ENEE 381 Problem Set #2

9/16/04 - due 9/28/04

(1) Calculate the electric field and magnetic field amplitudes produced 1km from a radio transmitter whose output is 4W at 100MHz. The waves coming from the transmitter are spherical, but to a good approximation they are plane far enough away from the transmitter. Compare these field amplitudes with those produced by a laser beam whose intensity is 10GW/cm^2 at a wavelength of 1μm.

(2) Calculate the electric and magnetic field amplitudes produced 50mm from a cellular phone that is isotropically transmitting 1W at 850 MHz. Use a plane wave approximation to calculate the fields. If all this power is absorbed in a spherical region of radius 100mm whose specific heat is 1 calorie/gram, what is the rate of heating expected?

(3) RWD 3.13b

(4) RWD 3.17b

(5) RWD 3.18b

(6) RWD 3.19b

(7) RWD 3.20b

(8) Calculate the time it would take for a light sail powered space craft to accelerate to $0.1\times$ the velocity of light. Use a sail area of 100km^2, a space craft mass of 10,000 kg, and a light intensity on the sail of 1W/m^2.

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SOLUTIONS

(1) The waves from the radio transmitter are distributed in all directions over the surface of a sphere of radius R.

The power flux (Watts $/m^2$) at radius R is

$$I = \frac{P}{4\pi R^2}$$

The power flux is often called the Intensity, I. It is the average value of the magnitude of the Poynting vector

$$I = \overline{|\underline{S}|} = \overline{\underline{E} \times \underline{H}} = \frac{E^2}{2Z_0}$$

Therefore
$$E = \sqrt{2Z_0 I} = \sqrt{\frac{2Z_0 P}{4\pi R^2}} = \sqrt{\frac{Z_0 P}{2\pi R^2}}$$

Plug in $P = 4W$, $R = 10^3$ m, $Z_0 = 376.7$ ohm

$\frac{E}{H} = Z_0$ Therefore $\longrightarrow$

$$\boxed{E = 1.55 \times 10^{-2}\ V/m}$$
$$\boxed{H = 4.11 \times 10^{-5}\ A/m}$$

For a laser beam with $I = 10\,GW/cm^2 = 10^{14}\ W/m^2$

$E = \sqrt{2Z_0 I} = \underline{2.74 \times 10^8\ V/m}$

$H = \frac{E}{Z_0} = \underline{7.29 \times 10^5\ A/m}$

(2) $E = \sqrt{\dfrac{Z_0 P}{2\pi R^2}}$

Plug-in $Z_0 = 376.7$

$P = 1\ W$

$R = 50 \times 10^{-3}\ m$

$\underline{E = 155\ V/m}$

$\underline{H = 0.41 A/m}$

If we assume that 1W is absorbed in a sphere of radius 100 mm then the average energy absorption is

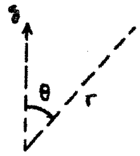
$$u = \frac{P}{\frac{4}{3}\pi R^3} = 23.9\ J\ s^{-1}\ m^{-3}$$

If we take a reasonable value for the density equal to the density of water $= 1 kg/m^3$ then the mass absorbs $0.024\ J/kg \equiv 2.39 \times 10^{-5}\ J/g$

Now 1 calorie = $4.2 J$, therefore the absorbed power is 5.69×10^{-6} calories/g.

Therefore the rate of heating is 5.69×10^{-6} degrees/sec

(3) 3.13b



$\vec{P}_{av} = \frac{1}{2} Re[\vec{E} \times \vec{H}^*] = \frac{1}{2} Re[\hat{\theta} E_\theta \times \hat{\phi} H_\phi^*] = \frac{1}{2} Re[\hat{r} E_\theta H_\phi^*]$

$= \hat{r} \frac{E_\theta E_\theta^*}{2\eta} = \hat{r} \frac{A^2}{2\eta r^2} \sin^2\theta$

$W = \int_0^{2\pi}\int_0^{\pi} (P_r)_{av} r^2 \sin\theta \, d\theta \, d\phi = \frac{2\pi A^2}{2\eta} \int_0^\pi \sin^3\theta \, d\theta$

