Modeling Video Traffic Using M/G/∞ Input Processes: A Compromise Between Markovian and LRD Models

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Abstract—Statistical evidence suggests that the autocorrelation function $\rho(k)$ ($k = 0, 1, \cdots$) of a compressed-video sequence is better captured by $\rho(k) = e^{-\beta \sqrt{k}}$ than by $\rho(k) = k^{-\beta} = e^{-\beta \log k}$ (long-range dependence) or $\rho(k) = e^{-\beta k}$ (Markovian). A video model with such a correlation structure is introduced based on the so-called M/G/$\infty$ input processes. In essence, the M/G/$\infty$ process is a stationary version of the busy-server process of a discrete-time M/G/$\infty$ queue. By varying $G$, many forms of time dependence can be displayed, which makes the class of M/G/$\infty$ input models a good candidate for modeling many types of correlated traffic in computer networks. For video traffic, we derive the appropriate $G$ that gives the desired correlation function $\rho(k) = e^{-\beta \sqrt{k}}$. Though not Markovian, this model is shown to exhibit short-range dependence. Poisson variates of the M/G/$\infty$ model are appropriately transformed to capture the marginal distribution of a video sequence. Using the performance of a real video stream as a reference, we study via simulations the queuing performance under three video models: our M/G/$\infty$ model, the fractional ARIMA model [9] (which exhibits LRD), and the DAR(1) model (which exhibits a Markovian structure). Our results indicate that only the M/G/$\infty$ model is capable of consistently providing acceptable predictions of the actual queuing performance. Furthermore, only $O(n)$ computations are required to generate an M/G/$\infty$ trace of length $n$, compared to $O(n^2)$ for an F-ARIMA trace.

Index Terms—Correlated variates, M/G/$\infty$ process, traffic modeling, VBR video.

I. INTRODUCTION

Recent indications of persistent correlations in various types of network traffic (including Ethernet LAN [7], [19], WAN [28], and variable-bit-rate (VBR) video traffic [2, 9]) have spurred an ongoing debate on the relevance of these correlations to the dimensioning of network resources. While there is a general agreement on the importance of traffic correlations, researchers tend to disagree on how much of them should be incorporated in a traffic model. Conventional traffic models are Markovian in nature, with an autocorrelation function (ACF) that drops off exponentially. They include many familiar models such as autoregressive models, Markov arrival processes (MAP), and Markov modulated processes (cf. [1], [8], and [23] for surveys). Markovian models exhibit short-range dependence (SRD), in that the ACF $\rho(k)$ ($k = 1, 2, \cdots$) is summable, i.e., $\sum_k \rho(k) < \infty$, implying a rapid decay of the ACF for large lags $k$. Note, however, that an SRD model is not necessarily Markovian. The persistence of traffic correlations and their presence at multiple time scales have prompted some researchers to consider instead long-range dependent (LRD) models. The ACF in LRD models drops off slowly (typically as a power function) to the extent that the correlations now have an infinite sum: $\sum_k \rho(k) = \infty$. The LRD phenomenon has long been observed in other domains such as hydraulics and economics (see [2] and the references therein). In teletraffic studies, advocates of LRD argue that such a phenomenon has significant impact on network performance, and thus must be taken into account when dimensioning network resources. On the other hand, supporters of Markovian modeling, while acknowledging the presence of such a phenomenon, argue that for networks with finite buffers it is sufficient to incorporate correlations up to some finite lag that is proportional to the buffer size [12], [10], [29].

As indicated above, the key difference between these two modeling approaches lies in the asymptotic behavior of the ACF: Markovian models give rise to an ACF of the form $\rho(k) \sim e^{-\beta k}$ ($\beta > 0$), whereas in LRD models we find $\rho(k) \sim k^{-\beta} = e^{-\beta \log k}$ ($\beta > 0$), which drops off much slower than its Markovian counterpart. These ACF’s represent two extremes, between which other forms can be envisioned, at least in principle. More generally, the ACF can have the general representation $\rho(k) \sim e^{-f(k)}$, for some monotone function $f: \mathbb{N} \rightarrow \mathbb{R}_+$ which increases no slower than $\log k$ but no faster than $k$.

The challenge for the traffic modeler is to identify a class of stochastic processes that can display forms of correlations as diverse as possible. One such class, which is considered here, is the class of M/G/$\infty$ input processes, which are obtained from the (correlated) busy-server process of a discrete-time M/G/$\infty$ queue. The viability of M/G/$\infty$ processes for modeling network traffic can be attributed to several factors [25]. First, they constitute a versatile class of processes, which can display various forms of time dependencies, the extent of which is governed by the service-time distribution $G$; in
fact, the M/G/∞ process was first mentioned by Cox [3] as an example of a process exhibiting LRD (which occurs when $G$ is a Pareto distribution). Second, the M/G/∞ model arises naturally in teletraffic as the limiting case for the aggregation of on/off sources [20]. Third, queuing performance for these processes is sometimes feasible, as demonstrated in [4], [27], [26], and [21]. Finally, when their queuing analysis in not tractable (as in the case of the video model presented in this paper), the computational complexity for generating synthetic M/G/∞ traces is only $O(n)$, with $n$ being the trace length. This low complexity allows for fast generation of these traces to be used in network simulations.

In this paper, we investigate the use of M/G/∞ processes in modeling VBR compressed video streams. We start by reexamining the empirical ACF of four VBR video sequences, which were generated by various video encoders. Statistical evidence suggests that the empirical ACF is better captured by $\rho(k) \sim e^{-\alpha \sqrt{k}}$ than by $\rho(k) \sim k^{-\alpha}$ (LRD) or $\rho(k) \sim e^{-\lambda k}$ (Markovian), where $k$ is the lag between frames. Accordingly, we introduce an M/G/∞-based video model with an ACF of the form $\rho(k) \sim e^{-\alpha \sqrt{k}}$. We determine the appropriate $G$ that provides such an ACF. Although non-Markovian, this model is SRD. The variates in the basic M/G/∞ process are Poisson distributed. To capture the frame-size distribution of a real video sequence, the Poisson marginal distribution is transformed into a hybrid Gamma/Pareto distribution, in line with the findings in [9]. This nonlinear transformation is shown to have negligible impact on the original correlation structure.

As a means of validating the appropriateness of our M/G/∞ model, we study its queuing performance via simulations and contrast it to two previously proposed video models: the F-ARIMA model [9] (an LRD model) and the discrete autoregressive of order one model (DAR(1)) [14] (a Markovian model). Using the queuing performance for the real video streams as a reference point, we evaluate the performance for the three models with respect to two measures: the cell loss rate due to buffer overflow and the frame error rate. The main conclusions drawn from our study are that 1) the M/G/∞ model provides acceptable performance predictions over a wide spectrum of traffic loads; 2) the performance of the F-ARIMA model is overly sensitive to the size of the buffer, which causes it in certain cases to underestimate the actual performance by several orders of magnitude; and 3) the DAR(1) model provides very good performance predictions at heavy loads, but performs poorly at light loads. The adequacy of the M/G/∞ video model is justified by the fact that it attempts to capture both short-term and long-term correlations, hence combining the goodness of Markovian models at small lags with that of LRD models at large lags. It is a compromise that incorporates the benefits of the two competing paradigms.

