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Shape optimization of distributed sensors and actuators for smart structure control

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ABSTRACT

We describe a design technique for optimal control in active structural vibration damping using smart materials. The vibration of a cantilever beam is stabilized by feedback the weighted integration of vibration velocity in the closed loop system through the application of distributed sensors and actuators made of smart materials. We model the beam by the Timoshenko beam model embedded with the distributed sensors and actuators to account for the shear and inertial effects on the structures. We propose an algorithm to find the optimal placement of the actuators and sensors so as to maximize the damping effect. An objective functional is defined based on the vibration energy of the system. The optimal shapes of the sensor and actuator are determined through minimizing the energy functional of the beam over the admissible shape function space subject to certain constraints. This approach can be generalized to cases of plate damping and active control of more complicated smart structures as well.

Keywords: flexible structure control, distributed control, adaptive structures, sensors, actuators, smart materials.

1 INTRODUCTION

An important issue in the control system design for flexible systems is the determination of the optimal number and location of the control system components: sensors and actuators as well as their backups. In general there is a larger number of candidate locations than available sensors and actuators. Based on experience and knowledge on structure dynamics and control objectives, a priori selection is usually available. However, this may not give the optimal effect on the closed loop system. Extensive experimental work is expected to justify the design. For discrete optimal sensor and actuator locations, a method based on the orthogonal projection of structural modes onto the intersection of the controllable and observable subspaces is introduced [1]. The controllability and observability Gramians are used to reflect the degrees of controllability and observability of an actuator/sensor pair. However, this method is based on a second order linear model. An objective function is defined [2] based on the elements of the actuator influence matrix, and an optimization study is performed to compare the system performance. This work suggests that a relative even distribution of the actuators can lead to satisfactory results. Again, this method treats the case of pointwise sensors and actuators. With the application of smart materials to flexible structures as distributed sensors and actuators, a natural need exists in system optimization and the design of the optimal layout of the distributed sensors and actuators.

The use of distributed sensors and actuators made of smart materials has several advantages and also need its own treatment in system integration. It allows the adjustment of geometry and dynamical behavior of flexible space structures and reduces the weight and complexity of such structures. It also provides means of signal processing by sensor's geometry. Bailey and Hubbard [3] used PVDF actuators to control the vibration of a cantilever beam. The control voltage applied across the actuator is the sign function of the tip rotation velocity multiplied by a constant so as to introduce active damping. Piezoelectric actuators were also used as elements of intelligent structures by Crawley and de Luis [4]. We studied the active damping problem with distributed actuator and sensor with the rotational inertial included in the beam model [5]. Modal sensors/actuators are proposed and developed [6] to be incorporated into flexible structures. The sensors and actuators can provide signals related to certain elastic modes. It has been pointed out that the location of distributed sensors and actuators need to be placed away from the nodes of the specified elastic modes to be sensed or controlled to achieve maximum effect. The choice of the sensor and actuator shapes is also an important factor in system's performance. An interesting question is how to choose

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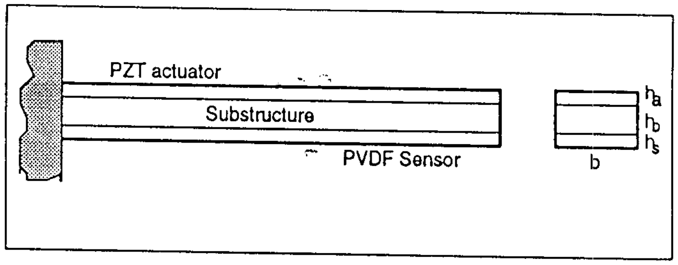

Figure 1: The composite beam

and design the shapes of the distributed sensors and actuators made from smart materials for the smart structures. This paper addresses the optimal design issue regarding the application of distributed sensors and actuators.

We know that the flexible beam is an infinite dimensional system. In order to faithfully measure and control the system without using a truncated model, there is a need in designing control algorithms directly from the partial differential equation model to avoid spillover. We can develop certain performance measures to carry out the optimal design.

