

ENEE 381 Problem Set #3

9/31/04- due 10/12/04

THE FIRST EXAMINATION IS ON NOVEMBER 2, 2004

Questions like (1) - (5) could be on the first examination.

(1) The electric vector of a wave propagating in the z -direction varies according to

$$E_y = E_0 \cos(\pi x/2a) e^{j(\omega t - kz)},$$

where $E_0=1\text{V/m}$, $a=1\text{m}$. The frequency of the wave is $\nu=100\text{MHz}$. How much energy flow per second passes through the region $-1 \leq x \leq 1$ (m), $-1 \leq y \leq 1$ (m). Hint: this is not a uniform plane wave.

(2) A point source transmitter at $(0,0,0)$ emits a total power of 5W . What is the value of the Poynting vector at the point $(10,10,10)$? What is the total power flux into the surface bounded by the two concentric spheres $R=5$, and $R=7$?

(3) A point source transmitter at $(0,0,0)$ emits a total power of 5W . What is the total power flux through the surface of a cube centered at $(0,0,0)$ with sides of length 1m ?

(4) A plane wave with magnetic field $H_y = 1\text{A/m}$ and electric field E_x traveling in the z -direction through a vacuum strikes an infinite planar copper medium. What is the value of the electric field and magnetic field at the surface of the sheet? What is the value of the surface resistance R_s ? How much energy is dissipated per unit area of the copper? For copper $\sigma=5.8 \times 10^7$ S/m.

(5) How are the answers to question (4) modified if the wave is traveling through a dielectric with $\epsilon_r=30$ when it strikes the copper

(6) RW&vD 4.5a

(7) RW&vD 4.5b

(8) RW&vD 3.17a

ENEE 381
SOLUTIONS

Problem Set # 3

(1)
$$E_y = E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)}$$

From the curl equation $\frac{\partial E_y}{\partial z} = \mu \frac{\partial H_x}{\partial t}$

so
$$-jk E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} = \mu j\omega H_x$$

$$H_x = -\frac{k E_0}{\mu \omega} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)}$$
$$= -\frac{E_0}{Z} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)}$$

$Z = \sqrt{\frac{\mu \nu}{\epsilon_0 \epsilon_r}}$. Assuming $\epsilon_r = \mu_r = 1$ $Z = 376.7 \text{ ohm}$

The Poynting vector is
$$S_z = -E_y H_x = \frac{E_0^2}{Z} \cos^2\left(\frac{\pi x}{2a}\right) e^{2j(\omega t - kz)}$$

Time averaging
$$\overline{S}_z = \frac{E_0^2}{2Z} \cos^2\left(\frac{\pi x}{2a}\right)$$

Power flow required is $\frac{E_0^2}{2Z} \int_{-1}^1 \int_{-1}^1 \cos^2\left(\frac{\pi x}{2a}\right) dx dy = \overline{S}_z$

$$\overline{S}_z = \frac{E_0^2}{2Z} \int_{-1}^1 \left(1 + \cos\left(\frac{\pi x}{a}\right)\right) dx$$

$$\overline{S}_z = \frac{E_0^2}{2Z} \frac{a}{\pi} \left(x + \sin\frac{\pi x}{a}\right) \Big|_{-1}^1, \text{ with } E_0 = 1 \text{ V/m, } a=1$$

$\overline{S}_z = 845 \mu\text{W}$

(2) The distance from $(0,0,0)$ to $(10,10,10)$ is $R = \sqrt{100+100+100} = \sqrt{300}$

The Poynting vector at this point is

$$S_r = \frac{P}{4\pi R^2} \hat{r} = \frac{5}{4\pi \times 300} = 1.33 \text{ mW/m}^2$$

\uparrow
 area of surface of sphere

Flux of energy into $R=5$ flows out of $R=7$ so no net flux into the volume between the 2 spheres

(3) all the power of 5 W flows outwards through the surface of the cube surrounding the origin. So, ANSWER IS 5 W

(4) Copper is a very good conductor so E_x (at the surface) ≈ 0

H_y at the surface is 2 A/m (incident wave + reflected wave)

