ENEE 381 Problem Set #4
4/1/04 due 4/13/04

(1) A length of loss-less transmission line is first short-circuited at one end and then open-circuited. The impedance measured at the other end in the first case is \( Z_1 \) and \( Z_2 \) in the second. Prove that \( Z_1 Z_2 = Z_0^2 \). This is a convenient way for measuring the characteristic impedance of an unknown line.

(2) A 50 ohm transmission line is terminated with an impedance of 20-j30. What is the magnitude and phase of the reflection coefficient?

(3) Repeat question (4) using the Smith chart.

(4) A 75 ohm transmission line is terminated with a load of 150 + j50 ohm. Compute \( \rho \) in terms of both amplitude \(|\rho|\) and \(\phi\). What fraction of incident power is absorbed in the load?

(5) Cheng problem 9.27

(6) Cheng problem 9.30

(7) Use Smith chart. A line with \( Z_0 = 1000\Omega \) is terminated with an unknown load. The SWR is found to be 3. A current maximum is observed 0.1\(\lambda\) from the load. What are:
   (a) the load?
   (b) the reflection coefficient \( \rho \), magnitude and angle?
   (c) how would you match the line without changing the load at the end of the line?

(8) Use Smith chart. A transmission line of characteristic impedance 75 ohm is terminated with an impedance 50+j125 ohm. 0.1\(\lambda\) from the load a 150ohm shorted stub 0.2\(\lambda\) long is connected in shunt to the main line. What are:
   (a) The reflection coefficient in magnitude and phase at this point?
   (b) The standing wave ratio?
   (c) Where is the nearest current minimum that is greater than 0.1\(\lambda\) from the load?
   (d) Where is the nearest point greater than 0.1\(\lambda\) from the load where the line can be matched with an open 75 ohm stub?

(9) Use Smith chart. Cheng problem 9.48


ENEE 381 Problem Set 4. SOLUTIONS

(1) The transformed impedance equation is

\[ Z_i = Z_0 \left( \frac{Z_L \cos(kl) + j \cdot Z_0 \sin(kl)}{Z_0 \cos(kl) + j \cdot Z_L \sin(kl)} \right) \]

For a shorted line \( Z_L = 0 \), so

\[ Z_{\text{short}} = j \cdot Z_0 \cdot \tan(kl) \]

For an open line \( Z_L \) is infinite, so

\[ Z_{\text{open}} = -j \cdot Z_0 \cdot \cot(kl) \]

\[ Z_{\text{short}} Z_{\text{open}} = Z_0^2 \]

Q.E.D.

(2) \( Z_0 := 50 \quad j := i \)

\[ Z_L := 20 - j \cdot 30 \]

\[ \rho := \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ \rho = -0.207 - 0.517i \]

\[ |\rho| = 0.557 \quad \text{magnitude} \]

\[ \frac{\arg(\rho)}{\text{deg}} = -111.801 \quad \text{phase angle in degrees} \]

(3) On chart the normalized impedance is

\[ \zeta_L := \frac{Z_L}{Z_0} \]

\[ \zeta_L = 0.4 - 0.6i \quad \text{Point A on chart} \]

(4) \( Z_0 := 75 \quad Z_L := 150 + j \cdot 50 \)

\[ \rho := \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ (|\rho|)^2 = 0.153 \quad \text{Fraction of power reflected} \]

\[ 1 - (|\rho|)^2 = 0.847 \quad \text{Fraction of power absorbed in load} \]
(3) \[ I_L = 0.4 - j0.6 \]

\[ |I| = \frac{OA}{OX} = 0.56 \quad \phi = -111^\circ \]
(5)\[ P.9-27 \text{ a) } |\Gamma| = \frac{S-i}{S+i} = \frac{2-i}{2+i} = \frac{1}{3}.\]

Eq. (9-133a): \[V(z) = \frac{V_0}{Z_0} (z + Z_0) e^{j\beta z}[1 + |\Gamma| e^{j\phi}].\]

