Broadcast Stability in Random Access

Brooke Shrader and Anthony Ephremides
Electrical and Computer Engineering Dept. and Institute for Systems Research
University of Maryland, College Park, MD 20742, USA
Email: {bshrader,tony}@eng.umd.edu

***Abstract*— We introduce and study the problem of broadcast stability in a network where nodes utilize the ALOHA protocol to gain random access to the channel. We make use of the dominating systems argument used in previous works and also develop a novel method for finding the region of stable arrival rates of packets. The stability region is obtained by analyzing the broadcast service process for a channel with multipacket reception. Our results exhibit the effect of probabilistic reception and multipacket reception on the broadcast stability region. We also show that the broadcast stability region is contained within the stability region for unicast transmission. Our work is applicable to the broadcast transmission that underlies communication in many wireless networks, including multihop networks. In addition, our new method for finding the stability region may be applicable to previously unsolved problems, including the stability region for arbitrarily many sources and destinations.**

I. INTRODUCTION

We are interested in the following scenario. Information packets arriving to a source node are intended to be sent to a group of nodes in a network. Packets that cannot immediately be sent are queued at the source. The source competes for random access to the channel through the ALOHA protocol and thus suffers interference from other sources that are attempting to broadcast their packets to the same group of nodes. This framework is applicable to network discovery mechanisms (e.g., route discovery) and more generally to broadcast transmission, which is inherent to the wireless medium.

We will examine the stability properties of this random access broadcast system for a finite number of source nodes. This reduces to a problem of interacting queues. The stability of interacting queues has been previously researched for unicast transmission to a single destination. Studies on the stability conditions of ALOHA were initiated in [1] using the transition probabilities of the corresponding Markov chain. Later work suggested the use of stochastic dominance (described below) as a means of achieving the same results. The dominating systems approach provided an exact characterization of the stability region for $N{=}2$ sources in [2] and $N{=}3$ sources in [3]. For $N{>}2$ only bounds on the stability region have been obtained (e.g. [4]).

An important aspect of previous work on ALOHA stability is the channel model. All works cited above make use of the *collision* channel model in which access to the channel by more than one source causes a collision and all packets are lost with probability one. Similarly, if a source accesses the channel without interference its packet is received correctly with probability one. Recent work in [5] better reflects the nature of the wireless channel by allowing for probabilistic reception and multipacket reception (MPR). That work showed that the ALOHA stability region for unicast transmission experiences a phase transition when MPR capability is accounted for.

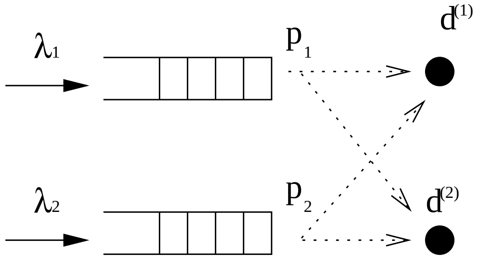

Fig. 1. Two sources broadcast to two destinations.

II. SYSTEM MODEL

We formulate the problem of broadcast stability as follows. We study the simple scenario depicted in Fig. 1 in which $N{=}2$ sources denoted by s_1 and s_2 wish to broadcast to $M{=}2$ destinations, $d^{(1)}$ and $d^{(2)}$. Packet arrivals to each source are Bernoulli distributed with rate λ_n packets/slot. We employ the *late arrival model* described in [6] in which packet arrivals occur near the end of the k^{th} slot and service can be started no earlier than in the $k{+}1^{st}$ slot. Packets that are not immediately transmitted are stored in a single buffer maintained at each source. Transmissions are regulated by the ALOHA protocol such that in any given slot, if source n has a packet waiting, it transmits with probability p_n.

