CHAPTER SEVEN

# Control of Laser Oscillators

# 7

# Control of Laser Oscillators

## 7.1 Introduction

During laser oscillation one or more distinct mode frequencies can be emitted, which in the general case behave as independent monochromatic oscillations. When more than one mode oscillates this multimode operation usually corresponds to simultaneous oscillation on more than one longitudinal mode of a single transverse mode of the cavity. It is also possible for more than one transverse mode to oscillate simultaneously, but this is not of great importance or interest to us here. In this chapter we will examine how multimode operation can be suppressed in favor of single mode operation. We shall also see how many simultaneously oscillating modes can become *locked* together in phase to produce a form of laser oscillation in which the laser output becomes a train of very short pulses. This *mode-locked* operation is of considerable fundamental and practical interest and we shall examine the various ways in which the phenomenon can be induced.

## 7.2 Multimode Operation

We have seen that in an inhomogeneously broadened laser simultaneous oscillation is possible on more than one longitudinal mode. These oscillations can be coupled to each other by applying an external perturbation, as we shall see shortly. However, in the absence of such perturbation, each mode oscillates independently of the others. We can represent the total electric field of the laser beam as

$$E(t) = \sum_{n=1}^{N} E_n(t)\cos[\omega_n t + \phi_n(t)], \tag{7.1}$$

where $\omega_n$ is the frequency of the $n$th of $N$ modes and $\phi_n(t)$ is its phase. The frequences $\omega_n, \omega_{n+1}$ are separated by approximately $\pi c/\ell$, but successive adjacent modes are not exactly evenly spaced because of mode-pulling. The phase term fluctuates with time in a random way because the coherence time of the beam is finite. These fluctuations can be attributed to three main causes:

(i) spontaneous emission, of which there is always some even into the narrow solid angle occupied by the laser beam,
(ii) fluctuation in the index of refraction of the amplifying medium,
(iii) vibrations of the mechanical structure of the laser resonator.

The amplitude $E_n(t)$ changes with time if the frequency $\omega_n$ drifts relative to the center of the gain profile. Such drift can be made very small by stabilizing the frequency of the laser. This is most simply done by stabilizing the optical length $L$ of the resonator. The optical length can be defined in the most general way as

$$L = \int_0^{\ell} n(x)dx, \tag{7.2}$$

where $n(x)$ is the refractive index of the medium (or media) between the resonator mirrors placed at the geometric spatial locations 0 and $\ell$.

If the laser intensity is monitored with a fast optical detector, it will be observed to fluctuate with time because of the beating together of the various longitudinal modes. We saw in Chapter 5 that these beats allow the number of oscillating modes to be determined. Although these beats might generally be expected to give only high frequency fluctuation this is not so. The beat signals from different pairs of modes also beat with each other to give fluctuations down to quite low frequencies. The laser intensity also fluctuates because of variations in the mode amplitudes $E_n(t)$ caused by fluctuations in the gain of the amplifying medium.

Fluctuations in amplitude and laser frequency can be controlled in several ways. The simplest approach to ideal behavior is to cause the laser to oscillate in only a single longitudinal and transverse mode. The transverse mode most desirable in this context is the so-called *fundamental* mode in which the distribution of intensity across the laser beam

Fig. 7.1.

is Gaussian[†]. Oscillation in the fundamental transverse mode can be controlled by appropriate selection of the radii of curvature of the laser mirrors and the size of apertures in the laser cavity – for example, the apertures provided by the lateral size of the amplifying medium itself.

## 7.3 Single Longitudinal Mode Operation

Single longitudinal mode operation in a homogeneously broadened laser should be automatic. Surprisingly, sometimes this is not so. The strongest mode should suppress all its neighbors, but because of the standing wave character of the in-cavity field this may not occur if the maxima of field amplitudes of two different modes do not overlap in space. This phenomenon is called *spatial hole-burning.* Fig. (7.1) shows schematically how it happens. Mode 1 draws its gain from, for example, particles located near a field maximum such as $A$. Mode 2 draws its gain from particles located near field maximum $B$. The two modes will not compete strongly unless particles can move rapidly from $A$ to $B$. This phenomena is called *spatial cross-relaxation.* Clearly, such effects are more important in a gas than a solid-state laser.

