(1) A transmission line of characteristic admittance \(Y_0\) is terminated with an admittance \(Y_L\). Prove that the reflection coefficient on the line is
\[
\rho = \frac{Y_0 - Y_L}{Y_0 + Y_L}
\]
and the transmission coefficient is
\[
\tau = \frac{2Y_0}{Y_0 + Y_L}
\]
What is the reflection coefficient for the current?

A line with \(Z_0 = 50\Omega\) is terminated with an unknown load. 0.2\(\lambda\) from this load the reflection coefficient is \(|\rho| = 0.6, \phi = 50^\circ\). Use the Smith Chart to find the load impedance and standing wave ratio.

What is the measured input impedance 0.4\(\lambda\) from the load?

(2) Use Maxwell’s equations to prove that electromagnetic waves should exist.

A plane, linearly polarized electromagnetic wave is travelling through a lossy dielectric - one for which we can represent the dielectric constant as \(\varepsilon = \varepsilon' - i\varepsilon''\). Derive an equation for the attenuation coefficient of the wave in the case where \(\varepsilon'' \ll \varepsilon'\).

In a conductor \(\varepsilon'' \ll \varepsilon'\) and \(\varepsilon'' = \frac{\sigma}{\omega\varepsilon_0}\). Use these facts to derive an equation for the skin depth.

[Hints: curl curlE = grad divE - \(\nabla^2 E\).
If \(x \ll 1\) then \((1 + x)^n = nx\).]

(3) Where are the pass-bands of the generalized transmission line shown below?

Assume \(L_2C_2 > L_1C_1\).

Draw an \(\omega - \beta (\omega - k)\) diagram for the pass-band(s) and show the attenuation coefficient in the stop-band (where \(\gamma\) is real)

[Hint: \(\gamma = \sqrt{2Y} \)]

(4) A transmission line with \(Z_0 = 50\Omega\) is connected to a 100MHz generator. The phase velocity on the line is \(1.8 \times 10^8\) m/s. A load of \(80 + i30\Omega\) is connected to the end of the line.

Calculate the location closest to the load where matching can be achieved with a shorted 100\(\Omega\) stub connected in shunt to the main line. What length (m) of shorted stub is needed? The phase velocity on the stub is \(2.4 \times 10^8\) m/s.

Repeat for an open stub.
ENEE 381 First Examination
Tuesday, March 2, 1993, 11:00 am - 12:15 pm

ANSWER 3 QUESTIONS
(if more than 3 are answered best 3 will count)

(1) State Maxwell's equations, and explain briefly where each comes from in terms of experimental observations. From Maxwell's equations prove that 3-D waves exist in free space and calculate their velocity. Prove that for linearly polarized plane waves propagating in the z direction

\[ \frac{E_z}{H_y} = -\frac{E_y}{H_x} = \sqrt{\frac{\mu}{\varepsilon}}. \]

What is the average magnitude of the Poynting vector for a plane wave with

\[ E_z = \cos(\omega t - kz + 2\pi/3), \]
\[ H_y = \cos(\omega t - kz + \pi/6)? \]

(2) A transmission line of characteristic admittance \( Y_0 \) is terminated with an admittance \( Y_L \). Prove that the input admittance a distance \( l \) from the load is

\[ Y_I = Y_0 \left[ \frac{Y_L \cos kl + jY_0 \sin kl}{Y_0 \cos kl + jY_L \sin kl} \right] \]

A line with \( Z_0 = 100\Omega \) is terminated with an unknown load. The SWR is found to be 2. A current minimum is observed 0.15\( \lambda \) from the load. What are:

(a) the load?
(b) the reflection coefficient \( \rho \), magnitude and angle?
(c) where can you match the line with an inductor in parallel?

(3) A transmission line of characteristic impedance 50 ohm has a 0.1\( \lambda \), 50 ohm open stub placed 0.2\( \lambda \) from the load. At a distance of 0.1\( \lambda \) further from the load \( |\rho| = 0.5, \phi = 90^\circ \). What is the value of the load? What is the standing wave ratio, and where is the nearest voltage maximum to the load?

