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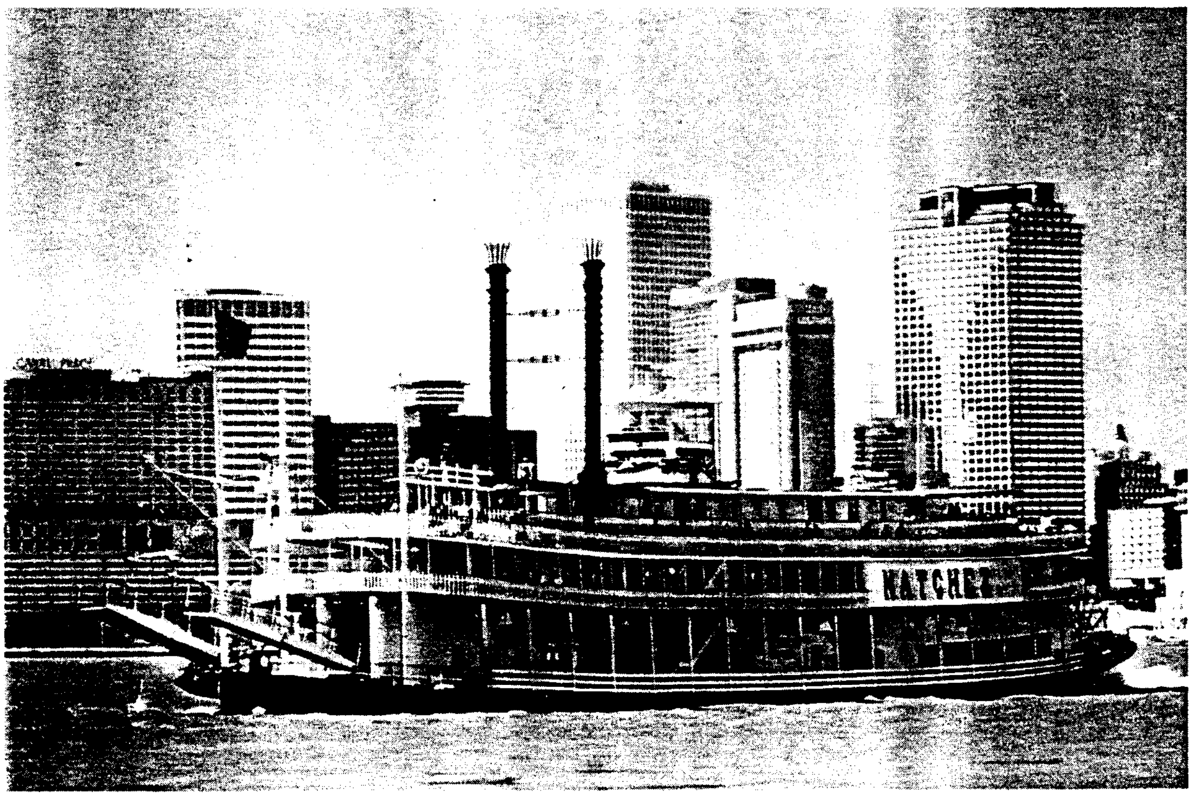

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Some results on risk-sensitive control with partial information

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Abstract

We consider in this presentation the risk-sensitive control problem with partial observation. Risk-sensitive Zakai and Kushner equations are established and studied. They are explicitly solved in all situations where the classical Zakai and Kushner equations can be solved. In these cases the solution of the control problem can be charactized nicely. Also, the small noise case is considered. The limit problem is a differential game with output control. The full justification of all the results is still a long range plan.

1 Introduction

The solution of the risk-sensitive control problem with partial observation is well-known in the case called "linear exponential quadratic gaussian model", [5]. See [2] for a more complete treatment. Concerning the general case the first issue is to derive a modified Zakai equation, whose solution represents an infinite dimensional observable state, with respect to which the cost functional can be expressed. This equation is given in [6] and in [3]. We present here, which is not surprising, a corresponding modified Kushner equation. The full rigorous treatment of these equations, in terms of existence and uniqueness, with growth conditions applicable to the LEQG model, is yet to be done. This concerns weak and strong formulations. One can then write formally the dynamic programming equation. In some cases, exact solutions involving finite dimensional statistics, can be obtained. They correspond to the cases for which the result is known, with respect to the classical Zakai and Kushner equations. See [3], [4] and the developments of this article.