The rest of the paper is structured as follows. In Section II, we give an overview of M/G/∞ input processes. In Section III we present the fitting results for the ACF’s of four video sequences. The M/G/∞-based video model is introduced in Section IV. Issues related to generating synthetic M/G/∞ traces are discussed in Section V. In Section VI we present simulations of the queuing performance under the three video models. Section VII concludes the paper.

II. M/G/∞ INPUT PROCESSES

In this section, we formally introduce the class of M/G/∞ processes, and summarize some of their properties as they relate to our modeling efforts; additional information can be found in [24] and [26].

A. Stationary M/G/∞ Input Processes

Consider a discrete-time system with an infinite number of servers. During time slot $[n,n+1)$ ($n = 0,1,\ldots$), $\xi_{n+1}$ new customers arrive into the system. Customer $j, j = 1,\ldots, \xi_{n+1}$, is presented to its own server, which begins its service by the start of slot $[n+1,n+2)$. Let $b_0$ denote the number of busy servers or, equivalently, the number of customers present in the system at the beginning of time slot $[n,n+1)$, with $b_0$ being the initial number of customers present in the system. It is assumed that the $N$-valued random variables (rv’s) $\{\xi_{n+1}, n = 0,1,\ldots\}, \{\sigma_{n,j}, n = 1,2,\ldots; j = 1,2,\ldots\}$, and $\{\sigma_{0,j}, j = 1,2,\ldots\}$ satisfy the following assumptions: 1) they are mutually independent; 2) $\{\xi_{n+1}, n = 0,1,\ldots\}$ are i.i.d. Poisson rv’s with parameter $\lambda > 0$; 3) $\{\sigma_{n,j}, n = 1,2,\ldots; j = 1,2,\ldots\}$ are i.i.d. rv’s with common pmf $G$ on $\{1,2,\ldots\}$. Let $\sigma$ be a generic $\mathbb{N}$-valued rv distributed according to the pmf $G$; assume that $\mathbb{E}[\sigma] < \infty$. Then, the M/G/∞ input process is simply the busy-server process $b_n = 0,1,\ldots$.

For $n = 0,1,\ldots$, let $b_n$ denote the $\mathbb{N}^{n+1}$-valued rv $(b_0,b_1,\ldots,b_n)$. The fact that the M/G/∞ process $\{b_n, n = 0,1,\ldots\}$ exhibits some form of positive dependence is indicated by the following result [27].

Proposition 1: For any choice of the initial condition rv $b_0$ and of the service times $\{\sigma_{0,i}, i = 1,2,\ldots\}$, the rv’s $\{b_{n+k}, n = 0,1,\ldots\}$ are associated in the following sense: For any $n = 0,1,\ldots$, and any pair of nondecreasing mappings $f,g: \mathbb{N}^{n+1} \rightarrow \mathbb{R}$, it holds that

\[
\mathbb{E}[f(b^n)g(b^n)] \geq \mathbb{E}[f(b^n)\mathbb{E}[g(b^n)]]
\]

providing the expectations exist and are finite. From (1), we already conclude that

\[
\text{cov} [b_n,b_{n+k}] \geq 0, \quad n,k = 0,1,\ldots.
\]

The notion of association used above was introduced in [5], and has been found useful in many contexts when formalizing the idea of positive dependence.

Thus far, no additional assumptions are made on the rv’s $\{\sigma_{0,j}, j = 1,2,\ldots\}$, which represent the service durations of the $b_0$ customers initially present in the system. Various scenarios can, in principle, be accommodated. If the initial customers start their service at time $n = 0$, then it is appropriate to assume that the rv’s $\{\sigma_{0,j}, j = 1,2,\ldots\}$ are also i.i.d. rv’s with common pmf $G$. On the other hand, if we take the viewpoint that the system has been in operation for some time, then these rv’s $\{\sigma_{0,j}, j = 1,2,\ldots\}$ may be interpreted as the residual work (expressed in time slots) that the $b_0$ “initial” customers require from their respective servers.
before service is completed. In general, the statistics of the
rv's \( \{ \sigma_{0,j} \mid j = 1, 2, \cdots \} \) cannot be specified in any meaningful
way, except for the situation when the system is in steady state.

Although the busy server process \( \{ b_{n}, n = 0, 1, \cdots \} \) is
in general not a (strictly) stationary process, it does admit
a stationary and ergodic version. The existence of this sta-
tionary regime emerges very naturally through the following
proposition. We use \( \Rightarrow \) to indicate weak convergence.

**Proposition 2:** There exists a stationary and ergodic \( \mathbb{N} \)-valued process \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) such that
\[
\{ b_{n+k}, n = 0, 1, \cdots \} \Rightarrow \{ b_{n}^{*}, n = 0, 1, \cdots \} \quad \text{as } k \to \infty
\]  
for any choice of the initial condition rv \( b_{0} \) and of the service
times \( \{ \sigma_{0,i} \mid i = 1, 2, \cdots \} \).

This stationary version \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) admits an explicit
construction, which corresponds to taking \( b_{0} \) to be Poisson
distributed with parameter \( \lambda \mathbb{E}[\sigma] \); \( \{ \sigma_{0,j} \mid j = 1, 2, \cdots \} \) to be i.i.d. rv's distributed according to the forward recurrence
time \( \hat{\sigma} \) associated with \( \sigma \). The pmf of \( \hat{\sigma} \) is given by
\[
P[\hat{\sigma} = r] \triangleq \frac{P[\sigma \geq r]}{\mathbb{E}[\sigma]}, \quad r = 1, 2, \cdots.
\]  
Based on the above construction, several useful properties of
the stationary version \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) are readily obtained
[24].

**Proposition 3:** The stationary and ergodic version \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) of the busy-server process has the following properties.

1) For each \( n = 0, 1, \cdots \), the rv \( b_{n}^{*} \) is a Poisson rv with
parameter \( \lambda \mathbb{E}[\sigma] \).
2) It holds that
\[
\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n} b_{k}^{*} = \lambda \mathbb{E}[\sigma] \quad \text{a.s.}
\]  
3) The covariance structure of \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) is given by
\[
\Gamma(k) \equiv \text{cov}\left[ b_{n}^{*}, b_{n+k}^{*} \right] = \lambda \mathbb{E}\left[ (\sigma - k)^{+} \right], \quad n, k = 0, 1, \cdots
\]  
Henceforth, by an M/G/\( \infty \) input process, we mean its
stationary version \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \), as described above.
This stationary process, which is fully characterized by the pair \( (\lambda, \sigma) \), will be used here as the basis for traffic modeling.