To force a laser into single longitudinal mode operation several methods have been used:

(i) By making the laser sufficiently short that only one cavity resonance lies under the gain profile and above the loss line. This generally requires $c/2L > \Delta\nu_D$.

(ii) With an intracavity etalon of thickness $t << L$, as shown in Fig.

† To be discussed in detail in Chapter 16.

Fig. 7.2.

(7.2). The etalon is usually tilted slightly to prevent complications caused by reflections between the etalon faces and laser mirrors. The transmission maxima of such an etalon are relatively wide in frequency compared with the transmission maxima of the main cavity, but they are separated by a much greater frequency, $\Delta\nu_{etalon} >> \Delta\nu_{cavity}$,

$$\Delta\nu_{etalon} = \frac{c_0}{2nd}, \tag{7.3}$$

where $n$ is the refractive index of the etalon.

For example, for a 10 mm thick fused silica etalon $\Delta\nu_{etalon} \simeq 10$ GHz. If this etalon is used inside the cavity of a laser with a gain linewidth of 1 GHz (say), then it is likely that only one of the main cavity resonances will be within a transmission maximum of the in-cavity etalon. This situation is shown schematically in Fig. (7.3).

(iii) By placing a very thin absorbing film inside the cavity. The position of the film, which could be a thin layer of metal evaporated onto a glass substrate, is adjusted to be at a node of the intracavity standing wave of a strong longitudinal mode. Other modes suffer loss at the film and can be suppressed.

(iv) By making the laser homogeneously broadened. This is sometimes possible in the case of gas lasers by making them operate at high gas pressure.

(v) With a Fox–Smith interferometer as shown in Fig. (7.4), which acts as a frequency-dependent[7.1] cavity mirror. Laser oscillation can use either mirror $A$ or mirror $B$, the combination of which behaves like a Michelson interferometer. A longitudinal mode that divides into two parts at the beamsplitter will suffer high loss from the

Fig. 7.3.

Fig. 7.4.

cavity unless both parts reflect in phase towards the amplifying medium.

(vi) If one of the main cavity mirrors is an etalon, both faces of which are reflective, then a frequency-dependent cavity mirror results and can discriminate between longitudinal modes.

(vii) In homogeneously broadened lasers, multimode oscillation frequently results because the standing waves corresponding to different modes deplete the population inversion at different spatial locations. This is the spatial hole-burning already mentioned. Single longitudinal mode operation can be obtained if the periodicity of the intracavity field can be smoothed out. This requires the wave inside the cavity to look like a travelling rather than a standing wave, which can be accomplished by building the laser in the form of a ring. In Fig. (7.5) waves can travel around the three-mirror cavity in both directions, if one of these waves can be suppressed, then a travelling wave exists inside the laser medium and no spatial

Fig. 7.5.

hole-burning results. To explain how this might be done, it is actually simplest to describe a clever scheme that uses a conventional cavity with two mirrors but with elements inside the cavity that change the polarization state of the wave. The laser configuration shown in Fig. (7.6) has a Brewster window in the laser cavity and two quarter-wave plates. These optical components, which will be described in detail in Chapter 18, change linearly polarized wave light passing through them into circularly polarized light. At point $A$ inside the cavity the wave that oscillates with minimum loss will be linearly polarized in the direction shown. With respect to $x$ and $y$ axes oriented at 45° to the plane of the diagram we can represent a wave travelling to the right as

$$\begin{aligned}(E_x)_A &= E_0 \cos(\omega t - kz),\\ (E_y)_A &= E_0 \cos(\omega t - kz).\end{aligned} \tag{7.4}$$

After passage through quarter-wave plate $L_1$, which is oriented appropriately, the electric fields at point $B$ are

$$\begin{aligned}(E_x)_B &= E_0 \cos(\omega t - kz + \pi/2),\\ (E_y)_B &= E_0 \cos(\omega t - kz).\end{aligned} \tag{7.5}$$

This is left hand circularly polarized light (LHCP). The second quarter-wave plate $L_2$ is oriented to produce a $\pi/2$ phase shift of the $y$-directed electric vector, so at point $C$ the right travelling wave can be written as

$$\begin{aligned}(E_x)_C &= E_0 \cos(\omega t - kz + \pi/2),\\ (E_y)_C &= E_0 \cos(\omega t - kz + \pi/2),\end{aligned} \tag{7.6}$$

which is once again linearly polarized. This wave reflects from mirror $M_2$ and returns through $L_2$. At point $D$ this left travelling wave can be

Fig. 7.6.

written as

$$(E_x)_D = E_0 \cos(\omega t + kz + \pi/2),$$
$$(E_y)_D = E_0 \cos(\omega t + kz + \pi), \tag{7.7}$$

which is right hand circularly polarized light (RHCP). After a second pass through $L_1$ this wave becomes linearly polarized in its original direction once more.