It is very interesting to introduce a large deviation approach of risk-sensitive control problems with partial observation. This is linked with deterministic game problems related to H_1 or robust control, with output feedback. The partial observation case is much more involved than the full information case, since formally it amounts to passing to the limit with respect to a small parameter in an infinite dimensional PDE. The formal treatment is made in [6]. The case when a finite dimensional statistics exists is of course easier, since the problem reduces to passing to the limit in an ordinary PDE.

2 Modified Zakai and Kushner equations

2.1 Setting of the problem

Let us consider functions $g(x,t), \sigma(x,t)$ such that

$$\begin{array}{l} g, \sigma \text{ measurable from } R^n \times (0,T) \to R^n,\ L(R^n;R^n) \\ \text{respectively} \\ |g(x,t) - g(x^0,t)| + ||\sigma(x,t) - \sigma(x^0,t)|| \leq k|x - x^0| \end{array} \tag{1}$$

Let now $\Omega, \mathcal{A}, P, F^t$ be a probability space with a filtration, and w_t, z_t be two F^t independent Wiener processes, with values in R^n, R^m and with covariance matrices $Q(t), R(t)$ respectively. We first define the solution x_t of the Ito equation

$$\begin{array}{l} dx = g(x_t,t)\,dt + \sigma(x_t,t)dw_t \\ x_0 = \xi \end{array} \tag{2}$$

where

$$\begin{array}{l} \xi \text{ is } F^0 \text{ measurable, independent of } w_t, z_t \\ \text{the distribution of } \xi \text{ is } \Pi_0 \end{array} \tag{3}$$

Consider next a function h such that

$$\begin{array}{l} h(x,t) \text{ is measurable from } R^n \times (0,T) \to R^m \\ |h(x,t)| \leq k(1+|x|) \end{array} \tag{4}$$

We then define

$$\Lambda_t = \exp\{\int_0^t h'(x_s,s)R_s^{-1}dz_s \tag{5}$$

$$-\frac{1}{2}\int_0^t h'(x_s,s)R_s^{-1}h(x_s,s)\,ds\}$$

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Let next a function ℓ such that

$$\begin{array}{l} \ell(x,t) \text{ is measurable from } R^n \times (0,T) \to R \\ |\ell(x,t)| \le k(1+|x|^2) \end{array} \tag{6}$$

We define

$$D_t = \exp(\theta \int_0^t \ell(x_s, s)\, ds) \tag{7}$$

For $\phi(x)$ Borel we define

$$\sigma(t)(\phi) = E[\Lambda_t D_t \phi(x_t) | Z^t] \tag{8}$$

where Z^t is the σ-algebra generated by $z_s, s \le t$.

Remark 2.1 *We take $\theta > 0$. For $\theta = 0$, the operator $\sigma(t)$ reduces to the operator of nonlinear filtering, solution of the Zakai equation.*

2.2 The modified Zakai equation

We give here the formal equation that $\sigma(t)$ is solution of, called the modified Zakai equation. Introduce the 2nd order differential operator

$$A(t) = -g_i(x,t)\frac{\partial}{\partial x_i} - a_{i,j}(x,t)\frac{\partial^2}{\partial x_i \partial x_j}$$

where as usual

$$a(x,t) = \frac{1}{2}\sigma(x,t)Q(t)\sigma'(x,t).$$

Let $\phi(x,t) \in C_b^{2,1}(R^n \times [0,T])$, then writing

$$\phi(t)(x) = \phi(x,t)$$

and similar notation for other functions of x, t, we can state the

Proposition 2.1 *The operator $\sigma(t)$ satisfies*

$$\begin{array}{l} d\sigma(t)(\phi(t)) = \sigma(t)(\frac{\partial \phi}{\partial t} - A(t)\phi(t) \\ +\theta\phi(t)\ell(t)) + \sigma(t)(\phi(t)h'(t))R_t^{-1}dz_t \\ \sigma(0) = \Pi_0 \end{array} \tag{9}$$

