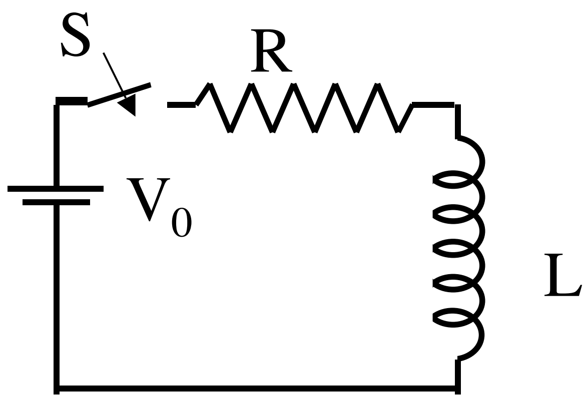
Chapter 36 Review

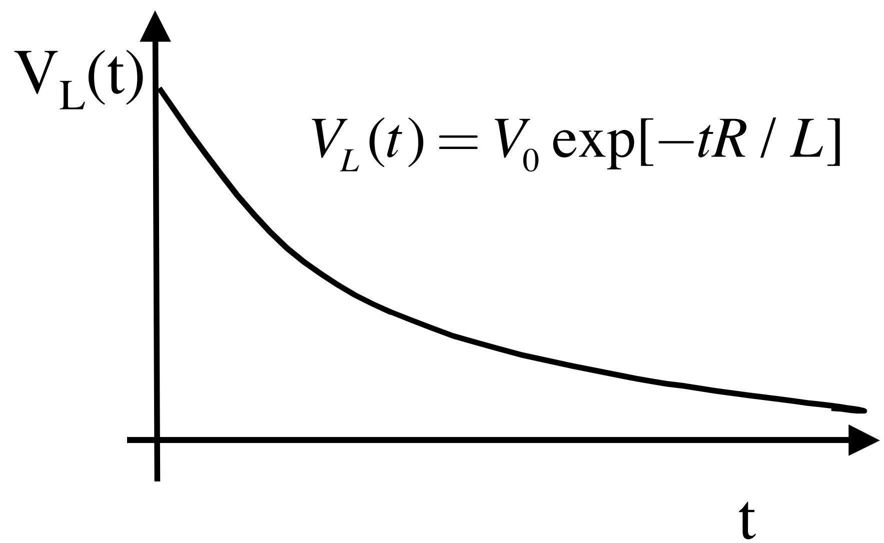
AC Circuits

Three categories of time behavior

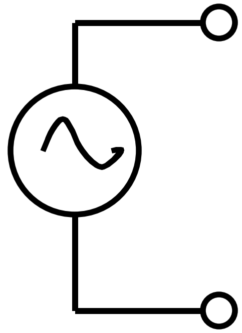
1. Direct Current (DC) Voltages and currents are constants in time. Example: batteries - circuits driven by batteries

2. Transients Voltages and currents change in time after a switch is opened or closed. Changes diminish in time and stop if you wait long enough.





3. <u>Alternating Current (AC).</u> The voltages and currents continually change sinusoidally in time.



$$V(t) = \underbrace{V_0}_{\text{amplitude}} \cos[\overbrace{\omega}^{\text{frequency}} t + \underbrace{\theta}_{\text{phase}}]$$

Examples: our power grid when it is on. f=60 Hz, V=110 V (RMS)
- audio signals
- communication signals
- Power in microwave ovens
- Power in MRI machines

Real Life voltages involve DC, AC and Transients

AC - Circuits

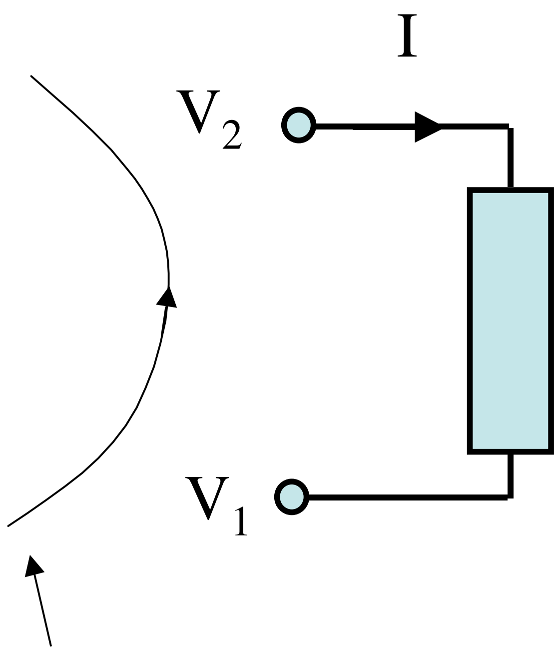
<u>First Rule of AC - Circuits</u> - everything oscillates at the same frequency

The problem then becomes: Find the amplitude and phase of each voltage and current.

<u>Phasors</u> - Everything you learned about DC circuits can be applied to AC circuits provided you do the following:

1. Replace all voltages and currents by their complex phasor amplitudes. In practice this means putting a hat on each letter.
2. Treat inductors as resistors with "resistance" $j\omega L$
3. Treat capacitors as resistors with "resistance" $1/(j\omega C)$

Foolproof sign convention for two terminal devices



KVL Loop

Contribution to voltage sum = +V

1. Label current going in one terminal (your choice).
2. Define voltage to be potential at that terminal wrt the other terminal

$$V = V_2 - V_1$$

3. Then <u>no</u> minus signs

$$V = RI$$

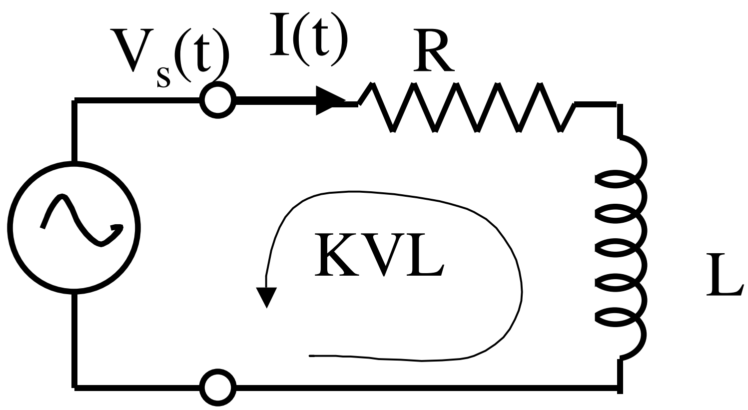
$$V = L\frac{dI}{dt}$$

$$I = C\frac{dV}{dt}$$

Power <u>to</u> device

$$P = VI$$

Phasors for R-L circuit



Write currents and voltages in phasor form

$$V_s(t) = \mathrm{Re}\left[\hat{V}_0 e^{j\omega t}\right] \quad V_L(t) = \mathrm{Re}\left[\hat{V}_L e^{j\omega t}\right]$$

$$I(t) = \mathrm{Re}\left[\hat{I} e^{j\omega t}\right] \quad V_R(t) = \mathrm{Re}\left[\hat{V}_R e^{j\omega t}\right]$$

