ENEE 381 Problem Set #3
10/1/02- due 10/15/02

Questions like (1) - (6) could be on the first examination.

(1) The electric vector of a wave propagating in the z-direction varies according to

\[ E_y = E_0 \cos(\pi x/2a) e^{j\omega t}, \]

where \( E_0 = 1 \text{V/m} \), \( a = 1 \text{m} \). The frequency of the wave is \( \nu = 100 \text{MHz} \). How much energy flow per second passes through the region \(-1 \leq x \leq 1 \text{ (m)}\), \(-1 \leq y \leq 1 \text{ (m)}\).

(2) A point source transmitter at (0,0,0) emits a total power of 5W. What is the value of the Poynting vector at the point (10,10,10)? What is the total power flux into the surface bounded by the two concentric spheres \( R=5 \), and \( R=7 \)?

(3) A point source transmitter at (0,0,0) emits a total power of 5W. What is the total power flux through the surface of a cube centered at (0,0,0) with sides of length 1m?

(4) A plane wave with magnetic field \( H_y = 1 \text{A/m} \) and electric field \( E_z \) traveling in the z-direction through a vacuum strikes an infinite planar copper medium. What is the value of the electric field and magnetic field at the surface of the sheet? What is the value of the surface resistance \( R_s \)? How much energy is dissipated per unit area of the copper? For copper \( \sigma = 5.8 \times 10^7 \text{ S/m} \).

(5) How are the answers to question (4) modified if the wave is traveling through a dielectric with \( \varepsilon_r = 30 \) when it strikes the copper?

(6) A 50 ohm transmission line is terminated with an impedance of 20-j30. What is the magnitude and phase of the reflection coefficient?

(7) A 75 ohm transmission line is terminated with a load of 150 + j50 ohm. Compute \( \rho \) in terms of both amplitude |\rho| and \( \phi \). What fraction of incident power is absorbed in the load?

(8) Repeat (7) with the Smith Chart

The following questions are easiest with the Smith chart

(9) A 50 ohm transmission line is terminated with an inductor of 10\( \mu \)H and a capacitor of 0.005\( \mu \)F in series. The line is driven at 1MHz. Compute |\rho|, \( \phi \), the standing wave ratio, \( S \), and the location of the nearest voltage maximum to the load.

(10) A 100 ohm transmission line is terminated with an inductor of 10\( \mu \)H and a capacitor of 10nF in parallel. Compute |\rho|, \( \phi \), \( S \) and the location of the nearest current maximum to the load. The line is being operated at 1MHz.

(11) A line of impedance 50 ohm is terminated in an impedance of 25 + j75 ohm. Where on the line nearest to the load can the line be matched with a pure inductor in series, and what is the value of the inductor if the line is operated at 100MHz.

(12) Repeat (5) except where can the line be matched with an inductor in parallel, and what is the value of the inductor at 100MHz.

(13) Repeat (5) except do the matching with a capacitance in parallel.

(14) If the capacitance calculated in (7) is being determined by a length of shorted 50 ohm line, what is the minimum length that it must be.

(15) A 75 ohm transmission line is terminated with a load that gives an impedance of 50 + j50 measured 0.3\( \lambda \) from the load. What is the load?
\[ E_y = E_0 \cos \left( \frac{\pi x}{2a} \right) e^{j(ut-kz)} \]

From the curl equation \[ \nabla \times \mathbf{E} = -j \mu_0 \varepsilon \frac{\partial \mathbf{H}}{\partial t} \]
so
\[ -j k E_0 \cos \left( \frac{\pi x}{2a} \right) e^{j(ut-kz)} = \mu j \omega H_x \]

\[ H_x = -\frac{k E_0}{\mu_0 \omega} \cos \left( \frac{\pi x}{2a} \right) e^{j(ut-kz)} \]
\[ = -\frac{E_0}{Z} \cos \left( \frac{\pi x}{2a} \right) e^{j(ut-kz)} \]

\[ Z = \sqrt{\mu_0 \varepsilon_0 \omega} \]
Assuming \( \varepsilon_r = \mu_r = 1 \)
\[ Z = 376.7 \text{ ohm} \]

The Poynting vector is
\[ S_z = -E_y H_x = \frac{E_0^2}{Z} \cos \left( \frac{\pi x}{2a} \right) e^{j(ut-kz)} \]

Time averaging
\[ \overline{S_z} = \frac{E_0^2}{2Z} \cos \left( \frac{\pi x}{2a} \right) \]