Success of transmissions is determined by the channel model with notation adapted from [5] for reception probabilities. For $N{=}2$ sources these probabilities are given below.

$$\begin{aligned} q^{(m)}_{n|n} &= Pr\{\text{packet from source } n \text{ is received} \\ &\qquad \text{at destination } m|\text{ only source } n \text{ transmits}\} \\ q^{(m)}_{n|1,2} &= Pr\{\text{packet from source } n \text{ is received} \\ &\qquad \text{at destination } m|\text{ both sources transmit}\}, n{=}1,2 \end{aligned}$$

For example, if both s_1 and s_2 have packets waiting to be transmitted, the probability that a packet transmitted from source s_1 is successfully received at both destinations in a certain slot is given by

$$\tau_1 = \overline{p_2} q^{(1)}_{1|1} q^{(2)}_{1|1} + p_2 q^{(1)}_{1|1,2} q^{(2)}_{1|1,2} \tag{1}$$

where $\overline{p_2} = 1 - p_2$ and generally $\tau_n, n{=}1,2$ is the probability that a packet from s_n is received at both destinations when that source attempts transmission. Upon successful reception, a destination sends an acknowledgement (ACK) packet to the source. It is assumed that every source-destination pair has an orthogonal control channel for ACKs. If an ACK is not received, the packet is retransmitted. In this work we study the scenario in which sources retransmit their packets until they receive an ACK from all destinations.

The stability region we would like to characterize corresponds to the stability conditions of the two-dimensional Markov chain formed by the pair of queue lengths at s_1 and s_2. (Note that each queue does not form a one-dimensional Markov chain since the service rate is dependent upon the state of the other queue.) Precisely, the stability region is the region of values $[\lambda_1, \lambda_2]$ such that this Markov chain is ergodic.

III. DOMINATING SYSTEMS APPROACH

The concept of dominating systems is key to solving for the broadcast stability region because it allows us to decouple the interaction of the queues to find average service rates. This approach is used in previous work to find the stability region. The basic idea is the following. We can introduce a system S^2 that differs from the original system S in that whenever the buffers at s_1 and s_2 become empty, the sources transmit 'dummy' packets that can cause interference but do not increase the throughput of the source. In system S^2 the service rates are constants which we denote by μ_{nb}, $n{=}1,2$, where the subscript b denotes the backlogged state of the other source. System S^2 dominates system S in the sense that the queue lengths in S^2 can never be smaller than those in S. Thus stability of the queues in S^2 implies stability in S. We can find the stability conditions for S^2 by applying Loynes' result [7] which tells us that ergodicity holds if the average arrival rate λ_n is less than the average service rate.

We can find the exact stability region in our problem by considering the dominating systems $S_n^1, n{=}1,2$, in which only s_n transmits dummy packets when it empties while the other source behaves as in the original system S. We note that if the queues in S_n^1 and S begin in the same initial state and never empty, the two systems will be indistinguishable.

First, consider S_1^1 in which p_1 and p_2 are fixed. In this system, s_1 transmits dummy packets when it empties and is always in the backlogged state. As a result, the queue at s_2 behaves as an independent M/G/1 queue, and the sufficient stability condition is $\lambda_2 < \mu_{2b}$. Assuming that this condition is satisfied, we can find the stability condition for s_1 by considering the two possible states of s_2: empty and backlogged. The probability of each state is determined by the utilization factor of s_2, where ρ_2 denotes the probability that s_2 is backlogged. (Our use of the late arrival model [6] makes this so.) The utilization is the arrival rate divided by the service rate, $\rho_2 = \lambda_2/\mu_{2b}$. The service rate of s_1 is given by

$$\rho_2\mu_{1b} + (1-\rho_2)\mu_{1e}$$

where the subscript e in μ_{1e} denotes the state in which s_2 is empty. Taken together with the condition for stability of s_2, the stability condition for the system S_1^1 is given by

$$\lambda_1 < \rho_2\mu_{1b} + (1-\rho_2)\mu_{1e}, \quad \lambda_2 < \mu_{2b}. \tag{2}$$

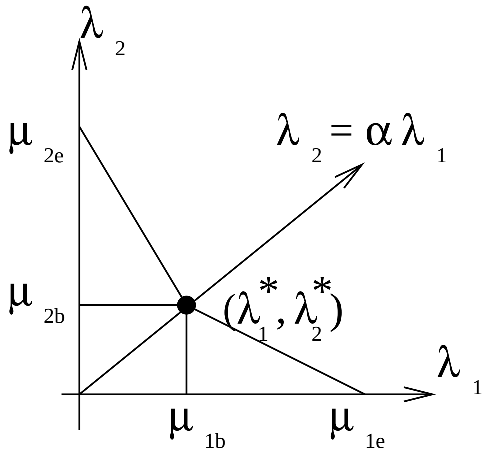

Fig. 2. Stability region for fixed p_1 and p_2.