Inside the amplifying medium the total electric field resulting from the right and left travelling waves is

$$E_x = E_0 \cos(\omega t - kz + \pi/2) + E_0 \cos(\omega t + kz + \pi/2)$$
$$= -2E_0 \sin \omega t \cos kz, \tag{7.8}$$

$$E_y = E_0 \cos(\omega t - kz) + E_0 \cos \omega t + kz + \pi$$
$$= 2E_0 \sin \omega t \sin kz. \tag{7.9}$$

The intracavity intensity is proportional to

$$E_x^2 + E_y^2 = 4E_0^2 \sin^2 \omega t, \tag{7.10}$$

which is independent of $z$. No standing wave distribution of energy density exists in the cavity and single longitudinal mode operation can be obtained.

## 7.4 Mode-Locking

So far in this chapter we have discussed ways in which to cause a laser to generate one or more CW oscillating frequencies corresponding to cavity modes. However, it is also possible to cause the laser to generate a train of regularly spaced, generally very short, pulses. A laser that operates in this way is said to be *mode-locked*. This kind of behavior often occurs spontaneously and is then referred to as *self-mode-locking*.

A fundamental mode-locked pulse train consists of a series of pulses separated by the cavity round-trip time $2\ell/c$. That this kind of behavior could occur should not seem surprising. Early in the development of the laser, Fleck[7.2] showed that as a laser oscillation built up from spontaneous emission there was a natural tendency for the laser output to show strong fluctuations on a time scale that corresponded to the round-trip time in the cavity. A single spontaneous photon emitted into a high $Q$ mode of the cavity will bounce back and forth in the resonator, and be amplified leading to a pulse train of spacing $2\ell/c$. Whether such a group of photons will succeed in competing with other growing oscillations will determine whether the laser ends up operating CW or mode-locked.†

There are two ways of looking at mode-locking. The more common, but less physically satisfying, is to treat the independent oscillating cavity modes whose electric fields are given in Eq. (6.61) as modes that have had their phases locked together, so that each mode has $\phi_n(t) = \phi_0$. By a shift of our choice of time origin we can arbitrarily set $\phi_0 = 0$. The total electric field of the laser can then be represented as

$$E(t) = \mathcal{R}\left[\sum_{n=1}^{N} E_n(t)e^{i\omega_n t}\right]. \tag{7.11}$$

To simplify the analysis we take $E_n(t) = 1$ and

$$\omega_n = \omega_0 + \left[n - \frac{(N+1)}{2}\right]\Delta\omega_c, \tag{7.12}$$

where $\Delta\omega_c = \pi c/\ell$.

Substituting Eq. (7.12) into Eq. (7.11) and summing the resultant geometric series gives

$$E(t) = \cos\omega_0 t \frac{\sin(N\Delta\omega_c t/2)}{\sin(\Delta\omega_c t/2)}. \tag{7.13}$$

This is a pure oscillation at frequency $\omega_0$ modulated with the *envelope* function

$$f(t) = \frac{\sin(N\Delta\omega_c t/2)}{\sin(\Delta\omega_c t/2)}. \tag{7.14}$$

The average power corresponding to this envelope is

$$P(t) \propto \frac{\sin^2(N\Delta\omega_c t/2)}{\sin^2(\Delta\omega_c t/2)}. \tag{7.15}$$

† In this context the term CW is used to include lasers in which the laser excitation is pulsed, but in which the output radiation consists of one or more waves at longitudinal mode frequencies that last for the period of excitation.

Fig. 7.7.

Eq. (7.15) represents a periodic train of pulses that have the following properties:

(a) the pulse spacing is $\Delta T = 2\pi/\Delta\omega_c = 2\ell/c$;
(b) the peak power in the train is $N$ times the average power, and the peak field is $N$ times the average for a single mode;
(c) the pulses within the train become narrower as $N$ increases, and for large $N$ approach a value $\tau = \Delta T/N$.

