Cryptography

Lecture 12

Announcements

- Homework 4 due Monday 3/9
- Midterm Coming up on Wednesday 3/11
- Study Materials
  - Review sheet will be posted on course webpage, solutions posted on Canvas
    - Includes list of topics and corresponding textbook chapters
  - Cheat Sheet posted on Canvas
  - Extra practice folder on Canvas with additional class exercises and solutions

Agenda

- Last time:
  - Domain Extension for MACs (K/L 4.4) and Class Exercise solutions
  - CCA security (K/L 3.7)
  - Unforgeability for Encryption (K/L 4.5)
- This time:
  - Authenticated Encryption (K/L 4.5)
  - Collision-Resistant Hash Functions (K/L 5.1)
  - Hash-and-Mac
  - Domain extension for CRHF

Chosen Ciphertext Security

CCA Security

Consider a private-key encryption scheme $\Pi = (Gen, Enc, Dec)$, any adversary A, and any value n for the security parameter.

Experiment $PrivK_{A,\Pi}^{cca}(n)$

Adversary $A(1^n)$ — Challenger

$k \leftarrow Gen(1^n)$

$A^{Enc_k(\cdot),Dec_k(\cdot)}$

m'/c' →

← c'/m'

⋮

m_0, m_1 →

$b \leftarrow \{0,1\}$

$c \leftarrow Enc_k(m_b)$

← c

$A^{Enc_k(\cdot),Dec_k^*(\cdot)}$

m'/c' →

← c'/m'

⋮

b' →

A may not query challenge ciphertext c to the decryption oracle

$PrivK_{A,\Pi}^{cca}(n) = 1$ if $b' = b$ and $PrivK_{A,\Pi}^{cca}(n) = 0$ if $b' \neq b$.

CCA Security

The CCA Indistinguishability Experiment $PrivK^{cca}{}_{A,\Pi}(n)$:

1. A key k is generated by running $Gen(1^n)$.
2. The adversary A is given input 1^n and oracle access to $Enc_k(\cdot)$ and $Dec_k(\cdot)$, and outputs a pair of messages m_0, m_1 of the same length.
3. A random bit $b \leftarrow \{0,1\}$ is chosen, and then a challenge ciphertext $c \leftarrow Enc_k(m_b)$ is computed and given to A.
4. The adversary A continues to have oracle access to $Enc_k(\cdot)$ and $Dec_k(\cdot)$, but is not allowed to query the latter on the challenge ciphertext itself. Eventually, A outputs a bit b'.
5. The output of the experiment is defined to be 1 if $b' = b$, and 0 otherwise.

CCA Security

A private-key encryption scheme $\Pi = (Gen, Enc, Dec)$ has indistinguishable encryptions under a chosen-ciphertext attack if for all ppt adversaries A there exists a negligible function $negl$ such that

$$\Pr\left[PrivK^{cca}{}_{A,\Pi}(n) = 1\right] \leq \frac{1}{2} + negl(n),$$

where the probability is taken over the random coins used by A, as well as the random coins used in the experiment.

Authenticated Encryption

The unforgeable encryption experiment $EncForge_{A,\Pi}(n)$:

1. Run $Gen(1^n)$ to obtain key k.
2. The adversary A is given input 1^n and access to an encryption oracle $Enc_k(\cdot)$. The adversary outputs a ciphertext c.
3. Let $m \coloneqq Dec_k(c)$, and let Q denote the set of all queries that A asked its encryption oracle. The output of the experiment is 1 if and only if (1) $m \neq \perp$ and (2) $m \notin Q$.

Authenticated Encryption

Definition: A private-key encryption scheme Π is unforgeable if for all ppt adversaries A, there is a negligible funcion neg such that:

$$\Pr\left[EncForge_{A,\Pi}(n) = 1\right] \leq neg(n).$$

Definition: A private-key encryption scheme is an authenticated encryption scheme if it is CCA-secure and unforgeable.

Generic Constructions

Encrypt-and-authenticate

Encryption and message authentication are computed independently in parallel.

$$c \leftarrow Enc_{k_E}(m) \qquad t \leftarrow Mac_{k_M}(m)$$

$$\langle c, t \rangle$$

Is this secure?