(4) For a pair of parallel, plane conductors that form a transmission line show that the telegraphist's equations

\[ \frac{\partial V}{\partial z} = -L \frac{\partial I}{\partial t}, \]

and

\[ \frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}, \]

can be obtained from either a voltage-current analysis of the line or an electric field-magnetic field analysis.

Hint: For a line of unit width, spacing \( a \) the capacitance and inductance per unit length are, respectively,

\[ C = \epsilon /a, \]
\[ L = \mu a. \]

What is the average power flow at 1MHz in the z direction along a lossless line with \( a = 0.1 \), width=1m, if \( E = E_z = 100V/m \)? Hint: \( Z_0 \) (free space)=376.7 ohm, \( \mu_0 = 4\pi \times 10^{-7} \) Henry/m.
ENEE 381  Second Examination  
March 30, 1993. 11:00am - 12:15pm

ANSWER 3 QUESTIONS - If more than 3 are answered best 3 will count

(1) Prove that the attenuation coefficient of a two-conductor transmission line with series inductance and resistance per unit length L and R, respectively, and shunt capacitance and conductance per unit length C and G, respectively is

$$\alpha = \frac{1}{Z} \left[ G Z_o + \frac{R}{Z_o} \right]$$

where $Z_o$ is the characteristic impedance of the line. (6 points)
It is wise to prove initially that

$$\alpha = \frac{W_L}{2W_T}$$

where $W_L$ is the power dissipated per unit length at a particular position where $W_T$ is the power being transferred along the line.

A parallel conductor line has conductors of surface resistance 100 ohm, width 100\mu m, spacing 10\mu m, characteristic impedance 10 ohm, and uses a lossless dielectric. What is the attenuation coefficient for waves on the line? (4 points)

(2) A transmission line can be represented per unit length as

\[ \begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{schematics/line.png}}
\end{array} \]

Calculate
(a) $k$ (2 points)
(b) $V_F$ (2 points)
(c) $V_J$ (2 points)
(d) The value of the cutoff frequencies (if any) (2 points)

Sketch the $\omega-k$ diagram. (2 points)
ADER 3 QUESTIONS

(1) Where are the pass-bands of the following transmission line? Per unit length the line can be represented as

Sketch an $\omega - \beta$ diagram the behaviour of the propagation constant $\beta$ in the pass-bands and the attenuation constant $\alpha$ in the stop-band. In what frequency range is this a backward wave line?

(2) Determine the input impedance, magnitude and phase of the reflection coefficient and SWR of the following transmission line arrangement, measurements being made across AB. Where is the closest current maximum to the left of AB?

(3) Derive expressions for the attenuation and propagation constants $\alpha$ and $\beta$, respectively, in a lossy dielectric with $\varepsilon''/\varepsilon' < 1$. A plane electromagnetic wave has intensity 1 W m$^{-2}$ just inside a dielectric with $\varepsilon'' = 3.78$, $\varepsilon''/\varepsilon' = 10^{-4}$. What is its intensity 1 km inside the dielectric? What is the phase angle between its electric and magnetic fields. Assume that $\mu = 1$.

(4) Show that the effective impedance for plane electromagnetic waves polarized orthogonal to the plane of incidence and incident from a medium of characteristic impedance $Z$ at angle $\Theta$ is $Z/\cos \Theta$. If such a wave strikes a boundary at Brewster's angle calculate the fraction of its energy which is reflected. Take $n_1 = 1$, $n_2 = 1.5$

(5) A microwave television link has a transmitter power of 100W at 3GHz and can be assumed to transmit uniformly into a cone of 0.1 str. The receiver is situated 10km away and is protected by a plexiglass ($\varepsilon'' = 2.6$) radome 3cm thick. Calculate the received intensity on a rainy day when the intensity attenuation coefficient of the atmosphere is $2 \times 10^{-4}$ m$^{-1}$. 

$\varepsilon_0 = 8.85 \times 10^{-12}$ farad m$^{-1}$

$\mu_0 = 4\pi \times 10^{-7}$ henry m$^{-1}$
ANSWER THREE QUESTIONS

(1) A 50 ohm transmission line is terminated with a load of 30 + j 70 ohm.

