ENEE 381 Problem Set #5

Solutions. Worked out using Mathcad

\[ C_1 := 100 \cdot 10^{-12} \quad L_1 := 10 \cdot 10^{-6} \quad C_2 := 75 \cdot 10^{-12} \quad L_2 := 15 \cdot 10^{-6} \]

\[ \omega_c := \sqrt{\frac{1}{L_1 \cdot C_1}} \quad \omega_c := \sqrt{\frac{1}{L_2 \cdot C_2}} \quad \text{cutoff frequencies} \]

\[ \omega_c = 3.162 \times 10^7 \quad \omega_c = 2.981 \times 10^7 \]

\[ i := 1, 2, \ldots, 1000 \quad \text{Running integer to give different frequencies on plots} \]

Use designations 1 for the low frequency band, 2 for the middle band, 3 for the high frequency band

\[ \omega_1 := \frac{5 \cdot 10^8}{1000} \quad \omega_{1i} := \frac{\omega_2}{1001} \quad \omega_{2i} := \omega_2 + i \frac{(\omega_1 - \omega_2)}{1001} \quad \omega_{3i} := \omega_1 + i \frac{\omega_1}{1001} \]

In a pass band \( \gamma \) is imaginary, in a stop band it is real

(1) SERIES L1 and C1 in series

\[ Z_i := i \cdot \omega_1 \cdot L_1 + \frac{1}{i \cdot \omega_1 \cdot C_1} \quad Y_i := \frac{1}{i \cdot \omega_1 \cdot L_2 + \frac{1}{i \cdot \omega_1 \cdot C_2}} \]

\[ Z_{1i} := i \cdot \omega_{1i} \cdot L_1 + \frac{1}{i \cdot \omega_{1i} \cdot C_1} \quad Y_{1i} := \frac{1}{i \cdot \omega_{1i} \cdot L_2 + \frac{1}{i \cdot \omega_{1i} \cdot C_2}} \]

\[ Z_{2i} := i \cdot \omega_{2i} \cdot L_1 + \frac{1}{i \cdot \omega_{2i} \cdot C_1} \quad Y_{2i} := \frac{1}{i \cdot \omega_{2i} \cdot L_2 + \frac{1}{i \cdot \omega_{2i} \cdot C_2}} \]

\[ Z_{3i} := i \cdot \omega_{3i} \cdot L_1 + \frac{1}{i \cdot \omega_{3i} \cdot C_1} \quad Y_{3i} := \frac{1}{i \cdot \omega_{3i} \cdot L_2 + \frac{1}{i \cdot \omega_{3i} \cdot C_2}} \]

\[ \gamma_i := \sqrt{Z_i \cdot Y_i} \quad \gamma_{1i} := \sqrt{Z_{1i} \cdot Y_{1i}} \quad \gamma_{2i} := \sqrt{Z_{2i} \cdot Y_{2i}} \quad \gamma_{3i} := \sqrt{Z_{3i} \cdot Y_{3i}} \]
alpha in the stop bands

\[ \frac{\omega_1}{\omega_3} \]

Re(\(\gamma_1\)), Re(\(\gamma_3\))

beta in the pass band

\[ \frac{\omega_2}{\omega_3} \]

Im(\(\gamma_2\))
(1) SERIES L1 and C1 in parallel

SHUNT L2 and C2 in parallel

\[
Z_1 := \frac{1}{i \omega_1 C_1 + \frac{1}{i \omega_1 L_1}} \\
Y_1 := i \omega_1 C_2 + \frac{1}{i \omega_1 L_2}
\]

\[
Z_2 := \frac{1}{i \omega_2 C_1 + \frac{1}{i \omega_2 L_1}} \\
Y_2 := i \omega_2 C_2 + \frac{1}{i \omega_2 L_2}
\]

\[
Z_3 := \frac{1}{i \omega_3 C_1 + \frac{1}{i \omega_3 L_1}} \\
Y_3 := i \omega_3 C_2 + \frac{1}{i \omega_3 L_2}
\]

\[
\gamma_1 := \sqrt{Z_1 Y_1} \\
\gamma_1 := \sqrt{Z_1 Y_1} \\
\gamma_2 := \sqrt{Z_2 Y_2} \\
\gamma_3 := \sqrt{Z_3 Y_3}
\]

alpha in the stop bands

\[
\omega_1, \omega_3
\]

\[
4 \cdot 10^7, 2 \cdot 10^7
\]

\[
\text{Re}(\gamma_1, \text{Re}(\gamma_3))
\]
(3) SERIES L1 and C1 in series

\[ Z_i := i\cdot\omega_1 L_1 + \frac{1}{i\cdot\omega_1 C_1} \quad Y_i := (i\cdot\omega_1 C_2) + \frac{1}{i\cdot\omega_1 L_2} \]