We consider the design issue associated with the vibration damping control of a cantilever beam. The beam is modeled as the Timoshenko beam. Both sides of the beam are covered by PVDF and PZT materials for sensing and actuation. Using the control algorithm developed in [7], the closed loop system can be asymptotically stabilized. Based on this result, we want to further determine the optimal layout of the continuous distributed sensors and actuators for the system based on minimizing the vibration energy of the system. We hope that this can lead to a general design methodology or at least provide a design guideline.

2 SYSTEM MODELS

We model the cantilever beam with the Timoshenko beam model which accounts for shear effects and rotary inertial. The Timoshenko model describes the physical behavior better than the Euler-Bernoulli model does especially for the high frequency vibration components. The actuator and sensor are the layers made of piezoelectric ceramic (PZT) and piezoelectric polymer polyvinylidene fluoride (PVDF) materials attached to both sides of the beam. Figure 1 shows the structure of the composite beam. In this figure, h stands for the thickness of the different layers of the composite beam. The subscripts s, b and a denote sensor, beam and actuator respectively.

2.1 Model of substructure

We use the Timoshenko beam model [8] to describe the dynamical behavior of the beam. Unlike the Euler-Bernoulli beam model, the Timoshenko model contains the rotational inertial and shear effect of the actual beam. The analysis of the latter is more complicated. The beam model is given as

$$\rho A \frac{\partial^2 w}{\partial t^2} = kAG(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \Phi}{\partial x}), \tag{1}$$

$$\rho I \frac{\partial^2 \Phi}{\partial t^2} = EI\frac{\partial^2 \Phi}{\partial x^2} + kAG(\frac{\partial w}{\partial x} - \Phi). \tag{2}$$

Here t is the time variable and x is the space coordinate along the beam in its equilibrium configuration. $w(x,t)$ is the displacement of the centroid from its equilibrium line which is described by $w = 0$. Φ denotes the deflection curve when the shearing force is neglected. The total slope of deflection is

$$\frac{dw}{dx} = \Phi + \beta,$$

where β is the angle of shear. ρ, I and E are mass density, moment of inertial of cross section and Young's modulus respectively. k is a numerical factor depending on the shape of the beam while A and G are area of cross section and modulus of elasticity in shear.

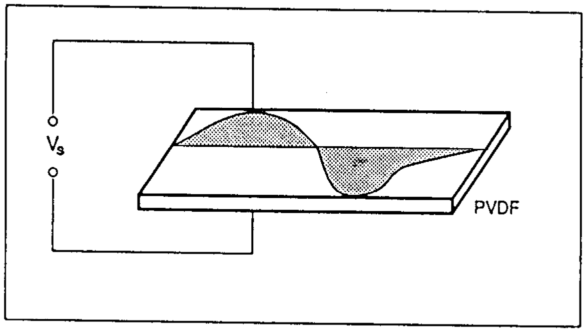

Figure 2: The PVDF sensor

2.2 Actuator model

Piezoelectric actuators is made based on the constitutive property of the PZT material. The stress-strain relation for the piezoelectric material is similar to that of the thermoelastic materials, with the thermal strain term replaced by the piezoelectric strain Λ. The constitutive equation of the actuator is given by

$$\sigma = E_a(\epsilon - \Lambda) \tag{3}$$

where Λ is the actuation strain due to the external electric field, and ϵ is the strain without external electric field. E_a is the Youngs' modulus of the actuator, σ is the stress of the actuator. The actuation strain is given by

$$\Lambda(x,t) = \frac{d_{31}}{h_a} V(x,t) \tag{4}$$

where d_{31} is the piezoelectric field and strain field constant. $V(x,t)$ is the distributed voltage. Suppose the bonding between the actuator layer and the beam is perfect, i.e., there is no shear lag layer in between them, the induced strain has two effects on the beam. One effect is that it introduces a longitudinal strain to insure a force equilibrium along the axial direction. The steady state value of the induced longitudinal strain can be derived by solving a force equilibrium equation. The second effect is that the net force in each layer acts through the moment arm with the length from the midplane of the layer to the neutral plane of the beam. The result of the actuation produces the bending moment which is introduced as the control mechanism. Taking a similar approach as in [3], the actuation moment can be expressed as

$$M_a(x,t) = K_a \Lambda(x,t) \tag{5}$$

where K_a is a constant depending on the geometry and the materials of the beam. Equation (5) and (4) describe the relationship between the applied voltage and the actuation moment of the PZT actuator.

