Solutions

(1) \[ J = \frac{0.1e^{-6t}}{r} \]

At \( t = 1 \mu s \), \[ J = 0.1e^{-1} \frac{A}{m} \]

\( J \) is independent of \( \theta \) or \( \phi \). Therefore at \( r = 5 \)

\[ I = 4\pi r^2 \left( 0.1e^{-1} \right) = 4\pi r \times 0.1e^{-1} = 2.441A \]

(2) \[ J = 5.2A/m^2 \text{ for } |z| < 2 \]

(a) The field from a surface current \( K = H = \frac{1}{2} K \times \hat{n} \),

where \( \hat{n} \) is a surface normal.

For \( |z| > 2 \), the observation point is on one side or the

other of the region of current flow. Therefore the effective

\( K = 20 \frac{A}{m} \)

For \( z > 2 \) \[ H = 10 \hat{z} \times \hat{k} = 10 \hat{z} \]

For \( z < 2 \) \[ H = 10 \hat{z} \times (-\hat{k}) = -10 \hat{z} \]
If observation point has \( |z| < 2 \), then it is inside
the current region.

At point \((x, y, z)\) the surface current above the
point is \( 5(2-z) \, \text{A/m} \), the surface current below is
\( 5(2+z) \, \text{A/m} \).

The normal field is \( \frac{-5}{2} (2-z) \hat{z} + \frac{5}{2} (2+z) \hat{z} = 5z \hat{z} \).

(b) For \( |z| < 2 \), \( H = 5z \hat{z} \), \( B = 5\mu z \hat{z} \).

\[ B = \text{curl} \, A \Rightarrow B_z = (\text{curl} \, A)_z \]

Therefore \( \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{z} = 5\mu z \hat{z} \).

Because \( J_y \) is in the y direction, \( A_y \) is only in
y direction, and only varies with \( z \).

\[ \frac{\partial A_y}{\partial z} = -5\mu z \]

\[ A_y = -\frac{5\mu z^2}{2} \]
Flux through coil when it makes an angle \( \theta \) with \( B \) is

\[
\Phi = \frac{\pi a^2}{\text{area}} N B \sin \theta
\]

Number of turns

\[
\frac{d\Phi}{dt} = -\pi a^2 NB \cos \theta \frac{d\theta}{dt} = -\pi a^2 NB \omega \cos \theta
\]

Now

\[
\frac{d\Phi}{dt} = -\int \mathbf{E} \cdot d\mathbf{l} = V
\]

potential difference across coil

Therefore

\[
V = \pi a^2 NB \omega \cos \theta
\]

Current in coil \( I = \frac{V}{R} = \frac{\pi a^2 NB \omega \cos \theta}{R} \)

Ohmic heating \( \mathbf{V} \mathbf{I} = \frac{N \pi a^4 B^2 \omega^2 \cos^2 \theta}{R} \)

\[
= \frac{N^2 \pi^2 a^4 B^2 \omega^2}{2R} (1 + \cos^2 \theta)
\]

Average ohmic heating \( \frac{N \pi a^4 B^2 \omega^2}{2R} \)
The current in the coil produces a magnetic dipole in that is oriented as shown — according to Lenz’s law it acts to oppose the change in external flux

\[ \vec{M} = \mu I N A \]
\[ \text{area of coil} \]

The torque that acts is \( \vec{\Gamma} = \vec{m} \times \vec{H} = \vec{m} \times \vec{B} \)

Therefore \( \vec{\Gamma} = \mu I a^2 N \cdot \frac{\pi a^2 NBw \cos \theta}{R} \cdot \frac{B \sin (90 - \theta)}{\mu} \)

\[ = \frac{\pi^2 a^4 N^2 B^2 w \cos \theta}{2R} \]

The average work done against the torque in one revolution is \( 2\pi \vec{\Gamma} \)

\[ \vec{\Gamma} = \frac{\pi^2 a^4 N^2 B^2 w}{2R} \]

Work done per second is \( 2\pi \vec{\Gamma} \cdot \frac{\omega}{2\pi} \)

\[ \text{No of revolutions per second} \]

Power = \( \frac{\pi^2 a^4 N^2 B^2 w}{2R} \) = ohmic heating

For \( N = 100 \), \( a = 50 \text{ mm} \), \( B = 0.2 \text{ T} \), \( \omega = 100 \text{ rad/s} \), \( R = 1 \)

Power = ohmic heating = \( 2467 \) Watts
Two magnetic fields act on the small magnetic needle – the external field $H_1$ and the field that results from the current induced in the coil, $H_i$.

