Energy-Based Control of a Flexible Beam

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**Introduction**

- In this work we are interested to stabilize the flexible beam under disturbances.
- The control effort is the horizontal motion of the cart.
- The system is modeled in a port-controlled Hamiltonian (PCH) framework.
  - Both the beam and the cart are mechanical energy storing elements. They exchange energy between themselves and the control objective has been achieved by regulating this process.
  - We adopt the energy Casimir method to achieve this goal.

**Modeling**

- Hamiltonian (total energy) for the flexible beam (\(H\)):
  \[
  \frac{1}{2} \int_0^L \left( \frac{\partial w}{\partial t} + \frac{\partial \Pi}{\partial \xi} \right) + \frac{1}{2} \left( \frac{\partial \Pi}{\partial \xi} \right) + K \left( \frac{\partial \Pi}{\partial \xi} \right) \text{ d}L
  \]
  - To model this infinite-dimensional system we need to introduce the following differential 1-forms:
    - Translational Momentum: \(p_x(t) = \frac{\partial}{\partial \xi} \frac{\partial \Pi}{\partial \xi} \text{ d}L\)
    - Rotational Momentum: \(p_{\theta}(t) = \frac{\partial}{\partial \xi} \frac{\partial \Pi}{\partial \xi} \text{ d}L\)
    - Translational Strain: \(\varepsilon(t) = \frac{\partial}{\partial \xi} \frac{\partial \Pi}{\partial \xi} \text{ d}L\)
    - Rotational Strain: \(\gamma(t) = \frac{\partial}{\partial \xi} \frac{\partial \Pi}{\partial \xi} \text{ d}L\)
  
  where, \(w(t, \xi) = w(t, \xi_0 + \xi) \forall \xi\)
- Now we’ll define a Dirac structure (\(\mathbb{D}\)), essentially a notion of orthogonality to express power conservation in the system, over the space of flows and efforts of the system.
- Once the Dirac structure has been defined the port-Hamiltonian formulation will automatically follow.

- Exploiting the underlying Dirac structure, the infinite dimensional port Hamiltonian model for the flexible beam is obtained as:
  \[
  \begin{align*}
  \dot{p}_x & = 0 & \dot{p}_{\theta} & = 0 & \dot{\xi} & = \frac{\partial R}{\partial \xi} - \frac{\partial U}{\partial \xi} & \text{ and, } & \dot{\theta} & = \frac{\partial R}{\partial \theta} - \frac{\partial U}{\partial \theta} & \text{ Hodge Star Operator} \\
  \frac{\partial \Pi}{\partial \xi} & = 0 & \frac{\partial \Pi}{\partial \theta} & = 0 & \frac{\partial \Pi}{\partial \xi} & = 0 & \text{ Exterior Derivative} \\
  \end{align*}
  \]

- Cart: a typical second order system governed by a single configuration variable (the displacement \(x\))
- Controller: one configuration variable (\(\theta\))
- The controller and the cart is considered as an integrated system.

**Motivation:**

Separation of the finite dimensional and infinite dimensional parts of the overall system

- Combined Hamiltonian: \(H_c = H_{\text{cart}} + H_{\text{controller}}\)

**Dynamics of integrated system:**

- \(\dot{q}_c = G_c^T \partial_{q_c} H_c\) for \(c < 0\)
- \(f_{c2} = 0\)

**Power Conserving Interconnection:**

**Stabilization**

- Equilibrium Configuration: Upright position of the beam with stationary cart.
- Energy-Casimir approach is adopted to stabilize the relative equilibrium.
- Casimir functionals are invariant of the trajectories of the system and relates the states of the controller with those of the cart and the beam.
- Number of Casimir functional for this system: 2

- Casimir Functional:
  \[\mathcal{C}_c = q_0 + \int_0^L \left[ (k_1 \xi + k_2 \theta) \psi + k_3 \psi \phi + k_4 \theta \right] \text{ d}L\]

  - By suitable choice for \(\epsilon_c\), we get: \(\mathcal{C}_c = \left[ \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right] \]
  - \(H_c\) will be chosen in such a way that the Hamiltonian for the closed loop system will be invariant under the translation of the cart and the controller variable.
  - The stability has been proved using the convexity condition.
  - The presence of dissipation in the finite dimensional part of the system guarantees the asymptotic stability.

**Controller Structure:**

- Controller Hamiltonian:
  \[H_{\text{controller}} = \frac{1}{2} \int_0^L \dot{q}_c^T \dot{q}_c + \frac{1}{2} K_1 (q_c - q_0)^2 + \frac{1}{2} K_2 (q_c - q_0)^2\]

- Cart and Controller Dynamics:
  - \(\dot{q}_c = f_c\)

- Feedback Control Law:
  - \(F = -K_1 (q_c - q_0) - a_1 \dot{q}_c - a_2 \dot{q}_c + f_{c2}\)

**Discussion**

- An important feature of this control methodology is that it is solution free; that is: the solution of the PDEs defining the system is not required to obtain a stabilizing controller.

- This theoretical formulation and extraction of control law can be applied to solve issues related to stabilization of flexible structures.

**References:**