Writing, formally

$$\sigma(t)(\phi) = \int q(x,t)\phi(x)\, dx \tag{10}$$

then q appears as the solution of the stochastic PDE

$$\begin{array}{l} dq + (A'(t)q - \theta q\ell)\, dt = qh R_t^{-1}dz_t \\ q(x,0) = p_0(x) \end{array} \tag{11}$$

where

$$\Pi_0(\phi) = \int p_0(x)\phi(x)\, dx.$$

2.3 The modified Kushner equation

We introduce the normalized operator

$$\zeta_t(\phi) = \frac{\sigma(t)(\phi)}{\sigma(t)(1)} \tag{12}$$

then we can state the

Proposition 2.2 *The operator $\zeta(t)$ satisfies*

$$\begin{array}{l} d\zeta(t)(\phi(t)) = \zeta(t)(\frac{\partial \phi}{\partial t} - A(t)\phi(t))\, dt \\ +\theta(\zeta(t)(\phi(t)\ell(t)) - \zeta(t)(\phi(t))\zeta(t)(\ell(t)))\, dt \\ +(\zeta(t)(\phi(t)h*(t)) - \zeta(t)(\phi(t))\zeta(t)(h*(t)))R_t^{-1} \\ (dz_t - \zeta(t)(h(t))\, dt) \\ \zeta(0) = \Pi_0 \end{array} \tag{13}$$

3 Exact solutions

We consider here several models where explicit solutions are available. These cases are exactly those where the classical Zakai and Kushner equations have solutions.

3.1 LEQG case

We assume here

$$g(x,t) = F_t x + f_t, \quad \sigma(x,t) = I \tag{14}$$

$$h(x,t) = H_t x + h_t \tag{15}$$

$$\ell(x,t) = \frac{1}{2}x' M_t x + m_t' x + N_t \tag{16}$$

$$p_0(x) = \frac{1}{(2\Pi)^{\frac{n}{2}}|P_0|^{\frac{1}{2}}}\exp(-\frac{1}{2}(x-x_0)'P_0^{-1}(x-x_0)) \tag{17}$$

then we have the

Proposition 3.1 *The operator $\sigma(t)$ has a density $q(x,t)$ given by the formulas*

$$q(x,t) = \nu_t \rho_t \frac{1}{(2\Pi)^{\frac{n}{2}}|\Pi_t|^{\frac{1}{2}}}\exp(-\frac{1}{2}(x-r_t)'\Pi_t^{-1}(x-r_t)) \tag{18}$$

where Π_t is the solution of

$$\begin{array}{l} \dot{\Pi}_t = F_t\Pi_t + \Pi_t F_t' + \Pi_t(\theta M_t - H_t' R_t^{-1} H_t)\Pi_t + Q_t \\ \Pi_0 = P_0 \end{array} \tag{19}$$

r_t is the solution of

$$\begin{array}{l} dr_t = (F_t r_t + f_t + \theta\Pi_t(M_t r_t + m_t))\, dt \\ \quad + \Pi_t H_t' R_t^{-1}(dz_t - (H_t r_t + h_t)\, dt) \\ r_0 = x_0 \end{array} \tag{20}$$

and ν_t, ρ_t are defined by the formulas

$$\begin{array}{l} \nu_t = \exp\{\int_0^t (H_s r_s + h_s)' R_s^{-1} dz_s - \frac{1}{2}\int_0^t \\ \quad (H_s r_s + h_s)' R_s^{-1}(H_s r_s + h_s)\, ds\} \end{array} \tag{21}$$

$$\rho_t = \exp\{\theta \int_0^t [\frac{1}{2}r_s' M_s r_s + m_s' r_s + N_s + \frac{1}{2}\ tr\ \Pi_s M_s]\, ds\} \tag{22}$$

