text{s},$$
$$\tau_0 = 10^{-8}\ \text{s},$$
$$r, = 2,$$

Eq. (8.79) gives the frequency of the relaxation oscillation as 1.82×10^5 s^{-1} and its period ($= 2\pi/\xi$) as 3.45×10^{-5}s. Such a damped oscillation in the output of a Nd^{3+}/calcium tungstate laser is shown in Fig. (8.25). It would appear from Eq. (8.80) that for pumping far enough above threshold, when $r \gg 1$ and $r > 4\tau/\tau_0$, the output laser intensity would cease to be oscillatory. Such an assumption cannot be made with any certainty, however, because under these circumstances, q' and N' no longer satisfy the conditions $q' \ll q_0,\ N' \ll N_0$, and our solution of the Statz–deMars equations breaks down.

8.14 Undamped Relaxation Oscillations

Under normal circumstances relaxation oscillations are damped, although the relaxation oscillation may become undamped if the pumping rate is varying with time. Suppose, $R = R_0 + r'(t)$, in which case Eq. (8.68) becomes

$$\frac{dN'}{dt} = r' - R_0 B\tau_0 N' - \frac{q'}{\tau_0} \tag{8.81}$$

Fig. (8.23).

and Eq. (8.61) takes the form

$$\frac{d^2q'}{dt^2} + \frac{r}{\tau}\frac{dq'}{dt} + \frac{1}{\tau\tau_0}(r-1)q' = -\frac{1}{\tau}(r-1)r'. \tag{8.82}$$

This is now the differential equation describing a damped oscillator that is being driven with a "force" $-(1/\tau)(r-1)r'$, and as such can have an oscillatory solution of large amplitude if the driving force has a frequency component near the natural frequency ξ.

Suppose that $r'(t)$ is oscillatory and can be written as

$$r'(t) = R'e^{i\xi t},$$

then Eq. (8.82) becomes

$$\frac{d^2q'}{dt} + \frac{r}{\tau}\frac{dq'}{dt} + \frac{1}{\tau\tau_0}(r-1)q' = -\frac{1}{\tau}(r-1)r_0e^{i\xi' t}. \tag{8.83}$$

We try as a solution $q' = Qe^{i\xi' t}$.

Substitution in Eq. (8.83) gives

$$-\xi'^2Q + i\xi'\frac{r}{\tau}Q + \frac{1}{\tau\tau_0}(r-1)Q = -\frac{1}{\tau}(r-1)r_0 \tag{8.84}$$

and

$$Q = \frac{(r-1)r_0}{\tau\xi'^2 - ir\xi' - (r-1)/\tau_0}. \tag{8.85}$$

From Eqs. (8.78) and (8.80), Eq. (8.85) can be written as

$$Q = \frac{(r-1)r_0/\tau}{(\xi' - \xi - i\eta)(\xi' + \xi - i\eta)}. \tag{8.86}$$

This amplitude becomes large as $\xi' \to \xi$, particularly if the damping factor η is small.

Undamped relaxation oscillations are quite common in real lasers. A component in the pumping process that fluctuates at a natural resonance frequency of the laser system will tend to drive a relaxation oscillation

Fig. (8.24).

at, or near, the resonance frequency. Fig. (8.26) shows just such an oscillation, and its frequency spectrum observed in the output of a Nd:YAG laser pumped by a semiconductor laser diode. Such relaxation oscillations can be suppressed by minimizing fluctuations in the pump through feedback control, by increasing the damping factor η, or by channeling the fluctuations in the pump into a frequency region far from a natural resonance frequency of the laser.

8.15 Giant Pulse (Q-Switched) Lasers

In a pulsed solid-state laser the time dependence of the optical pumping leads naturally to "spiking" in the laser output. The pseudo-random nature of this light output is generally undesirable but can be controlled by a technique called *Q-switching*[8.7]–[8.9]. If the feedback process within the oscillator is blocked, for example by preventing reflection from one of the laser mirrors, then the cavity has low Q. If the lamp is fired inversion builds up within the laser, but oscillation does not start. If the obstacle to feedback is suddenly removed, the Q of the cavity switches to a higher value and oscillation begins. Because the laser now finds itself with an inversion much above threshold the intracavity intensity grows very rapidly and the laser delivers a giant or *Q-switched* pulse of high intensity and short duration. This duration is typically on the order of the natural lifetime τ_0 of the laser cavity described in Section 4.4.

$$\tau_0 = \frac{\ell}{c(1-R)}. \tag{8.87}$$

This giant pulse may so deplete the inversion in the laser that no subsequent laser emission occurs for that particular firing of the flashlamp.