2.3 Sensor model

Piezoelectric polymer polyvinylidene fluoride (PVDF) is also strain sensitive and relies on the applied strain to produce electrical charge. The amount of electrical charge is proportional to the amount of strain induced by the structure. This process is the reverse to piezoelectric actuation. It is also based on the constitutive property of the sensor material. Figure 2 shows the structure of the PVDF sensor. The shaded area illustrates the area of electrodes covering the surfaces of the sensor material.

The induced charge per unit length from the strain is

$$q(x,t) = -E_s d_{31} \epsilon_s. \tag{6}$$

where E_s is Young's modulus of the sensor material.

The output of the sensor is given by [7]

$$V_s(x,t) = -K_s \int_0^x F(x) \frac{\partial \Phi}{\partial x} dx, \tag{7}$$

where

$$K_s = \frac{E_s d_{31}(h_b + h_s)}{2C} \tag{8}$$

is a constant determined by the sensor geometry, material property and the capacitance C between the electrodes of both sides of the sensor surfaces. When the sensor covers the whole beam, the sensor output becomes

$$V_s(t) = -K_s \int_0^L F(x)\frac{\partial \Phi}{\partial x} dx. \tag{9}$$

Equation (9) is the sensor output equation. Function $F(x)$ is the weight function or shape function of the sensor; it is the local width of the electrodes covering both sides of the sensor. The output voltage is the weighted integral of the beam curvature. Integrating by parts the right hand side of the above expression once in the spatial variable yields another expression of the sensor output,

$$V_s(t) = -K_s\Phi(L,t)F(L) + K_s \int_0^L \Phi(x,t)\frac{\partial F(x)}{\partial x} dx. \tag{10}$$

This first term is the measurement of rotation of the beam on the free end. The format of Equation (10) can be manipulated to yield different sensory data for controling and monitoring the system.

2.4 System model

The equations of motion are given by embedding the actuator model into the Timoshenko beam model as discussed in our previous work [7],

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} = kAG\left(\frac{\partial^2 w(x,t)}{\partial x^2} - \frac{\partial \Phi(x,t)}{\partial x}\right), \tag{11}$$

$$\rho I \frac{\partial^2 \Phi(x,t)}{\partial t^2} = EI\frac{\partial^2 \Phi(x,t)}{\partial x^2} + kAG\left(\frac{\partial w(x,t)}{\partial x} - \Phi(x,t)\right) + c\frac{\partial V(x,t)}{\partial x}, \tag{12}$$

with boundary conditions

$$\begin{aligned} w(0,t) &= 0, \\ \Phi(0,t) &= 0, \\ \frac{\partial w(L,t)}{\partial x} - \Phi(L,t) &= 0, \\ EI\frac{\partial \Phi(L,t)}{\partial x} &= 0, \end{aligned} \tag{13}$$

where

$$EI = E_a I_a + E_b I_b + E_s I_s, \tag{14}$$

$$c = \frac{d_{31}}{h_a} K_a.$$

The constant c is determined by the PZT material property and the manufacturing process. The functions $w(x,t)$ and $\Phi(x,t)$ are the displacement of the centroid and orientation of the cross section of the beam. G is the Young's modulus in shear. EI is the bending rigidity with the subscripts b, a and s denoting beam, sensor and actuator layers respectively . A is the area of cross section. The distributed control $V(x,t)$ appears in (12) as the distributed bending moment.

3 A SHAPE OPTIMIZATION PROBLEM

Our goal is to suppress the structural vibration due to poorly damped flexible structure and to increase the nature frequencies of the structure through the optimal design of distributed sensors and actuators made of smart materials. Unlike point sensors and actuators, the geometry of the spatially distributed sensor and actuator is capable of preprocessing the sensor output signals and allocating control weight. A judicious choice of the shapes can extract the desired signals and implement the control algorithm. We discussed in [7] a method of choosing the appropriate sensor and actuator shapes for active damping control by means of modal analysis. We would like to develop a systematic approach to deal with this problem here.