\[ H_i = \frac{NI}{2a} = \frac{\pi a N^2 B \omega \cos \theta}{2R} \quad \text{(field at center of circular coil)} \]

\[ B_i = \mu H_i = \frac{\mu \pi a N^2 \omega B \sin \theta}{2R} = k B \cos \theta \]

\[
(k = \frac{\mu \pi a N^2 \omega}{2R})
\]

Two forces act on the magnetic needle. In equilibrium:

\[ \frac{M \times H_i}{M} = M \times H \]

\[ \frac{M B_i \sin(90-(\phi-\theta))}{\mu} = MB \sin \phi \]

Therefore

\[ k \cos \theta (\cos \phi \cos \theta + \sin \phi \sin \theta) = \sin \phi \]
\[ k \cos \phi (\cos \theta) + k \sin \phi \sin \theta \cos \theta = \sin \phi \]

Given:
\[ \frac{k \cos \phi}{z} = \sin \phi \]

\[ \cot \phi = \frac{z}{k}, \quad \tan \phi = k \frac{z}{2} = \frac{\pi N \rho \omega}{4R} \]

With \( N = 100, \quad \omega = 100 \text{ rad/s}, \quad R = 1, \quad a = 50 \text{ mm}, \quad \rho = 4\pi \times 10^{-7} \)

\[ \phi = 2.83^\circ \]


\[ \mathbf{F} = e \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \]

\[ F_x = e \left( E_x + v_y B_z \right) = e \left( E_x + v_x B_y + v_y B_x \right) \]

\[ F_y = -e v_x B = -e B \frac{dx}{dt} \]

\[ F_z = 0 \]

If particle mass is \( m \) then we have

\[ m \frac{d^2 \mathbf{r}}{dt^2} = e \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \]

\[ m \frac{dv_x}{dt} = e \left( E_x + v_y B \right) \quad (1) \]

\[ m \frac{dv_y}{dt} = -e B v_x \quad (2) \]

Therefore

\[ \frac{d^2 v_x}{dt^2} = -\left( \frac{eB}{m} \right)^2 v_x = -\omega^2 v_x \quad \omega = \frac{eB}{m} \]
Therefore \( V_x = A \sin \omega t + B \cos \omega t \)

At \( t = 0 \), \( V_x = 0 \) therefore \( B = 0 \)

\[ V_x = A \sin \omega t \Rightarrow \frac{dx}{dt} = A \sin \omega t \quad x = -A \cos \omega t + \text{constant} \]

At \( t = 0 \), \( x = 0 \), therefore \( x = A(1 - \cos \omega t) \)

\[ \frac{d^2 x}{dt^2} = +A \omega^2 \cos \omega t \quad (\frac{d^2 x}{dt^2})_{t=0} = +A \omega^2 = \frac{eE}{m} \]

Therefore \( A = +\frac{eE}{m \omega^2} \quad x = \frac{eE}{m \omega^2} (1 - \cos \omega t) = \frac{E}{\omega B} (1 - \cos \omega t) \)

Therefore \( m \frac{d^2 y}{dt^2} = -\frac{eE}{w} (\omega \sin \omega t) \)

\[ \frac{d^2 y}{dt^2} = -\frac{eE}{m} \sin \omega t \quad \frac{dy}{dt} = +\frac{eE}{m \omega} \cos \omega t + D \]

At \( t = 0 \) \( \frac{dy}{dt} = 0 \) therefore \( D = -\frac{eE}{m \omega} \)

\[ \frac{dy}{dt} = -\frac{eE}{m \omega} (1 - \cos \omega t) \quad y = -\frac{eE}{m \omega} (t - \sin \omega t) + E \]

At \( t = 0 \), \( y = 0 \) therefore \( E = 0 \)

\[ y = \frac{eE}{m \omega^2} (\omega t - \sin \omega t) = -\frac{E}{\omega B} (\omega t - \sin \omega t) \]
(6) From (5) \[ x_{\text{max}} = \frac{2E}{\omega B} \]

\[ \omega = \frac{eB}{m} \]

\[ x_{\text{max}} = \frac{2mE}{eB^2} \]

Therefore, charge will not reach opposite capacitor plate if \[ d > \frac{2mE}{eB^2} \]

(7)

\[ \omega = 2\pi f \]

Along a radius of the rotating disc, the motional electric field is \((\mathbf{v} \times \mathbf{B}) = E_m\)

At radius \(x\)

\[ \mathbf{v} \times = \omega \mathbf{x} \]

\[ E_m = \mathbf{B} \mathbf{w} \times \hat{r} \]

\[ V = -\int E \cdot dl = \int \mathbf{B} \mathbf{w} \times \mathbf{d}x = \frac{\mathbf{B} \mathbf{w} \mathbf{a}^2}{2} \]

\[ = \frac{2\pi f \mathbf{a}^2}{2} \]

\[ V = \frac{fB \pi a^2}{2} \]