Fig. (8.25).

There are several ways of Q-switching[8.19]–[8.21]. They all rely on removal of an obstacle to reflection from one laser mirror after a substantial population has been stored in the upper laser level.

(a) A spinning wheel with a hole or slot placed inside the laser cavity[8.22]. This method is largely of historic interest, but it is the easiest to understand. The lamp is synchronized to fire before the spinning wheel brings the hole into alignment. To provide a rapid transition from low to high Q, the hole in the wheel should be of small size, with a pair of lenses to focus the intracavity laser beam, as shown in Fig. (8.27).

(b) One of the end reflectors is spun about an axis orthogonal to the alignment axis of the laser cavity. This is most easily accomplished if one of the laser mirrors is replaced with a spinning roof prism, as shown in Fig. (8.28a). A roof prism has the property of reflecting any ray incident orthogonal to the ridge line of the roof back parallel to itself. Therefore, if such a prism is spun about an axis perpendicular to the ridge, alignment of the laser cavity is guaranteed twice per rotation. Alternatively, the roof prism can remain fixed, or be fabricated directly onto the end of the laser rod. A conventional laser mirror spun about its diameter perpendicular to the ridge of the roof prism as shown in Fig. (8.28b) is guaranteed to align twice per rotation.

(c) If a glass cell containing a dilute solution of absorbing dye such as cryptocyanine is placed inside the cavity then the oscillation threshold predicted by Eq. (5.9) is raised to

$$\gamma_t = \alpha - \frac{1}{\ell}(\alpha_d d + \ell n r_1 r_2), \tag{8.88}$$

Fig. (8.26).

Fig. (8.27).

Fig. (8.28).

where α_d is the absorption coefficient of the dye and d its thickness. The laser will not start to oscillate until this threshold is reached. When oscillation does start the increasing intracavity intensity "bleaches" the dye by exciting ground state dye molecules into excited states. Solid materials with absorbing impurities can be used in a similar way[8.23]. This is the reverse of the gain saturation process described in Section 2.6. In describing this process theoretically, provided the Q-switched pulse is of short duration, spontaneous emission of the dye molecules can be neglected[8.24].

(d) With an optically active element in the laser cavity that changes the polarization state of the light travelling through it by either the Kerr effect†, Faraday effect‡ or by electro-optic activity in a crystal*. In a Kerr-active medium or electro-optic crystal a transverse electric field of sufficient magnitude will change linearly polarized light to circularly polarized or rotate its plane of linear polarization by 90°. In a Faraday-active medium an axial magnitude field causes the plane of linear polarization of a wave travelling along the field to rotate. The general way in which these phenomena can be used to build a Q-switch is best illustrated with Fig. (8.29), which shows an electro-optic crystal used with a transverse electric field as the switching element. The crystal has sufficient voltage applied to it that the electric field causes input linearly polarized light to be converted to circularly polarized light on output. Exactly why this occurs in certain materials is explained in detail in Chapters 18 and 19. The circularly polarized light strikes mirror M_2, and returns through the crystal. In this second pass through the crystal the circularly polarized light is converted back to linearly polarized, but with its polarization direction orthogonal to its original direction. This light cannot pass the linear polarizator, and reflection from mirror M_2 is effectively blocked. Fast removal of the voltage applied to the crystal opens up mirror M_2 for reflection and laser oscillation occurs.

8.16 Theoretical Description of the Q-Switching Process

In a simple model of the Q-switching process the Q-switched pulse is assumed to be short enough that spontaneous emission processes during the pulse can be neglected and the changeover from low Q to high Q is instantaneous. For a resonator of volume V the total number of photons

† See Chapter 20.

‡ See Chapter 14.