3.2 Arbitrary initial density

Here we assume (14), (15),(16) but not (17), i.e. $p_0(x)$ is an arbitrary density. We need some notation. Define the functions

$$b(\eta, S) = \frac{\int \xi p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi}{\int p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi} \quad (23)$$

$$B(\eta, S) = \frac{\int \xi\xi' p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi}{\int p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi} \quad (24)$$

and

$$C(\eta, S)_{i,j,k} = \frac{\int \xi_i\xi_j\xi_k p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi}{\int p_0(\xi)\exp[-\frac{1}{2}(\xi' S\xi - 2\xi'\eta)]\,d\xi} \quad (25)$$

Introduce next Π_t, Φ_t, S_t defined by

$$\dot\Pi_t = F_t\Pi_t + \Pi_t F_t' + \Pi_t(\theta M_t - H_t' R_t^{-1} H_t)\Pi_t + Q_t \qquad \Pi_0 = 0 \quad (26)$$

$$\dot\Phi_t = (F_t + \Pi_t(\theta M_t - H_t' R_t^{-1} H_t))\Phi_t \qquad \Phi_0 = I \quad (27)$$

$$\dot S_t = \Phi_t'(H_t' R_t^{-1} H_t - \theta M_t)\Phi_t \qquad S_0 = 0 \quad (28)$$

To simplify a little the notation we write

$$b_t(\eta) = b(\eta, S_t)$$

and similarly $B_t(\eta), C_t(\eta)$. We then define the pair r_t, η_t by the equations

$$\begin{aligned} dr_t = \Big[&F_t r_t + f_t + \frac{\theta}{2}\,\mathrm{tr}\,\Phi_t C_t(\eta_t)\Phi_t' M_t\Phi_t \\ &-\theta\Phi_t B_t(\eta_t)\Phi_t' M_t\Phi_t b_t(\eta_t) \\ &-\frac{\theta}{2} b_t(\eta_t)\,\mathrm{tr}\,\Phi_t' M_t\Phi_t B_t(\eta_t) \\ &+\theta\Phi_t b_t(\eta_t) b_t(\eta_t)'\Phi_t' M_t\Phi_t b_t(\eta_t) + \theta\Pi_t(M_t r_t + m_t) \\ &+ \theta\Phi_t(B_t(\eta_t) - b_t(\eta_t)b_t(\eta_t)')\Phi_t'(M_t r_t + m_t)\Big]\,dt \\ &+ [\Pi_t + \Phi_t(B_t(\eta_t) - b_t(\eta_t)b_t(\eta_t)')\Phi_t'] H_t' R_t^{-1}(dz_t \\ &-(H_t r_t + h_t)dt) \\ r_0 = x_0& \end{aligned} \quad (29)$$

$$\begin{aligned} d\eta_t &= \theta\Phi_t'(M_t r_t + m_t - M_t\Phi_t b_t(\eta_t))\,dt \\ &+\Phi_t' H_t' R_t^{-1}(dz_t - (H_t r_t + h_t)dt) \\ \eta_0 &= 0 \end{aligned} \quad (30)$$

We can then state the following

Proposition 3.2 *The operator $\sigma(t)$ has a density $q(x,t)$ given by the formulas*

$$\begin{aligned} q(x,t) &= \nu_t\rho_t\frac{1}{(2\Pi)^{\frac{n}{2}}|\Pi_t|^{\frac{1}{2}}} \\ &\int p_0(\xi)\exp(-\frac{1}{2}(\xi' S_t\xi - 2\xi'\eta_t)) \\ &\exp(-\frac{1}{2}(x - r_t - \Phi_t(\xi - b_t(\eta_t)))' \\ &\Pi_t^{-1}(x - r_t - \Phi_t(\xi - b_t(\eta_t))))\,d\xi \end{aligned} \quad (31)$$