Write circuit equations for phasor amplitudes

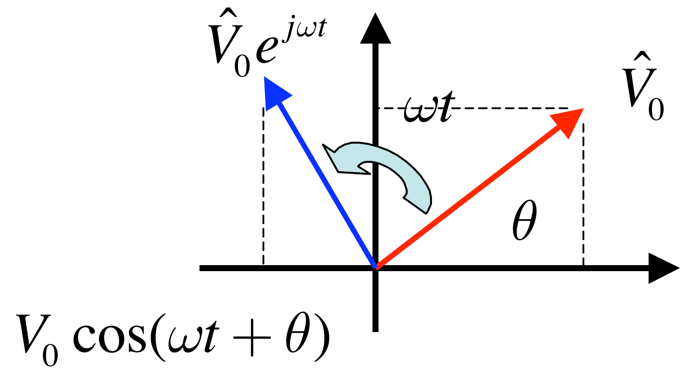
KVL: $0 = \hat{V}_L + \hat{V}_R - \hat{V}_0$

$$\hat{V}_L = j(\omega L)\hat{I} = jX_L\hat{I} \qquad \hat{V}_R = R\hat{I}$$

Result: $\hat{I} = \dfrac{\hat{V}_0}{Z}$

Impedance $Z = R + jX_L$

$$V(t) = \mathrm{Re}\left[\hat{V}_0 e^{j\omega t}\right]$$



Multiplying $\hat{V}_0$ by $e^{j\omega t}$ rotates the angle of the product by ωt

Remember:

$$|Z_3| = |Z_1||Z_2|$$

$$\theta_3 = \theta_1 + \theta_2$$

How to use in circuits:

1. Every voltage and current is written in phasor form:

$$V_s(t) = \mathrm{Re}\left[\hat{V}_0 e^{j\omega t}\right]$$

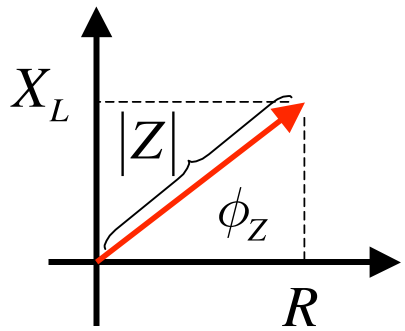
$$I(t) = \mathrm{Re}\left[\hat{I} e^{j\omega t}\right]$$

$$V_L(t) = \mathrm{Re}\left[\hat{V}_L e^{j\omega t}\right]$$

Result: $\hat{I} = \dfrac{\hat{V}_0}{Z}$

Impedance $Z = R + jX_L$

Impedance has a magnitude and phase



$$Z = |Z| e^{j\phi_Z}$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\tan\phi_Z = X_L / R$$

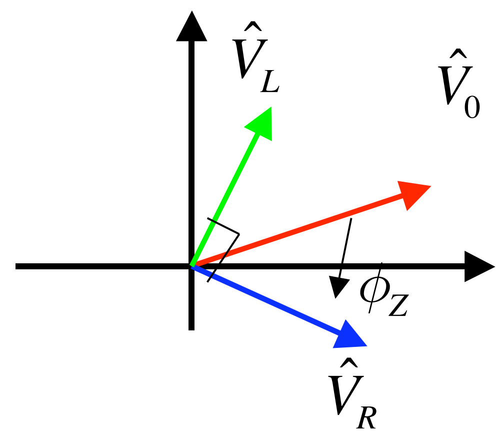
Resistor Voltage

$$\hat{V}_R = R\hat{I} = \hat{V}_0 \frac{R}{Z} = \hat{V}_0 \frac{R}{|Z|} e^{-j\phi_Z}$$