By similar arguments, we can write down the stability conditions for system S_2^1 and fixed $[p_1, p_2]$ as follows.

$$\lambda_1 < \mu_{1b}, \quad \lambda_2 < \rho_1\mu_{2b} + (1-\rho_1)\mu_{2e}. \tag{3}$$

The union of the two regions given by Eqns. 2 and 3 provides the stability region of system S for fixed transmission probabilities p_n. A diagram of the boundaries given by these equations is shown in Fig. 2.

Once the stability region for fixed $[p_1, p_2]$ has been found, the stability region over all $[\lambda_1, \lambda_2]$ can be found as the union of all such regions as $[p_1, p_2]$ varies over $[0,1]^2$. There are two equivalent formulations for achieving the final result. First, one can compute the geometric envelope of the lines given by Eqns. (2) and (3) for parameters $[p_1, p_2]$. Alternatively, the boundary of the region can be found by solving a constrained optimization problem in which λ_1 is fixed and λ_2 is maximized over all values $[p_1, p_2] \in [0,1]^2$ with constraints given by Eqns. (2) and (3). This is the approach used in [5].

IV. A NOVEL APPROACH

In this section we outline a novel approach to determining the stability region for a system of interacting queues. This approach finds the point along the line $\lambda_2 = \alpha\lambda_1$, $0{<}\alpha{<}\infty$ that coincides with the boundary of the stability region. We can then examine various values of α to trace out the boundary of the stability region. This new approach has merit in its simplicity and independence from the arrival process. Also, it may be applicable to the case of arbitrary number of sources N.

The idea behind this approach is as follows. Consider an operating point $[l_1, l_2]$ in the region of arrival rates that is known to be unstable. (For example, consider arrival rates greater than 1 packet/slot which must be unstable for a system in which a source can transmit only 1 packet/slot.) At this point, the queues at the sources will grow to infinity and both sources will remain in the backlogged state. Thus, the average service rate at source s_n will be μ_{nb}. Now, consider the line $\lambda_2 = \alpha\lambda_1$ that connects point $[l_1, l_2]$ to the origin. As we move from $[l_1, l_2]$ towards the origin along this line, we will eventually encounter the point $[\lambda_1^*, \lambda_2^*]$ that intersects with the boundary of the stability region as shown in Fig. 2. At point $[\lambda_1^*, \lambda_2^*]$ the service rates will still be μ_{nb}, but as soon as we move past this point towards the origin, the queues at the two sources will become stable simultaneously. Thus, the queues will empty occasionally and the departure of packets will occur at rates less than μ_{nb}. We can therefore identify point $[\lambda_1^*, \lambda_2^*]$ as the point along $\lambda_2 = \alpha\lambda_1$ at which μ_{nb} has a stationary point. This stationary point corresponds to some $[p_1^*, p_2^*]$.

For a given α, we can find $[p_1^*, p_2^*]$ through a sequence of computations. First, since λ_n^* equals μ_{nb} evaluated at $[p_1^*, p_2^*]$, we use the equation $\mu_{2b} = \alpha\mu_{1b}$ to solve for p_1 as a function of p_2. Using this expression, we can eliminate p_1 and write the service rate μ_{1b} as a function of only p_2. Next, we find p_2^* such that μ_{1b} is maximized. In many cases, this is achieved by setting the derivative of μ_{1b} with respect to p_2 to zero. However, there are instances in which μ_{1b} is maximized at a boundary, $p_2^* = 0$ or $p_2^* = 1$, and this possibility is accounted for as well. Next, we substitute p_2^* into our earlier expression to obtain p_1^*. Finally, using $[p_1^*, p_2^*]$, we compute the average service rates μ_{nb} that correspond to the point $[\lambda_1^*, \lambda_2^*]$. An example of the use of this method on the unicast collision channel model can be found in the Appendix.