Calculate

(a) |ρ|

(b) ϕ

(c) The SWR

(d) The position of the closest voltage minimum to the load

Where is the closest point to the load where the line could be matched with a shunt inductor?

(2) A transmission line can be represented per unit length as

What are the cut off frequencies? Sketch an ω-β diagram to show the behavior of β and α in their appropriate frequency regions.
What would be the zero frequency attenuation constant if the transmission line, per unit length, were

Assume that this is a low-loss line.

Derive expressions for the total electric and magnetic field components of a plane electromagnetic wave polarized perpendicular to the plane of incidence striking a planar, perfect conductor at angle $\theta$. Prove that the effective impedance in the normal direction in this case is $Z/\cos \theta$.

How far from the surface is the first minimum if $\lambda = 3\text{cm}$ and $\theta = 60^\circ$?

A planar, parallel-sided slab of thickness 0.5cm has dielectric constant 4. A plane electromagnetic wave of frequency 40 GHz strikes the slab normally. What fraction of the energy is transmitted? The medium on both sides of the slab is air.
THIRD EXAMINATION

ANSWER 3 QUESTIONS

(1) Prove that

\[ H_x = \frac{1}{\gamma^2 + k^2} \left( \omega \epsilon_0 \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x} \right) \]

\[ H_y = \frac{1}{\gamma^2 + k^2} \left( \omega \epsilon_0 \frac{\partial E_z}{\partial x} + \gamma \frac{\partial H_z}{\partial y} \right) \]

\[ E_x = \frac{1}{\gamma^2 + k^2} \left( \gamma \frac{\partial E_z}{\partial x} + \omega \mu_0 \frac{\partial H_z}{\partial y} \right) \]

\[ E_y = \frac{1}{\gamma^2 + k^2} \left( -\gamma \frac{\partial E_z}{\partial y} + \omega \mu_0 \frac{\partial H_z}{\partial x} \right) \]

\[ \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = -\left( \gamma^2 + k^2 \right) E \]

Thereby, determine the field components of a TM\textsubscript{m0} mode between parallel, perfectly-conducting planes.

(2) Show that TM modes propagating inside a closed perfect conductor always exhibit cut-off behavior. Why can TEM modes not propagate in such a system?

(3) A TE\textsubscript{21} mode propagating at twice its cut-off frequency inside an air-filled waveguide strikes a planar dielectric of dielectric constant 4 which fills the rest of the guide. What are the magnitude and phase of the reflection coefficient? A quarter-wavelength thick layer is applied to the boundary to reduce the reflection to zero. What should its dielectric constant be?
The waveguide impedance of a TE mode is

\[ Z_{TE} = \frac{Z_{\text{medium}}}{\sqrt{1 - (\omega_c/\omega)^2}} \]

(4) Derive expressions for all the field components and impedance of a TM\(_{mn}\) mode in a rectangular waveguide of dimensions a x b. (You may use information given in question (1)). What are the waveguide impedance, phase velocity and group velocity of a \(CEH_2\) \(TM_{mn}\) mode at three times its cut-off frequency?
ENEE 381 Final Examination
Saturday, May 21, 1988, 10:30 -12:30

ANSWER 4 QUESTIONS
(if more than 4 are answered best 4 will count)

(1) Prove that on a shorted section of lossless transmission line $\lambda/4$ long that the total stored energy at any time is

$$U = \frac{V^2 L \lambda}{4 Z_0^2} = \frac{V^2 C \lambda}{4}$$

where $L$ and $C$ are the inductance and capacitance per unit length, respectively, of the line.

If the line has series resistance per unit length $R$ and shunt conductance per unit length $G$, show that the $Q$ of a shorted $\lambda/4$ section is

$$Q = \frac{\omega_0 C Z_0}{G Z_0 + (R/Z_0)}.$$  

[Hint: the $Q$ is defined as

$$Q = \frac{\omega_0 U}{W_L},$$

where $W_L$ is the power dissipated on the section of line and $\omega_0$ is the drive angular frequency]

(2) On the lossless transmission line shown below, the line is matched at AB. calculate the lengths $l_1$ and $l_2$. The generator frequency is 10GHz and the phase velocity on the line is $1.8 \times 10^8$ m/s.