### III. CORRELATION STRUCTURE OF VBR VIDEO SOURCES

#### B. Correlation Properties of M/G/\( \infty \) Input Processes

We note from (6) that
\[
\Gamma(k) = \lambda \sum_{\epsilon=0}^{\infty} P\left[ (\sigma - k)^{+} > \epsilon \right]
\]  
\[
= \lambda \sum_{\epsilon=0}^{\infty} P[\sigma > k + \epsilon]
\]  
\[
= \lambda \sum_{\epsilon=k+1}^{\infty} P[\sigma \geq \epsilon]
\]  
\[
= \lambda \mathbb{E}[\sigma] \sum_{\epsilon=k+1}^{\infty} P[\sigma = \epsilon]
\]  
\[
= \lambda \mathbb{E}[\sigma] P[\sigma > k], \quad k = 0, 1, \cdots
\]  
Thus, the ACF for an M/G/\( \infty \) process is given by
\[
\rho(k) \triangleq \frac{\Gamma(k)}{\Gamma(0)} = \frac{\mathbb{E}[\sigma] P[\sigma > k]}{\mathbb{E}[\sigma]} \quad \text{a.s., for } k = 0, 1, \cdots
\]  
since \( \Gamma(0) = \lambda \mathbb{E}[\sigma] \) by (6). By varying \( \sigma \), the process
\( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) can display various forms of positive
autocorrelations, the extent of which is controlled by the tail
behavior of \( \sigma \).

To close this section, we point out that the process \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) can induce both SRD and LRD behaviors. From (8),
it follows readily [27] that
\[
\sum_{k=0}^{\infty} \Gamma(k) = \lambda \mathbb{E}[\sigma] \mathbb{E}[\hat{\sigma}] = \frac{\lambda}{2} \mathbb{E}[\sigma(\sigma + 1)]
\]  
whence
\[
\sum_{k=0}^{\infty} \rho(k) \frac{1}{k+1} = \mathbb{E}[\hat{\sigma}] = \frac{1}{2} \mathbb{E} \left[ \frac{\sigma^{2}}{2 \mathbb{E}[\sigma]} \right].
\]  
Consequently, the process \( \{ b_{n}^{*}, n = 0, 1, \cdots \} \) is LRD (re-
respectively, SRD) if and only if \( \mathbb{E}[\sigma^{2}] \) is infinite (respectively,
finite). In particular, the M/G/\( \infty \) input traffic will be LRD
when \( \sigma \) is Pareto, with a shape parameter in the interval
\( (1, 2) \) [3].

#### III. CORRELATION STRUCTURE OF VBR VIDEO SOURCES

In our study, we examined four public-domain VBR video
traces (Table I). These traces were generated using three
different encoding mechanisms (see references for further
details). Each trace represents an integer-valued sequence
of number of cells per frame for a given movie.

While a model is expected to capture some statistical
properties of the underlying empirical data, its goodness is

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**TABLE I**

**SUMMARY OF THE FOUR VBR TRACES USED IN THE STUDY**

<table>
<thead>
<tr>
<th>Movie</th>
<th>Source</th>
<th>Trace Length (frames)</th>
<th>Compression Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star Wars</td>
<td>M. Garrett [9]</td>
<td>174,000</td>
<td>DCT (intra-coding)</td>
</tr>
<tr>
<td>Beauty and the Beast</td>
<td>W. Feng [6]</td>
<td>143,442</td>
<td>JPEG</td>
</tr>
<tr>
<td>Wizard of Oz</td>
<td>M. Krunz [18]</td>
<td>12,600</td>
<td>MPEG-2 (I sequence)</td>
</tr>
</tbody>
</table>
ultimately determined based on its ability to achieve the goal it was designed for. In teletraffic studies, the goal of a model is to predict accurately network performance for the purpose of dimensioning network resources. Thus, the queueing performance is the crucial factor that determines the appropriateness of a traffic model. Since traffic correlations are known to have a profound impact on queueing behavior, preliminary indications of the goodness of a model can be obtained by examining its correlation structure.

The ACF’s for the four traces are shown in Fig. 1. Each empirical ACF was fitted by three functions: 1) \( \rho(k) = e^{-\beta k} \) (Markovian), 2) \( \rho(k) = k^{-\beta} \) (LRD), and 3) \( \rho(k) = e^{-\sqrt{\beta} k} \). The last fit was chosen because its drop-off behavior is similar to that of the empirical ACF (but other forms are also possible). For fits 1) and 3), \( \beta \) is obtained by least-square fitting. For the LRD fit of Star Wars trace, \( \beta = 0.4 \) was obtained from the estimated value of the Hurst parameter \( H = 1 - \beta/2 \approx 0.8 \), which was reported in [9]. For the other traces, the
Hurst parameter was estimated by several methods, including variance-time plots, R/S analysis, and Whittle’s approximation (see [2] and [31] for a discussion of these tests). In the interest of brevity, we only display the estimated values for the various parameters in Fig. 1. Clearly, the Markovian fit drops off much faster than the real ACF, so it only captures the short-term correlations. The LRD fit is not adequate either since it underestimates the correlations at lags 1–1000, and even beyond. Only at very large lags, the LRD fit becomes acceptable. In contrast, the choice \( \rho(k) = e^{-2k/\kappa} \) provides a very good fit at both small and large lags, particularly for the first three traces. Note that using a larger value for \( H \) would not improve the LRD fit, since \( k^{-\beta} \) always drops off fast and then maintains almost a flat appearance. Hence, it always underestimates the correlations up to some lag, and overestimates them beyond that lag.
IV. M/G/∞-BASED MODEL FOR VIDEO TRAFFIC

As indicated in Fig. 1, the ACF of a video sequence is adequately captured by
\[ \rho(k) = e^{-\beta \sqrt{k}}, \quad k = 0, 1, 2, \ldots \]  
for some constant \( \beta > 0 \). A model with such an ACF can be constructed using M/G/∞ input processes. In teletraffic modeling studies, a common practice is to try to capture the first two moments, the autocorrelation structure, and the general shape of the marginal distribution. More recently, researchers have realized the importance of capturing the tail of the marginal distribution (e.g., [9], [10], [22]), which is especially important for computing the buffer overflow probability at a multiplexer.

The parameters of the M/G/∞ process that can be used in the fitting are the service distribution \( G \) and the arrival rate \( \lambda \). While \( G \) can be chosen to provide a given autocorrelation structure [via (6)], the arrival rate \( \lambda \) can only be fitted to one moment (mean or variance). To capture the complete marginal distribution (including the mean and variance) as well as the correlation structure, we proceed in two steps. First, we choose \( G \) in the M/G/∞ model that provides the target ACF. Then, we identify a pointwise transformation that transforms the Poisson marginal distribution of the original M/G/∞ process into a more appropriate distribution. These steps are described next.