V. BROADCAST SERVICE RATES

We need to find expressions for the service rates μ_{1b}, μ_{1e}, μ_{2b}, and μ_{2e} which characterize the stability region. For unicast transmission, service times are geometrically distributed and the computation is trivial. For broadcast transmission, these computations are more involved. For ease in expressing these computations, we introduce the placeholders ϕ_n and σ_n that are similar to τ_n introduced in Eqn. 1. Let ϕ_n denote the probability that a successful reception occurs at $d^{(1)}$ when s_n transmits. Similarly, σ_n denotes the probability that $d^{(2)}$ successfully receives when s_n transmits. As an example ϕ_1 and σ_1 are defined for s_1 as follows.

$$\begin{aligned} p_1\phi_1 &= p_1\overline{p_2}q_{1|1}^{(1)}+p_1p_2q_{1|1,2}^{(1)} &(4)\\ p_1\sigma_1 &= p_1\overline{p_2}q_{1|1}^{(2)}+p_1p_2q_{1|1,2}^{(2)} &(5)\end{aligned}$$

With this notation in place, we can express the probabilities of different reception scenarios on the channel. These expressions are used later in computation of the average service rate. First, when both sources are backlogged, the probability that s_1 attempts transmission and is successful at $d^{(1)}$ but not at $d^{(2)}$ is given by

$$p_1\left(\phi_1-\tau_1\right)=p_1\overline{p_2}q_{1|1}^{(1)}\overline{q_{1|1}^{(2)}}+p_1p_2q_{1|1,2}^{(1)}\overline{q_{1|1,2}^{(2)}} \qquad (6)$$

Similarly, the probability that s_1 attempts transmission and is successful at $d^{(2)}$ but not at $d^{(1)}$ is given by

$$p_1\left(\sigma_1-\tau_1\right)=p_1\overline{p_2}\,\overline{q_{1|1}^{(1)}}q_{1|1}^{(2)}+p_1p_2\overline{q_{1|1,2}^{(1)}}q_{1|1,2}^{(2)} \qquad (7)$$

Finally, the probability that s_1 attempts transmission and its packet is received at neither $d^{(1)}$ nor $d^{(2)}$ is given by

$$p_1\left(\tau_1+1-\phi_1-\sigma_1\right)=p_1\overline{p_2}\,\overline{q_{1|1}^{(1)}}\,\overline{q_{1|1}^{(2)}}+p_1p_2\overline{q_{1|1,2}^{(1)}}\,\overline{q_{1|1,2}^{(2)}} \qquad (8)$$

A. Service time distribution

One way to compute the average service rates is by using the probability mass function (pmf) for the service time. The pmf $f_n(k)$ represents the probability that s_n must transmit k times before its packet is successfully received at both destinations. When both sources have a packet to transmit (i.e. both are backlogged), we can write $f_1(k)$ for s_1 as follows.

$$\begin{aligned} &f_1(k) = p_1\tau_1\left(\overline{p_1}+p_1\left(\tau_1+1-\phi_1-\sigma_1\right)\right)^{k-1} + \\ &p_1\phi_1\sum_{l=1}^{k-1}\binom{k-1}{l}\left(p_1\left(\sigma_1-\tau_1\right)\right)^l\left(\overline{p_1}+p_1\left(\tau_1+1-\phi_1-\sigma_1\right)\right)^{k-1-l} + \\ &p_1\sigma_1\sum_{j=1}^{k-1}\binom{k-1}{j}\left(p_1\left(\phi_1-\tau_1\right)\right)^j\left(\overline{p_1}+p_1\left(\tau_1+1-\phi_1-\sigma_1\right)\right)^{k-1-j} \end{aligned} \qquad (9)$$

