ENEE 381 Problem Set #3

9/31/04- due 10/12/04

THE FIRST EXAMINATION IS ON NOVEMBER 2, 2004

Questions like (1) - (5) could be on the first examination.

(1) The electric vector of a wave propagating in the z-direction varies according to

\[ E_y = E_0 \cos(\pi x / 2a) e^{j(\omega t-kz)}, \]

where \( E_0 = 1 \text{V/m} \), \( a = 1 \text{m} \). The frequency of the wave is \( \nu = 100 \text{MHz} \). How much energy flow per second passes through the region \(-1 \leq x \leq 1 \) (m), \(-1 \leq y \leq 1 \) (m). Hint: this is not a uniform plane wave.

(2) A point source transmitter at \((0,0,0)\) emits a total power of 5W. What is the value of the Poynting vector at the point \((10,10,10)\)? What is the total power flux into the surface bounded by the two concentric spheres \( R = 5 \), and \( R = 7 \)?

(3) A point source transmitter at \((0,0,0)\) emits a total power of 5W. What is the total power flux through the surface of a cube centered at \((0,0,0)\) with sides of length 1m?

(4) A plane wave with magnetic field \( H_y = 1 \text{A/m} \) and electric field \( E_x \) traveling in the z-direction through a vacuum strikes an infinite planar copper medium. What is the value of the electric field and magnetic field at the surface of the sheet? What is the value of the surface resistance \( R_s \)? How much energy is dissipated per unit area of the copper? For copper \( \sigma = 5.8 \times 10^7 \text{S/m} \).

(5) How are the answers to question (4) modified if the wave is traveling through a dielectric with \( \varepsilon_r = 30 \) when it strikes the copper

(6) RW&vD 4.5a

(7) RW&vD 4.5b

(8) RW&vD 3.17a
\[ E_y = E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} \]

From the wave equation \( \frac{\partial^2 E_y}{\partial t^2} = \mu \frac{\partial^2 H_x}{\partial z^2} \)

so
\[-j k E_0 \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} = \mu j \omega H_x \]

\[ H_x = -\frac{k E_0}{\mu \omega} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} \]

\[ = -\frac{E_0}{2} \cos\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} \]

\[ Z = \sqrt{\frac{\mu \omega}{\rho c}} \quad \text{Assuming} \quad \epsilon_r = \mu = 1 \quad Z = 376.7 \Omega \text{m} \]

The Poynting vector is
\[ S_z = -E_y H_x = \frac{E_0^2}{Z} \cos^2\left(\frac{\pi x}{2a}\right) e^{j(\omega t - kz)} \]

Time averaging \( S_z = \frac{E_0^2}{Z} \cos^2\left(\frac{\pi x}{2a}\right) \)

Power flux required is \( \frac{E_0^2}{Z} \int_{-1}^{1} \int_{-1}^{1} \cos^2\left(\frac{\pi x}{2a}\right) dx \, dy = \bar{S}_z \)

\[ \bar{S}_z = \frac{E_0^2}{Z} \int_{-1}^{1} \left(1 + \cos^2\left(\frac{\pi x}{a}\right)\right) dx \]

\[ \bar{S}_z = \frac{E_0^2}{Z} \frac{a}{\pi} \left(\pi \cos \frac{\pi a}{2}\right)^2 \]

with \( E_0 = 14 \text{V/m}, a = 1 \)

\[ \bar{S}_z = 84.5 \mu \text{W} \]
(2) The distance from \((0, 0, 0)\) to \((10, 10, 10)\) is 
\[ R = \sqrt{100 + 100 + 100} = \sqrt{300} \]

The Poynting vector at this point is 
\[ S = \frac{P}{4\pi R^2} \]
\[ = \frac{5}{4\pi \times 300} \]
\[ \approx 1.33 \text{ mW/m}^2 \]

Flux of energy into \(R = 5\) flows out of \(R = 7\) so no net flux into the volume between the 2 spheres.