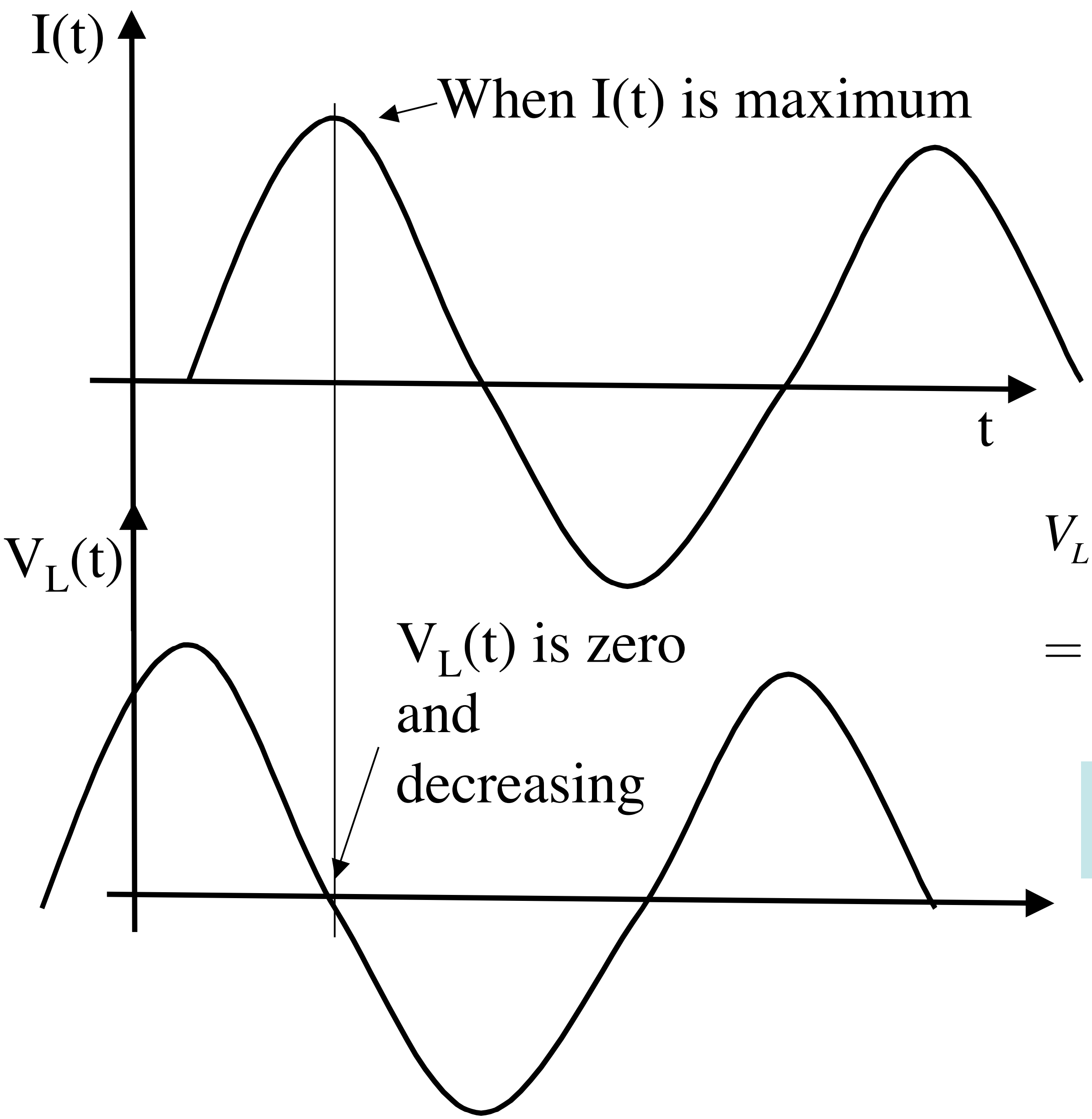
Inductor Voltage

$$\hat{V}_L = jX_L\hat{I} = \hat{V}_0 \frac{jX_L}{Z} = \hat{V}_0 \frac{X_L}{|Z|} e^{j(\frac{\pi}{2} - \phi_Z)}$$

Note: $j = e^{j\frac{\pi}{2}}$



$$\hat{V}_0 = \hat{V}_L + \hat{V}_R$$



$$I(t) = I_0 \cos[\omega t + \theta_I]$$

$$V_L = -\omega L I_0 \sin[\omega t + \theta_I]$$
$$= \omega L I_0 \cos[\omega t + \theta_I + \frac{\pi}{2}]$$

V_L leads I by $\pi/2$

Power Dissipated in Resistor

Current

$$I(t) = I_R \cos[\omega t]$$

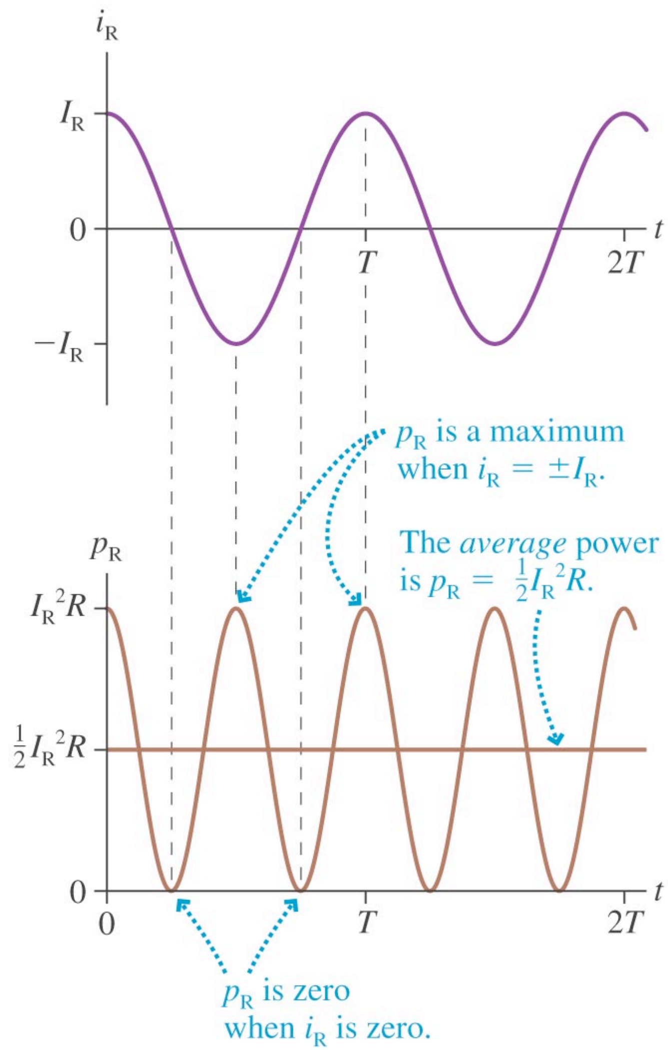
Instantaneous Power

$$p(t) = RI^2 = RI_R^2 \cos^2[\omega t]$$

Average over time is 1/2

Average Power

$$P = \frac{1}{2} R I_R^2$$



Root Mean Square (RMS) Voltage and Current

Current $\mathrm{I}(t) = I_R \cos[\omega t]$

Peak current

Average Power $P = \frac{1}{2} R I_R^2$

What would be the equivalent DC current as far as average power is concerned?

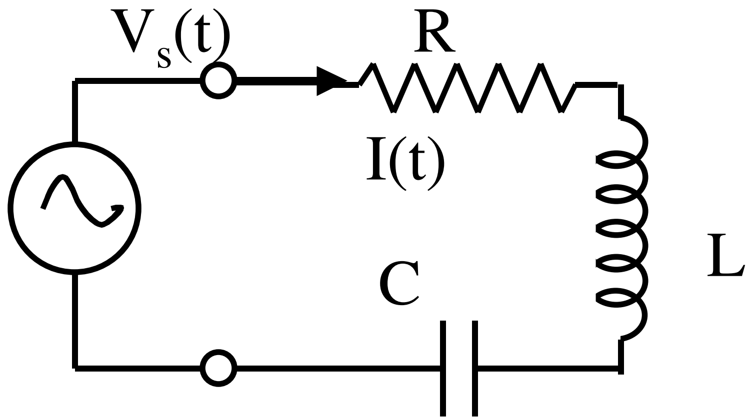
$$I_{RMS} = \frac{I_R}{\sqrt{2}}$$

Average Power $P = R I_{RMS}^2 \longleftarrow$ No pesky 2

What is the peak voltage for 110 V-AC- RMS?