This is a sum of three terms with probabilities corresponding to Eqns. 6, 7, and 8. The first term represents the probability that both destinations receive the packet on the k^{th} attempt and no earlier. The second term states that $d^{(1)}$ receives the packet on the k^{th} attempt and no earlier while $d^{(2)}$ receives the packet on at least one of the first $k-1$ attempts. The third term accounts for the situation in which $d^{(2)}$ receives the packet on the k^{th} attempt and no earlier whereas $d^{(1)}$ receives the packet on at least one of the first $k-1$ attempts. A similar expression can be written for $f_2(k)$ by replacing p_1, τ_1, σ_1, and ϕ_1 with p_2, τ_2, σ_2, and ϕ_2, respectively. We can find the expected service time from this pmf by taking $\sum_{k=1}^{\infty} kf_n(k)$. (Note the requirements that $\phi_1 > 0$, $\sigma_1 > 0$, and $\sigma_1 + \phi_1 > \tau_1$ for the sum to converge.) Finally, $(\sum_{k=1}^{\infty} kf_n(k))^{-1}$ corresponds to the average backlogged service rate. The exact expression is given below.

$$\mu_{nb} = \frac{p_n\phi_n\sigma_n(\phi_n+\sigma_n-\tau_n)}{(\phi_n+\sigma_n)(\phi_n+\sigma_n-\tau_n)-\phi_n\sigma_n} \qquad (10)$$

To find the empty service rates, we can write the pmf $f_1(k)$ with p_2 set to 0 and $f_2(k)$ with p_1 set to 0. In doing so, we see that the service rate μ_{1e} can be found by evaluating our expressions for the backlogged case at $p_2 = 0$. Similarly, we find μ_{2e} by evaluating μ_{2b} at $p_1 = 0$.

$$\mu_{1e} = \mu_{1b}|_{p_2=0}, \quad \mu_{2e} = \mu_{2b}|_{p_1=0} \qquad (11)$$

B. Markovian receiver state model

Another method of computing the average service rates of the broadcast system is through the use of a Markovian model for the status of the receivers. We include a description of this

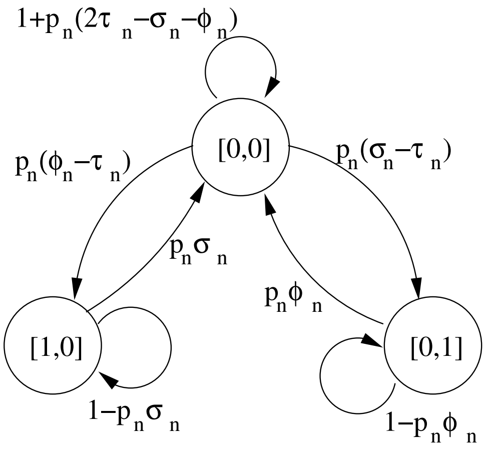

Fig. 3. Markov chain and transition probabilities for receiver state.

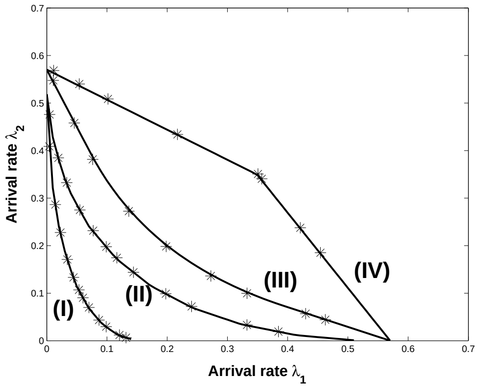

Fig. 4. Broadcast stability regions for different channels: (I) asymmetric sources, No MPR (II) symmetric sources, No MPR (III) symmetric sources, weak MPR (IV) symmetric sources, strong MPR . Results of our novel method are shown as *.

method as it provides additional insight into the behavior of the broadcast system. This method computes average service rate by taking the expected value of B_n, which denotes the probability that the packet currently in service at s_n completes service and is removed from the buffer. Clearly, B_n takes different values depending upon which of the destinations $d^{(m)}, m{=}1,2$, have already received the packet.