$= \frac{\pi A^2}{\eta} [\int_0^\pi -(1-\cos^2\theta) d(\cos\theta)] = -\frac{\pi A^2}{\eta} [\cos\theta - \frac{\cos^3\theta}{3}]_0^\pi$

$= \frac{\pi A^2}{\eta} [1 - \frac{1}{3} + 1 - \frac{1}{3}] = \frac{4}{3} \frac{\pi A^2}{\eta}$

(4)

3.17b $E_z = \frac{J_0}{\sigma} e^{-(1+j)x/\delta}$

$\nabla \times \vec{E} = -j\omega\mu \vec{H} \Rightarrow -\frac{\partial E_z}{\partial x} = -j\omega\mu H_y = \frac{(1+j)J_0}{\sigma\delta} e^{-(1+j)x/\delta}$

so $H_y = \frac{j(1+j)J_0}{\sigma\delta\omega\mu} e^{-(1+j)x/\delta} = \frac{(1-j)J_0\delta}{2} e^{-(1+j)x/\delta}$

but $\frac{(1-j)}{2} = \frac{1}{1+j}$ so this is the same as Eq. 3.17 (1) at $x=0$.

(5)

3.18b If incident wave $H_y = H_0 e^{-jkz}$

$(P_z)_{av} = \frac{1}{2}\eta H_0^2$, $H_0 = [\frac{2(P_z)_{av}}{\eta}]^{\frac{1}{2}} = [\frac{2\times 10^6}{377}]^{\frac{1}{2}} = 72.8 \frac{A}{m}$

so if magnetic field at surface is doubled,

$W_L = \frac{R_s}{2}(2H_0)^2$; for Al @ 1GHz, $R_s = 3.26\times 10^{-7}\sqrt{10^9} = 1.03\times 10^{-2}\Omega$

so $W_L = \frac{1.03\times 10^{-2}}{2}(2\times 72.8)^2 = 109.2 \frac{W}{m^2}$

thus $\frac{W_L}{(P_z)_{av}} = \frac{109.2}{10^6} = 1.09\times 10^{-4}$

(6)

3.19b $\nabla \times \vec{E} = \nabla \times [\nabla(\nabla\cdot\vec{\Pi}) - \mu\epsilon \frac{\partial^2 \vec{\Pi}}{\partial t^2}] = -\frac{\partial \vec{B}}{\partial t} = -\mu\epsilon \frac{\partial^2 (\nabla\times\vec{\Pi})}{\partial t^2}$

this is satisfied since $\nabla\times\nabla(\nabla\cdot\vec{\Pi}) \equiv 0$

$\nabla\times\vec{H} = \epsilon\frac{\partial}{\partial t}(\nabla\times\nabla\times\vec{\Pi}) = \vec{J} + \epsilon\frac{\partial\vec{E}}{\partial t} = \frac{\partial\vec{P}}{\partial t} + \epsilon\frac{\partial}{\partial t}\nabla(\nabla\cdot\vec{\Pi}) - \epsilon(\mu\epsilon\frac{\partial^2\vec{\Pi}}{\partial t^2})$

$\epsilon\frac{\partial}{\partial t}[-\nabla^2\vec{\Pi} + \nabla(\nabla\cdot\vec{\Pi}) - \frac{\vec{P}}{\epsilon} - \nabla(\nabla\cdot\vec{\Pi}) + \mu\epsilon\frac{\partial^2\vec{\Pi}}{\partial t^2}] = 0$

or $\nabla^2\vec{\Pi} - \mu\epsilon\frac{\partial^2\vec{\Pi}}{\partial t^2} = -\frac{\vec{P}}{\epsilon}$, in agreement with diff. eqn. given

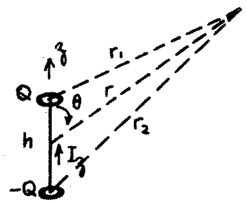
$\nabla\cdot\vec{D} = \epsilon\nabla\cdot\vec{E} = \epsilon\nabla^2(\nabla\cdot\vec{\Pi}) - \mu\epsilon^2\frac{\partial^2\nabla\cdot\vec{\Pi}}{\partial t^2} = \rho = -\nabla\cdot\vec{P}$

but $\nabla\cdot(\nabla^2\vec{\Pi}) = \nabla\cdot[\nabla(\nabla\cdot\vec{\Pi}) - \nabla\times\nabla\times\vec{\Pi}] = \nabla^2(\nabla\cdot\vec{\Pi})$ so

$\epsilon\nabla\cdot[\nabla^2\vec{\Pi} - \mu\epsilon\frac{\partial\vec{\Pi}}{\partial t} + \frac{\vec{P}}{\epsilon}] = 0$ which also agrees with diff. eqn. for $\vec{\Pi}$.

