Cryptography

Lecture 7

Announcements

- HW3 up on course webpage, due Wednesday, 2/22

Agenda

- Last time:
  - SKE secure against eavesdroppers from PRG (K/L 3.3)
- This time:
  - Stream Ciphers
  - CPA Security (K/L 3.4)
  - Pseudorandom Functions (PRF) (K/L 3.5)

Stream Cipher

Sender

State s_i after sending the i-th message:

$$s_0 := k$$
$$s_{i+1} := G(s_i)_2, \ldots, G(s_i)_{n+1}$$
$$pad_{i+1} := G(s_i)_1$$

$$c_{i+1} := m_{i+1} \oplus pad_{i+1} \longrightarrow$$

Receiver

State s_i after receiving the i-th message:

$$s_0 := k$$
$$s_{i+1} := G(s_i)_2, \ldots, G(s_i)_{n+1}$$
$$pad_{i+1} := G(s_i)_1$$

$$m_{i+1} := c_{i+1} \oplus pad_{i+1}$$

CPA Security

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cpa}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

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$k \leftarrow Gen(1^n)$

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$A^{Enc_k(\cdot)}$

Challenger

$k \leftarrow Gen(1^n)$

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m'

$A^{Enc_k(\cdot)}$ → $k \leftarrow Gen(1^n)$

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$k \leftarrow Gen(1^n)$

$A^{Enc_k(\cdot)}$

m'

c'

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Experiment $PrivK^{cpa}_{A,\Pi}(n)$

Adversary $A(1^n)$

Challenger

$k \leftarrow Gen(1^n)$

$A^{Enc_k(\cdot)}$

m' $\longrightarrow$

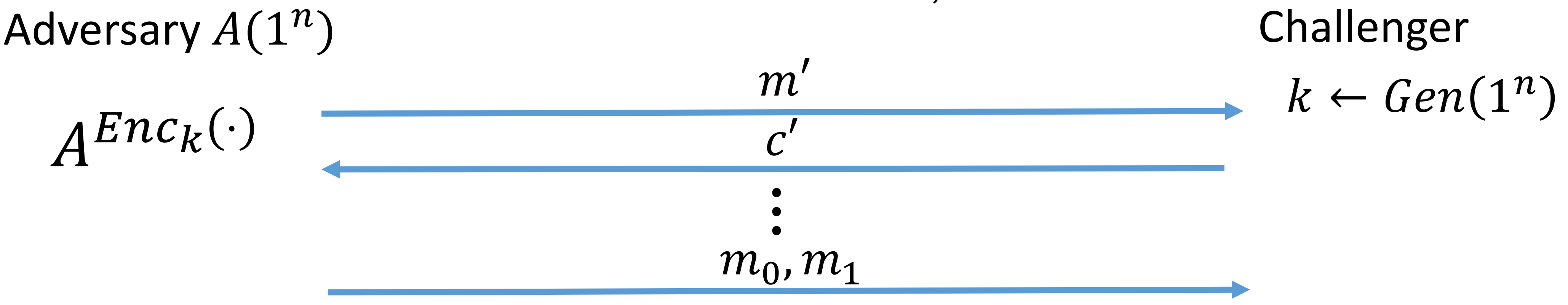
$\longleftarrow$ c'

