ENEE 459E/CMSC 498R: Introduction to Cryptology
PRG Class Exercise 2/16/17

Let G be a pseudorandom generator where $|G(s)| = |s| + 1$

1. Define $G'(s) = G(s||\bar{s})$, where $\bar{s}$ is the bit-wise negation of s. Is G' necessarily a pseudorandom generator? No.

Let G^* be a PRG from inputs of length n to length $2n+1$
Define G in terms of G^* as follows: $G(s=s_1||s_2) := G^*(s_1 \oplus s_2)$
G is a PRG from n to $n+1$. G is secure b/c $s_1 \oplus s_2$ is unif. dist.
Note $G'(s) = G(s||\bar{s}) = G^*(s \oplus \bar{s}) = G^*(1^{n/2}) =$ constant.
Distinguisher for G':
$D(w)$:
If $w == G^*(1^{n/2})$ output 1
Else output 0

Need to show $|Pr[D(r)=1] - Pr[D(G'(s))=1]|$ is high.

2. Define $G'(s) = G(s)||G(\bar{s})$, where $\bar{s}$ is the bit-wise negation of s. Is G' necessarily a pseudorandom generator? No.

Let G^* be a PRG from inputs of length n to $n+2$.
Define G in terms of G^* as follows.
$G(s=s_1 s')$: [where s_1 is a single bit]
If $s_1=0$, output $G^*(s')$
If $s_1=1$, output $G^*(\bar{s'})$

G is PRG from n to $n+1$.
G is secure b/c s', $\bar{s'}$ unif. dis-

Note $G'(s_1 s') = G(s_1 s')||G(\bar{s_1}, \bar{s'}) = G^*(s)||G^*(s)$
Distinguisher checks if 1st and 2nd half of w are the same.

Need to show $|Pr[D(r=1)] - Pr[D($ is high

3. Define $G'(s) = G(s)_1||G(G(s)_2, \dots, G(s)_{|s|+1})$, where $G(s)_i$ denotes the i-th output bit of $G(s)$. Is G' necessarily a pseudorandom generator? Yes.

Use a <u>hybrid argument</u>. Consider 3 distributions H_0, H_1, H_2
In order to prove $G'(s)$ is a PRG, need to show distinguisher cannot dist. H_0, H_2

H_0: $G(s)_1||G(G(s)_2 \dots G(s)_{|s|+1})$ where $s \xleftarrow{R} \{0,1\}^n$

Indistinguishable due to security of PRG G.

H_1: $r_1||G(r_2 \dots r_{|s|+1})$ where $r \xleftarrow{R} \{0,1\}^{n+1}$

Indistinguishable due to security of PRG G.

H_2: $r_1||r'_1 \dots r'_{n+1}$ where $r_1 \xleftarrow{R} \{0,1\}$, $r' \xleftarrow{R} \{0,1\}^{n+1}$

Note G' has stretch 2. Takes inputs of length n, produces outputs of length $n+2$.