![Diagram oflossless transmission line](image)

(3) Derive expressions for all the non-zero field components of the TE$_{10}$ in a rectangular waveguide of cross-section axb.

Prove that the attenuation of the TE$_{10}$ mode caused by imperfect conductors is

$$\alpha_e = \frac{R_s}{bZ_0 \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} \left[ 1 + \frac{2b}{a} \left(\frac{\omega_c}{\omega}\right)^2 \right].$$
(4) For the TE modes propagating between parallel planes prove that the phase velocity \( v_p \), group velocity \( v_g \), mode impedance \( Z_{TE} \), and guide wavelength \( \lambda_g \) are:

\[
\begin{align*}
  v_p &= \frac{c}{\sqrt{1 - \left(\frac{\omega_e}{\omega}\right)^2}} \\
  v_g &= c\sqrt{1 - \left(\frac{\omega_e}{\omega}\right)^2} \\
  Z_{TE} &= \frac{Z_0}{\sqrt{1 - \left(\frac{\omega_e}{\omega}\right)^2}} \\
  \lambda_g &= \frac{\lambda}{\sqrt{1 - \left(\frac{\omega_e}{\omega}\right)^2}}.
\end{align*}
\]

What is the attenuation per unit length of a TE\(_{10} \) mode with \( \omega = \omega_e/2 \) in an air-filled guide with \( a = 10 \)mm?

Hint: For a TE wave between parallel planes

\[
E_y = \frac{-i\omega\mu}{K}B \sin Kx
\]

\[K^2 = \gamma^2 + k^2.\]

(5) Discuss three of the following:

(a) The interpretation of \( \mathbf{E} \times \mathbf{H} \) as a vector describing energy flow.

Hint: \( \text{div}(\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \text{curl} \mathbf{E} - \mathbf{E} \cdot \text{curl} \mathbf{H} \)

(b) The boundary condition for the TM\(_{m1} \) mode in a circular waveguide.

(c) The effect of filling a rectangular waveguide completely with a lossless dielectric of dielectric constant 4 if the guide was originally designed to propagate only the TE\(_{10} \) mode in air.

(d) How to prevent unwanted modes from propagating in a waveguide.

(6) A length of lossless transmission line is first short-circuited and then open-circuited. In the first case the input impedance is measured as \( Z_1 \), in the second case it is \( Z_2 \). Prove that

\[Z_1Z_2 = Z_0^2\]

Prove any equations that you use, where \( Z_0 \) is the characteristic impedance of the line.

In an infinite transmission line, a shunt leak develops that is equal to the characteristic impedance of the line. Prove that 1/9 of the incident power is reflected, 4/9 is transmitted, and 4/9 is dissipated in the leak.
1. A 75 ohm transmission line is terminated with a load of 50+j125 ohm. Calculate:
   (a) $|\xi|$ (2 pts.)
   (b) $\phi$ (2 pts.)
   (c) the standing wave ratio (1 pt.)
   (d) the nearest current maximum to the load (1 pt.)

0.9$\lambda$ from the load an open 75 ohm stub 0.2$\lambda$ long is connected in shunt to the main line. Where is the nearest point (moving towards the generator) where a second, shorted stub can be connected to match the line? (2 pts.) If this second stub is a 150 ohm stub, how long should it be for matching? (2 pts.)

2. A lossy transmission line can be represented per unit length as

   ![Diagram showing transmission line parameters](image)

   Calculate:
   (a) Attenuation per unit length from lossy conductors (4 pts.)
   (b) Attenuation per unit length from the lossy dielectric (4 pts.)
   (c) Characteristic impedance of the line (this will be complex) (2 pts.)

   The following information may be useful:

   Surface impedance is $Z_s = \sqrt{Z_c Z_n}$, where $Z_n$ is internal inductance.