$\vdots$

CPA Security

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK^{cpa}_{A,\Pi}(n)$



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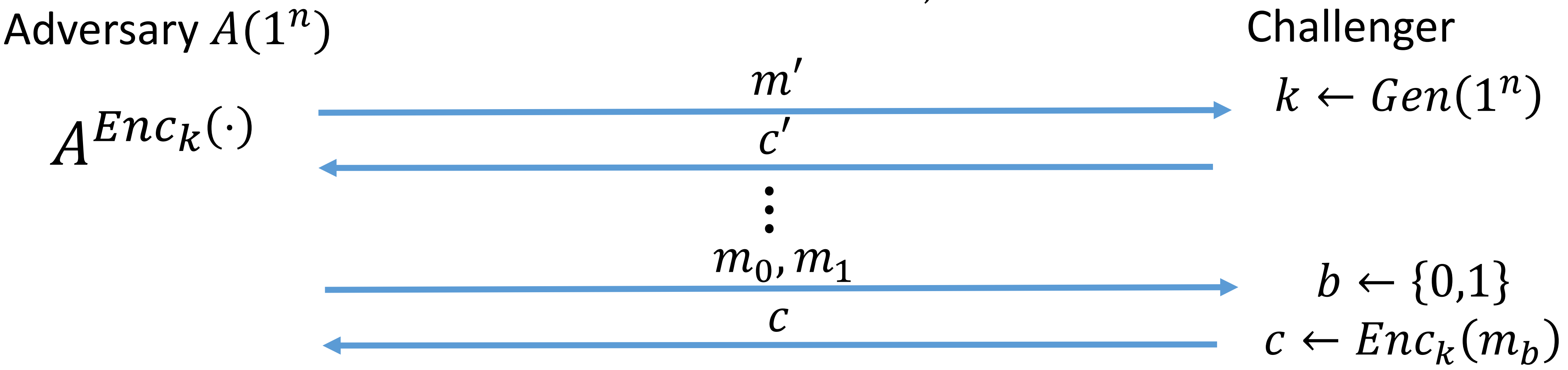

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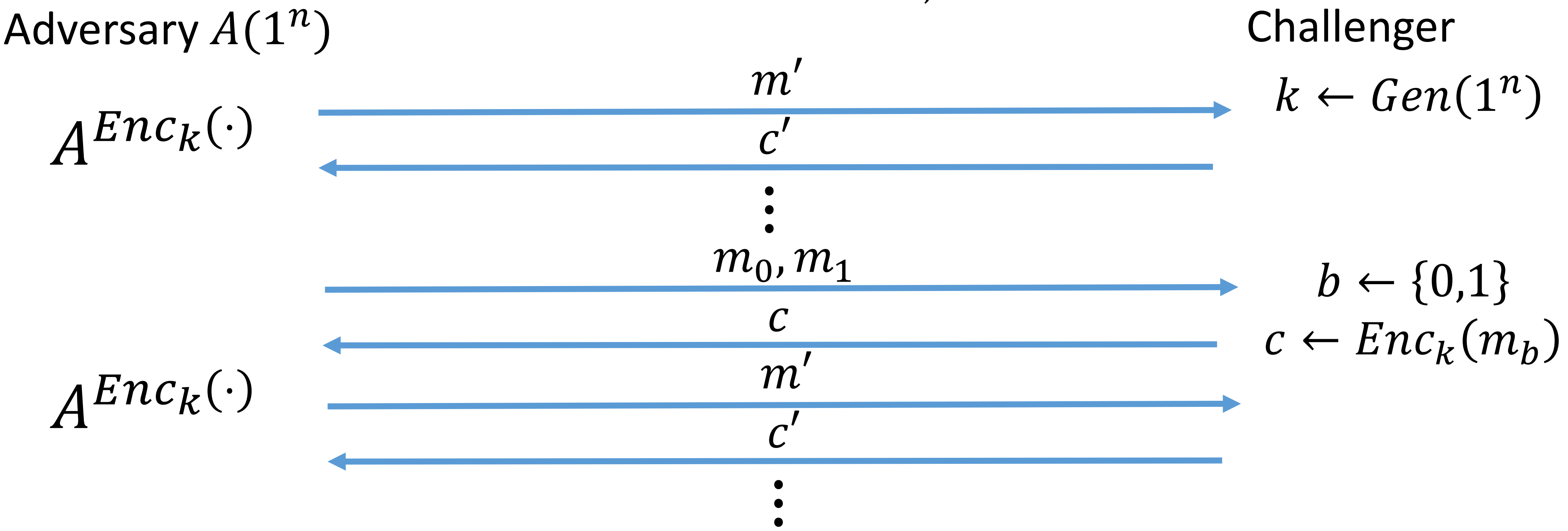