Eq. (9-134): \[\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta}, \quad \phi = \frac{\theta}{2} - \frac{\pi}{2}.\]

Voltage is a minimum when \[\phi = \pm \pi \longrightarrow \theta = 2 \left(\frac{2\pi}{\lambda}\right) x 0.3 \lambda - \pi = 0.2 \pi.\]

\[\Gamma = \frac{1}{3} e^{j1.2 \pi}.\]

b) \[Z_L = Z_0 \left(\frac{1+i\pi}{1-i\pi}\right) = 466 + j206 (\Omega).\]

c) Terminating resistance \[R_m = \frac{R_0}{S} = \frac{380}{2} = 190 (\Omega),\]

\[L_m = \frac{\lambda_0}{2} - z_m = (0.5 - 0.3) \lambda = 0.2 \lambda.\]

Another set of solution is: \[Z_m' = 5R_0 = 600 (\Omega) \text{ & } L_m' = 0.45 \lambda.\]

(6) \[P.9-30 \text{ a) Given: } V_g = 0.1 \text{ kV} (V), \ Z_g = Z_0 = 50 (\Omega), \ R_L = 25 (\Omega) \]

\[V_i = \frac{Z_i}{Z_0 + Z_i} V_g, \quad I_i = \frac{V_i}{Z_0 + Z_i},\]

where \[Z_i = Z_0 \left(\frac{0.5 Z_0 + j \frac{Z_0 \tan \beta L}{2} + j \frac{Z_0 \tan \beta L}{2}}{Z_0 + j \frac{0.5 Z_0 \tan \beta L}{2}}\right) = \frac{Z_0}{2} + j \frac{Z_0}{2} \tan \beta L.\]

\[V_i = \frac{1 + j \frac{2 \tan \beta L}{3 (1 + j \frac{2 \tan \beta L}{3})}}{V_g} = \frac{1}{30} \left(\frac{1 + j \frac{2 \tan \beta L}{3}}{1 + j \frac{2 \tan \beta L}{3}}\right) (V),\]

\[I_i = \frac{2}{3 \left(1 + j \frac{2 \tan \beta L}{3}\right)} V_g = \frac{2}{3} \left(\frac{2 + j \frac{2 \tan \beta L}{1 + j \frac{2 \tan \beta L}{3}}}{1 + j \frac{2 \tan \beta L}{3}}\right) (mA).\]

Setting \[Z_0 = Z_0 \text{ and } \Gamma_0 = 0 \text{ in Eqs. (9-120a) and (9-120b), we have }\]

\[V_L = V(z=0) = \frac{V_0 Z_i}{Z_0 + Z_i} e^{j2 \beta L (1 - i \Gamma)} \quad (i \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{4}{3});\]

\[= \frac{1}{30} e^{j2 \beta L} (V),\]

\[I_L = I(z=0) = \frac{V_0 I_0}{Z_0 + Z_i} e^{j2 \beta L (1 - i \Gamma)} = \frac{4}{3} e^{j2 \beta L} (mA).\]

b) \[S = \frac{i + |r|}{i - |r|} = 2.\]

c) \[(P_{av})_L = \frac{1}{2} P_a (V_L I_L^*) = \frac{1}{2} \left(\frac{1}{30}\right) \left(\frac{4}{3} \times 10^{-3}\right) = 2.22 \times 10^{-3} (W) = 0.0222 (mW).\]

\[\text{If } R_L = 50 (\Omega), \quad V_L = \frac{V_0}{2} e^{j2 \beta L}, \quad I_L = \frac{V_0}{2} e^{j2 \beta L}\]

\[\rightarrow \text{Max} (P_{av})_L = \frac{V_0^2}{8 Z_0} = 2.50 \times 10^{-3} (W).\]
(7) \( Z_0 := 100 \) \hspace{1cm} S := 3

On chart mark point on real axis for S=3, draw circle about center of chart \( I_{\text{max}} \) is on left hand side. Load is 0.1\( \lambda \) away at point 0.5-j0.6

\[ |\rho| = 0.49 \hspace{1cm} \phi = 108 \text{ degrees} \]