We introduce an energy functional to describe and measure the amount of vibration of the composite beam. The energy functional is defined as

$$E(t) = \frac{1}{2}\int_0^L \left\{ \rho A[\frac{\partial w}{\partial t}]^2 + \rho I[\frac{\partial \Phi}{\partial t}]^2 + K[\frac{\partial w}{\partial x} - \Phi]^2 + EI[\frac{\partial \Phi}{\partial x}]^2 \right\} dx, \tag{15}$$

where

$$K = kAG.$$

The first two terms in the integral are kinetic energy of the beam due to the displacement of the centroid and rotation of the cross section. The third term is the energy due to shear modeled in the Timoshenko beam model. The last term represents the amount of the stored energy from bending. The vibration energy defined above is time dependent.

An asymptotic stablizing feedback control law is given by [7]

$$V(x,t) = K_s v(x) \int_0^L F(x) \frac{\partial^2 \Phi}{\partial t \partial x} dx, \tag{16}$$

to introduce active damping to the system, in which $v(x)$ is the actuator shape function. Similarly to the sensor shape function, $v(x)$ is the width of the electrodes covering the surfaces of the PZT actuator.

Given the system and the control algorithm, we need to maximize the damping effect by optimization of the shape functions of both the sensors and the actuators. The problem becomes finding shape functions $v_0(x)$ and F_0 such that, for a given time $t = T$,

$$J[T, v_0, F_0] = \min_{v\in\mathcal{V}, F\in\mathcal{F}} \frac{1}{2}\int_0^L \left\{ \rho A[\frac{\partial w}{\partial t}]^2 + \rho I[\frac{\partial \Phi}{\partial t}]^2 + K[\frac{\partial w}{\partial x} - \Phi]^2 + EI[\frac{\partial \Phi}{\partial x}]^2 \right\} dx, \tag{17}$$

where $\mathcal{V}$ and $\mathcal{F}$ are the sets of all the admissible actuator and sensor shape functions. This is the resulting shape optimization problem.

4 OPTIMAL CONTROL AND A NUMERICAL ALGORITHM

The optimal control is identified with the above shape optimization. The control (16) is a functional of sensor and actuator shape functions and the weighted integral of the beam curvature. We have proved that the control (16) asymptotically stabilizes the system [7], i.e., we have

$$\lim_{t\to\infty} E(t) = 0. \tag{18}$$

We seek the optimal control in the sense of minimizing the vibration energy over all the possible sensor and actuator shapes at time T. The task here is to find the optimal sensor and actuator shape functions $v(x)$ and $F(x)$ so as to minimize the energy functional (15). The admissible shape functions depend on geometry of the structure. For beam and plate like structures, the geometry is usually simple. Since the region of the beam which can be covered with smart materials is assumed to have length L and width b, the sets $\mathcal{V}$ and $\mathcal{F}$ contain the collection of all the piecewise continuous curves within this region. The optimization hence has a geometric constraint. The functions $v(x)$ and $F(x)$ denote the width of the electrodes covering the smart materials for actuators and sensors; we have $0 \leq v(x) \leq b$ and $0 \leq F(x) \leq b$. We are interested in observing the amount of vibration energy at time T.

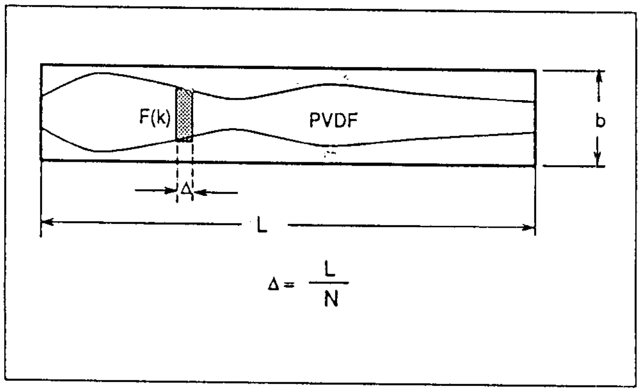

Figure 3: Discretizing the sensor and actuator layout

For numerical solution of a shape optimization problem, one typically starts by guessing an initial design. One then discretizes the elastic problem using finite elements or using difference method or a Galerkin procedure. After discretization, the optimal design problem becomes a large nonlinear programming problem. Different routines are available for working on the later.