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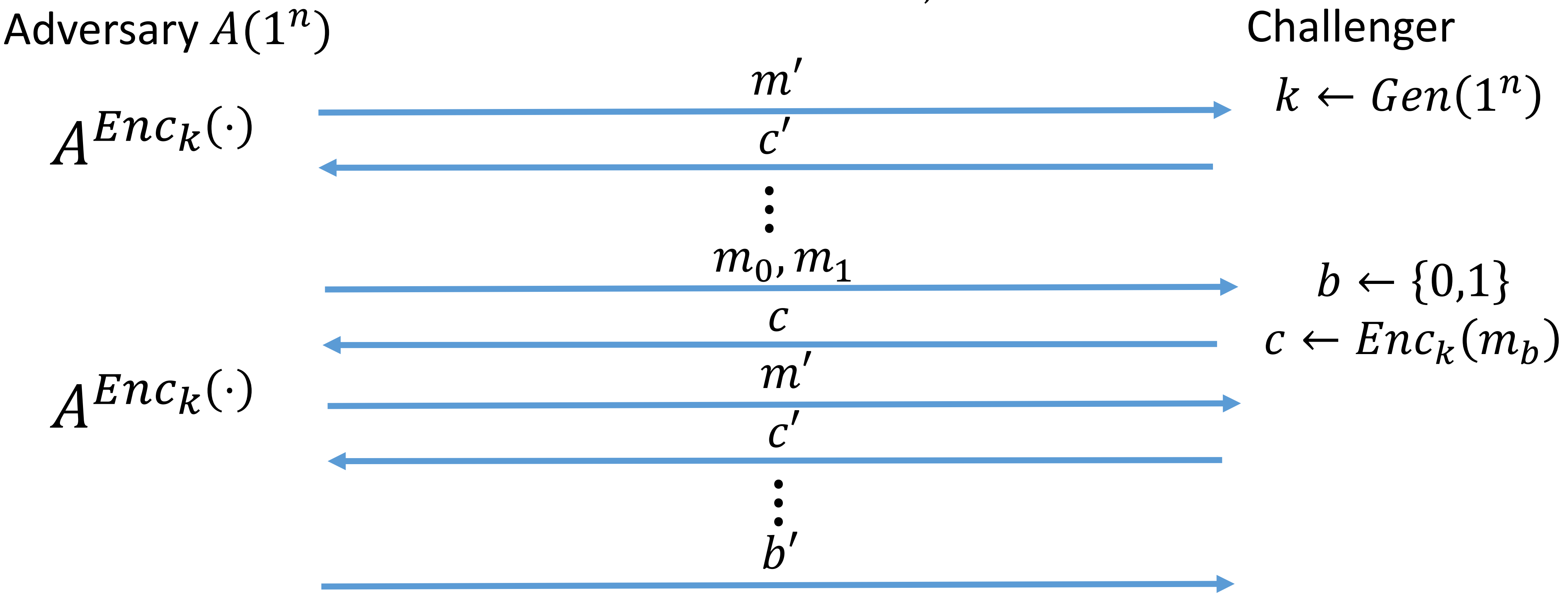
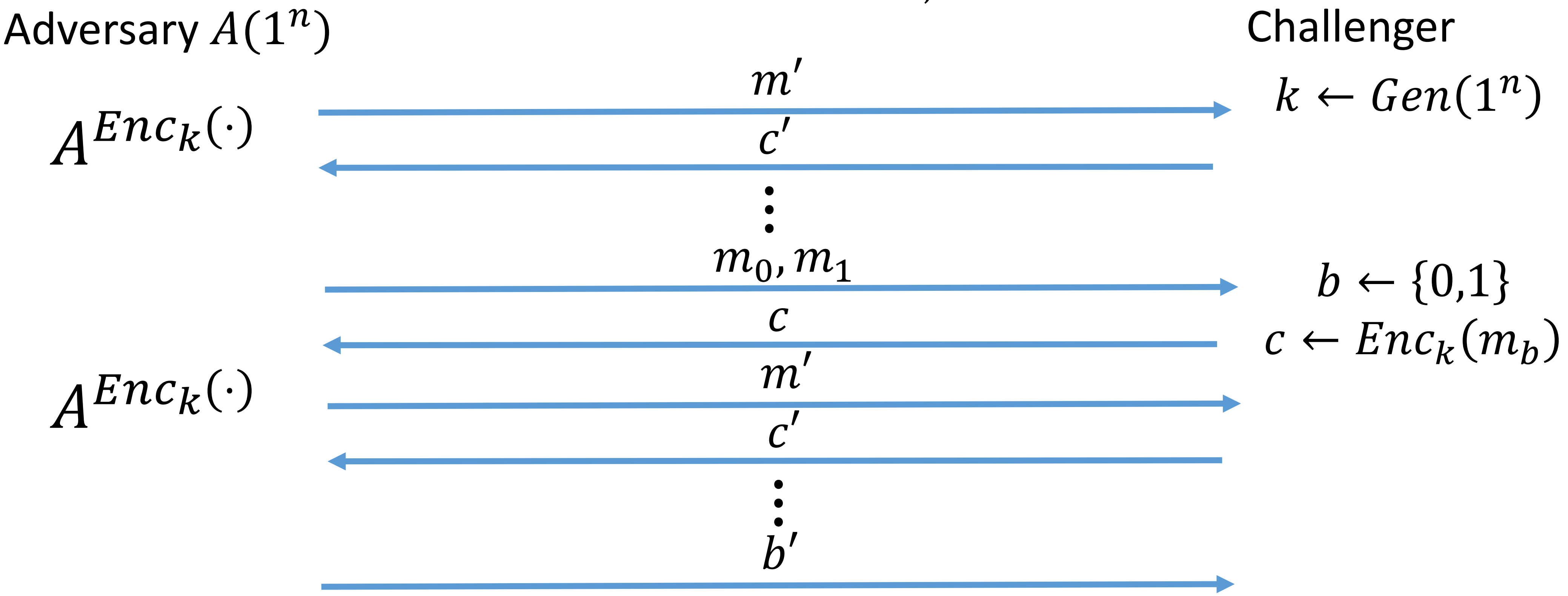