and ν_t, ρ_t are defined by the formulas

$$\begin{aligned} \nu_t = \exp\{&\int_0^t (H_s(r_s - \Phi_s b_s(\eta_s)) + h_s)' R_s^{-1} dz_s \\ &-\frac{1}{2}\int_0^t (H_s(r_s - \Phi_s b_s(\eta_s)) + h_s)' R_s^{-1} \\ &(H_s(r_s - \Phi_s b_s(\eta_s)) + h_s)\,ds\} \end{aligned} \quad (32)$$

$$\begin{aligned} \rho_t = \exp\{\theta&\int_0^t [\frac{1}{2}(r_s - \Phi_s b_s(\eta_s))' M_s(r_s \\ &-\Phi_s b_s(\eta_s)) + m_s'(r_s - \Phi_s b_s(\eta_s)) + \\ &N_s + \frac{1}{2}\,tr\,\Pi_s M_s]\,ds\} \end{aligned} \quad (33)$$

3.3 Nonlinear drift

We assume now

$$g(x,t) = F_t x + f_t + g_0(x,t), \quad \sigma(x,t) = I \quad (34)$$

with an additional assumption on g_0 to be made explicit later. We also assume (15) and

$$\ell(x,t) = \frac{1}{2}x' M_t x + m_t' x + N_t + \phi(x,t) \quad (35)$$

The initial probability p_0 is general. Consider the PDE

$$\begin{aligned} &\frac{\partial\zeta}{\partial t} + \frac{1}{2}\,\mathrm{tr}\,Q_t D^2\zeta + \frac{1}{2}D\zeta' Q_t D\zeta + (F_t x + f_t)' D\zeta = \\ &\theta(\frac{1}{2}x'\Lambda_t x + \lambda_t' x + \chi_t + \phi(x,t)) \end{aligned} \quad (36)$$

with an initial value $\zeta(x,0)$ such that

$$p_0(x) = \exp(\zeta(x,0) - \frac{1}{2}x' P_0 x + x_0' x) \quad (37)$$

and we assume that

$$g_0(x,t) = Q_t D\zeta(x,t) \quad (38)$$

Construct then P_t to be the solution of

$$\begin{aligned} \dot P_t &= \theta(\Lambda_t - M_t) + H_t' R_t^{-1} H_t - F_t' P_t \\ &-P_t F_t - P_t Q_t P_t \\ P_0 &= P_0 \end{aligned} \quad (39)$$

and r_t to be the solution of

$$\begin{aligned} dr_t &= -[(F_t' + P_t Q_t)r_t + \theta(\lambda_t - m_t) \\ &-P_t f_t + H_t' R_t^{-1} h_t]dt \\ &+ H_t' R_t^{-1} dz_t \\ r_0 &= x_0 \end{aligned} \quad (40)$$

Define finally

$$\begin{aligned} \beta_t = -&\int_0^t [f_s' r_s - \frac{1}{2} r_s' Q_s r_s + \theta(\chi_s - N_s) \\ &+\tfrac{1}{2} h_s' R_s^{-1} h_s + \tfrac{1}{2}\,\mathrm{tr}\,Q_s P_s + \mathrm{tr}\,F_s]ds \\ &+ \int_0^t h_s' R_s^{-1} dz_s \end{aligned} \quad (41)$$

We then state the following

Proposition 3.3 *We assume (15),(34),(35),(38), then the operator $\sigma(t)$ has a density $q(x,t)$ given by the formula*

$$q(x,t) = \exp(\zeta(x,t) - \frac{1}{2}x'P_t x + x'r_t + \beta_t) \quad (42)$$

4 The control problem

4.1 Setting of the problem

We introduce a control v_t, which will be a process adapted to $Z^t = \sigma(z_s, s \leq t)$, with values in $\mathcal{V}$, a Borel subset of R^k. Skipping a few steps of justifications, we replace in the above

$$\begin{aligned} g(x,t) &= b(x,t) + g(x,v_t,t) \\ \ell(x,t) &= f(x,t) + \ell(x,v_t,t) \end{aligned} \quad (43)$$

