Introduction to Cryptology

Lecture 11

Announcements

- HW4 due today
- Optional HW5 up on course webpage, due 3/13
- Midterm coming up in class on 3/15
- Midterm Review Sheet up on course webpage
- There will be a cheat sheet on the midterm. Cheat sheet will be posted on Canvas.

Agenda

- Last time:
  - Block Ciphers (K/L 3.5)
  - Modes of Operation (K/L 3.6)
  - Started MAC (K/L 4.2)
- This time:
  - Security Definition for MAC (K/L 4.2)
  - Constructing MAC from PRF (K/L 4.3)
  - Domain Extension for MACs (K/L 4.4)

Message Authentication Codes

Definition: A message authentication code (MAC) consists of three probabilistic polynomial-time algorithms $(Gen, Mac, Vrfy)$ such that:

1. The key-generation algorithm Gen takes as input the security parameter 1^n and outputs a key k with $|k| \geq n$.
2. The tag-generation algorithm Mac takes as input a key k and a message $m \in \{0,1\}^*$, and outputs a tag t.
   $t \leftarrow Mac_k(m)$.
3. The deterministic verification algorithm $Vrfy$ takes as input a key k, a message m, and a tag t. It outputs a bit b with $b = 1$ meaning valid and $b = 0$ meaning invalid.
   $b \coloneqq Vrfy_k(m, t)$.

It is required that for every n, every key k output by $Gen(1^n)$, and every $m \in \{0,1\}^*$, it holds that $Vrfy_k\big(m, Mac_k(m)\big) = 1$.

Security of MACs

The message authentication experiment $MACforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m, t). Let Q denote the set of all queries that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m, t) = 1$ and (2) $m \notin Q$. In that case, the output of the experiment is defined to be 1.

Security of MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is existentially unforgeable under an adaptive chosen message attack if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACforge_{A,\Pi}(n) = 1\right] \leq neg(n)\,.$$

Strong MACs

The strong message authentication experiment $MACsforge_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Mac_k(\cdot)$. The adversary eventually outputs (m, t). Let Q denote the set of all pairs (m, t) that A asked its oracle.
3. A succeeds if and only if (1) $Vrfy_k(m, t) = 1$ and (2) $(m, t) \notin Q$. In that case, the output of the experiment is defined to be 1.

Strong MACs

Definition: A message authentication code $\Pi = (Gen, Mac, Vrfy)$ is a strong MAC if for all probabilistic polynomial-time adversaries A, there is a negligible function neg such that:

$$\Pr\left[MACsforge_{A,\Pi}(n) = 1\right] \leq neg(n).$$

Constructing Secure Message Authentication Codes

A Fixed-Length MAC

Let F be a pseudorandom function. Define a fixed-length MAC for messages of length n as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message $m \in \{0,1\}^n$, output the tag $t \coloneqq F_k(m)$.
- $Vrfy$: on input a key $k \in \{0,1\}^n$, a message $m \in \{0,1\}^n$, and a tag $t \in \{0,1\}^n$, output 1 if and only if $t = F_k(m)$.

Security Analysis

Theorem: If F is a pseudorandom function, then the construction above is a secure fixed-length MAC for messages of length n.

Security Analysis

Let A be a ppt adversary trying to break the security of the construction. We construct a distinguisher D that uses A as a subroutine to break the security of the PRF.

Distinguisher D:

D gets oracle access to oracle O, which is either F_k, where F is pseudorandom or f which is truly random.

1. Instantiate $A^{Mac_k(\cdot)}(1^n)$.
2. When A queries its oracle with message m, output $O(m)$.
3. Eventually, A outputs (m^*, t^*) where $m^*, t^* \in \{0,1\}^n$.
4. If $m^* \in Q$, output 0.
5. If $m^* \notin Q$, query $O(m^*)$ to obtain output z^*.
6. If $t^* = z^*$ output 1. Otherwise, output 0.

Security Analysis

Consider the probability D outputs 1 in the case that O is truly random function f vs. O is a pseudorandom function F_k.

- When O is pseudorandom, D outputs 1 with probability $\Pr\left[MACforge_{A,\Pi}(n) = 1\right] = \rho(n)$, where ρ is non-negligible.
- When O is random, D outputs 1 with probability at most $\frac{1}{2^n}$. Why?

Security Analysis

D's distinguishing probability is:

$$\left|\frac{1}{2^n} - \rho(n)\right| = \rho(n) \ - \ \frac{1}{2^n}.$$

Since, $\frac{1}{2^n}$ is negligible and $\rho(n)$ is non-negligible, $\rho(n) \ - \ \frac{1}{2^n}$ is non-negligible.

This is a contradiction to the security of the PRF.

Domain Extension for MACs

CBC-MAC

Let F be a pseudorandom function, and fix a length function ℓ. The basic CBC-MAC construction is as follows:

- Mac: on input a key $k \in \{0,1\}^n$ and a message m of length $\ell(n) \cdot n$, do the following:
  1. Parse m as $m = m_1, \ldots, m_\ell$ where each m_i is of length n.
  2. Set $t_0 \coloneqq 0^n$. Then, for $i = 1\ to\ \ell$:

     Set $t_i \coloneqq F_k(t_{i-1} \oplus m_i)$.

  Output t_ℓ as the tag.
- $Vrfy$: on input a key $k \in \{0,1\}^n$, a message m, and a tag t, do: If m is not of length $\ell(n) \cdot n$ then output 0. Otherwise, output 1 if and only if $t = Mac_k(m)$.

CBC-MAC

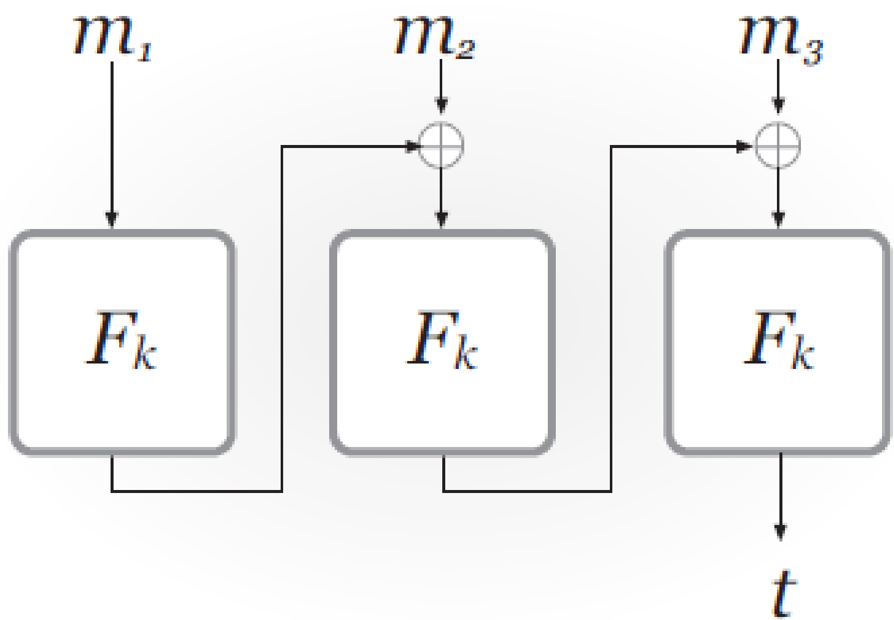

FIGURE 4.1: Basic CBC-MAC (for fixed-length messages).