These characteristics are evident from Fig. (7.7), which is a plot of Eq. (7.15). For an inhomogeneously broadened laser with linewidth $\Delta\nu_D$, the number of independent oscillating modes will be

$$N \sim \frac{2\pi\Delta\nu_D}{\Delta\omega_c}, \tag{7.16}$$

so the mode-locked pulse width will approach a value $\tau \simeq 1/\Delta\nu_D$.

For a homogeneously broadened laser, which ideally oscillates in a single longitudinal mode, the frequency explanation of the generation of short "mode-locked" pulses seems less than satisfactory. This brings us to a more internally self-consistent model of mode-locking as a mode of operation that the laser finds *energy advantageous*, and which does not require the *a priori* assumption of independent oscillating cavity modes that become locked. This argument is illuminated by considering one of the most common ways in which mode-locking is accomplished – passive locking with a saturable dye cell in the laser cavity as shown in Fig. (7.8). The dye cell is frequently placed in close proximity to one of the cavity mirrors.

The oscillation threshold for an individual cavity mode in this arrangement will be high because of the intracavity loss presented by the absorbing dye. However, if the laser oscillates in a "bouncing-pulse"

Fig. 7.8.

mode in which the intracavity intensity is concentrated, then this pulse can "bleach" the dye.†

In this situation a bouncing pulse is more energy efficient in overcoming the intracavity loss presented by the dye. The actual shape of the pulse will be determined by a self-consistency condition. The pulse train $f(t)$ will have a Fourier transform $(FT)$ $F(w)$ that describes the frequency content of the bouncing pulse. Each of the frequency components of this effective pulse train will be amplified by the gain medium according to a saturated gain profile $g_s(w)$. It is clear that the *narrowest* width that the bouncing pulse can have is $\sim 1/\Delta\nu$, where $\Delta\nu$ is the width of the gain profile. In common with any amplifier, frequencies outside the gain bandwidth are not amplified. Schematically, we could write a self-consistency equation

$$FT[f(t)] = F(\omega)g_s(\omega)\alpha_d(\omega), \tag{7.17}$$

where $\alpha_d(\omega)$ is the effective absorption spectrum of the bleached dye. This explanation does not impose an inhomogeneously broadened description of the gain medium. Any broadband gain profile can, in principle, amplify a short pulse. Of course, once we accept the bouncing-pulse model of mode-locking we can examine $F(\omega)$ and will find that the principal components of the Fourier transform will correspond to longitudinal mode frequencies.‡

† A dilute dye becomes less absorbing at high light intensities, as stimulated absorption and emission of dye molecules become balanced as the ground state of the dye is depleted by excitation.

‡ In a sense this is an example of what comes first – *the chicken or the egg?*

## 7.5 Methods of Mode-Locking

In general, mode-locked behavior is caused to occur by modulating the gain or loss of the laser cavity in a periodic way, usually at a frequency $f_m = c/2L$. In amplitude modulation (AM) mode-locking the magnitude of cavity loss (or gain) is modulated; in phase (or frequency) modulation (FM) mode-locking only the complex part of the gain is modulated. In *active* mode-locking the modulation is introduced by external means, for example, with an intracavity modulator. In *passive* or *self*-mode-locking the modulation is created by the bouncing mode-locked pulse itself.

### *7.5.1 Active Mode-Locking*

In the simplest form of AM mode-locking the intracavity loss is switched periodically with an intracavity shutter as shown schematically in Fig. (7.9). The precise time variation of the periodic intracavity loss is not very important, except that its period should equal the round-trip time, $\Delta T$, for an optical pulse bouncing back and forth in the cavity. It is energy advantageous for a bouncing pulse to develop, rather than one or more CW longitudinal modes, as the pulse will adjust its arrival time at the intracavity modulator to correspond to the time of maximum transmission. An alternative way of viewing what happens is to realize that the modulator generates new frequency components in a longitudinal mode of frequency $\nu_m$ passing through it. These new frequencies are called *side-bands*† and have values

$$\nu_k = \nu_m + k f_m \qquad k = 0, \pm 1, \pm 2 \ldots . \tag{7.18}$$

Each of these side-bands also corresponds to another longitudinal cavity mode with frequency $\nu_{(m+k)}$.

