Introduction to Cryptology

Lecture 6

Announcements

- HW2 due today
- Readings/Quizzes on Canvas due today (11:59pm)
- HW3 up on course webpage, due 2/21

Agenda

- Last time:
  - Class exercise on intractibility
- This time:
  - The Computational Approach (K/L 3.1)
  - Defining computationally secure SKE (K/L 3.2)
  - Defining PRG (K/L 3.3)
  - Constructing computationally secure SKE (K/L 3.3)
    - i.e. a Stream Cipher

The Computational Approach

Two main relaxations:

1. Security is only guaranteed against efficient adversaries that run for some feasible amount of time.
2. Adversaries can potentially succeed with some very small probability.

Security Parameter

- Integer valued security parameter denoted by n that parameterizes both the cryptographic schemes as well as all involved parties.
- When honest parties initialize a scheme, they choose some value n for the security parameter.
- Can think of security parameter as corresponding to the length of the key.
- Security parameter is assumed to be known to any adversary attacking the scheme.
- View run time of the adversary and its success probability as functions of the security parameter.

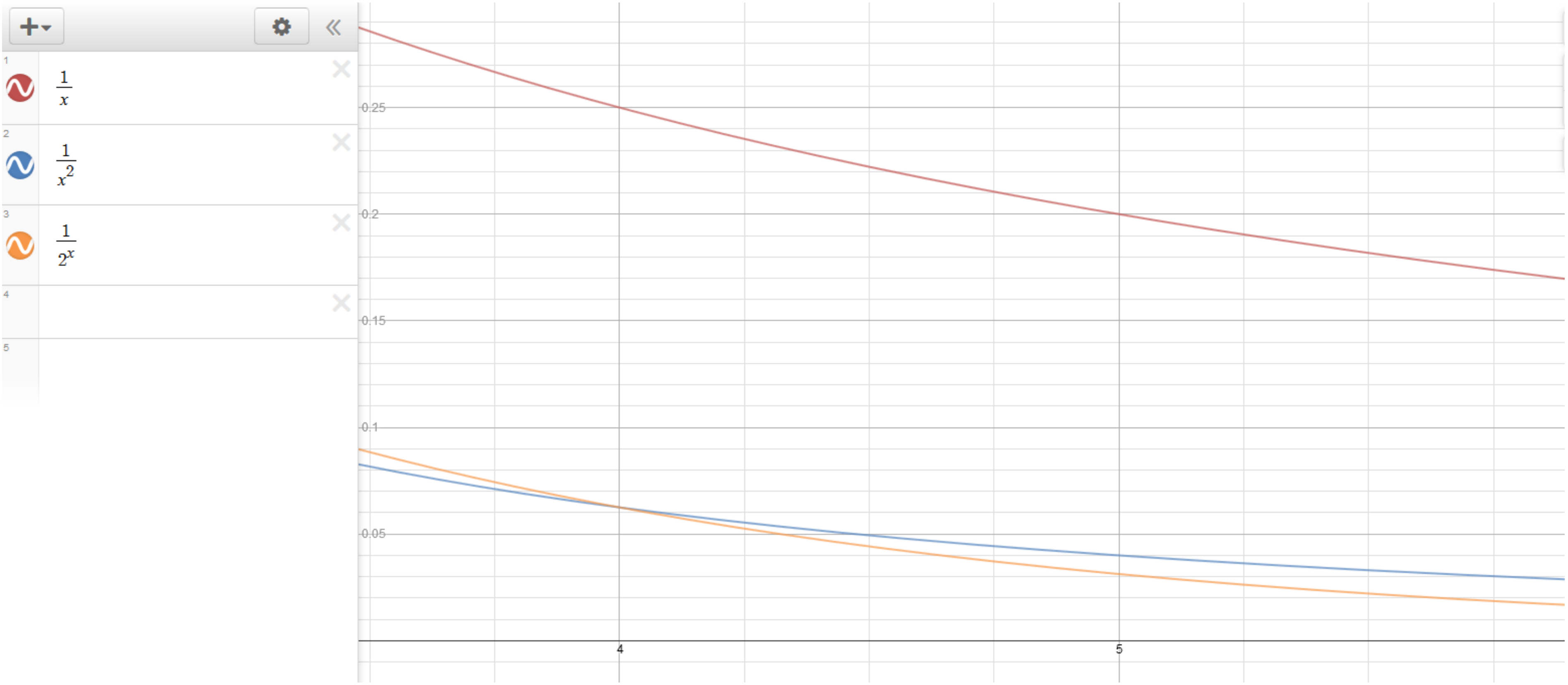
Polynomial Time

- Efficient adversaries = Polynomial time adversaries
  - There is some polynomial p such that the adversary runs for time at most $p(n)$ when the security parameter is n.
  - Honest parties also run in polynomial time.
  - The adversary may be much more powerful than the honest parties.