Sanity check

\[ \zeta_L := 0.5 - j0.6 \]

\[ Z_L := Z_0 \cdot \zeta_L \]

\[ \rho := \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ |\rho| = 0.483 \hspace{1cm} \frac{\text{arg}(\rho)}{\text{deg}} = -108.004 \]

Matching points are at B or C. You can match with series or parallel components but the distances from the load vary depending on whether you use the chart in impedance or admittance

(8) \( Z_0 := 75 \) \hspace{1cm} Z_L := 50 + j125

\[ \zeta_L := \frac{Z_L}{Z_0} \hspace{1cm} \text{Normalized load} \]

\[ \zeta_L = 0.667 + 1.667i \hspace{1cm} \text{point A} \]

\[ y_L := \frac{1}{\zeta_L} \]

\[ y_L = 0.207 - 0.517i \hspace{1cm} \text{Normalized admittance of load, point B} \]

Shorted stub 0.2\( \lambda \) long starts at point C and has its normalized admittance at point D

\[ y_{\text{stub150}} := -j0.32 \]

Renormalized \[ Y_{\text{stub}} := \frac{1}{150} \cdot y_{\text{stub150}} \]

Multiplying by admittance of 150 ohm line

Normalize again but this time to 75 ohm line

\[ y_{\text{stub75}} := \frac{Y_{\text{stub}}}{1/75} \]

\[ y_{\text{stub75}} = -0.16i \]
0.1λ from the load is point E, where 

\( y_1 := 0.17 + j \cdot 0.14 \)

After adding stub

\[ Y_{\text{total}} := Y_i + Y_{\text{stub75}} \]

\[ y_{\text{total}} = 0.17 - 0.02i \] \quad \text{Point F}

See chart for rest of problem
(7) \( s_L = 0.5 - j0.6 \)  \( z_L = 50 - j60 \)

\[ |e| = 0.49 \quad \phi = -108^\circ \]  MATCH AT B or C

EXAMPLE  \( B = 1 + j1.16 \)  ADD normalized

\(-j1.16\) IN SERIES
At \( F \) \( |e| = 0.71 \ \phi = -178^\circ \ \ \ S = 6 \\
S \) is also the location of \( I_{\text{Max}}, I_{\text{Min}} \) is just past point \( F \). The nearest \( I_{\text{Min}} > 0.1\lambda \) from load is \( 0.078\lambda + 0.5\lambda = 0.578\lambda \) from load.
\( z_l = 0.5 + j0.5 \).
\( y_l = 1 - j \).

a) See construction.
\( \rho_1 : z_l = 0.5 + j0.5 \).
\( \rho_2 : y_l = 1 - j \to d_1 = 0. \)
\( \rho_3 : y_l = 1 - j \to d_2 = 0. \)

\[ d_2 = 0.162 \lambda \cdot (0.5 + 0.118) \lambda = 0.324 \lambda. \]

\[ b_k = j \to d_k = 0.375 \lambda. \]

\[ b_k = -j \to d_k = 0.375 \lambda. \]

b) For \( z_0' = 0.5 = 1.5 z_0 \), \( y_0' = 0.667 y_0 \).

The required normalized stub admittances are \( b_k' = \frac{2}{0.667} = 3.05 \).

<table>
<thead>
<tr>
<th>( z_l = 0.5 + j0.5 )</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_l = 0.5 + j0.5 )</td>
<td>( d_1 = 0 ), ( \rho_1 = 0.375 \lambda )</td>
<td>( d_2 = 0 ), ( \rho_2 = 0.406 \lambda )</td>
<td></td>
</tr>
<tr>
<td>( y_l = 1 - j )</td>
<td>( d_2 = 0.324 \lambda ), ( \rho_3 = 0.118 \lambda )</td>
<td>( d_3 = 0.324 \lambda ), ( \rho_3 = 0.093 \lambda )</td>
<td></td>
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</tbody>
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\( P.9-49 \)

\( z_l = 0.5 + j0.5 \)

Use Smith chart as an impedance chart. Some construction as that in problem P.9-34 except \( \rho_9 \) would be on the extreme left (marked by a dot) and \( g = 1 \) circle becomes a red circle.