   $Z_c = \sqrt{\frac{\mu}{\varepsilon}}$, where $\mu = \mu_0 \mu_\epsilon$ H/m, and $\varepsilon = \varepsilon_0 \varepsilon_\epsilon$ F/m.

   Hint: $Y = \frac{1}{\omega C}$

3. A charged particle starts from rest at the origin of coordinates in a region where there is a uniform electric field $E$ parallel to the x-axis, and a uniform magnetic flux density $B$ parallel to the z-axis. Show that the coordinates of the particle at a time $t$ later will be

   $x = \left(\frac{E}{\omega B}\right) \left(1 - \cos(\omega t)\right)$

   $y = \left(\frac{E}{\omega B}\right) \left(\omega t - \sin(\omega t)\right)$

   $z = 0$

   where $\omega = \frac{E}{B}$ (This path is called a cycloid)
(4) A plane wave polarized with its electric vector perpendicular to the plane of incidence (an S-wave) is incident at angle $\theta_i$ on the planar boundary between two lossless dielectrics of refractive index $n_1$ and $n_2$, respectively. Derive all the field components of incident (2 pts.), reflected (2 pts.), and transmitted (3 pts.) waves. 
Prove that angle of reflection = angle of incidence (1 pts.)
Prove that $n_1 \sin \theta_i = n_2 \sin \theta_2$, where $\theta_2$ is the angle of refraction.

Hint: the characteristic impedance for S-waves is 
\[ Z' = \frac{Z}{\cos \theta} \]
\[ \tau = \frac{Z}{Z + \frac{Z}{2}} \]

Write the fields in terms of $\mathbf{E}$ and $\mathbf{H}$.

(5) From the following equations derive equations for all five field components of a TE$_{11}$ mode in a rectangular waveguide of dimensions $a \times b$. (7 pts.)
\[ \frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} = -k_c^2 \mathbf{E} \]
\[ \frac{\partial^2 \mathbf{H}}{\partial x^2} + \frac{\partial^2 \mathbf{H}}{\partial y^2} = -k_c^2 \mathbf{H} \]
\[ \mathbf{E}_x = -\frac{1}{(b^2 + k^2)} \left( \frac{\partial}{\partial y} \right) \left( \omega_0 \frac{\partial \mathbf{H}_z}{\partial y} \right) \]
\[ \mathbf{E}_y = \frac{1}{(b^2 + k^2)} \left( \omega_0 \frac{\partial \mathbf{H}_z}{\partial x} \right) \]
\[ \mathbf{H}_x = \frac{1}{(b^2 + k^2)} \left( -b \frac{\partial \mathbf{H}_z}{\partial x} \right) \]
\[ \mathbf{H}_y = -\frac{1}{(b^2 + k^2)} \left( \frac{\partial \mathbf{H}_z}{\partial y} \right) \]

What is the cutoff frequency of the TE$_{11}$ mode in an air-filled rectangular waveguide of dimensions 5mm X 2mm? (3 pts.)

(6) A TE$_{01}$ mode is propagating in a cylindrical air-filled waveguide of internal radius 5mm at a frequency twice its cutoff frequency. The propagating wave strikes a slab of material that fills the guide cross section and is 7mm thick. The dielectric constant of the slab material is $\varepsilon_r = 5$. Calculate:
(a) The fraction of power that is transmitted. (6 pts.)
(b) The standing wave ratio to the left of the slab. (1 pt.)
(c) The location of the nearest transverse magnetic field maximum to the left of the slab. (1 pt.)
(d) The smallest change in thickness of the slab that would lead to 100% power transmission. (2 pts.)