* See Chapter 19.

is

$$\phi = Vq \tag{8.89}$$

and the total inversion is

$$n = \left(N_2 - \frac{g_2}{g_1} N_1 \right) V. \tag{8.90}$$

The gain of the amplifying medium is γ. We neglect the additional complexity of gain saturation. Radiation of intensity I passing through the amplifier grows according to

$$\frac{dI}{dz} = \gamma I \tag{8.91}$$

and as a function of time

$$\frac{dI}{dt} = \frac{dI}{dz}\frac{dz}{dt} = c\gamma I, \tag{8.92}$$

where c is the velocity of light within the laser medium. For a laser cavity of length ℓ containing an amplifying medium of length L, only a fraction L/ℓ of all the light is being amplified. We can describe the average intensity increase within the cavity by Eq. (8.92) with a scale factor of L/ℓ. The total number of photons changes according to

$$\frac{d\phi}{dt} = \phi \left(\frac{c\gamma L}{\ell} - \frac{1}{\tau_0} \right), \tag{8.93}$$

where the cavity time constant τ_0 includes all passive loss effects such as mirror transmission and scattering within the amplifying medium. If we introduce a normalized time unit $\tau = t/\tau_0$, Eq. (8.93) becomes

$$\frac{d\phi}{d\tau} = \phi \frac{\gamma}{\ell/cLT_0} - 1. \tag{8.94}$$

Oscillation will not occur if $\gamma < \gamma_t = \ell/(cL\tau_0)$; in this case $d\phi/d\tau = 0$. Therefore we can write

$$\frac{d\phi}{d\tau} = \phi \left(\frac{\gamma}{\gamma_t} - 1 \right) \tag{8.95}$$

and since gain is proportional to population inversion

$$\frac{d\phi}{d\tau} = \phi \left(\frac{n}{n_t} - 1 \right). \tag{8.96}$$

Each stimulated emission reduces the upper laser level population by one and increases the lower level by the same number. Consequently,

$$\frac{dn}{d\tau} = -2\phi \frac{n}{n_t}. \tag{8.97}$$

Eqs. (8.96) and 8.97) can be solved numerically very easily, for example by a Runge–Kutta[8.26] method. Before giving examples of such solutions we can learn quite a lot about the way the population inversion and photon density behave by examining the equations. If we divide Eq. (8.96) by Eq. (8.97) we get

$$\frac{d\phi}{dn} = \frac{1}{2}\left(\frac{n_t}{n} - 1\right), \tag{8.98}$$

which has the solution

$$\phi = \frac{1}{2}(n_t \ln n - n) + \text{ constant.} \tag{8.99}$$

If the initial photon density and inversion are ϕ_0, n_0, respectively, then

$$\phi_0 = \frac{1}{2}(n_t \ell \mathrm{n} n_0 - n_0) + \text{ constant.} \tag{8.100}$$

Combining Eqs. (8.99) and (8.100)

$$\phi - \phi = \frac{1}{2}\left[n_t \ell \mathrm{n}\left(\frac{n}{n_0}\right) - (n - n_0)\right]. \tag{8.101}$$

Since there is negligible photon density before laser action starts, we set $\phi_0 = 0$. Therefore,

$$\phi = \frac{1}{2}\left[n_t \ell \mathrm{n}\left(\frac{n}{n_0}\right) - (n - n_0)\right]. \tag{8.102}$$

After the Q-switched pulse is over $\phi \to 0$ and the final inversion n_f satisfies

$$n_t \ell \mathrm{n}\left(\frac{nf}{n_0}\right) - (n_f - n_0) = 0, \tag{8.103}$$

or

$$\frac{n_f}{n_0} = e^{-(n_0 - n_f)/n_t}. \tag{8.104}$$

The quantity $(n_0 - n_f)/n_0$ represents the fraction of the original inversion that is converted to laser energy, which is

$$\left(\frac{n_0 - n_f}{n_0}\right) = 1 - e^{-(n_0 - n_f)/n_t}. \tag{8.105}$$

This approaches unity as $n_0/n_t \to \infty$.

If n_f is very small this means the Q-switched pulse has very efficiently depleted the inversion. The value of n_f can swing below the threshold value n_t, analogous to the underdamped discharge of a capacitor.

The output power of the laser is

$$P = \frac{\phi h\nu}{\tau_0}, \tag{8.106}$$

since the rate at which instantaneous energy stored is being lost from the cavity depends only on the cavity lifetime. Therefore from Eq. (8.102)

$$P = \frac{h\nu}{2\tau_0}\left[n_t \ell \mathrm{n}\left(\frac{n}{n_0}\right) - (n - n_0)\right]. \tag{8.107}$$

Fig. (8.29).

Fig. (8.30).