A. Modeling the Correlation Structure

We seek the pmf \( P \) which results in a correlation sequence of the form (11). To that end, we note from Proposition 3 and (8) that the correlation structure of the stationary M/G/∞ input process (which is parameterized by \( \lambda \) and \( G \)) is completely determined by the pmf of \( \hat{\sigma} \) (thus of \( \sigma \)). It turns out that the inverse is also true, as we now show. Indeed, if \( \rho(k), k = 0, 1, \ldots, \) is the ACF of the stationary M/G/∞ input process \((\lambda, G)\), then (4) and (8) together imply
\[ \rho(k) - \rho(k+1) = P[\sigma > k] - P[\sigma > k+1] = \frac{1}{E[\sigma]} P[\sigma > k], \quad k = 0, 1, \ldots \]  
so that the mapping \( k \rightarrow \rho(k) \) is necessarily decreasing and integer-convex. Taking into account the facts \( \rho(0) = 1 \) and \( P[\sigma > 0] = 1 \), we conclude from (12) (with \( k = 0 \)) that
\[ E[\sigma]^{-1} = 1 - \rho(1) \]  
with \( \rho(1) < 1 \) necessarily by the finiteness of \( E[\sigma] \). Combining (12) and (13) we find that
\[ P[\sigma > k] = \frac{\rho(k) - \rho(k+1)}{1 - \rho(1)}, \quad k = 0, 1, \ldots \]  
Note also from (14) that
\[ E[\sigma] = \sum_{k=0}^{\infty} P[\sigma > k] = \frac{1 - \lim_{k \to \infty} \rho(k)}{1 - \rho(1)} \]  
and (13) then imposes \( \lim_{k \to \infty} \rho(k) = 0 \). A moment of reflection readily yields the following invertibility result.

Proposition 4: An \( \mathbb{R}_+ \)-valued sequence \( \{\rho(k), k = 0, 1, \ldots\} \) is the autocorrelation function of the stationary M/G/∞ process with integrable \( \sigma \) if and only the corresponding mapping \( k \rightarrow \rho(k) \) is decreasing and integer-convex with \( \rho(0) = 1 \) and \( \lim_{k \to \infty} \rho(k) = 0 \), in which case the pmf \( G \) of \( \sigma \) is given by (14).

Differentiating (14) yields the pmf of \( \sigma \)
\[ P[\sigma = k] = \frac{\rho(k-1) - 2\rho(k) + \rho(k+1)}{1 - \rho(1)}, \quad k = 1, 2, \ldots \]  
The mapping \( x \rightarrow e^{-\beta \sqrt{x}} \) is decreasing and convex on \( \mathbb{R}_+ \), so that the sequence \( k \rightarrow e^{-\beta \sqrt{k}} \) is automatically decreasing and integer-convex on \( \mathbb{N} \). Proposition 4 can thus be applied to the correlation sequence (11). Upon substitution into (13) and (16), we find that the desired pmf for \( \sigma \) is simply
\[ P[k] = e^{-\beta \sqrt{k+1}} - 2e^{-\beta \sqrt{k}} + e^{-\beta \sqrt{k-1}}, \quad \frac{1 - e^{-\beta}}{1 - e^{-\beta}} \]  
and its mean service time is given by
\[ E[\sigma] = (1 - e^{-\beta})^{-1}. \]  
The value of \( \beta \) used in (17) and (18) is obtained by fitting the empirical ACF. It might be suggested that in determining the pmf of \( \sigma \), the empirical ACF be used directly in (16) instead of an analytical fit. However, the empirical ACF is not always monotone, and thus there is no \emph{a priori} guarantee that \( P[\sigma > k] \geq 0 \) in (14) for all \( k = 1, 2, \ldots \).

To conclude, we observe by an elementary comparison that
\[ \sum_{k=0}^{\infty} \rho(k) = 1 + \sum_{k=1}^{\infty} e^{-\beta \sqrt{k}} \leq 1 + \int_{0}^{\infty} e^{-\beta \sqrt{t}} \, dt = 1 + \frac{2}{\beta^2} < \infty \]  
and the correlation structure (11) indeed gives rise to an SRD model.

B. Modeling the Marginal Distribution

By Proposition 3, the M/G/∞ model produces correlated variates with a Poisson marginal distribution \( P_{\text{Poisson}} \), whose tail drops faster than that of the empirical distribution of a real video sequence. This is illustrated in Fig. 2 for the \emph{Star Wars} sequence where the parameter of the Poisson distribution (of the M/G/∞ fit) is obtained by matching the sample mean to \( \lambda E[\sigma] \), and setting \( \lambda \) accordingly \( E[\sigma] \) is estimated from the empirical ACF via (18). Indeed, the sample mean provides a natural estimate of \( \lambda E[\sigma] \) owing to the ergodic property (5) of M/G/∞ processes. The tail of the marginal distribution plays an important role in determining the buffer overflow probability at a multiplexer [10]. Hence, we need to provide a better fit to the empirical tail than the Poisson fit. To do that, we transform the Poisson distribution of the M/G/∞ process into a more appropriate distribution. The key idea
here resides in the following well-known observation. For a frame-size distribution $F$, a transformation $T: \mathbb{R} \rightarrow \mathbb{R}$ can always be constructed so that if the $\mathbb{R}$-valued $X$ is distributed according to some distribution $H$, then the $\mathbb{R}$-valued $Y = T(X)$ is distributed according to $F$. Indeed, it suffices to take

$$T(x) \overset{\Delta}{=} F^{-1}(H(x)), \quad x \in \mathbb{R} \quad (20)$$

where $F^{-1}$ denotes the (generalized) inverse of $F$.

The program is now clear. Consider a (stationary) $M/G/\infty$ process $\{b_n, n = 0, 1, \cdots\}$ characterized by the pair $(\lambda, G)$. The common distribution $H$ of these variates is Poisson with parameter $\lambda \mathbb{E}[\sigma]$. For any frame-size distribution $F$, define the transformed process $\{a_n, n = 0, 1, \cdots\}$ as

$$a_n \overset{\Delta}{=} T(b_n) = F^{-1}(H(b_n)) = F^{-1}(F_{\text{Poisson}}(b_n)), \quad n = 1, 2, \cdots \quad (21)$$

For each $n = 0, 1, \cdots$, the rv $a_n$ will be distributed according to $F$. In fact, the transformed process $\{a_n, n = 0, 1, \cdots\}$ is still stationary and ergodic. In general, the covariance structures of the two processes will not be exactly the same. The best one may hope for is that these covariance structures are approximately equal, i.e.,

$$\text{cov}[a_{n+k}, a_n] \simeq \text{cov}[b_{n+k}, b_n], \quad n, k = 0, 1, \cdots \quad (22)$$

Next, we need to select an appropriate distribution $F$. Several theoretical fits have been suggested for the frame-size distribution of a video sequence, including Gamma [14], log-normal [13], [17], and hybrid Gamma/Pareto distributions [9]. The last fit was found quite appropriate for Star Wars data. Accordingly, we use it here to model the frame-size distribution. As explained in [9], the Gamma distribution is used to capture the general shape of the empirical distribution, whereas the Pareto distribution is used to capture the tail of the empirical distribution. Let $F_G$ and $F_P$ denote the cumulative probability functions for the Gamma and Pareto distributions, respectively. Although $F_G$ has no closed-form expression, its derivative is given simply by

$$F_G(x) = \frac{\omega^s}{\Gamma(s)} x^{s-1} e^{-\omega x}, \quad x \geq 0 \quad (23)$$

where the parameters $s > 0$ and $\omega > 0$ are the shape and scale parameters, respectively, and the standard Gamma function $\Gamma(s)$ is given by

$$\Gamma(s) \overset{\Delta}{=} \int_0^\infty x^{s-1} e^{-x} dx, \quad s > 0 \quad (24)$$

The Pareto distribution we use has the explicit form

$$F_P(x) = \begin{cases} 1 - \left(\frac{a}{x}\right)^\alpha & \text{if } x \geq a \\ 0 & \text{if } x < a \end{cases} \quad (25)$$

with parameters $\alpha > 0$ and $a > 0$ which are both determined by fitting.