To describe B_n, we introduce the receiver state variables $r_n^{(m)}$ for $n, m = 1, 2$. Let $\mathbf{r_n} = [\mathbf{r_n^{(1)}}\mathbf{r_n^{(2)}}]$ be a vector of binary values indicating whether the packet currently in service at s_n has been received at each destination. The possible receiver states are $\mathbf{r_n} = \{[\mathbf{0},\mathbf{0}], [\mathbf{1},\mathbf{0}], [\mathbf{0},\mathbf{1}]\}$. We do not allow for the state $[1, 1]$ since upon reaching that state, the source will immediately either begin serving the next packet or become idle. With the receiver states defined, we can write the conditional pmf of B_n. When both sources are backlogged, B_1 satisfies the equation below.

$$P\{B_1 = 1|\mathbf{r_1}\} = \begin{cases} p_1\tau_1, & \mathbf{r_1}\text{=[0,0]} \\ p_1\phi_1, & \mathbf{r_1}\text{=[0,1]} \\ p_1\sigma_1, & \mathbf{r_1}\text{=[1,0]} \end{cases} \tag{12}$$

We can write a similar expression for B_2 when both sources are backlogged. To find the expected value of B_n we must first find the probability of each receiver state.

The set of receiver states can be modeled by the Markov chain with transition probabilities depicted in Fig. 3. We can then compute the steady-state probabilities of the receiver states $\mathbf{r_n}$ and use them to find the expected value of B_n.

As an example, we compute the steady-state probabilities of $\mathbf{r_1}$ and the expected value of B_1 when both sources are backlogged. Let $\pi_{0,0}$, $\pi_{0,1}$, and $\pi_{1,0}$ denote the steady-state probabilities of $\mathbf{r_1}$ when both sources are backlogged. Through use of the balance equations for the Markov chain and the equation $\pi_{0,0} + \pi_{0,1} + \pi_{1,0} = 1$ we find the steady-state probabilities to be as follows.

$$\begin{aligned} \pi_{0,0} &= \frac{\phi_1\sigma_1}{(\phi_1+\sigma_1)(\phi_1+\sigma_1-\tau_1)-\phi_1\sigma_1} \\ \pi_{0,1} &= \frac{\sigma_1(\sigma_1-\tau_1)}{(\phi_1+\sigma_1)(\phi_1+\sigma_1-\tau_1)-\phi_1\sigma_1} \\ \pi_{1,0} &= \frac{\phi_1(\phi_1-\tau_1)}{(\phi_1+\sigma_1)(\phi_1+\sigma_1-\tau_1)-\phi_1\sigma_1} \end{aligned}$$

Then the service rate μ_{1b} can be obtained as

$$\mu_{1b} = E[B_1] = p_1\tau_1\pi_{0,0} + p_1\phi_1\pi_{0,1} + p_1\sigma_1\pi_{1,0}. \tag{13}$$

The result for μ_{1b} is the same as given in Eqn. 10. We can employ a similar strategy to find the empty service rates μ_{ne} (i.e., the Markov chain for $\mathbf{r_1}$ has the same transition probabilities evaluated at $p_2 = 0$). As expected, the expressions for μ_{ne} are the same as given in Eqn. 11.

VI. RESULTS

The broadcast stability region with varying degrees of MPR is shown in Fig. 4 and corresponds to the reception probabilities shown in Table I. Channel model (I) has asymmetric sources (i.e., one source has a better channel than the other) and the result is a stability region that is truncated for the source with the poorer channel. Channels (III) and (IV) show the effect of MPR, which is the phase transition of the stability region. Though it's difficult to discern, the edges of the stability region for (III) are straight lines. This is caused by the introduction of a small non-zero probability for reception in the presence of interference. Channel (IV) has higher MPR probabilities which give rise to a concave stability region that's bounded by straight lines. The behavior of these curves agrees with results in [5]. This indicates that in relation to MPR capabilities, the broadcast channel behaves in a similar fashion to the unicast channel.

The performance of our novel approach for finding the stability region is also shown in Fig. 4. All points generated by the proposed approach fall on the boundary found using the dominating systems approach. This validates the effectiveness

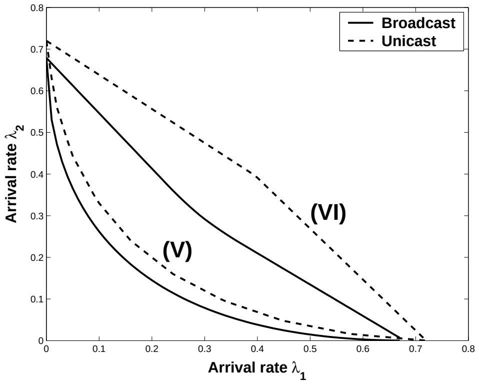

Fig. 5. Comparison of broadcast and unicast stability regions for symmetric destinations and (V) No MPR (VI) MPR.