Negligible

- Small probability of success = negligible probability
  - A function f is negligible if for every polynomial p and all sufficiently large values of n it holds that $f(n) < \frac{1}{p(n)}$.
  - Intuition, $f(n) < n^{-c}$ for every constant c, as n goes to infinity.

Negligible



Practical Implications of Computational Security

- For key size n, any adversary running in time $2^{n/2}$ breaks the scheme with probability $1/2^{n/2}$.
- Meanwhile, Gen, Enc, Dec each take time n^2.
- If $n = 128$ then:
  - Gen, Enc, Dec take time 16,384
  - Adversarial run time is $2^{64} \approx 10^{18}$
- If n = 256 then:
  - Gen, Enc, Dec quadruples--takes time 65,536
  - Adversary run time is multiplied by 2^{64}. Becomes $2^{128} \approx 10^{38}$

Defining Computationally Secure Encryption

A private-key encryption scheme is a tuple of probabilistic polynomial-time algorithms (Gen, Enc, Dec) such that:

1. The key-generation algorithm Gen takes as input security parameter 1^n and outputs a key k denoted $k \leftarrow Gen(1^n)$. We assume WLOG that $|k| \geq n$.
2. The encryption algorithm Enc takes as input a key k and a message $m \in \{0,1\}^*$, and outputs a ciphertext c denoted $c \leftarrow Enc_k(m)$.
3. The decryption algorithm Dec takes as input a key k and ciphertext c and outputs a message m denoted by $m := Dec_k(c)$.

Correctness: For every n, every key $k \leftarrow Gen(1^n)$, and every $m \in \{0,1\}^*$, it holds that $Dec_k\big(Enc_k(m)\big) = m$.

Indistinguishability in the presence of an eavesdropper

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

The eavesdropping indistinguishability experiment ${PrivK^{eav}}_{A,\Pi}(n)$:

1. The adversary A is given input 1^n, and outputs a pair of messages m_0, m_1 of the same length.
2. A key k is generated by running $Gen(1^n)$, and a random bit $b \leftarrow \{0,1\}$ is chosen. A challenge ciphertext $c \leftarrow Enc_k(m_b)$ is computed and given to A.
3. Adversary A outputs a bit b'.
4. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise. If ${PrivK^{eav}}_{A,\Pi}(n) = 1$, we say that A succeeded.

Indistinguishability in the presence of an eavesdropper

Definition: A private key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions in the presence of an eavesdropper if for all probabilistic polynomial-time adversaries A there exists a negligible function $negl$ such that

$$\Pr\left[{PrivK^{eav}}_{A,\Pi}(n) = 1\right] \leq \frac{1}{2} + negl(n),$$

Where the prob. Is taken over the random coins used by A, as well as the random coins used in the experiment.

Semantic Security

- The full definition of semantic security is even more general.
- Consider arbitrary distributions over plaintext messages and arbitrary external information about the plaintext.

Semantic Security

Definition: A private key encryption scheme $\Pi = (Gen, Enc, Dec)$ is semantically secure in the presence of an eavesdropper if for every ppt adversary A there exists a ppt algorithm A' such that for all efficiently sampleable distributions $X = (X_1, \dots,)$ and all poly time computable functions f, h, there exists a negligible function $negl$ such that

$$|\Pr[A(1^n, Enc_k(m), h(m)) = f(m)] - \Pr[A'(1^n, h(m)) = f(m)]| \le negl(n),$$

where m is chosen according to distribution X_n, and the probabilities are taken over choice of m and the key k, and any random coins used by A, A', and the encryption process.

Equivalence of Definitions

Theorem: A private-key encryption scheme has indistinguishable encryptions in the presence of an eavesdropper if and only if it is semantically secure in the presence of an eavesdropper.