(i) \( NA = \sin \theta_0 = \sqrt{n_1^2 - n_2^2} \)

\[
NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}
\]

\[
= \sqrt{\left(\frac{n_1 - n_2}{n_1}\right)2n_1^2}
\]

\[
= n_1 \sqrt{2 \Delta}
\]

\[
= 1.46 \sqrt{0.044}
\]

\[
= 0.0923 \text{ rad} = 0.09243 \text{ rad}
\]

\[
= 5.3^\circ
\]

\[
\theta = \arctan \frac{25}{20000}
\]

\[
= 0.0716^\circ
\]

since \( \theta < NA' \) all light reaching core will be guided

solid angle \( \frac{\pi \times 25}{400 \times 10^{-6}} \times 10^{-12} \approx 4.909 \times 10^{-6} \text{ rad} \)

power in \( \frac{10^{-2} \times 4.909 \times 10^{-6}}{4\pi} \approx 3.906 \text{ nW} \)
When point source is against end of fiber the fraction of light entering the fiber is determined by the solid angle associated with $\theta_0$.

The area of the sphere part of which $AB$ is limiting is

$$
\int_0^{\theta_0} \int_0^{\pi} r^2 \sin \theta \, d\theta \, d\phi
$$

$$
= -2\pi r^2 \cos \theta \bigg|_0^{\theta_0}
$$

$$
= 2\pi r^2 (1 - \cos \theta_0)
$$

The power fraction is

$$
P = \frac{2\pi r^2 (1 - \cos \theta_0)}{4\pi r^2} = \frac{\rho}{2} (1 - \cos \theta_0)
$$

$$
= \frac{10^{-2}}{2} (1 - \cos \theta_0) = 21.94 \mu W
$$
\[
L = \frac{z_2 - z_1}{z_2 + z_1} = \frac{1}{n_1} - 1 = \frac{1 - n_1}{1 + n_1}
\]

\[
R = |e|^2 = \left( \frac{1 - n_1}{1 + n_1} \right)^2 = 0.0343
\]

\[
\theta_1 = 8^\circ
\]

1.955 \sin 8^\circ = 1.35 \sin \theta_2

\[
\theta_2 = 8.96^\circ
\]

1.3 \sin \theta_2 = \sin \theta_3

\[
\theta_3 = 11.68^\circ
\]

P wave impedance one of the form

\[
z = \frac{z_0 \cos \theta}{n}
\]

Ignore the \( z_0 \), it cancels.

\[
z_1, z_2 \approx 0.6806
\]

\[
z_1, z_2 \approx 0.7598
\]

\[
z_3, z_4 \approx 0.9793
\]

\[
k_2 d' = k_2 d_2 \cos \theta_2 = \frac{2\pi}{\lambda} \frac{\lambda_2 \cos \theta_2}{4}
\]

\[
\theta_2 = \frac{\pi}{2} \cos \theta_2 = 1.552
\]

\[
z_3 \approx 0.7598 \left( 0.9793 \cos(1.552) + 0.7598 \sin(1.552) \right)
\]

\[
0.7598 \cos(1.552) + 0.9793 \sin(1.552)
\]

\[
z = \frac{0.01399 + j 0.5777}{0.01428 + j 0.9791}
\]

\[
0.5894 - 0.00561
\]
\[ P = \frac{0.5894 - j0.00569 - 0.6806}{0.5894 - j0.00569 + 0.6806} \]
\[ = \frac{-0.0912 - j0.00569}{1.27 - j0.00569} \]
\[ R = |P| = 0.00518 \]

The power at post 5 is determined by \( R \), the reflection of a cleared, uncontaminated forest.

\[ R = 0.00518 \]

\[ P_2 = \frac{P_2 \left( \frac{1}{2} \right) R \left( \frac{1}{2} \right) (1-R) + P_{51} \left( \frac{1}{2} \right) R \left( \frac{1}{2} \right) (1-R)}{(1-R)} \]
\[ = (1-R) \left[ \frac{R}{8} + \frac{R^2}{32} + \frac{R^3}{128} + \cdots \right] P \]

additional contribution

\[ P_2 \left( 1-R \right) \left[ \frac{R}{128} + \frac{R^3}{512} + \cdots \right] + \text{smaller term} \]
\[ = 0.00415 \text{ mW} \]
The following Bessel function zero
apply:
\[
J_{01}, J_{02}
\]
\[
J_{11}, J_{12}
\]
\[
J_{21}
\]
\[
J_{31}
\]

The cutoff of a \( LP \) mode is determined by the condition

\[
U \frac{J_{m-1}(n)}{J_{m}(n)} = 0
\]
as then \( U = V \) and \( W = 0 \). The mode spreads into the cladding and is no longer guided.

For \( m = 0 \), \( J_0(0) = 1 \) so \( u = 0 \), no solution, hence \( LP_{01} \).

\( LP_{01}, LP_{02}, LP_{03} \) propagate for \( m = 1 \), zero of \( J_1 \), apply.

\( LP_{11}, LP_{12} \) propagate for \( m = 0 \), zero of \( J_1 \), apply.

For \( m = 2 \), zero of \( J_2 \), apply.

\( LP_{21}, LP_{22} \) propagate.
For \( m = 3 \) zero \( J_3 \) apply 
\( LP_{31} \) propagates

For \( m = 9 \) zero \( J_3 \) apply 
\( LP_{91} \) propagates

\( LP \) modes \hspace{1.5cm} \text{modes} \hspace{1.5cm} \text{degeneracy}

\( LP_{01} \hspace{1.5cm} HE_{11} \hspace{1.5cm} 2 \)
\( LP_{02} \hspace{1.5cm} HE_{12} \hspace{1.5cm} 2 \)
\( LP_{03} \hspace{1.5cm} HE_{13} \hspace{1.5cm} 2 \)
\( LP_{11} \hspace{1.5cm} TE_{01}, TM_{01}, HE_{21} \hspace{1.5cm} 4 \)
\( LP_{12} \hspace{1.5cm} TE_{02}, TM_{02}, HE_{22} \hspace{1.5cm} 4 \)
\( LP_{21} \hspace{1.5cm} HE_{31}, EH_{11} \hspace{1.5cm} 4 \)
\( LP_{22} \hspace{1.5cm} HE_{32}, EH_{12} \hspace{1.5cm} 4 \)
\( LP_{31} \hspace{1.5cm} HE_{41}, EH_{21} \hspace{1.5cm} 4 \)
\( LP_{41} \hspace{1.5cm} HE_{51}, EH_{31} \hspace{1.5cm} 4 \)

9 \( LP \) modes \hspace{1.5cm} 17 \( \text{modes} \hspace{1.5cm} 30 \text{mode} \hspace{1.5cm} \text{inclusion} \hspace{1.5cm} \text{degeneracy}

\[ \frac{V^2}{2} \] \hspace{1.5cm} \text{predicts 25 modes} \]