Encrypt-and-authenticate

Encryption and message authentication are computed independently in parallel.

$$c \leftarrow Enc_{k_E}(m) \qquad t \leftarrow Mac_{k_M}(m)$$

$$\langle c, t \rangle$$

Is this secure? NO! Tag can leak info on m

Authenticate-then-encrypt

Here a MAC tag t is first computed, and then the message and tag are encrypted together.

$$t \leftarrow Mac_{k_M}(m) \qquad c \leftarrow Enc_{k_E}(m||t)$$

c is sent

Is this secure?

Authenticate-then-encrypt

Here a MAC tag t is first computed, and then the message and tag are encrypted together.

$$t \leftarrow Mac_{k_M}(m) \qquad c \leftarrow Enc_{k_E}(m||t)$$

c is sent

Is this secure? NO! Encryption scheme may not be CCA-secure.

Encrypt-then-authenticate

The message m is first encrypted and then a MAC tag is computed over the result

$$c \leftarrow Enc_{k_E}(m) \quad t \leftarrow Mac_{k_M}(c)$$

$$\langle c, t \rangle$$

Is this secure?

Encrypt-then-authenticate

The message m is first encrypted and then a MAC tag is computed over the result

$$c \leftarrow Enc_{k_E}(m) \quad t \leftarrow Mac_{k_M}(c)$$

$$\langle c, t \rangle$$

Is this secure? YES! As long as the MAC is strongly secure.

Collision Resistant Hashing

Collision Resistant Hashing

Definition: A hash function (with output length ℓ) is a pair of ppt algorithms (Gen, H) satisfying the following:

- Gen takes as input a security parameter 1^n and outputs a key s. We assume that 1^n is implicit in s.
- H takes as input a key s and a string $x \in \{0,1\}^*$ and outputs a string $H^s(x) \in \{0,1\}^{\ell(n)}$.

If H^s is defined only for inputs $x \in \{0,1\}^{\ell'(n)}$ and $\ell'(n) > \ell(n)$, then we say that (Gen, H) is a fixed-length hash function for inputs of length ℓ'. In this case, we also call H a compression function.

The collision-finding experiment

$Hashcoll_{A,\Pi}(n)$:

1. A key s is generated by running $Gen(1^n)$.
2. The adversary A is given s and outputs x, x'. (If Π is a fixed-length hash function for inputs of length $\ell'(n)$, then we require $x, x' \in \{0,1\}^{\ell'(n)}$.)
3. The output of the experiment is defined to be 1 if and only if $x \neq x'$ and $H^s(x) = H^s(x')$. In such a case we say that A has found a collision.

Security Definition

Definition: A hash function $\Pi = (Gen, H)$ is collision resistant if for all ppt adversaries A there is a negligible function neg such that

$$\Pr\left[Hashcoll_{A,\Pi}(n) = 1\right] \leq neg(n).$$

Message Authentication Using Hash Functions

Hash-and-Mac Construction

Let $\Pi = (Mac, Vrfy)$ be a MAC for messages of length $\ell(n)$, and let $\Pi_H = (Gen_H, H)$ be a hash function with output length $\ell(n)$. Construct a MAC $\Pi' = (Gen', Mac', Vrfy')$ for arbitrary-length messages as follows:

- Gen': on input 1^n, choose uniform $k \in \{0,1\}^n$ and run $Gen_H(1^n)$ to obtain s. The key is $k' := \langle k, s \rangle$.
- Mac': on input a key $\langle k, s \rangle$ and a message $m \in \{0,1\}^*$, output $t \leftarrow Mac_k(H^s(m))$.
- $Vrfy'$: on input a key $\langle k, s \rangle$, a message $m \in \{0,1\}^*$, and a MAC tag t, output 1 if and only if $Vrfy_k(H^s(m), t) = 1$.

Security of Hash-and-MAC

Theorem: If Π is a secure MAC for messages of length ℓ and Π_H is collision resistant, then the construction above is a secure MAC for arbitrary-length messages.

Proof Intuition

Let Q be the set of messages m queried by adversary A.

Assume A manages to forge a tag for a message $m^* \notin Q$.

There are two cases to consider:

1. $H^s(m^*) = H^s(m)$ for some message $m \in Q$. Then A breaks collision resistance of H^s.
2. $H^s(m^*) \neq H^s(m)$ for all messages $m \in Q$. Then A forges a valid tag with respect to MAC Π.