Useful information: $K_a = 3.832$,
\[ \gamma = \frac{a}{\lambda} \]
\[ Z_{TE} = \frac{Z}{\sqrt{1 - \left( \frac{\omega_c}{\omega} \right)^2}} \]
ANSWER 4 QUESTIONS

1. Show that in order for plane TEM waves to propagate there can be no losses in the system, either from imperfect conductors or dielectrics. Show also that in an unbounded free space these waves are the only kind of wave that can propagate in a specific direction. Prove that the impedance relations for these waves are

\[ \frac{E_X}{H_y} = - \frac{E_y}{H_x} = \sqrt{\frac{\mu\mu_0}{\varepsilon\varepsilon_0}} \]

2. Show that the vector \( \mathbf{P} = \mathbf{E} \times \mathbf{H} \) can legitimately be regarded as the vector representing energy flow at a point in an e.m. field. Derive an expression for the radiation pressure on a totally absorbing surface illuminated normally by a plane wave of frequency \( \nu \), in terms of the average Poynting vector. What is this pressure if \( \mathbf{P} = 10^{18} \text{ watt cm}^{-2} \) and \( \lambda = 1.06 \mu\text{m} \)?

You may assume that the momentum of a photon is \( h/\lambda \).

3. A transmission line of impedance \( Z_0 \) is terminated with an impedance \( Z_L \). Prove that the voltage reflection coefficient on the line is

\[ \rho = \frac{Z_L - Z_0}{Z_L + Z_0} \]

What is the current reflection coefficient?

A line with \( Z_0 = 100 \text{ ohm} \) and an unknown \( Z_L \) can be perfectly matched with a shorted 100 ohm line 0.2\( \lambda \) long connected in shunt with the main line at 0.35\( \lambda \) from the load. What is \( Z_L \)? If \( Z_L \) were doubled and the shunt line removed, what would be \( |\rho| \phi \), and the SWR?
4. Determine the total fields when a TEM plane wave is incident on a perfect planar conductor at angle $\theta$ with its $E$ vector in the plane of incidence. Show that the impedance in this case is $\frac{E}{H_y} = Z_0 \cos \theta$. If the conductor is in the $z = 0$ plane, show that the electric field component parallel to the conductor will be zero at distances $nd$ in front of the conductor where

$$d = \frac{\lambda}{2 \cos \theta}$$

Use this result to determine the cutoff frequency of a TM wave propagating between parallel planes spaced by $1\text{ mm}$ and spaced by a dielectric with $\varepsilon = 2.5$.

5. A plane TEM wave with its $E$ vector perpendicular to the plane of incidence strikes a planar dielectric slab of dielectric constant 5 at $45^\circ$. If the slab is $1\text{ cm}$ thick and the wave has a frequency of $25\text{ GHz}$ what fraction of the incident energy is transmitted? (Hint, you may find information in question 4 helpful) What are the phase angle of the reflected wave, the SWR and the location of the closest maximum of the $E$ field to the boundary?

6. By deriving an expression for the $E$ field as a function of distance from the surface of a non-perfect, but good, conductor show that the ratio of very high frequency resistance to D.C. resistance of a round conductor of radius $r_0$ can be written

$$\frac{R_{h.f}}{R_0} = \frac{r_0}{2\delta}$$

where $\delta$ is the skin depth.
7. Derive expressions for all the field components of a TM mode in a rectangular waveguide of dimensions \( a \times b \).

Show that the impedance for these modes is

\[
Z_{TM} = Z_0 \sqrt{1 - \left( \frac{\omega_c}{\omega} \right)^2}
\]

if \( a = 2 \text{ cm} \) \( b = 1 \text{ cm} \)

determine the phase and group velocity of a \( TM_{22} \) mode at three times its cutoff frequency.

8. A \( TE_{11} \) mode of frequency 30 GHz is propagating in a perfectly conducting circular waveguide of radius 1 cm which is filled with air (\( \varepsilon = 1 \)) when it strikes a planar dielectric slab (\( \varepsilon = 3 \)) of thickness 1 cm which fills the cross-section of the guide. Calculate the thickness and dielectric constant of the layers which must be placed on the glass slab to reduce the reflected wave in the guide to zero. What is the fraction of energy reflected without these layers?