The hybrid Gamma/Pareto distribution $F_{GP}$ is then given by

$$F_{GP}(x) = \begin{cases} F_G(x) & \text{if } x \leq x^* \\ F_P(x) & \text{if } x > x^* \end{cases} \quad (26)$$

for some $x^* > 0$. As in [9], the parameters of the Gamma distribution are obtained by matching the first and second moments of the empirical sequence to those of a Gamma rv.

Once the Gamma part is fitted, $x^*$ can be estimated graphically by inspecting the tail of the empirical distribution, and determining where it starts to deviate from the tail of the Gamma fit (Fig. 2). Using the continuity condition $F_G(x^*) =
Table II

<table>
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<tr>
<th>Trace</th>
<th>Mean (cells)</th>
<th>Std. Dev. (cells)</th>
<th>$\omega$ (1/cells)</th>
<th>$s$</th>
<th>$x^*$ (cells)</th>
<th>$a$ (cells)</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
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<td>10.7</td>
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<td>12.55</td>
<td>398</td>
<td>215</td>
<td>5.31</td>
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<td>225.0</td>
<td>48.7</td>
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<td>21.4</td>
<td>355</td>
<td>224</td>
<td>10.1</td>
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</tbody>
</table>

Fig. 3. Impact of transformation on the autocorrelation structure.

$F_T(x^*)$ along with least-square fitting of the Pareto tail, estimates of $\alpha$ and $\alpha$ can be obtained. Table II gives the estimated parameters for three traces (frame sizes are in 48-byte cells). Since the fourth video trace is relatively short, accurate fitting of its extreme tail is not possible.

Thus, we select $F = F_{\gamma/P}$ and the Poisson variates of the M/G/$\infty$ process can now be transformed into Gamma/Pareto variates. Let $\{\xi_{kn}^*, n = 0, 1, \ldots\}$ denote the M/G/$\infty$ process with $\lambda = 1$ and service time distribution (17), so that its correlation structure is given by (11). The sequence $\{\xi_{kn}^*, n = 0, 1, \ldots\}$ is transformed into a new sequence $\{a_n, n = 0, 1, \ldots\}$ through the transformation

$$a_n = F_{\gamma/P}^{-1}(F_{\text{Poisson}}(\xi_{kn}^*)), \quad n = 1, 2, \ldots$$  (27)

where $F_{\text{Poisson}}$ is the cumulative probability function of a Poisson rv with parameter $E[\sigma]$ [given by (18)] and

$$F_{\gamma/P}^{-1}(y) = \begin{cases} F_T^{-1}(y) = a/(1-y)^{1/\alpha} & \text{if } y > F_T(x^*) = 1 - (a/x^*)^\alpha \\ F_T^{-1}(y) & \text{otherwise} \end{cases}$$  (28)

with $F_T^{-1}$ obtained numerically.

Since only the Gamma part is used in fitting the mean and variance, the mean and variance of $a_n$ will be slightly different from their empirical counterparts. For example, the mean frame size in a synthetic trace is given by

$$E[a_n] = \int_0^{x^*} x f_T(x) \, dx + \int_{x^*}^{\infty} f_T(x) \, dx$$  (29)

while the empirical mean is fitted to $\int_0^{\infty} x f_T(x) \, dx$. However, this slight discrepancy is of no significance.

As pointed out above, this transformation does not, in general, preserve the original correlation structure. However, in all our experiments, the effect of transformation was barely noticeable. An example of the average ACF of transformed M/G/$\infty$ traces is shown in Fig. 3 based on Star Wars fitting. The average ACF is almost indistinguishable from the theoretical ACF of the nontransformed M/G/$\infty$ process.

V. SYNTHETIC TRACE GENERATION AND COMPUTATIONAL ISSUES

Ideally, we would like to analytically determine the queueing performance for a traffic model so that control decisions related to call admission and resource allocation can be done on-line. However, there is a natural tradeoff between the complexity of a model and the relative accuracy of its queueing predictions. A detailed video model, such as the one considered in this paper, does not easily lend itself to
queueing analysis, but can be used to drive network simulations. Performance evaluation by means of simulations is useful in off-line dimensioning problems (e.g., buffer sizing under a fixed quality of service). The simulation time can sometimes be reduced by employing certain problem-specific techniques (some of which are discussed in the next section). Separating the issue of model construction from that of queueing tractability allows highly accurate models to be developed. It should also be mentioned that models with analytically tractable performance are not always usable in on-line traffic control problems, particularly when extensive numerical computations are needed to obtain the results. While network simulations can be driven by “real” data, such data sets are often not available or very difficult to obtain. A stochastic model, on the other hand, encompasses many realizations that represent independent yet structurally similar (i.e., homogeneous) streams, which are ideal for statistical multiplexing studies.

A. Simulation Models

To verify the appropriateness of the M/G/∞-based model, we investigate its queueing performance and contrast it with the performance of two popular video models: the F-ARIMA model [9] (which exhibits LRD), and the DAR(1) model [14] (which exhibits a Markovian structure). By a suitable transformation, we ensure that all models share the same hybrid Gamma/Pareto marginal distribution, thereby eliminating the impact of the marginal frame-size distribution. In all three models, the hybrid Gamma/Pareto distribution is discretized to obtain integer-valued frame sizes.

Synthetic realizations from the three video models were generated and used in the queueing simulations described in the next section. Each of the M/G/∞ and DAR(1) traces consists of 1 million data points, while each F-ARIMA trace consists of 500,000 data points (a data point corresponds to a frame size measured in cells). The F-ARIMA traces are shorter than their M/G/∞ and DAR(1) counterparts since generating F-ARIMA traces of length 1 million is computationally prohibitive. More specifically, it requires $O(n^2)$ computations to generate an F-ARIMA trace of length $n$ using Hosking’s algorithm [15] (before transformation). In contrast, both the M/G/∞ and DAR(1) models require only $O(n)$ computations per trace. To generate an F-ARIMA trace of only 100,000 points using Hosking’s algorithm, it took about three days of execution on a Sparc-10 workstation. To generate 500,000-long F-ARIMA traces, we used an approximation due to Haslett and Raftery [11], which was incorporated in the S-Plus package. Even with this approximation, it took about two days to obtain one 500,000-long F-ARIMA trace, compared to less than one minute for a 1 million long M/G/∞ or DAR(1) trace. Extensive simulations based on the three models were conducted. For brevity, we show the results for one real trace (the Star Wars) and its corresponding models.

B. F-ARIMA and DAR(1) Models

The F-ARIMA model [9] used here is constructed by transforming a fractional ARIMA process with a standard normal marginal distribution into one with a hybrid Gamma/Pareto distribution. An example of the sample ACF of a synthetic F-ARIMA realization for the Star Wars trace is shown in Fig. 4.