and we define the cost function

$$J(v) = E\Lambda_t D_t \exp\theta f_0(x_T) \quad (44)$$

where Λ_t, D_t have been defined in (5),(7) . Clearly

$$J(v) = E\sigma(T)(\exp\theta f_0)$$

and if $\sigma(t)$ has a density $q(x,t)$

$$J(v) = E\int q(x,T)\exp\theta f_0(x)\,dx.$$

4.2 Expression of the cost in the case of finite dimensional statistics

We limit ourselves to the first case, section 3.1. We take

$$\begin{aligned} b = 0;\ & g(x,v,t) = F_t x + f_t(v) \\ f = 0;\ & \ell(x,v,t) = \frac{1}{2}x'M_t x + m_t(v)'x + N_t(v) \end{aligned} \quad (45)$$

then introducing

$$\begin{aligned} &\dot{\Pi}_t = F_t\Pi_t + \Pi_t F_t' + \Pi_t(\theta M_t - H_t'R_t^{-1}H_t)\Pi_t + Q_t \\ &\Pi_0 = P_0 \end{aligned} \quad (46)$$

r_t is the solution of

$$\begin{aligned} dr_t &= (F_t r_t + f_t(v_t) + \theta\Pi_t(M_t r_t + m_t(v_t)))\,dt \\ &\quad + \Pi_t H_t' R_t^{-1}(dz_t - (H_t r_t + h_t)\,dt) \\ r_0 &= x_0 \end{aligned} \quad (47)$$

and ν_t, ρ_t are defined by the formulas

$$\begin{aligned} \nu_t = \exp\{&\int_0^t (H_s r_s + h_s)'R_s^{-1}dz_s \\ &-\frac{1}{2}\int_0^t (H_s r_s + h_s)'R_s^{-1} \\ &(H_s r_s + h_s)\,ds\} \end{aligned} \quad (48)$$

$$\begin{aligned} \rho_t = \exp\{\theta\int_0^t &[\frac{1}{2}r_s'M_s r_s + m_s(v_s)'r_s + N_s(v_s) \\ &+\frac{1}{2}\text{ tr }\Pi_s M_s]\,ds\} \end{aligned} \quad (49)$$

then the function $q(x,t)$ is given by

$$q(x,t) = \nu_t\rho_t\frac{1}{(2\Pi)^{\frac{n}{2}}|\Pi_t|^{\frac{1}{2}}}\exp(-\frac{1}{2}(x-r_t)'\Pi_t^{-1}(x-r_t)) \quad (50)$$

Setting

$$\exp\theta\overline{f_0^\theta}(x) = \int_{R^n}\exp[\theta f_0(\xi) - \frac{1}{2}(x-\xi)'\Pi_t^{-1}(x-\xi)]\,d\xi \quad (51)$$

we can express the cost function as

$$J(v) = E\left(\nu_T\rho_T\frac{1}{(2\Pi)^{\frac{n}{2}}|\Pi_T|^{\frac{1}{2}}}\exp\theta\overline{f_0^\theta}(r_T)\right) \quad (52)$$

Writing

$$d\hat{b}(t) = R_t^{-\frac{1}{2}}(dz_t - (H_t r_t + h_t)dt) \quad (53)$$

and performing a change of probability

$$\frac{dP^v}{dP} = \nu_T \quad (54)$$

then $\hat{b}(t)$ becomes a Wiener process, which is observable. The problem is reduced to a risk-sensitive optimal control problem with full observation. Therefore we can assert that

$$\begin{aligned} &\inf J(v) = \frac{1}{(2\Pi)^{\frac{n}{2}}|\Pi_T|^{\frac{1}{2}}}\exp\{\theta\int_0^T\frac{1}{2}\text{ tr }\Pi_t M_t\,dt\} \\ &\exp\theta\mathcal{X}(x_0,0) \end{aligned} \quad (55)$$

where $\mathcal{X}(x,t)$ is the solution of

$$\begin{aligned} &\frac{\partial\mathcal{X}}{\partial t} + \inf_v[D\mathcal{X}.(f_t(v) + \theta\Pi_t m_t(v)) + x.m_t(v) + N_t(v)] \\ &\quad + D\mathcal{X}.(F_t + \theta\Pi_t M_t)x + \text{ tr } D^2\mathcal{X}\Pi_t H_t R_t^{-1}H_t'\Pi_t \\ &\quad + \frac{1}{2}x'M_t x + \frac{\theta}{2}D\mathcal{X}'\Pi_t H_t R_t^{-1}H_t'\Pi_t D\mathcal{X} = 0 \\ &\mathcal{X}(x,T) = \overline{f_0^\theta}(x) \end{aligned} \quad (56)$$