You may assume that the cutoff frequency for the \( TE_{11} \) mode is a circular guide of radius \( a \) is

\[
\nu_c = \frac{1.84}{2\pi a \sqrt{\mu \mu_0 \varepsilon \varepsilon_0}}
\]

The guide wavelength of the mode is

\[
\lambda_g = \frac{\lambda}{\sqrt{1 - (\nu_c/\nu)^2}}
\]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ henry m}^{-1} \]
\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ farad m}^{-1} \]
\[ c = 3 \times 10^8 \text{ m s}^{-1} \]
\[ z_0 \text{ (free space)} = 377 \text{ ohm} \]
\[ h \text{ (Planck's constant)} = 6.625 \times 10^{-34} \text{ Joule s} \]
(1) Show that Maxwell's equations predict the existence of waves in free space \( (\rho = 0, j = 0) \). (10 marks) How is the wave equation modified if the wave propagates in a medium with conductivity \( \sigma \)? (5 marks) From these considerations show that the skin depth of a semi-infinite planar conductor is, if \( \omega \varepsilon \varepsilon_0 < < \sigma \), \( \delta = \left( \frac{2}{\sigma \omega \varepsilon_0} \right)^{1/2} \) (7 marks)

(2) For a transmission line system which per unit length can be represented in the form

![Diagram of transmission line system](image)

where \( L_1 C_1 \gg L_2 C_2 \), in what frequency range is the wave on the line propagating rather than attenuated? (13 marks) Sketch the variation of the propagation constant \( \beta \) with frequency \( \omega \) in the propagating frequency range and also the variation of the attenuation constant \( \alpha \) with \( \omega \) outside this range. (12 marks)
(3) Show from the two Maxwell's equations involving the curl that for waves of the form

\[ \frac{E_0}{E} = E(x, y) e^{i\omega t - \gamma z} \]
\[ \frac{H_0}{H} = H(x, y) e^{i\omega t - \gamma z} \]

\[ H_x = \frac{1}{\gamma^2 + k^2} \left( i \omega \varepsilon \varepsilon_0 \frac{\partial E_z}{\partial y} - \gamma \frac{\partial H_z}{\partial x} \right) \quad (3 \text{ marks}) \]

\[ H_y = -\frac{1}{\gamma^2 + k^2} \left( i \omega \varepsilon \varepsilon_0 \frac{\partial E_z}{\partial x} + \gamma \frac{\partial H_z}{\partial y} \right) \quad (3 \text{ marks}) \]

\[ E_x = -\frac{1}{\gamma^2 + k^2} \left( \gamma \frac{\partial E_z}{\partial x} + i \omega \varepsilon \varepsilon_0 \frac{\partial H_z}{\partial y} \right) \quad (3 \text{ marks}) \]

\[ E_y = \frac{1}{\gamma^2 + k^2} \left( -\gamma \frac{\partial E_z}{\partial y} + i \omega \varepsilon \varepsilon_0 \frac{\partial H_z}{\partial x} \right) \quad (3 \text{ marks}) \]

where \( k^2 = \omega^2 \mu \mu_0 \varepsilon \varepsilon_0 \)

Show also that

\[ \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = - (\gamma^2 + k^2) E \quad (3 \text{ marks}) \]

\[ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = - (\gamma^2 + k^2) H \quad (3 \text{ marks}) \]

How can you use these last two equations to demonstrate that TEM modes cannot propagate in a closed waveguide with perfectly conducting walls (7 marks)
(4) Using the equations given in question (3) derive the field components of a TM mode propagating between parallel perfectly conducting infinite planes of spacing a. (15 marks) Show that the cutoff frequency for the TM$_{n0}$ mode is

$$\omega_c = \frac{n \pi c}{a} \quad \text{where} \quad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

(5 marks)

What is the impedance for this mode? (5 marks)

(5) A 75 ohm transmission line is terminated with an impedance of 225 - i75 ohm. 0.2λ from the load a shorted stub 5.1λ long is connected in shunt to the line. What is the reflection coefficient, phase angle and SWR on the line on the generator side of the stub? (15 marks) Where could you put a second short-circuited stub to obtain perfect matching?