Therefore the surface current $J_s = 2 \text{ A/m}$

The surface resistance is $R_s = \frac{1}{\sigma \delta}$

$\delta = \frac{1}{\sqrt{\pi \omega \mu \sigma}}$. The electric field at the surface is $(E_x)_0 = (1+j) \frac{J_s}{\sigma \delta}$

The energy dissipated per unit area
is $\frac{1}{2} R_s J_s^2$

For copper $\delta = \frac{0.066}{\sqrt{\omega}}$

For $\omega = 100 \text{ MHz}$ $\delta = 6.6 \times 10^{-6} \text{ m}$

$\frac{1}{\sigma \delta} = 2.6 \times 10^{-3} \text{ ohm} = R_s$

energy dissipated is 5.225 mW/m^2

$(E_x)_0 = (1+j) 5.25 \times 10^{-3} \text{ V/m}$

(see page 159 in text)

(5) If the wave travels through a dielectric
with $\epsilon_r = 30$, the magnetic field is
still $H_y = 1 \text{ A/m}$ (incident wave) $H_y = 2 \text{ A/m}$
at the surface

so J_s is the same, R_s is the same
and $(E_x)_0$ is the same. In other words

answer is same as question (4).

(6)

$$4.5a: R_{hf} = \frac{R_s}{2\pi r_0} = \frac{1}{\sigma \delta 2\pi r_0}, \quad R_o = \frac{1}{\pi r_0^2 \sigma}$$

$$\text{so } \frac{R_{hf}}{R_o} = \frac{1}{r \delta 2\pi r_0} \cdot \pi r_0^2 \sigma = \frac{r_0}{2\delta} \quad \text{Q.E.D.}$$

(7)

$$4.5b: R_{lf} = \frac{1}{\pi r_0^2 \delta} \left[1 + \frac{1}{48} \left(\frac{r_0}{\delta} \right)^2 \right]; \quad \frac{1}{48} \left(\frac{r_0}{\delta} \right)^2 = .02 \quad \text{for } \frac{r_0}{\delta} = .98$$

$$\text{From table 3.17, } \delta_{Cu} = \frac{.066}{\sqrt{f}} = 6.6 \times 10^{-4} \text{ m at } 10 \text{ kHz}$$

$$= 6.6 \times 10^{-5} \text{ m at } 1 \text{ MHz}$$

$$\text{so } r_0 \approx .65 \text{ mm at } 10 \text{ kHz, } .065 \text{ mm at } 1 \text{ MHz.}$$

$$\text{For brass, } \delta_{brass} = \frac{.127}{\sqrt{f}} = 1.27 \times 10^{-3} \text{ m at } 10 \text{ kHz}$$

$$= .127 \times 10^{-3} \text{ m at } 1 \text{ MHz}$$

$$\text{so } r_0 = 1.24 \text{ mm at } 10 \text{ kHz, } .124 \text{ mm at } 1 \text{ MHz}$$

(8)

$$3.17a \quad R_s = \left[\frac{\pi f \mu}{\sigma} \right]^{\frac{1}{2}}, \quad \mu = \mu_r (4\pi \times 10^{-7})$$

Tin:

$$R_s = \left[\frac{4\pi^2 \times 10^{-7} f}{10^7} \right]^{\frac{1}{2}} = 2\pi \times 10^{-7} \sqrt{f} = \begin{cases} 4.87 \times 10^{-6} \Omega @ 60 \text{ Hz} \\ 1.99 \times 10^{-5} \Omega @ 1 \text{ kHz} \\ 6.28 \times 10^{-4} \Omega @ 1 \text{ MHz} \end{cases}$$

Iron:

$$\mu_r = 500, \text{ so } \sqrt{500} \times \text{above} = \begin{cases} 1.09 \times 10^{-4} \Omega @ 60 \text{ Hz} \\ 4.44 \times 10^{-4} \Omega @ 1 \text{ kHz} \\ 1.4 \times 10^{-2} \Omega @ 1 \text{ MHz} \end{cases}$$