Two dimensional solutions of Laplace's equations in Cartesian coordinates are easy to come by. Let \( z = x + iy \) be a complex number, and \( f(z) \) any complex analytic function of \( z \). Examples of analytic functions are: \( z^n, \sin z, e^z \ldots \) The complex function \( f \) will have a real part \( f_R(x,y) \) and an imaginary part \( f_I(x,y) \) each of which depend on \( x \) and \( y \). Both \( f_R \) and \( f_I \) can be regarded as the real functions of \( x \) and \( y \) as well as the real and imaginary parts of the complex function \( f = f_R + i f_I \).

Show that \( f_R \) and \( f_I \) are both solutions of Laplace's Equation. Along the way you must first show the following (Cauchy-Riemann) equations,

\[
\frac{\partial f_R}{\partial x} = -\frac{\partial f_I}{\partial y}, \quad \frac{\partial f_R}{\partial y} = \frac{\partial f_I}{\partial x}.
\]

Take \( f(z) \) to be \( \arcsin(z) \). Make contour plots of the potential corresponding to the real part of \( f \). What problem is this the potential for?