A: 156 V

RLC Circuit



KVL

$$\hat{V}_0 = \hat{V}_R + \hat{V}_L + \hat{V}_C$$

$$\hat{V}_R = R\hat{I} \qquad \hat{V}_L = j\omega L\hat{I} \qquad \hat{V}_C = 1/(j\omega C)\hat{I}$$

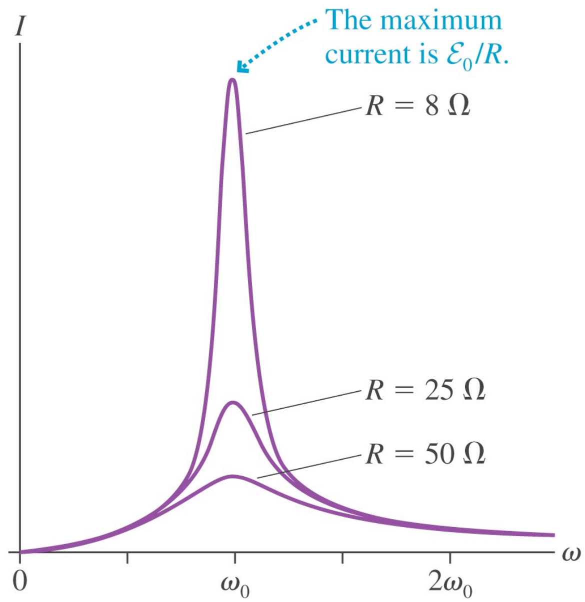
Current phasor $$\hat{I} = \frac{\hat{V}_0}{R + j[\omega L - 1/(\omega C)]} = \frac{\hat{V}_0}{Z}$$

Complex Impedance $$Z = R + j[\omega L - 1/(\omega C)]$$

Magnitude of Impedance $$|Z| = \sqrt{R^2 + [\omega L - 1/(\omega C)]^2}$$

Phase of Impedance $$\tan\phi = [\omega L - 1/(\omega C)]/R$$

Resonance: At what frequency is the amplitude of the current maximum?



Complex Amplitude

$$\hat{I} = \left|\hat{I}\right| e^{j\theta} = \frac{\hat{V}_0}{R + j[\omega L - 1/(\omega C)]} = \frac{\hat{V}_0}{Z}$$

Current Amplitude

$$\left|\hat{I}\right| = \frac{\left|\hat{V}_0\right|}{|Z|} = \frac{\left|\hat{V}_0\right|}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$$

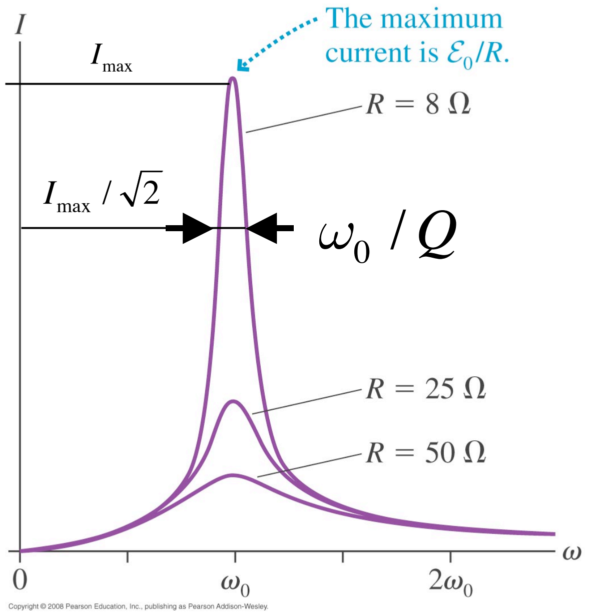
Current is largest when this term is zero

$$\omega = \omega_0 = 1/\sqrt{LC}$$

Resonant frequency

At resonance:

$$\left|\hat{I}\right| = \frac{\left|\hat{V}_0\right|}{R}$$



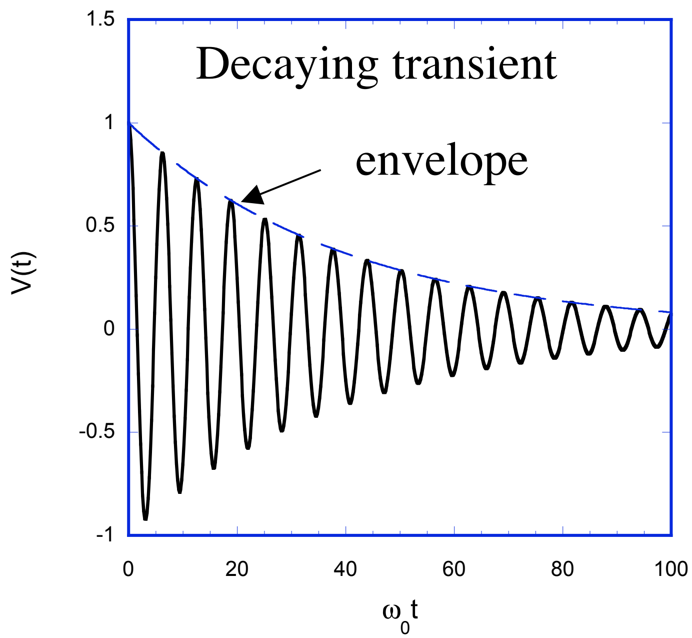
How narrow is the Resonance?

$$\left|\hat{I}\right| = \frac{\left|\hat{V}_0\right|}{|Z|} = \frac{\left|\hat{V}_0\right|}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$$

Width of resonance determined by when these two are equal

$$Q = \sqrt{\frac{L}{C}} / R$$

Quality Factor



Quality factor determines rate of decay of transient

$$\text{envelope} \quad = e^{-\omega_o t/(2Q)}$$

$$\frac{\textit{Power dissipated in } R}{\textit{Energy stored in } L \,\&\, C} = \frac{\omega_0}{Q}$$