Maximum power output results when $d\phi/d\tau = 0$, which from Eq. (8.96) occurs when $n = n_t$. The maximum power output is

$$P_{max} = \frac{h\nu}{2\tau_0}\left[n_t \ell\text{n}\left(\frac{n_t}{n_0}\right) - \left(n_t - n_0\right)\right]. \qquad (8.108)$$

If the laser is Q-switched from far above threshold then $n_0 \gg n_t$ and

$$P_{max} \simeq \frac{n_0 h\nu}{2\tau_0} \qquad (8.109)$$

and the maximum number of photons stored in the cavity is

$$\phi_{max} = \frac{P_{max} h\nu}{\tau_0} = \frac{n_0}{2}. \qquad (8.110)$$

All these predictions are borne out by actual numerical solution of Eqs. (8.96) and (8.97), as can be seen from Figs. (8.30) and (8.31). To summarize: these simulations demonstrate that:

(i) The Q-switched pulse has higher amplitude and turns on faster as the initial inversion increases from the threshold value – see Fig. (8.30).

(ii) The inversion falls more rapidly at the onset of Q-switching as the initial inversion increases from the threshold value – see Fig. (8.30).
(iii) The peak photon density occurs when the inversion passes through the threshold value – see Figs. (8.29) and (8.30).
(iv) After the inversion has fallen to a low value the remaining photon density decays with the characteristic lifetime of the cavity.
(v) The final inversion falls increasingly below threshold as the initial inversion rises from the threshold value – see Fig. (8.30).

8.16.1 Example Calculation of Q-Switched Pulse Characteristics

We take as an example a Nd:YAG laser with a 1% Nd^{3+} doping density. The density of YAG is 4550 kg m^{-3} so a 1% by weight doping level of Nd^{3+} corresponds to 1.9 $\times 10^{26}$ ions m^{-3}. For the Nd:YAG laser crystal considered earlier in this chapter the threshold inversion was 9.3×10^{21} ions m^{-3}, which corresponds to an excited Nd^{3+} ion population within the crystal volume ($\simeq 10^{-6}$ m^3) of 9.3×10^{15}. For this laser, with $R_{avg} = 0.96$ the cavity lifetime is

$$t_0 = \frac{\ell}{c(1 - R_{avg})} = \frac{50 \times 10^{-3}}{2.997 \times 10^8 \times 0.04} = 4.17 \text{ ns}.$$

If we can pump this crystal to 100 times above threshold then the small signal gain will be 88.3 m^{-1}. The peak Q-switched power will be, from Eq. (8.118)

$$P_{max} \simeq \frac{9.3 \times 10^{17} \times 6.626 \times 10^{-34} \times 2.947 \times 10^8}{2 \times 4.17 \times 10^{-9} \times 1.06 \times 10^{-6}} = 2.09 \times 10^7 \text{W}.$$

The total output pulse energy is

$$U \simeq \frac{n_0 h\nu}{2} \simeq 87 \text{ mJ},$$

where the factor of 2 in the denominator results from the "bottleneck" imposed by the lower laser level. Since the Q-switched pulse is very fast, ions that reach the lower laser level can be assumed to remain there during the pulse so the inversion will be lost when half the initially excited ions have emitted.

8.17 Problem

(8.1) Consider a ruby laser rod whose pump band absorption characteristics correspond to the data in Fig. 3.3, i.e., the concentration of Cr^{3+} is 1.88$\times 10^{25}$ m^{-3}. Use the data given for $E \perp c$. Assume that the laser rod is rectangular (10 mm$\times$ 10 mm$\times$ 100 mm) and is closed-coupled to the flashlamp so that 25% of the flashlamp radiation enters the rod. Calculate the optical flux requirement from the lamp (W m^{-2} Hz^{-1}) to exceed laser threshold by 100% on the R_1 transition.

(a) The two cavity mirror reflectances are 90% and 100%.
(b) The flashlamp pulse is a square pulse of 10^{-4} s duration with a flat spectral output from 300 nm to 700 nm.
(c) The efficiency η for transfer of energy to the upper laser level is 50% for both pump bands.
(d) For the laser transition $\tau_2 \simeq 1/A_{21} = 3 \times 10^{-3}$ s, $\Delta\nu = 200$ GHz chomogeneously broadened).
(e) $T = 300$ K.

Take both the $2\overline{A}$ and $\overline{E}$ upper ruby laser states into consideration. The ratio of degeneracy factors for the upper/lower levels is 1/2. Make a suitable approximation in evaluating the intergral for the population of the upper laser levels.

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