CPA Security

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK_{A,\Pi}^{cpa}(n)$



$PrivK_{A,\Pi}^{cpa}(n) = 1$ if $b' = b$ and $PrivK_{A,\Pi}^{cpa}(n) = 0$ if $b' \neq b$.

CPA-Security

The CPA Indistinguishability Experiment $PrivK^{cpa}{}_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Enc_k(\cdot)$, and outputs a pair of messages m_0, m_1 of the same length.
3. A random bit $b \leftarrow \{0,1\}$ is chosen, and then a challenge ciphertext $c \leftarrow Enc_k(m_b)$ is computed and given to A.
4. The adversary A continues to have oracle access to $Enc_k(\cdot)$, and outputs a bit b'.
5. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise.

CPA-Security

Definition: A private-key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions under a chosen-plaintext attack if for all ppt adversaries A there exists a negligible function $negl$ such that

$$\Pr\left[PrivK^{cpa}{}_{A,\Pi}(n) = 1\right] \leq \frac{1}{2} + negl(n),$$

where the probability is taken over the random coins used by A, as well as the random coins used in the experiment.

CPA-security for multiple encryptions

Theorem: Any private-key encryption scheme that has indistinguishable encryptions under a chosen-plaintext attack also has indistinguishable multiple encryptions under a chosen-plaintext attack.

CPA-secure Encryption Must Be Probabilisitic

Theorem: If $\Pi = (Gen, Enc, Dec)$ is an encryption scheme in which Enc is a deterministic function of the key and the message, then Π cannot be CPA-secure.