The theoretical ACF of an F-ARIMA process is given by

$$\rho(k) = \frac{d(1+d)\cdots(k-1+d)}{(1-d)(2-d)\cdots(k-d)},$$

$$k = 1, 2, \cdots, \quad 0 < d < 0.5$$

(30)
which behaves as $k^{-3}$ only asymptotically ($d = H - 1/2$).

In fact, the ACF of the F-ARIMA model underestimates the short-term correlations of the real data even more than $k^{-3}$.

We have transformed the normally distributed variates of the standard F-ARIMA model into Gamma/Pareto variates. Here, as with the M/G/$\infty$-based model, inspection of Fig. 4 suggests that the transformation has almost no impact on the correlation structure of the original F-ARIMA process. This is in keeping with the work in [16] where, under mild conditions, a transformed LRD Gaussian process is shown to maintain its Hurst value.

The DAR(1) model is obtained as follows [1]. Let $\{V_n, n = 0, 1, \ldots\}$ and $\{U_n, n = 0, 1, \ldots\}$ be two mutually independent processes of i.i.d. rv's. For $n = 0, 1, \ldots$, the rv $V_n$ is Bernoulli with $P[X_n = 1] = 1 - P[V_n = 0] = r$, and the rv $U_n$ is an $N$-valued rv distributed according to the pmf $\pi(i) \triangleq P[U_n = i]$, $i = 0, 1, \ldots$. A DAR(1) process $\{X_n, n = 0, 1, \ldots\}$ is defined through the recursion

$$X_n = V_nX_{n-1} + (1 - V_n)U_n, \quad n = 1, 2, \cdots \tag{31}$$

with given $X_0$. The sequence $\{X_n, n = 0, 1, \ldots\}$ is a Markov chain with the same marginal distribution as $\pi = (\pi(0), \pi(1), \cdots)$, i.e., $P[X_n = i] = \pi(i), i = 0, 1, \ldots$, and with an ACF of the form $\rho_k = r^k$, similar to that of the familiar AR(1) process. In [14], the DAR(1) model was used to characterize video-teleconferencing streams, with the marginal distribution taken as a negative binomial distribution—the discrete analog of a Gamma distribution. Here, instead, we use a hybrid Gamma/Pareto marginal distribution, consistent with our choice for the other two models examined in the paper.

C. Generation of M/G/$\infty$ Traces

A CSIM program\(^1\) was written to generate synthetic M/G/$\infty$ traces. The program simulates an M/G/$\infty$ queue with infinite servers. Time is slotted in frame periods. At the start of a time slot, a batch of arrivals is generated according to a Poisson distribution with $\lambda = 1$. Each arrival is kept for a random time $\tau$ whose pmf is given by (17). A synthetic M/G/$\infty$ trace is obtained from the number of remaining customers at the beginning of each time slot. This trace is then transformed into one with a Gamma/Pareto marginal distribution.

The computational complexity for generating an M/G/$\infty$ trace of length $n$ (before transforming the marginals) is $O(n)$. The computational complexity for the generation of DAR(1) traces is also $O(n)$ (see [14] for details on how to generate DAR(1) traces).

We note that due to the correlated nature of cell losses, extremely long traces are needed to obtain meaningful results under small cell loss probabilities. In fact, we first tried using shorter traces of length 100,000, and found that for realistic loss rates, losses occur in only few frames, e.g., in one particular experiment, a loss rate of $8.3E-6$ (484 cells) came from five errored frames only. Intuitively, correlations make it more likely that large frames follow each other, thus causing correlated periods of buffer overflow. Moreover, 100,000-long realizations may not be long enough to display the extreme tail of the frame-size distribution, causing the loss performance to be underestimated. For example, the maximum frame size in the Star Wars trace is 894 cells. In order to display this value in a transformed M/G/$\infty$ trace, the corresponding value before transformation is 33, i.e., $F_{\tau}^{-1}(F_{\text{Poisson}}(33)) = 894$. An M/G/$\infty$ trace before transformation is a realization of $n$ identically distributed rv’s $b_{1\cdot n}$, which are associated (Proposition 1). By the well-known properties of associated rv’s [5], we have

$$P\left[ \max_{i=1,\cdots,n} b_i > x \right] \leq 1 - \prod_{i=1}^{n} P[b_i < x]$$

$$= 1 - F_{\text{Poisson}}(x)^n, \quad x \in \mathbb{R}. \tag{32}$$

Thus, for $n = 100,000$, we find that

$$P\left[ \max_{i=1,\cdots,n} b_i > 32 \right] < 1 - (F_{\text{Poisson}}(32))^{100,000} = 0.4745 \tag{33}$$

i.e., there is less than a 50% chance that the 100,000-long realization reaches the real maximum frame size.

VI. Queueing Performance

To verify the appropriateness of the M/G/$\infty$ model, we investigate its queueing performance and compare it to the

\[^1\]CSIM is a C-based discrete-event simulation language [30].
The cell loss rate (CLR) and the frame error rate (FER) are examined at three loads: \( U = 80\% \) (heavy load), \( 60\% \) (moderate load), and \( 40\% \) (light load). A summary of the simulation results to two significant digits is given in Table III. The depicted results for the three models represent the averages of ten independent runs.

For \( U = 80\% \) and \( 60\% \), the buffer size is varied from 100 to 2500 cells. As expected, CLR and FER for a real stream are quite high at \( U = 80\% \). Adding extra buffer barely provides any improvement in performance. In contrast, reducing the load from 80\% to 60\% (i.e., increasing bandwidth by 33\%) improves the CLR by about an order of magnitude. The buffer size seems to have a bigger impact on the FER than on the CLR. At both \( U = 80\% \) and \( U = 60\% \), the FER for the real stream decreases by about 50\% when \( B \) is increased from 100 to 2500 cells.

Contrasting the performances under the three models with reference to the performance under the real stream, we observe the following: In the heavy-load regime, both M/G/\( \infty \) and DAR(1) models provide acceptable predictions of CLR and FER, with DAR(1) being slightly more accurate. Under the F-ARIMA model, the performance is overly sensitive to the buffer size, to the extent that it underestimates the actual CLR and FER by orders of magnitude when \( B \) is large. This is clearly a consequence of not sufficiently capturing the short-term correlations. Going to the moderate-load regime, we observe that once again both M/G/\( \infty \) and DAR(1) models provide significantly more accurate predictions of CLR and FER than the F-ARIMA model. In this regime, DAR(1) and M/G/\( \infty \) models give comparable results (particularly, with respect to the CLR measure).

Interestingly, in the light-load regime (\( U = 40\% \)), the DAR(1) model is no more capable of providing acceptable performance predictions. In fact, no losses were observed in any of the DAR(1) simulations (although ten independent simulations were used, each with a 1 million long trace). The M/G/\( \infty \) model is quite accurate in this regime. The F-ARIMA model is still overly sensitive to the buffer size, although the gap between its performance and the real performance is now smaller (when \( B \) is small the F-ARIMA model overestimates CLR and FER, but as \( B \) increases the model starts to underestimate both performance measures). The main conclusion to be drawn from Table III is that of the three examined models, only the M/G/\( \infty \) model is observed to consistently provide acceptable performance predictions at various traffic loads. The performance of the M/G/\( \infty \) is always within an order of magnitude of the real performance. The capability of the M/G/\( \infty \) model of providing acceptable results can be attributed to the fact that it incorporates the good aspects of Markovian and LRD models. Similar to Markovian models, it incorporates the short-term correlations; and similar to LRD models, it captures the slowly decaying nature of the correlation structure of a VBR video sequence.