Power Delivered to a Capacitor

Voltage $V(t) = V_C \cos[\omega t]$

Current $I(t) = C dV(t)/dt$

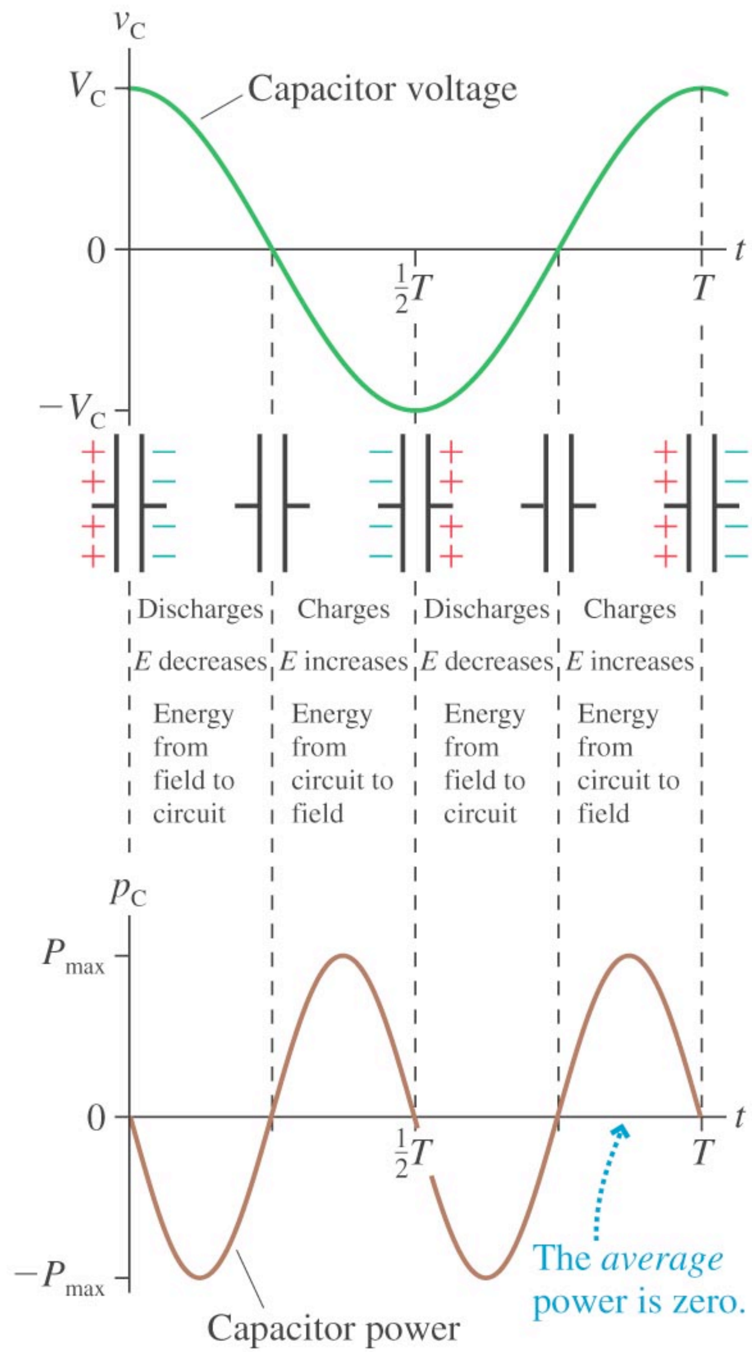
$$I(t) = -\omega C V_C \sin[\omega t]$$

Instantaneous Power

$$p(t) = IV = -\omega C V_C^2 \cos[\omega t] \sin[\omega t]$$

$$p(t) = -\frac{\omega C V_C^2}{2} \sin[2\omega t]$$

Average Power $P = 0$



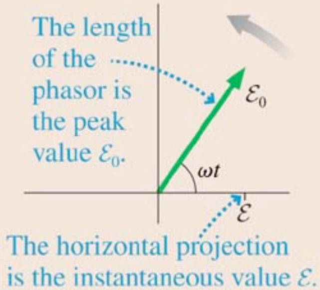
Important Concepts

AC circuits are driven by an emf

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

that oscillates with angular frequency $\omega = 2\pi f$.

Phasors can be used to represent the oscillating emf, current, and voltage.



Basic circuit elements

Element	i and v	Resistance/ reactance	I and V	Power
Resistor	In phase	R is fixed	$V = IR$	$V_{rms}I_{rms}$
Capacitor	i leads v by 90°	$X_C = 1/\omega C$	$V = IX_C$	0
Inductor	i lags v by 90°	$X_L = \omega L$	$V = IX_L$	0

For many purposes, especially calculating power, the **root-mean-square** (rms) quantities

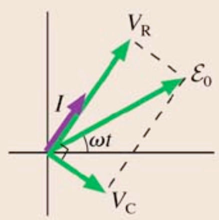
$$V_{rms} = V/\sqrt{2} \qquad I_{rms} = I/\sqrt{2} \qquad \mathcal{E}_{rms} = \mathcal{E}_0/\sqrt{2}$$

are equivalent to the corresponding DC quantities.

Key Skills

Phasor diagrams

- Start with a phasor (v or i) common to two or more circuit elements.
- The sum of instantaneous quantities is vector addition.
- Use the Pythagorean theorem to relate peak quantities.



For an RC circuit, shown here,

$$v_R + v_C = \mathcal{E}$$

$$V_R^2 + V_C^2 = \mathcal{E}_0^2$$

Kirchhoff's laws

Loop law The sum of the potential differences around a loop is zero.

Junction law The sum of currents entering a junction equals the sum leaving the junction.

Instantaneous and peak quantities

Instantaneous quantities v and i generally obey different relationships than peak quantities V and I.

Review of Waves

Properties of electromagnetic waves in vacuum:

Waves propagate through vacuum (no medium is required like sound waves)

All frequencies have the same propagation speed, c in vacuum.

Electric and magnetic fields are oriented transverse to the direction of propagation. (transverse waves)

Waves carry both energy and momentum.

Solution of the Wave equation

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

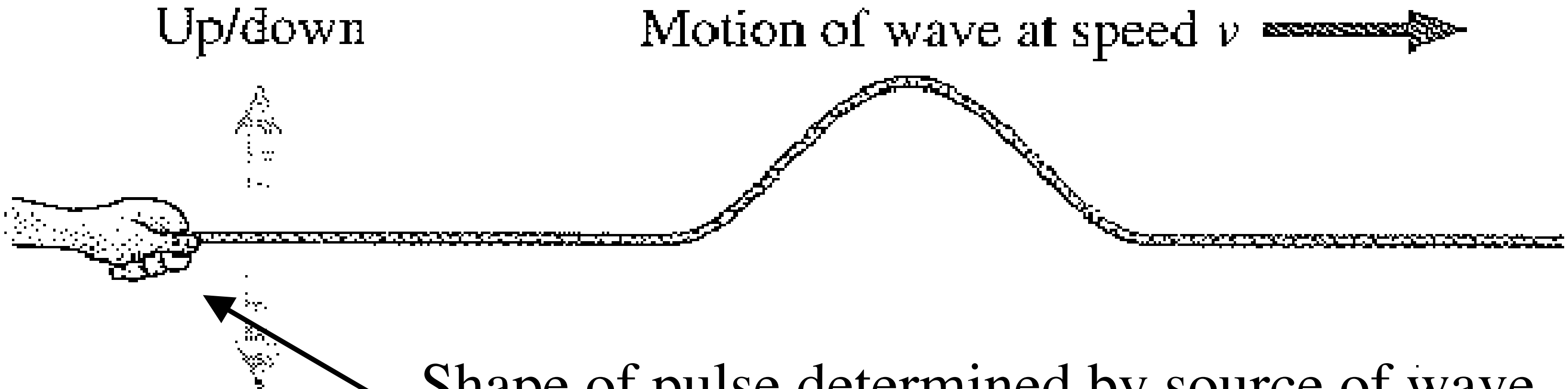
$$E_y(x,t) = f_+(x - v_{em} t) + f_-(x + v_{em} t)$$

Where $f_{+,-}$ are any two functions you like, and

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

v_{em} is a property of space. $v_{em} = 2.9979 \times 10^8 \ m/s$

$f_{+,-}$ Represent forward and backward propagating wave (pulses). They depend on how the waves were launched



Shape of pulse determined by source of wave.

$$E_y(x,t) = f_+(x - v_{em}t)$$

Speed of pulse determined by medium

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

What is the magnetic field of the wave?