(3) All the power of \(5\) W flows outward through the surface of the cube surrounding the origin. So, Answer is 5 W

(4) Copper is a very good conductor so \(E_x\) (at the surface) = 0

\(H_y\) at the surface is \(2\) A/m \((\text{incident wave} + \text{reflected wave})\)

Therefore, the surface current \(J_s = 2\) A/m

The surface resistance is \(R_s = \frac{1}{\sigma \nu} \)

\[ S = \frac{1}{\sqrt{\pi \nu \mu \sigma}} \] . The electric field at the surface is \((E_x)_0 = \frac{(1+5)J_s}{\sigma S} \)
The energy dissipated per unit area

\[ \frac{1}{2} R_s J_s \]

For copper, \( S = \frac{0.066}{\sqrt{W}} \)

For \( f = 100 \text{ MHz} \), \( S = 6.6 \times 10^{-6} \) m

\[ \frac{1}{\alpha S} = 2.6 \times 10^{-3} \Omega \text{m} = R_s \]

Energy dissipated is \( 5.225 \text{ mW/m}^2 \)

\[(E_x)_0 = (1 + j) 5.25 \times 10^{-3} \text{ V/m} \]

(See page 154 in text)

5) If the wave travels through a dielectric with \( \varepsilon_r = 30 \), the magnetic field in steel \( H_y = 1 \text{ A/m} \) (incident wave) \( H_y = 2 \text{ A/m} \) at the surface

so \( J_s \) is the same, \( R_s \) is the same, and \( (E_x)_0 \) is the same. In other words, answer is same as question (4).
4.5a: \( R_{hf} = \frac{R_s}{2\pi r_0} = \frac{1}{\sigma \delta 2 \pi r_0} \quad \) \( R_o = \frac{1}{\pi r^2 \sigma} \)

\[ \frac{R_{hf}}{R_o} = \frac{1}{r \cdot 2 \pi r_0 \cdot \pi r^2 \sigma} = \frac{R_s}{2\delta} \] Q.E.D.

4.5b: \( R_{hf} = \frac{1}{\pi r_0^2 \delta} \left[ 1 + \frac{1}{48} \left( \frac{R_s}{\delta} \right)^2 \right] \); \( \frac{1}{48} \left( \frac{R_s}{\delta} \right)^2 = 0.02 \) for \( \frac{R_s}{\delta} = 0.78 \)

From table 3.17, \( S_{Cu} = \frac{0.66}{10^6} = 6.6 \times 10^{-4} \) m at 10 kHz
\( = 6.6 \times 10^{-5} \) m at 1 MHz

\( \Rightarrow r_o = 0.65 \) mm at 10 kHz, \( 0.65 \) mm at 1 MHz.

For brass, \( S_{Brass} = \frac{1.27}{10^6} = 1.27 \times 10^{-3} \) m at 10 kHz
\( = 1.27 \times 10^{-4} \) m at 1 MHz

\( \Rightarrow r_o = 1.27 \) mm at 10 kHz, \( 1.27 \) mm at 1 MHz.

3.17a: \( R_s = \left[ \frac{\mu_s H}{2\pi} \right]^2 \), \( \mu_s = \mu_r \cdot (4\pi \times 10^{-7}) \)

Tin:
\( R_s = \left[ \frac{4\pi^2 \cdot 10^{-7}}{10^6} \right]^2 = 2.8 \times 10^{-9} \mu \)\( \Rightarrow \) \( \left\{ \begin{array}{l} 4.87 \times 10^{-6} \ \Omega \ @ \ 60 \ \text{Hz} \\ 1.99 \times 10^{-5} \ \Omega \ @ \ 1 \ \text{kHz} \\ 6.28 \times 10^{-4} \ \Omega \ @ \ 1 \ \text{MHz} \end{array} \right\} \)

Iron:
\( \mu_r = 500 \), so \( \mu_s \times \) above = \( \left\{ \begin{array}{l} 1.09 \times 10^{-4} \ \Omega \ @ \ 60 \ \text{Hz} \\ 4.44 \times 10^{-4} \ \Omega \ @ \ 1 \ \text{kHz} \\ 1.4 \times 10^{-3} \ \Omega \ @ \ 1 \ \text{MHz} \end{array} \right\} \)