TABLE I

CHANNEL MODELS: (I) ASYMMETRIC SOURCES, NO MPR (II) SYMMETRIC SOURCES, NO MPR (III) SYMMETRIC SOURCES, WEAK MPR (IV) SYMMETRIC SOURCES, STRONG MPR (V) SYMMETRIC DESTINATIONS, NO MPR (VI) SYMMETRIC DESTINATIONS, MPR

Channel	I	II	III	IV	V	VI
$q^{(1)}_{1\|1}$	0.3	0.9	0.9	0.9	0.8	0.8
$q^{(2)}_{1\|1}$	0.2	0.6	0.6	0.6	0.8	0.8
$q^{(1)}_{1\|1,2}$	0	0	0.3	0.6	0	0.4
$q^{(2)}_{1\|1,2}$	0	0	0.2	0.4	0	0.4
$q^{(2)}_{2\|2}$	0.9	0.9	0.9	0.9	0.8	0.8
$q^{(1)}_{2\|2}$	0.6	0.6	0.6	0.6	0.8	0.8
$q^{(2)}_{2\|1,2}$	0	0	0.3	0.6	0	0.4
$q^{(1)}_{2\|1,2}$	0	0	0.2	0.4	0	0.4

of our new method. The one anomaly we see is that under the influence of strong MPR, the point $[\lambda_1^*, \lambda_2^*]$ found by our approach does not always fall on the line $\lambda_2 = \alpha\lambda_1$. Thus our proposed method may need to be slightly modified for the strong MPR scenario.

Finally, we compare the broadcast stability with the unicast stability in Fig. 5. In order to make a fair comparison, we show results for a channel with symmetric destinations (i.e., the channels from a source to each destination are identical). The unicast stability regions are computed using results from [5] where one of the destinations is simply ignored. As expected, the broadcast stability region is always contained within the unicast stability region. Additionally, the transition from convex to concave regions occurs at different points for broadcast and unicast.

VII. CONCLUSION

We have studied the broadcast stability for random access in a small network of two sources and two destinations. We employ the dominating systems approach as well as a novel approach to compute stability conditions for the interacting queues at the source nodes. In solving for the stability region, we have completed an analysis of broadcast service through characterization of the service process itself and by considering the Markovian nature of the receiver status. Our results hold for a general wireless channel and can be extended to larger networks.

The problem considered in this work provides numerous opportunities for continuing research. Under the same setting, one might consider broadcast retransmission control and its effect on the stability of the network. The stability of a network with relay nodes is also a related and interesting open problem.

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APPENDIX

We can apply our proposed approach to find points on the boundary of the stability region for unicast transmission over a collision channel. In this scenario, the backlogged service rates are $\mu_{1b} = p_1\overline{p_2}$ and $\mu_{2b} = p_2\overline{p_1}$. By solving the equation $\mu_{2b} = \alpha\mu_{1b}$ we obtain

$$p_1 = \frac{p_2}{\alpha + p_2(1-\alpha)} \tag{14}$$

and can write the average backlogged service rate of s_1 as

$$\mu_{1b} = \frac{p_2(1-p_2)}{\alpha + p_2(1-\alpha)}. \tag{15}$$

Next we take the derivative of μ_{1b} in Eqn. 15 with respect to p_2 and set it equal to zero. The result is

$$p_2^* = \frac{-\alpha \mp \sqrt{\alpha}}{1-\alpha}. \tag{16}$$

After substituting back into Eqn. 14 we obtain

$$p_1^* = \frac{\pm\sqrt{\alpha}+1}{1-\alpha}. \tag{17}$$

Our final result satisfies the parametric equations

$$\lambda_1^* = p_1^*(1-p_2^*), \quad \lambda_2^* = p_2^*(1-p_1^*), \quad p_1^* + p_2^* = 1. \tag{18}$$

This is identical to the result obtained in [2] using the dominating systems approach.

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