The M/G/\( \infty \) model slightly underestimates the actual queueing performance, particularly at intermediate loss rates (i.e., 1.0E-3–1.0E-4) and large buffer sizes. An examination of the real trace reveals that much of the discrepancy is related to some “nonstationarity” in the real data, which is not accounted for in the M/G/\( \infty \) model. In particular, the first and last few thousand frames of the Star Wars trace exhibit stronger statistical correlations than the rest of the trace. We speculate these frames correspond to the compressed frames.
TABLE III
AVERAGE CELL LOSS AND FRAME ERROR RATES AT THREE DIFFERENT LOADS (Star Wars Trace). TEN INDEPENDENT REPLICATIONS ARE USED TO OBTAIN THE VALUES FOR EACH MODEL. (a) U = 80%, (b) U = 60%, and (c) U = 40%.

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<tr>
<th>Buffer Size (cells)</th>
<th>Real</th>
<th>M/G/∞</th>
<th>F-ARIMA</th>
<th>DAR(1)</th>
<th>Real</th>
<th>M/G/∞</th>
<th>F-ARIMA</th>
<th>DAR(1)</th>
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(a)

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<td>3.8E-6</td>
<td>3.6E-6</td>
<td>7.8E-7</td>
<td>0</td>
<td>2.3E-5</td>
<td>8.6E-6</td>
<td>8.0E-7</td>
</tr>
<tr>
<td>1000</td>
<td>2.8E-6</td>
<td>3.4E-6</td>
<td>6.7E-7</td>
<td>0</td>
<td>2.3E-5</td>
<td>7.9E-6</td>
<td>6.0E-7</td>
</tr>
</tbody>
</table>

(c)

in the credits (the portion that contains the names of actors, acknowledgment, etc.).

In the above simulations, the simulation time was significantly reduced by conducting the discrete-event simulation at the frame level (rather than the cell level). The algorithm that was used for these single-stream simulations is shown in Fig. 6. It exploits the fact that only the frame sizes, the service rate, the maximum buffer size, and the queue length at the beginning and end of each time slot are relevant to the computation of the CLR and FER measures.

B. Multiplexed Streams

In this subsection, we investigate the multiplexing performance under the three models for the purpose of contrasting their different behaviors. It is not our objective here to provide a thorough evaluation of the multiplexing gain and the associated resource allocation problem, which will be the topic of a future study. For simplicity, we assume that the frames’ boundaries of multiplexed streams are aligned in time, so the time axis is slotted in frame periods. This specialization allows us to significantly reduce the simulation time using the following optimization.

Consider a simulation experiment in which N video streams (indicated by their frame-size traces) are to be multiplexed. Assume that the N streams have the same number of frames, n. Let \(X_j^{(k)}\), \(j = 1, 2, \ldots, n\) be the frame-size sequence for the kth stream, \(k = 1, 2, \ldots, N\). To obtain the CLR and FER for the multiplexed N streams, we first compute an aggregate trace \(\bar{X}_j = \sum_{k=1}^{N} X_j^{(k)}\) for \(j = 1, \ldots, n\). For a time slot (i.e., a frame period) in which buffer overflow cannot occur, the aggregate trace can be used to update the buffer occupancy.
at the end of that slot. This updating is done on a frame-by-frame basis, using an algorithm similar to the one in Fig. 6. For time slots during which buffer overflow is possible (based on some sufficient conditions that will be introduced shortly), the individual traces are used to simulate the performance on a cell-by-cell basis.

Fortunately, buffer overflow occurs only in a small fraction of the total number of simulated time slots. Let \( Q_j \) denote the queue length at the beginning of the \( j \)th slot. It can be shown that either of the following two conditions guarantees no buffer overflow during the \( j \)th slot:

1) \( \sum \{X_j, C \} \leq B \); or
2) \( \sum \{X_j, C, B - Q_j \} \leq B - N \).

With this optimization, the simulation time for computing the queueing performance for \( N \) multiplexed streams is \( O(n + \alpha n_W N) \), where \( n \) is the trace length, \( \alpha \) is the fraction of slots for which neither of the above conditions is satisfied, and \( W \) is the average number of cells per frame per stream during buffer overflow. Typically, \( \alpha W \ll 1 \), making the complexity much less than \( O(N) \).

To give an idea of the efficiency of the above simulation approach, Table IV gives an example of the simulation times for ten multiplexed M/G/∞ streams with different buffer sizes (the results in the table were based on a single run). As the buffer size increases, both CLR and FER decrease, resulting in shorter simulation times. In this example, an order of magnitude lower buffer size than the other two models. Both DAR(1) and M/G/∞ models display comparable sensitivities to buffer size. However, the CLR performance for the DAR(1) model is more than an order of magnitude higher than that of the M/G/∞ model. Given the performance for the real streams when \( N = 10 \) and that for the DAR(1) model when \( N = 10 \), one could conclude that the DAR(1) is probably overestimating the CLR performance (realistically, we should expect an appreciable reduction in the CLR when going from \( N = 5 \) to \( N = 10 \)), Of course, a conclusive judgment would require obtaining the performance for ten multiplexed real streams.

VII. CONCLUDING REMARKS

In this paper, we investigated a new approach for characterizing VBR video streams based on M/G/∞ processes. These processes enjoy several attractive features that make them a viable approach for modeling various types of network traffic. Compelling statistical evidence from four different video traces suggests that the ACF of a VBR sequence is better captured by \( e^{-\lambda r} \) than by \( e^{-\lambda r} \) (Markovian) or \( e^{-\lambda r} \) (LRD). While Markovian models capture the short-term correlations and LRD models capture the long-term correlations, the fit \( e^{-\lambda r} \) is shown to sufficiently capture the empirical correlations at all

---

**Table IV**

<table>
<thead>
<tr>
<th>Buffer Size (cells)</th>
<th>Average CLR</th>
<th>Simulation Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>3.2E-5</td>
<td>512.84</td>
</tr>
<tr>
<td>200</td>
<td>3.0E-5</td>
<td>381.95</td>
</tr>
<tr>
<td>300</td>
<td>2.8E-5</td>
<td>362.72</td>
</tr>
<tr>
<td>400</td>
<td>2.7E-5</td>
<td>328.50</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3500</td>
<td>7.8E-6</td>
<td>108.13</td>
</tr>
<tr>
<td>4000</td>
<td>6.1E-6</td>
<td>95.47</td>
</tr>
<tr>
<td>4500</td>
<td>4.4E-6</td>
<td>88.39</td>
</tr>
<tr>
<td>5000</td>
<td>2.7E-6</td>
<td>63.52</td>
</tr>
</tbody>
</table>
TABLE V