$\nabla\cdot\vec{B} = \mu\nabla\cdot\vec{H} = \epsilon\frac{\partial}{\partial t}(\nabla\cdot\nabla\times\vec{\Pi}) \equiv 0$, also OK.

3.20 b



From continuity, $I_z = \frac{dQ}{dt} = j\omega Q$

$$\Phi = \frac{Q}{4\pi\epsilon r_1} e^{-jkr_1} - \frac{Q}{4\pi\epsilon r_2} e^{-jkr_2}, \quad k = \frac{\omega}{v} = \omega\sqrt{\mu\epsilon}$$

and if $\frac{h}{r} << 1$, $r_1 \simeq r - \frac{h}{2}\cos\theta$, $r_2 \simeq r + \frac{h}{2}\cos\theta$

so $\Phi = \frac{Q}{4\pi\epsilon r} e^{-jkr}\left[\frac{e^{jkh/2\cos\theta}}{1-\frac{h}{2r}\cos\theta} - \frac{e^{-jkh/2\cos\theta}}{1+\frac{h}{2r}\cos\theta}\right]$

Now to 1st order in kh and $\frac{h}{r}$.

$$\Phi \simeq \frac{Q}{4\pi\epsilon r}e^{-jkr}\left[\left(1+\frac{h}{2r}\cos\theta\right)\left(1+j\frac{kh}{2}\cos\theta\right) - \left(1-\frac{h}{2r}\cos\theta\right)\left(1-j\frac{kh}{2}\cos\theta\right)\right]$$

$$\simeq \frac{Qhe^{-jkr}}{4\pi\epsilon}\left[j\frac{k}{r}+\frac{1}{r^2}\right]\cos\theta$$

$$= \frac{I_0 h e^{-jkr}}{j\omega 4\pi\epsilon}\left[j\frac{k}{r}+\frac{1}{r^2}\right]\cos\theta$$

Now $\vec{A} = \frac{\mu I_0 h}{4\pi r}e^{-jkr}[\hat{r}\cos\theta - \hat{\theta}\sin\theta]$

so $\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, A_\theta)]$

$$= \frac{\mu I_0 h}{4\pi}e^{-jkr}\left[\left(\frac{1}{r^2} - \frac{jk}{r}\right)\cos\theta - \frac{2\sin\theta\cos\theta}{r^2\sin\theta}\right]$$

$$= -\frac{\mu I_0 h}{4\pi}e^{-jkr}\left[\frac{1}{r^2}+\frac{jk}{r}\right]\cos\theta$$

so $\nabla\cdot\vec{A} = -j\omega\mu\epsilon\Phi$, as per Lorentz condition.

(8) An intensity I (Watts/m^2) corresponds to a photon flux $\frac{I}{h\nu}$ photons/m^2s

Each photon carries momentum $\frac{h}{\lambda}$

For a reflective surface the momentum change per square meter per second is

$$p = \left(\frac{I}{h\nu}\right) 2 \left(\frac{h}{\lambda}\right) = \frac{2I}{c}$$

reflection reverses momentum

The force on area A is

$$F = \frac{2IA}{c}$$

The acceleration is $a = \frac{2IA}{mc}$

where m is the mass.

To reach velocity v takes a time t where $v = at$. Therefore for $v = \frac{c}{10}$

$$t = \frac{c}{10a} = \frac{mc^2}{20IA}$$

Plugin $c = 3 \times 10^8$ m/s. $M = 10^7$ kg, $A = 10^8 m^2$, $I = 1$

$t = \underline{4.5 \times 10^{11} \text{ secs}} = \underline{14,269 \text{ years}}$