We start our numerical scheme by discretizing the region along the longitudinal spatial variable x as in Figure 3. Let N be the total number of segments with equal size, then the width of each segment is L/N. The discretized shape functions $v(x)$ and $F(x)$ assume constant values $v(k)$ and $F(k)$ inside the k^{th} element. We thus have piecewise constant functions $v(k)$ and $F(k)$ with $k = 1, 2, ...N$. The $2k$ members of $v(x)$ and $F(k)$ become the optimization parameters. We then compute the distributed control $V(k,t)$, $k = 1, 2, ...N$, based on the initial conditions of the system and the discretized shape functions. The time response to the input can be computed through solving the equations of motion numerically. This procedure yields the value of the cost functional J at time T. An optimal routine shall be followed to search and adjust the piecewise constant sensor and actuator shape functions toward reducing the value of the cost functional (17). The new shape functions are then used to generate the system input $V(k,t)$ again. This procedure is repeated until the optimum criterion is met. The resulting $v(k)$ and $F(k)$ can be smoothed to give the final optimal shape functions $v_0(x)$ and $F_0(x)$ for the sensors and actuators. The algorithm is given in Figure 4.

The optimization scheme can be used to deal with more complicated structures. The sensors and actuators may be piecewise continuously distributed on the structures. Our formulation still holds when structures contain both distributed and pointwise sensors and actuators, . The corresponding shape functions $v(x)$ and $F(x)$ become both piecewise continuous and pointwise in the relevant regions.

In terms of computation, the bottleneck is the simulation of the systems governed by partial differential equations. Different methods can be implemented to solve the partial differential equations (11) and (12).

Modal sensors and actuators can be designed through optimization as well. This may reduce the influence of leak-through, i.e. the crossover effect among different modes, to improve the overall performance. Different performance measures and cost functions are required to formulate the optimization problems.

5 CONCLUSIONS

We have developed a method to facilitate the optimal design of active vibration damping using smart materials. The optimal algorithm described above can be expected to yield reasonable good design for the layout of the distributed sensors and actuators. Although the algorithm is developed based on the beam model, the method can be extended to the plane case. This procedure is expected to work for the cases with irregular geometry or nonuniform structural material as well.

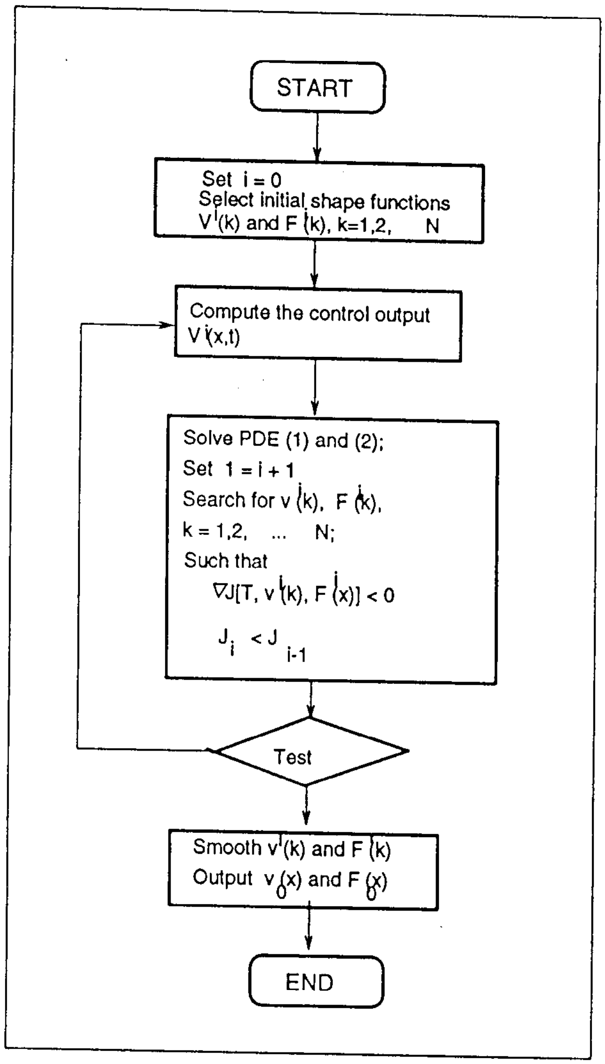

Figure 4: Optimization algorithm

6 ACKNOWLEDGEMENT

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