<table>
<thead>
<tr>
<th>Buffer Size (cells)</th>
<th>Average Cell Loss Rate</th>
<th>Frame Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>M/G/∞</td>
</tr>
<tr>
<td>200</td>
<td>2.6E-4</td>
<td>2.3E-4</td>
</tr>
<tr>
<td>400</td>
<td>2.4E-4</td>
<td>2.0E-4</td>
</tr>
<tr>
<td>600</td>
<td>2.3E-4</td>
<td>1.8E-4</td>
</tr>
<tr>
<td>800</td>
<td>2.2E-4</td>
<td>1.7E-4</td>
</tr>
</tbody>
</table>

(a)

<table>
<thead>
<tr>
<th>Buffer Size (cells)</th>
<th>Cell Loss Rate</th>
<th>Frame Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M/G/∞</td>
<td>F-ARIMA</td>
</tr>
<tr>
<td>100</td>
<td>7.4E-6</td>
<td>2.1E-5</td>
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<tr>
<td>200</td>
<td>6.3E-6</td>
<td>1.3E-5</td>
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<tr>
<td>300</td>
<td>5.6E-6</td>
<td>7.8E-6</td>
</tr>
<tr>
<td>400</td>
<td>4.8E-6</td>
<td>4.7E-6</td>
</tr>
</tbody>
</table>

(b)

lags. To display such a correlation structure, an M/G/∞-based model for video was developed, which exhibits short-range dependence (although not Markovian). The Poisson marginals of the M/G/∞ process were transformed into ones with a more appropriate distribution (due to Garrett and Willinger [9]). The impact of the transformation is shown to be negligible.

With the performance of a real stream taken as a reference, we examined the queueing performance under the M/G/∞ model and contrasted it to the performances for two popular video models: the F-ARIMA model (LRD) and the DAR(1) (Markovian). Our simulation results indicate that the M/G/∞ model consistently provides acceptable predictions of the actual cell loss and frame error rates at various traffic loads and buffer sizes. In contrast, the performance for the F-ARIMA model is overly sensitive to the buffer size, to the extent that it sometimes underestimates the real performance by several orders of magnitude. The DAR(1) model, while showing acceptable trends to changes in buffer size, sometimes gives unacceptably optimistic predictions (e.g., the case of a single stream with 40% traffic intensity), and at other times pessimistic predictions (case of multiplexed streams). An additional advantage of the M/G/∞ model over the F-ARIMA is that only computations are needed to generate a synthetic trace of size $n$, compared to $O(n^2)$ for an F-ARIMA trace. Our future work will focus on using the M/G/∞ model in on-line admission control and dynamic resource allocation. Toward this end, we have been working on analytically obtaining the queueing performance for multiplexed M/G/∞ sources and using such performance to compute the effective bandwidth.

Results of this research will be reported in future work.

APPENDIX

PROOF OF EQUATION (6)

The derivation of (6) is based on the following well-known result on random sums of i.i.d. rv’s.

Lemma 1: Let $\{X_n, n = 1, 2, \cdots\}$ be a sequence of $\mathbb{R}$-value i.i.d. rv’s which are independent of an $\mathbb{N}$-valued rv $\lambda$

For any two functions $f, g: \mathbb{R} \to \mathbb{R}$, we have

$$\text{cov} \left[ \sum_{j=1}^{\nu} f(X_j), \sum_{i=1}^{\nu} g(X_i) \right] = E[\nu]E[f(X)]E[g(X)] + \text{var}(\nu)E[f(X)]E[g(X)]$$

provided the expectations exist.

Consider the M/G/∞ input process $\{b_n, n = 0, 1, \cdots\}$. For each $n = 0, 1, \cdots$, we note that

$$b_n = b_n^{(0)} + b_n^{(\alpha)}$$

where the rv’s $b_n^{(0)}$ and $b_n^{(\alpha)}$ describe the contributions to the number of customers in the system at the beginning of slot $[n, n+1)$ from those initially present (at $n = 0$) and from the new arrivals, respectively. Under the enforced operational assumptions, we readily have

$$b_n^{(\alpha)} = \sum_{s=1}^{\nu} \sum_{i=1}^{\nu} 1[\sigma_{n+i} > n - s]$$

and

$$b_n^{(0)} = \sum_{i=1}^{\nu} 1[\sigma_{n+i} > n]$$

The stationary version $\{b_n^{*}, n = 0, 1, \cdots\}$ is obtained by assuming that 1) the rv $b_0$ is a Poisson rv with parameter $\lambda E[\sigma]$; 2) the rv’s $\{\sigma_{n+i}, j = 1, 2, \cdots\}$ are i.i.d. rv’s distributed according to the pmf (4) of the forward recurrence time associated with $\sigma$.

Fix $n = 0, 1, \cdots$ and $k = 1, 2, \cdots$. By independence, we have

$$\Gamma(k) = \text{cov} \left[ b_n, b_{n+k} \right] = \text{cov} \left[ b_n^{(0)}, b_n^{(0)} \right] + \text{cov} \left[ b_n^{(\alpha)}, b_n^{(\alpha)} \right]$$

(38)
First, we consider the term \( \text{cov} \left[ b_{n+k}^{(a)} \right] \). Under the enforced independence assumptions

\[
\text{cov} \left[ b_{n+k}^{(a)} \right] = \frac{1}{n} \sum_{s=1}^{n} \sum_{i=1}^{n} 1[\sigma_{s,i} > n + k - s] = 0
\]

so that

\[
\text{cov} \left[ b_{0}^{(a)}, b_{n+k}^{(a)} \right] = \text{cov} \left[ b_{0}^{(a)}, \sum_{s=1}^{n} \sum_{i=1}^{n} 1[\sigma_{s,i} > n + k - s] \right] = \sum_{s=1}^{n} \sum_{i=1}^{n} \text{cov} \left[ \sum_{j=1}^{n} 1[\sigma_{s,j} > n - r] \right]
\]

\[
\cdot \sum_{i=1}^{n} 1[\sigma_{s,i} > n + k - s] = \sum_{s=1}^{n} \text{cov} \left[ \sum_{j=1}^{n} 1[\sigma_{s,j} > n - r] \right] \cdot \sum_{i=1}^{n} 1[\sigma_{s,i} > n + k - s]
\]

\[
= \sum_{s=1}^{n} E[\xi_s] E[1[\sigma_{s,1} > n - s] \cdot 1[\sigma_{s,1} > n + k - s]] + \sum_{s=1}^{n} \left( E[\xi_s^2] - E[\xi_s]^2 \right) P[\sigma_{s,1} > n - s] \cdot P[\sigma_{s,1} > n + k - s] = \lambda \sum_{r=1}^{n} P[\sigma > r + k - 1].
\]

Next, we consider \( \text{cov} \left[ b_{0}^{(a)}, b_{n+k}^{(a)} \right] \). Again, making use of Lemma 1 under the enforced independence assumptions, we conclude that

\[
\text{cov} \left[ b_{0}^{(a)}, b_{n+k}^{(a)} \right] = \lambda \sum_{r=1}^{n} P[\sigma > r + k - 1].
\]

The proof of (6) is now completed.

### ACKNOWLEDGMENT

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### REFERENCES

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