Power flux required is
\[ \overline{ \int \int \cos \left( \frac{\pi x}{2a} \right) dx dy } = \overline{S_z} \]
\[ \overline{S_z} = \frac{E_0^2}{2Z} \int_{-1}^{1} \int_{-1}^{1} (1 + \cos \left( \frac{\pi x}{2a} \right) ) \ dx \]
\[ \overline{S_z} = \frac{E_0^2}{2Z} \int_{-1}^{1} \left( x^2 + \sin \left( \frac{\pi x}{2a} \right) \right) \ dx \]
\[ \overline{S_z} = \frac{E_0^2}{2Z} \frac{a}{\pi} \left( x + \sin \left( \frac{\pi x}{2a} \right) \right) \bigg|_{-1}^{1} \]
\[ \overline{S_z} = 845 \mu W \]
(2) The distance from \((0,0,0)\) to \((10,10,10)\) is 
\[ R = \sqrt{100 + 100 + 100} = \sqrt{300} \]

The Poynting vector at this point is 
\[ \mathbf{S} = \frac{P}{4\pi R^2} \mathbf{\hat{R}} = \frac{5}{4\pi \times 300} = 1.33 \text{ mW/m}^2 \]

Flux of energy into \(R = 5\) flows out of \(R = 7\) so no net flux into the volume between the 2 spheres.

(3) All the power of \(5\) flows outward through the surface of the cube surrounding the origin. So, answer is 5 W.

(4) Copper is a very good conductor so \(E_x\) (at the surface) \(= 0\).

\(H_y\) at the surface is \(2A/m\) (incident wave + reflected wave)

Therefore, the surface current \(J_s = 2A/m\).

The surface resistance \(R_s = \frac{1}{a S}\).

\[ S = \frac{1}{\sqrt{n_0 \mu_o}} \]

The electric field at the surface is \((E_x)_o = \frac{(1 + j)J_s}{\alpha S}\).
The energy dissipated per unit area
\[ \frac{1}{\nu} R_s J_s \]

For copper \( S = 0.066 \frac{1}{\sqrt{\nu}} \)

For \( \nu = 100 \text{ MHz} \) \( S = 6.6 \times 10^{-6} \text{ m} \)

\[ \frac{1}{\sigma S} = 2.6 \times 10^{-3} \text{ ohm} = R_s \]

Energy dissipated is \( 5.225 \text{ mW} / \text{m}^2 \)

\( (E_x)_0 = (1+j)5.25 \times 10^{-3} \text{ V/m} \)

(see page 154 in text)

(5) If the wave travel through a shell

with \( \varepsilon_r = 30 \), the magnetic field is

steel \( H_y = 1A/\text{m} \) (incident wave) \( H_y = 2A/\text{m} \)

at the surface

so \( J_s \) is the same, \( R_s \) is the same

and \( (E_x)_0 \) is the same. In other words

answer is same as question (9).

(6) \[ \nu = \frac{20 - j 30 - 50}{20 - j 30 + 50} = \frac{-30 - j 30}{70 - j 30} = \frac{-3 - j^3}{7 - j^3} \]

\[ \nu = \frac{(-3 - j^3)(7 + j^3)}{58} = \frac{-12 - 12j}{58} \]

\[ |\nu| = 0.293 \quad \phi = 225^\circ \]

\((-12,-12)\)
Solutions

\[ Z_L = 150 + j50 \]

\[ Z_0 = 75 \]

Normalized load \( Z_L = \frac{150 + j50}{75} = 2 + j0.67 \)

\[ e = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + j50}{225 + j50} = \frac{3 + j2}{9 + j2} = \frac{(3 + j2)(9 - j2)}{81 + 4} \]

\[ \rho = \frac{31 + 12j}{85} = |e|e^{\phi} \]

\[ |e| = \frac{1}{85} \sqrt{31^2 + 12^2} = 0.391 \quad \phi = \tan^{-1} \left( \frac{12}{31} \right) = 21.16^\circ \]

Fraction of energy reflected on \(|e|\), function into load \(d = 1 - |e| = 0.847\)

(8) See Chart

(9) \[ Z_L = j\omega L + \frac{1}{j\omega C} = \frac{1 - \omega^2 LC}{j\omega C} = -j \left[ \frac{1 - \omega L}{\omega C} \right] \]

\[ \omega = 2\pi \times 10^6 \quad \omega L = 2\pi \times 10^6 \times 10^{-5} = 20\pi = 62.83 \]

\[ \frac{1}{\omega C} = \frac{1}{2\pi \times 10^6 \times 5 \times 10^{-9}} = \frac{100}{\pi} = 31.83 \]

\[ Z_L = -j \left[ 31 \right] \quad \rho = -j \frac{31}{50} = 0.62 \]

