

Solutions

ENEE/CMSC/MATH 456: Cryptography PRF Class Exercise 2/14/24

Let F be a length-preserving pseudorandom function. For the following constructions of a keyed function $F': \{0,1\}^n \times \{0,1\}^{n-1} \rightarrow \{0,1\}^{2n}$, state whether F' is a pseudorandom function. If yes, prove it; if not, show an attack.

1. $F'_k(x) := F_k(0||x) || F_k(x||1)$. No.

Distinguisher $D^{\mathcal{F}}(1^n)$:

1. query \mathcal{Q} on input $x_1 = 0^{n-1}$, get back y_1

2. query \mathcal{Q} on input $x_2 = 0^{n-2}1$, get back y_2

Note $F'_k(x_1) = F'_k(0^{n-1}) = F_k(0^n) || F_k(0^{n-1}1)$

$F'_k(x_2) = F'_k(0^{n-2}1) = F_k(0^{n-1}1) || F_k(0^{n-2}1^2)$

3. If second half of y_1 is equal to first half of y_2 , output 1. o/w output 0.

$$\Pr[D^{F'_k(\cdot)}(1^n) = 1] = 1$$

$$\Pr[D^{f(\cdot)}(1^n) = 1] = \frac{1}{2^n}$$

Therefore

$$\left| \Pr[D^{F'_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| = 1 - \frac{1}{2^n}$$

contradicting security of F' .

Clearly, D is ppt.

2. $F'_k(x) := F_k(0||x) || F_k(1||x)$. Yes

Idea of proof: for any set of queries x_1, \dots, x_q

the responses $f(0||x_i) || f(1||x_i)$ are completely independent and uncorrelated when f is a truly random function.

Therefore, by security of PRF, $F'_k(\cdot)$ remains secure when truly random f is replaced with pseudorandom F_k .