\( \rho_1 : z_l = 0.5 + j0.5 \); \( \rho_2 : z_l = 1 + j1 \) with \( d_2 = 0.162 \cdot 0.088 \lambda = 0.074 \lambda \).

\( \rho_3 : z_l = 1 - j1 \) with \( d_3 = 0.338 \cdot 0.088 \lambda = 0.250 \lambda \).

To achieve a match with a series stub having \( \rho'_0 = \frac{35}{50} \rho_0 \), we need a normalized stub susctance \( -j \frac{35}{35} = -j1.43 \)
for solution corresponding to \( \rho_2 \). From Smith chart we obtain the required stub length \( l_0 = 0.347 \lambda \).

Similarly for solution corresponding to \( \rho_3 \), a stub with a normalized susctance \( +j1.43 \) is needed, which requires a stub length \( l_3 = 0.153 \lambda \).
\( z_L = 0.33 + j 0.33 \).

\( P_L : Y_L = 1.50 - j 1.50 \) (0.306 \( \lambda \) at \( P'_L \)).

\( P_{A1} : Y_{A1} = 1.50 - j 1.80 \) (0.304 \( \lambda \) at \( P'_{A1} \)).

\( P_{A2} : Y_{A2} = 1.50 - j 0.14 \) (0.269 \( \lambda \) at \( P'_{A2} \)).

\( P_{Al} : Y_{Al} = 1.00 + j 1.60 \) (0.179 \( \lambda \) at \( P'_{Al} \)).

\( P_{Bl} : Y_{Bl} = 1.00 + j 0.40 \) (0.144 \( \lambda \) at \( P'_{Bl} \)).

### Short-circuited Stubs

| \((Y_{SA})_L = Y_{Al} - Y_L = -j 0.30\) | \(L_{Al} = 0.203 \lambda\) | \(L_{Al} = 0.453 \lambda\) |
| \((Y_{SB})_L = Y_{Bl} - Y_L = j 1.36\) | \(L_{Bl} = 0.399 \lambda\) | \(L_{Bl} = 0.149 \lambda\) |
| \((Y_{SB})_L = -j 1.60\) | \(L_{Bl} = 0.089 \lambda\) | \(L_{Bl} = 0.339 \lambda\) |
| \((Y_{SB})_L = -j 0.40\) | \(L_{Bl} = 0.189 \lambda\) | \(L_{Bl} = 0.439 \lambda\) |

### Open-circuited Stubs

\( Y_L = \frac{200}{100 + j 50} = 2.4 - j 1.2 \).

Point \( P_L \) on Smith chart.

(0.280 \( \lambda \) at \( P'_L \)).

Since the rotated \( g = 1.0 \) circle is tangent to the \( g = 2.0 \) circle, an added line length \( d_L \) is needed to convert \( g_L (2.4) \) to 2.0.

Moving from \( P_L \) along the \(| \Gamma | \)-circle to \( P_{L1} \) (not shown) on the \( g = 2.0 \) circle (0.291 \( \lambda \) at \( P'_{L1} \)). Note that \( P_{L1} \) is different from \( P_A \), the point of tangency between the \( g = 2.0 \) and rotated \( g = 1.0 \) circles.

a) \( \text{Min. } d_L = 0.291 \lambda - 0.280 \lambda = 0.011 \lambda \).

b) \( P_A : Y_A = 2 - j 1 \) (0.287 \( \lambda \) at \( P' \)).

\( P_B : Y_B = 1 + j 1 \) (0.162 \( \lambda \) at \( P' \)).

\( Y_{SA} = Y_A - Y_{L1} = (2 - j 1) - (2 - j 1.35) = j 0.35 \rightarrow L_A = 0.304 \lambda \).

\( Y_{SB} = -j 1 \rightarrow L_B = 0.125 \lambda \).