SHUNT L2 and C2 in parallel

\[ Z_1 := i\cdot\omega_1 L_1 + \frac{1}{i\cdot\omega_1 C_1} \quad Y_1 := (i\cdot\omega_1 C_2) + \frac{1}{i\cdot\omega_1 L_2} \]
\[ Z_2 := i\cdot\omega_2 L_1 + \frac{1}{i\cdot\omega_2 C_1} \quad Y_2 := (i\cdot\omega_2 C_2) + \frac{1}{i\cdot\omega_2 L_2} \]
\[ Z_3 := i\cdot\omega_3 L_1 + \frac{1}{i\cdot\omega_3 C_1} \quad Y_3 := (i\cdot\omega_3 C_2) + \frac{1}{i\cdot\omega_3 L_2} \]
\[ \gamma_i := \sqrt{\frac{Z_i}{Y_i}} \quad \gamma_{1i} := \sqrt{Z_1 \cdot Y_1} \quad \gamma_{2i} := \sqrt{Z_2 \cdot Y_2} \quad \gamma_{3i} := \sqrt{Z_3 \cdot Y_3} \]
(4) SERIES L1 and C1 in parallel

SHUNT L2 and C2 in series

\[ Z_i := \frac{1}{(i\cdot\omega_i \cdot C1) + \frac{1}{i\cdot\omega_i \cdot L1}} \quad Y_i := \frac{1}{(i\cdot\omega_i \cdot L2) + \frac{1}{i\cdot\omega_i \cdot C2}} \]

\[ Z_{1i} := \frac{1}{(i\cdot\omega_1 \cdot L2) + \frac{1}{i\cdot\omega_1 \cdot C2}} \quad Y_{1i} := \frac{1}{(i\cdot\omega_1 \cdot C1) + \frac{1}{i\cdot\omega_1 \cdot L1}} \]

\[ Z_{2i} := \frac{1}{(i\cdot\omega_2 \cdot C1) + \frac{1}{i\cdot\omega_2 \cdot L1}} \quad Y_{2i} := \frac{1}{(i\cdot\omega_2 \cdot L2) + \frac{1}{i\cdot\omega_2 \cdot C2}} \]

\[ Z_{3i} := \frac{1}{(i\cdot\omega_3 \cdot C1) + \frac{1}{i\cdot\omega_3 \cdot L1}} \quad Y_{3i} := \frac{1}{(i\cdot\omega_3 \cdot L2) + \frac{1}{i\cdot\omega_3 \cdot C2}} \]

\[ \gamma_i := \sqrt{Z_i \cdot Y_i} \quad \gamma_{1i} := \sqrt{Z_{1i} \cdot Y_{1i}} \quad \gamma_{2i} := \sqrt{Z_{2i} \cdot Y_{2i}} \quad \gamma_{3i} := \sqrt{Z_{3i} \cdot Y_{3i}} \]

Beta in the pass bands
alpha in the stop band

\[ \omega^2 \]

\[ 2 \cdot 10^7 \]

\[ 3 \cdot 10^7 \]

\[ 3.1 \cdot 10^7 \]

\[ 3.15 \cdot 10^7 \]

\[ 3.2 \cdot 10^7 \]

\[ 0 \]

\[ 50 \]

\[ 100 \]

\[ 150 \]

\[ 200 \]

\[ 250 \]

Re(\gamma_2i)
\( \sigma := 4 \) conductivity of seawater

\[\begin{align*}
  i & := 1, 2, \ldots, 1000 \\
  v_i & := 10^4 + \frac{(i - 1) \cdot 10^9}{1000} \quad \text{different frequencies}
\end{align*}\]

\( \varepsilon_0 := 8.854 \cdot 10^{-12} \quad \mu_0 := 4 \cdot \pi \cdot 10^{-7} \) fundamental constants

\( \varepsilon_1 := 81 \quad \varepsilon_2 := \frac{\sigma}{2 \cdot \pi \cdot v_i \varepsilon_0} \) real and imaginary parts of dielectric constant

\( \varepsilon_r := \varepsilon_1 - i \cdot \varepsilon_2 \) complex dielectric constant

\[k_i := 2 \cdot \pi \cdot v_i \sqrt{1 - \mu_0 \cdot \varepsilon_0 \cdot \varepsilon_r} \] complex propagation constant

\[\alpha_i := -\text{Im}(k_i)\]

**Attenuation vs. frequency. Units m\(^{-1}\)**