The interaction between the fundamental longitudinal modes and side-bands causes the phases of the longitudinal modes to lock together and generate a mode-locked pulse train. In an inhomogeneously broadened laser the existence of several independent longitudinal modes whose phases can become locked is clear. In a homogeneously broadened laser in which only one such mode *should* oscillate, the side-bands of the dominant mode are not suppressed by gain-saturation: they compete for gain and a self-consistent bouncing pulse results. Almost all the lasers that are used to generate very short mode-locked pulses are (ide-

† For a more detailed discussion see Chapter 19.

Fig. 7.9.

ally) homogeneously broadened: for example Nd:YAG, Nd:glass, dye and semiconductor lasers.

A common method of intracavity AM mode-locking is to use an acousto-optic modulator.†. This was the method used in the first experimental observation of mode-locking using a helium–neon laser[7.3]. The acousto-optic modulator can be used to deflect light from the cavity. The undeflected beam is thereby amplitude modulated.

In mode-locking applications the acousto-optic modulator is used in a standing sound wave configuration; the acousto-optic modulator material is designed to resonate for sound waves at frequency $f_s$.‡ For an acousto-optic medium used in this way the variation of refractive index is

$$n(z,t) = n_0 + \Delta n \cos 2\pi f_s t \cos k_s z. \tag{7.19}$$

This standing wave acts like a phase diffraction grating that deflects light. The light travelling orthogonally to the sound wave is deflected at frequency $2f_s$, because the maximum index variation in Eq. (7.19) occurs twice per cycle of the standing sound wave. A typical experimental arrangement for mode-locking with an intracavity acoustic cell is shown in Fig. (7.10).

Mode-locking can also be induced through periodic variation of the gain of the amplifying medium. In a semiconductor laser (see Chapter 13) this can be simply done by modulating the drive current to the laser at frequency $f_m$. Alternatively, one periodically modulated laser can be

† A full discussion of these devices is given in Chapter 19.

‡ This is in contrast to acousto-optic frequency shifter applications where one end of the crystal is terminated in a matched acoustic load or absorber so that only a unidirectional travelling sound wave results.

Fig. 7.10.

used to mode-lock another. This approach is called *synchronous* pumping. For example, an argon laser that is itself mode-locked, but does not generate very short mode-locked pulses, can be used to periodically pump a dye laser (an optically excited laser that uses a dilute organic dye solution as its gain medium – see Chapter 12). The dye laser generates much shorter mode-locked pulses than were injected from the pump laser. The success of this scheme requires that the cavities of pump and sychronously mode-locked lasers be matched so that

$$f_m(\text{pump}) = \frac{1}{T}, \tag{7.20}$$

where $T$ is the cavity round-trip time in the pumped laser. Synchronous pumping allows the use of a convenient mode-locked pump, such as a Nd:YAG or argon ion laser, which do not themselves intrinsically generate the shortest mode-locked pulses, to pump dyes or doped glasses and crystals that support a large gain-bandwidth.

In the ideal case the length of the mode-locked pulses generated by any laser, whether it be homogeneously or inhomogeneously broadened, can approach a value $\tau \sim 1/\Delta\nu$, where $\Delta\nu$ is the width of the gain profile. Consequently gain media with large values of $\Delta\nu$ are desirable for the shortest mode-locked pulse generation, these include dye lasers,[7.4],[7.5] Nd:glass lasers[7.6],[7.7] and titanium–sapphire lasers[7.8]–[7.11]. Typical pulse lengths generated by such lasers are in the picosecond range, although special configurations achieve subpicosecond values. Fig. (7.11) shows a good example of an advanced mode-locked laser system that can generate sub-picosecond pulses. In this scheme a CW argon ion laser pump as acousto-optically mode-locked Nd:phosphate glass oscillator. The mode-locked pulses are then further amplified by causing

Fig. 7.11.

them to make multiple passes inside a Nd:phophate glass *regenerative* amplifier[7.7].†