(6) A plane electromagnetic wave with its electric vector in the plane of incidence strikes a parallel sided dielectric slab of refractive index 2 bounded on both sides by air at an angle of 45°. If the frequency of the input wave is $5 \times 10^{14}$ Hz and the thickness of the slab is 1μm what fraction of the input energy is transmitted? (15 marks) What thickness and refractive index of a coating on each face of the slab would you use to reduce reflections to zero? (10 marks)
(7) Starting from the equations given in question (3) derive the field components of a TE mode in a rectangular waveguide. (15 marks) Show that the characteristic impedance of the mode is

\[ Z_{TE} = \frac{Z_0}{\left[1 - \left(\frac{\psi_c}{\psi}\right)^2\right]^{1/2}} \]

where \( Z_0 = \sqrt{\frac{\mu_0}{\varepsilon \varepsilon_0}} \) (5 marks)

What is the cutoff frequency for the TE\(_{22}\) mode in a square guide of side 1 cm. (5 marks)

(8) A TE\(_{01}\) mode of angular frequency 300 GHz is propagating in a perfectly conducting air-filled circular waveguide of radius 1 cm when it strikes a planar parallel sided glass slab (refractive index = 1.5) of thickness 1 cm which extends right across the circular section of the guide 2 cm beyond the first slab is a second slab of thickness 0.1 cm.

The space between the two slabs and to the far side of the second slab is also filled with air. What fraction of the incident energy that reaches the first slab penetrates beyond the second slab and what is the S.W.R. on the generator side of the first slab. You may assume that the angular cutoff frequency for this TE\(_{01}\) mode in a circular waveguide of radius \( a \) is

\[ \psi_c = \frac{2\pi}{1.64a\sqrt{\frac{\mu_0}{\varepsilon \varepsilon_0}}} \]
(1) A transmission line of characteristic impedance 75 ohm is terminated with an impedance 50+j125 ohm. 0.1\lambda from the load a 150ohm shorted stub 0.2\lambda long is connected in shunt to the main line. What are:
(a) The reflection coefficient in magnitude and phase at this point?
(b) The standing wave ratio?
(c) Where is the nearest current minimum that is greater than 0.1\lambda from the load?
(d) Where is the nearest point greater than 0.1\lambda from the load where the line can be latched with an open 75 ohm stub?

(2) Where is the pass-band of the generalized transmission line shown below? Note that \( L_1C_1 > L_2C_2 \).

\[
\begin{array}{c}
L_1 \\
\hline
C_1 \\
\hline
\hline
L_2 \\
\hline
C_2 \\
\end{array}
\]

Draw an \( \omega - \beta (\omega - k) \) diagram for the pass-band and show the attenuation coefficient in the stop-bands (where \( \gamma \) is real)

[Hint: \( \gamma = \sqrt{ZY} \)] What is the value of \( \alpha \) at zero frequency?

(3) Explain the concept of group velocity by using the simple example of two wave of slightly different frequency travelling through a medium. Prove that

\[
v_g = \frac{v_p}{1 - (\omega/v_p)(dv_p/d\omega)}.
\]

If in a certain medium \( k = 3 - \omega/c_0 + 0.1(\omega/c_0)^2 \), what are the phase velocity and group velocity at 1GHz? (\( c_0 = 3 \times 10^8 \) m/s).

(4) By using the impedance concept for waves striking a boundary at angle \( \theta \) prove the following:
(a) a half-wavelength thick slab does not reflect
(b) there is an angle for which \( \rho = 0 \) provided the wave has its electric field vector in the plane of incidence. Derive an equation for this angle.
(c) what is the thickness and refractive index of a layer that will serve as a quarter-wave transformer on a boundary when the waves are incident at angle \( \theta \) and are polarized perpendicular to the plane of incidence. Take the refractive indices of the 2 media as \( n_1 \) and \( n_2 \).
(d) if \( n_1 = 1 \) and \( n_2 = 2 \), \( \theta = 45^\circ \), and the wave is polarized perpendicular to the plane of incidence, what is the thickness and refractive index of the quarter-wave transformer.

Hint: the effective thickness of a slab is \( d_2 \cos \theta_2 \) when the angle in the slab is \( \theta_2 \).