You can use \( e = \frac{Z_L - Z_0}{Z_L + Z_0} \) and \( S = \frac{1 + |e|}{1 - |e|} \)

However, see Chart
\[ Z_L = 2 + j0.667 \quad \phi = 21^\circ \]

\[ r = \frac{3.55}{9.1} \quad \text{(measured from)} \]

\[ = 0.39 \]
continued.

Clearly \( |e| = 1 \quad S = \infty \)

\( \phi = 116^\circ \)

Nearest \( V_{\text{MAX}} \) is at R.H. end of

Real Line \( \equiv 0.911 \lambda - 0.25 \lambda = \underline{0.661} \lambda \) from Load

\[
Y_L = j \omega C + \frac{1}{j \omega L}
\]

\( \omega C = 2 \pi \cdot 10^6 \cdot 10^{-8} = \frac{2 \pi}{100} \)

\( \omega L = 2 \pi \cdot 10^6 \cdot 10^{-5} = 20 \pi \)

\( Y_0 = \frac{1}{100} = 0.01 \)

\[
Y_L = j \left[ \omega C - \frac{1}{\omega L} \right] = j 0.0469
\]

Normalized load admittance is \( Y_L = \frac{j 0.0469}{0.01} = j 4.69 \)

See Chart

Once again \( |e| = 1 \quad S = \infty \)

\( \phi (\text{current}) = 29^\circ \quad \phi (\text{voltage}) = 207^\circ \)

Distance to current max = \( (0.25 - 0.27) \lambda = 0.033 \lambda \)
(11) \[ Z_L = 25 + j75 \quad Z_0 = 50 \Omega \]

\[ \frac{j}{Z_L} = \frac{25 + j75}{50} = 0.5 + j1.5 \]

To match with an inductor in series \( \equiv j \omega L = jX \)

Move to a point \( \frac{1}{Z_L} = 1 - jX \)

This is a point where \( X = 2.2 \).

Distance from load is \((0.3085 - 0.1615) \lambda = 0.147 \lambda \text{ AWG} \)

Therefore \( \omega L = 2.2 \times 50 \quad \omega \uparrow \quad L = \frac{2.2 \times 50}{2 \pi \times 10^8} = 0.175 \mu \text{H} \)

(12) Use same chart as (5)

Find \( y_L \) diametrically across from \( \frac{1}{Z_L} \)

\[ y_L = 0.2 - j0.6 \]

For an inductor in parallel \(-jS = \frac{1}{j \omega L} = -\frac{1}{\omega L} \)

Match at a point where \( y_n = 1 + jS \quad S = 2.2 \)

This is \((0.1907 + 0.088) \lambda \text{ AWG} = \frac{0.279 \lambda \text{ AWG}}{\text{From Load}} \)

To match \( \frac{2.2}{50} \quad \omega \downarrow \quad L = \frac{50}{2 \times 2} = 3.62 \mu \text{H} \)

(unnormalize \( (X, Y) \))
(13) Match at a point \( y_i = 1 - z_5 \)

This allows a capacitor to be added in parallel. The admittance of the capacitor is \( j \omega C \).

For matching, \( 5Y_0 = j \omega C \).

From chart the matching point is \((0.088 + 0.308) \lambda \) away \( = 0.396 \lambda \) away.

Matching point is \( 1 - j2.2 \) \( S = 2.2 \).

Renormalize \( \frac{2.2}{50} = j \omega C \) \( C = 70 \mu F \).

(14) To get a shorted line of 50 ohm to have an admittance of \( j2.2 \) move from R.H. end of chart, clockwise to point \( A = j2.2 \).

The stub is \((0.25 + 0.182) \lambda \) \( 10 \omega C = 0.432 \lambda \).

(15) \( Z_L = 75 \) ohm \( Z_i = 50 + j50 \) at 0.3 \( \lambda \) from load.

\( S_i = \frac{50 + j50}{75} = 0.667 + j0.667 \)

0.3 \( \lambda \) away towards load \( S_L = 1.28 - j1.0 \).

\( Z_L = 75 \left(1.28 - j1.0\right) = 96 - j75 \) ohm.