It is usual in a mode-locked system using a linear resonator to place the intracavity modulator or absorbing dye close to one end mirror to ensure that the laser pulse passes the modulator only once in every round trip. If this is not done, then more than one bouncing pulse can develop within the cavity and the output will not be a regular pulse train of spacing $2\ell/c$. In laser cavities configured in the form of a ring, the placement of the intracavity modulator or absorber is not so important. In this case the possibility exists for two counterpropagating pulses to develop in the ring. In ring and linear resonators where two bouncing pulses develop, it is possible to cause the pulses to overlap or *collide* as they pass through an intracavity saturable absorber (dye). Because of the nonlinear variation of absorption of the dye with intensity the superimposed counterpropagating short pulses actually become shorter in order to maximize bleaching of the dye. In this scheme, called colliding pulse mode-locking (CPM), the typical placement of the absorber is at the center of a linear cavity or one quarter perimeter away from the gain medium in a ring cavity as shown in Fig. (7.12). CPM lasers can directly generate mode-locked pulses of sub-picosecond generation[7.5],[7.12].

The shortest pulses generated to date in ring cavity CPM are 27 fs long[7.13]. There is also an approach called additive pulse mode-locking

† Synchronous mode-locking can also be achieved if the optical lengths of the *pump* laser cavity and the *pumped* laser cavity have a ratio that is of the form $L_{pump}/$
$L_{pumped} = p/p'$, where $p$ and $p'$ are small integers.

Fig. 7.12.

(APM). The basic idea involves two coherent pulses with a relative phase shift interfering so that their wings tend to cancel out[7.14],[7.15].

## 7.6 Pulse Compression

Any short pulse can be represented by its Fourier transform

$$F(w) = FT\{f(t)\}. \tag{7.21}$$

The Fourier transform describes the spectral composition of the pulse $f(t)$ in both amplitude and phase. For example,

$$F(w) = A(w)e^{i\phi(w)}. \tag{7.22}$$

If we pass our original short pulse through a system in which the optical delay varies with frequency as $\Delta\phi(\omega)$, then the emerging pulse will have a Fourier transform

$$F'(\omega) = F(\omega)e^{i\Delta\phi(\omega)}. \tag{7.23}$$

If the lower frequency components suffer a greater phase shift than the higher frequency components, this is called a negative group velocity delay. In this case the output pulse, $g(t)$ will be

$$g(t) = FT\{F'(\omega)\}. \tag{7.24}$$

If the original pulse is written as

$$f(t) = a(t)e^{-i[\omega_0 t + \phi(t)]}, \tag{7.25}$$

where $a(t)$ is a smooth, slowly varying function of time (the pulse envelope), then the *instantaneous* frequency of the pulse is

$$\omega(t) = \omega_0 + \frac{d\phi}{dt}. \tag{7.26}$$

If $\phi(t)$ is nonlinear, the pulse is said to be *chirped*. If

$$\phi(t) = bE^2, \tag{7.27}$$

Fig. 7.13.

Fig. 7.14.

the frequency chirping is linear, and with $b > 0$, is positive. The leading edge of the pulse has a lower instantaneous frequency than the trailing edge – it is red-shifted relative to the blue-shifted trailing edge. In a medium with a negative group velocity delay characteristic, the lower frequencies in the leading edge of the pulse travel more slowly than the higher frequencies in the trailing edge. The back of the pulse catches up with the front and the pulse is compressed, as shown schematically in Fig. (7.13).

A simple pulse compressor can be constructed with two diffraction gratings as shown in Fig. (7.14)[7.16]. The first order diffraction angle $\theta$ satisfies

$$\sin\theta = \frac{\lambda}{d} - \sin\theta_{in}. \tag{7.28}$$

The longer of two wavelengths diffracts at a larger angle and takes a longer path between the two gratings – thereby acquiring a larger phase shift and satisfying the condition for negative group delay.

An optical fiber can also be used to produce pulse compression. Because of the Kerr effect the refractive index of the fiber varies with intensity as

$$n = n_0 + \gamma I. \tag{7.29}$$

Accordingly, the peak of a pulse is slowed relative to its tail. There are additional effects on the pulse because of the dispersion in the fiber $n(\lambda)$. If $n(\lambda)$ lies in a region of negative group velocity dispersion (which for silica fibers lies beyond 1.3 $\mu$m) then the index nonlinearity and dispersion work together to make the pulse shorter.† To date pulses have been compressed to only 6 fs in length[7.17].

† An optical soliton (see Chapter 17) develops for similar reasons.

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