Why not?

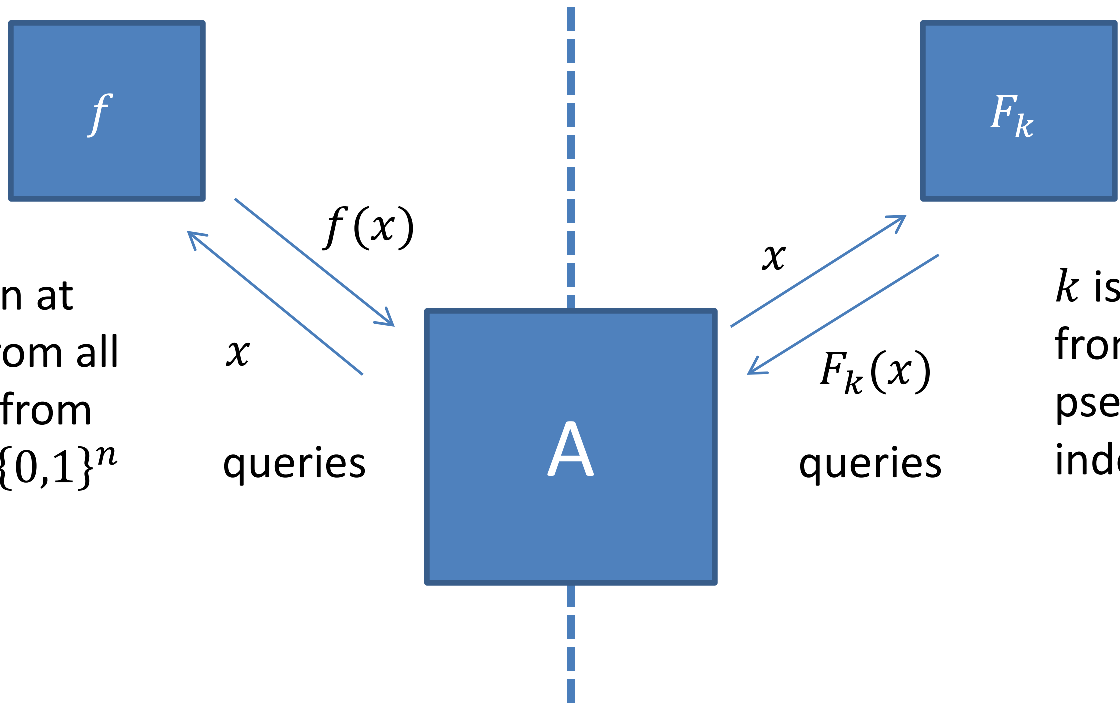
Constructing CPA-Secure Encryption Scheme

Pseudorandom Function

Definition: A keyed function $F: \{0,1\}^* \times \{0,1\}^* \rightarrow \{0,1\}^*$ is a two-input function, where the first input is called the key and denoted k.

Pseudorandom Function (PRF)

f is chosen at random from all functions from $\{0,1\}^n$ to $\{0,1\}^n$



k is chosen at random from $\{0,1\}^n$. F_k is the pseudorandom function indexed by k.

PRF: Any efficient A cannot tell which world it is in.

$$\left|\Pr[A^{f}() = 1] - \Pr[A^{F_k}() = 1]\right| \leq negligible$$

Pseudorandom Function

Definition: Let $F\colon \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$ be an efficient, length-preserving, keyed function. We say that F is a pseudorandom function if for all ppt distinguishers D, there exists a negligible function $negl$ such that:

$$\left|\Pr\left[D^{F_k(\cdot)}(1^n) = 1\right] - \Pr\left[D^{f(\cdot)}(1^n) = 1\right]\right| \leq negl(n).$$

where $k \leftarrow \{0,1\}^n$ is chosen uniformly at random and f is chosen uniformly at random from the set of all functions mapping n-bit strings to n-bit strings.