5 Singular perturbations

We briefly sketch the problem. We assume here

$$\begin{aligned} Q_t = R_t = P_0 &= \epsilon I \\ \theta &= \frac{\mu}{\epsilon} \end{aligned} \quad (57)$$

The solution Π_t^ϵ of (46) can be written as

$$\Pi_t^\epsilon = \epsilon\Pi_t$$

where Π_t is the solution of

$$\dot{\Pi}_t = F_t\Pi_t + \Pi_t F_t' + \Pi_t(\mu M_t - H_t'H_t)\Pi_t + I\Pi_0 = I \quad (58)$$

Moreover r_t^ϵ the solution of (47) appears as the solution of

$$\begin{aligned} dr_t^\epsilon &= (F_t r_t^\epsilon + f_t(v_t) + \mu\Pi_t(M_t r_t^\epsilon + m_t(v_t)))\,dt \\ &\quad + \Pi_t H_t' db^\epsilon(t) \\ r_0^\epsilon &= x_0 \end{aligned} \quad (59)$$

where $b^\epsilon(t)$ is a Wiener process with covariance ϵI. The cost function becomes

$$\inf J^{\mu,\epsilon}(v) = \frac{1}{(2\Pi\epsilon)^{\frac{n}{2}}|\Pi_T|^{\frac{1}{2}}}$$
$$\exp\{\mu \int_0^T \frac{1}{2} \text{ tr } \Pi_t M_t \, dt\} \exp \frac{\mu}{\epsilon}\mathcal{X}^\epsilon(x_0, 0) \quad (60)$$

where $\mathcal{X}^\epsilon(x,t)$ is the solution of

$$\begin{aligned} &\frac{\partial \mathcal{X}^\epsilon}{\partial t} + \inf_v [D\mathcal{X}^\epsilon.(f_t(v) + \mu\Pi_t m_t(v)) + x.m_t(v) + N_t(v)] \\ &+ D\mathcal{X}^\epsilon.(F_t + \mu\Pi_t M_t)x + \epsilon \text{ tr } D^2\mathcal{X}^\epsilon \Pi_t H_t H_t' \Pi_t \\ &+ \frac{1}{2}x' M_t x + \frac{\mu}{2} D\mathcal{X}^{\epsilon\prime} \Pi_t H_t H_t' \Pi_t D\mathcal{X}^\epsilon = 0 \\ &\mathcal{X}(x,T) = \overline{f_0^{\mu,\epsilon}}(x) \end{aligned} \quad (61)$$

We can check that

$$\overline{f_0^{\mu,\epsilon}}(x) \to \overline{f_0^{\mu}}(x)$$

where

$$\overline{f_0^{\mu}}(x) = \sup_\xi [f_0(x) - \frac{1}{2\mu}(\xi - x)' \Pi_T^{-1}(\xi - x)] \quad (62)$$

then we have

$$\mathcal{X}^\epsilon(x,t) \to \mathcal{X}(x,t)$$

where $\mathcal{X}(x,t)$ is the solution of

$$\begin{aligned} &\frac{\partial \mathcal{X}}{\partial t} + \inf_v [D\mathcal{X}.(f_t(v) + \mu\Pi_t m_t(v)) + x.m_t(v) \\ &+ N_t(v)] + D\mathcal{X}.(F_t + \mu\Pi_t M_t)x + \\ &\frac{1}{2}x' M_t x + \frac{\mu}{2} D\mathcal{X}' \Pi_t H_t H_t' \Pi_t D\mathcal{X} = 0 \\ &\mathcal{X}(x,T) = \overline{f_0^{\mu}}(x) \end{aligned} \quad (63)$$

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