$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

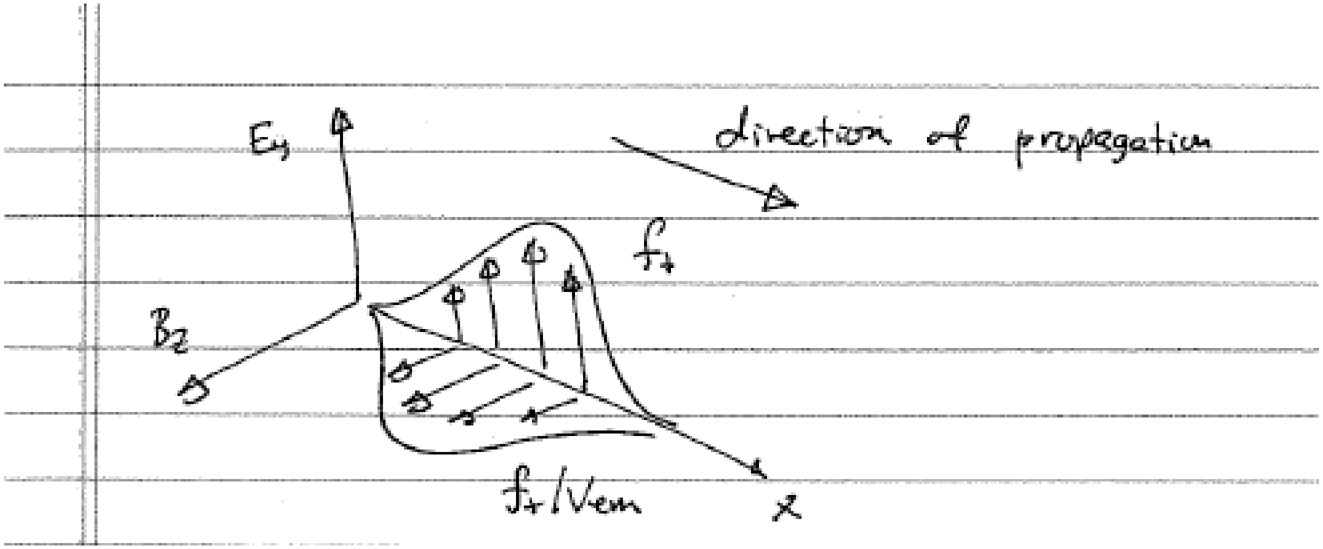
$$B_z(x,t) = \frac{1}{v_{em}}\left(f_+(x - v_{em}t) - f_-(x + v_{em}t)\right)$$

Notice minus sign

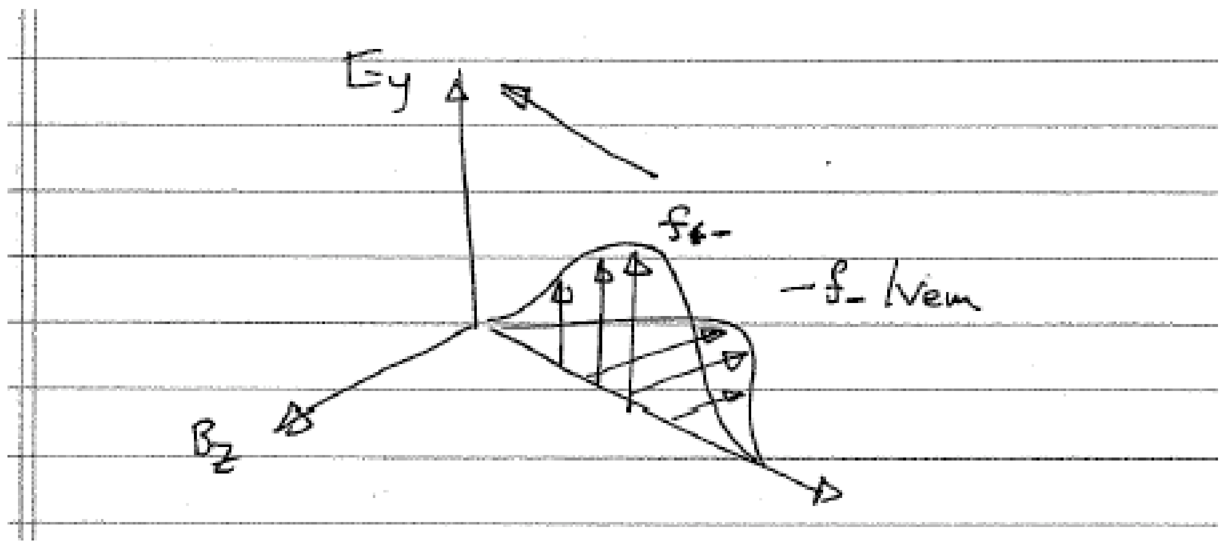
E and B fields in waves and Right Hand Rule:

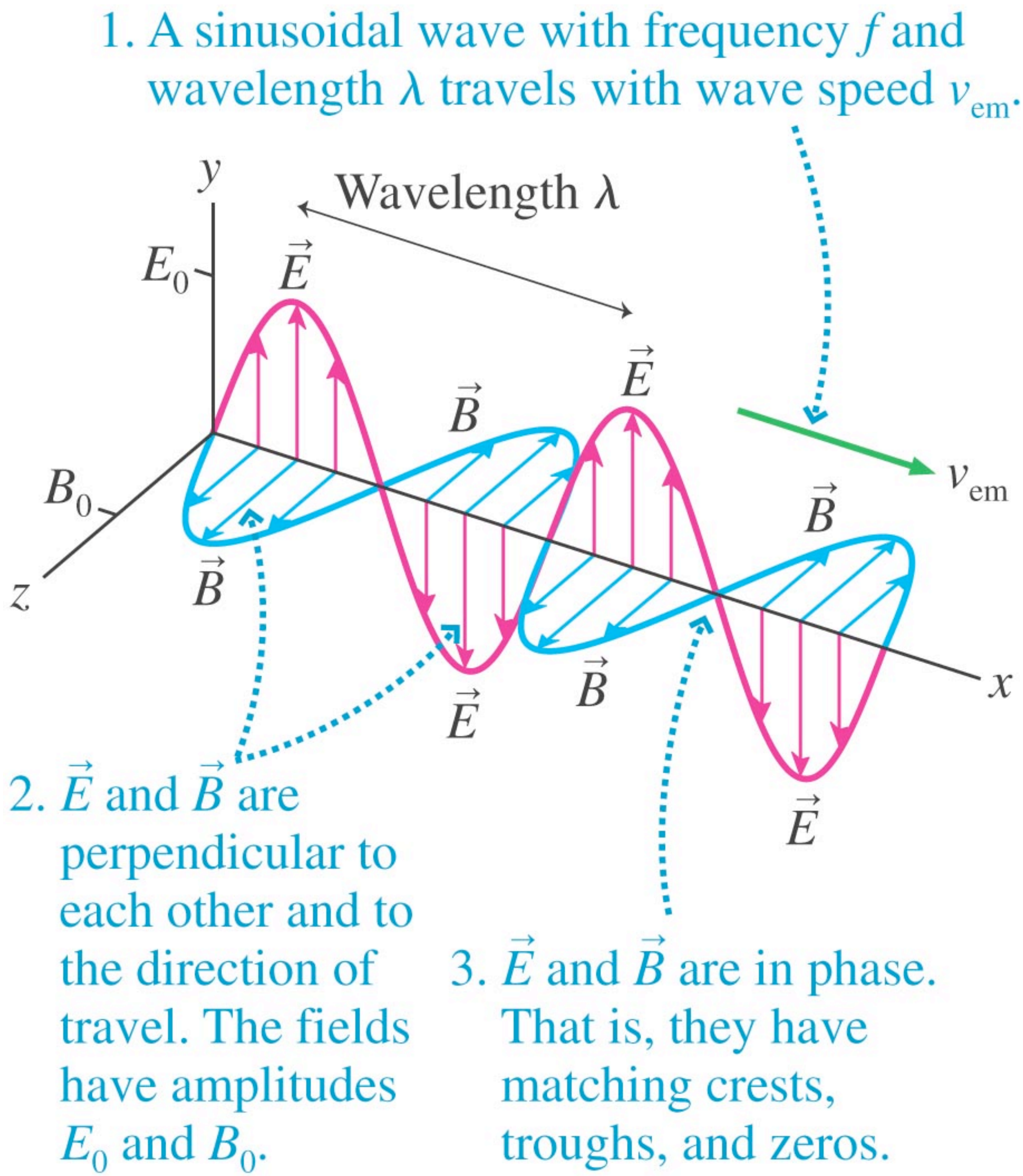
f+ solution

Wave propagates in **E** x **B** direction



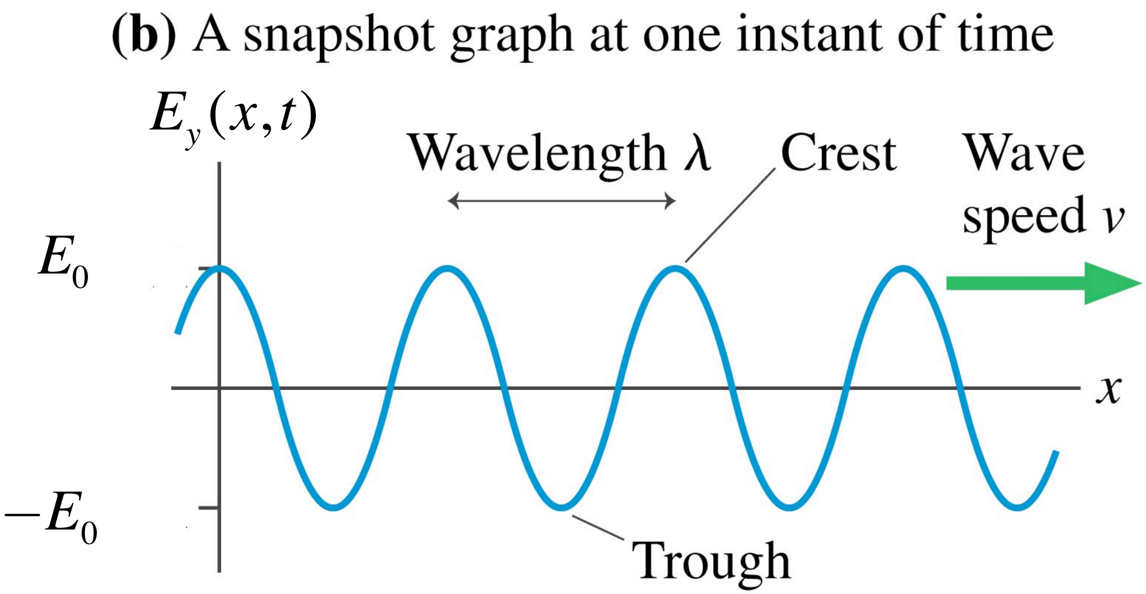
f- solution




1. A sinusoidal wave with frequency f and wavelength λ travels with wave speed v_{em}.
y
Wavelength λ
E_0
$\vec{E}$
$\vec{B}$
$\vec{E}$
v_{em}
B_0
$\vec{B}$
z
$\vec{B}$
$\vec{E}$
$\vec{B}$
x
$\vec{E}$
2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.
3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$



Wavenumber and wavelength

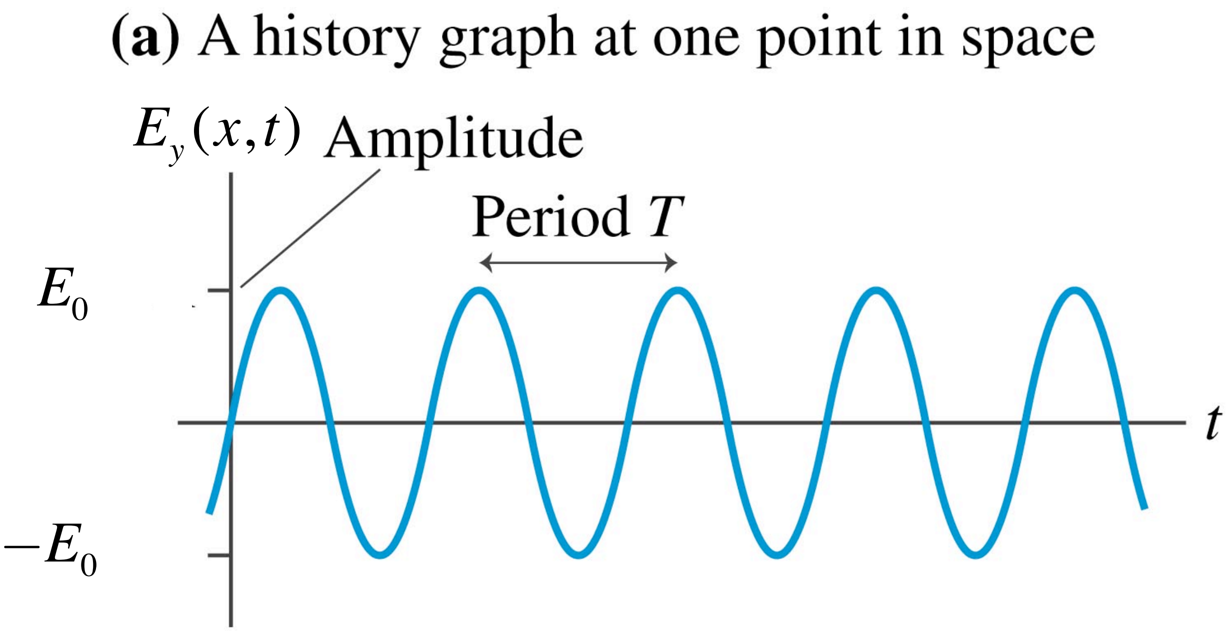
$$k = 2\pi / \lambda$$

$$\lambda = 2\pi / k$$

These two contain the same information

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$



$$2\pi = k v_{em} T$$

Introduce

$$\omega = 2\pi / T$$

$$f = 1 / T$$

Different ways of saying the same thing:

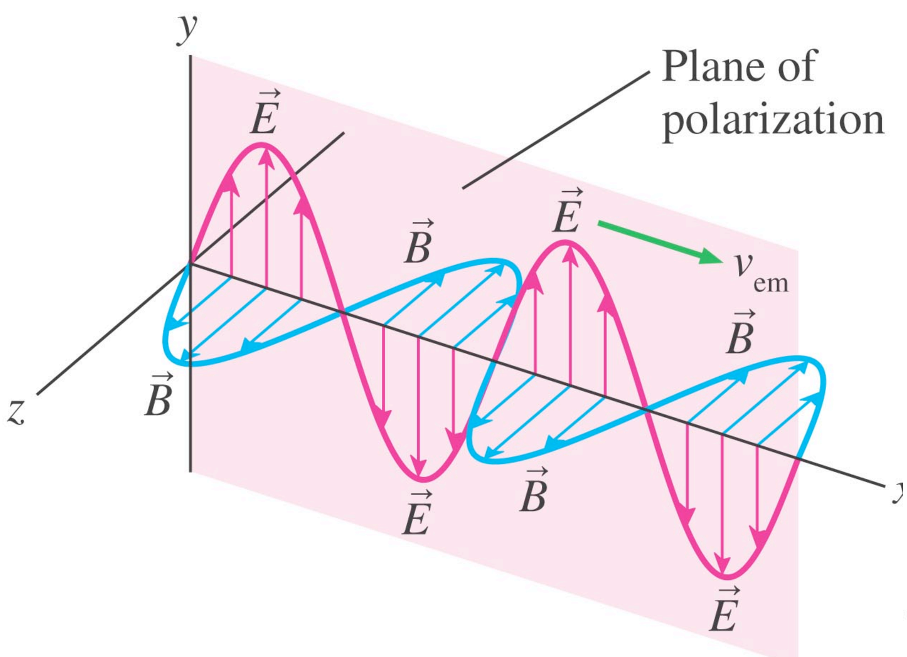
$$\omega / k = v_{em} \qquad f\lambda = v_{em}$$

Polarizations

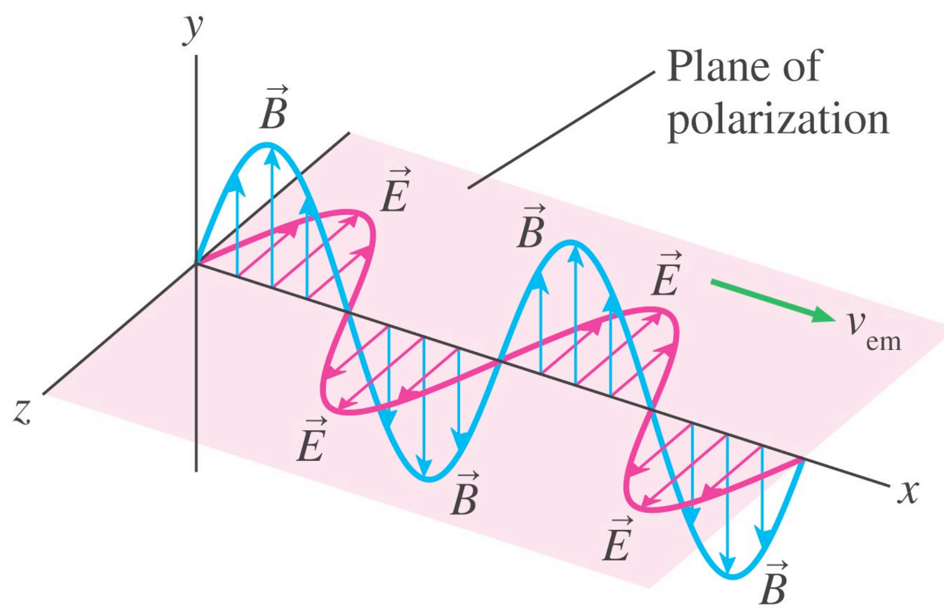
We picked this combination of fields: E_y - B_z

Could have picked this combination of fields: E_z - B_y

(a) Vertical polarization



(b) Horizontal polarization



These are called plane polarized. Fields lie in plane

Energy Density and Intensity of EM Waves

Energy density associated with electric and magnetic fields

$$u_E = \frac{\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2}{2} \qquad\qquad u_B = \frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}$$

For a wave: $$\left|\vec{\mathbf{B}}\right| = \frac{1}{v_{em}}\left|\vec{\mathbf{E}}\right| = \sqrt{\varepsilon_0\mu_0}\,\left|\vec{\mathbf{E}}\right|$$

Thus:

$$u_E = u_B$$

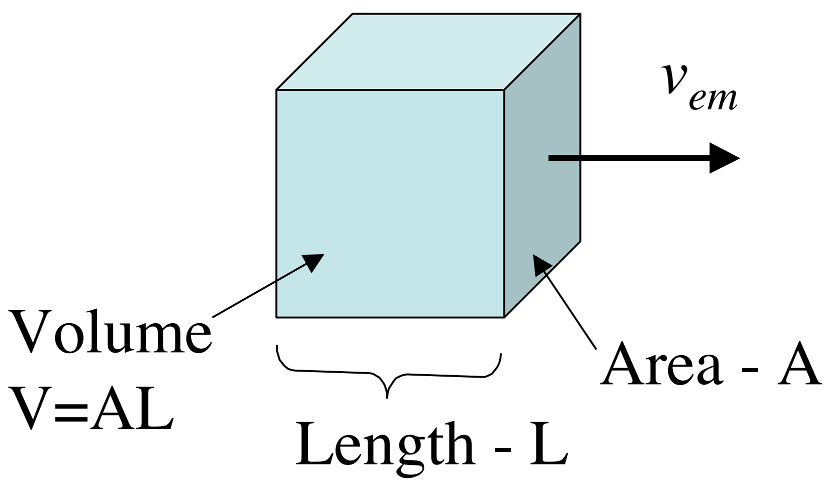
Units: J/m^3

Energy density in electric and magnetic fields are equal for a wave in vacuum.

Wave Intensity - Power/area

Energy density inside cube

$$u_E = \frac{\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2}{2} = u_B = \frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}$$



In time $\Delta t = L/v_{em}$ an amount of energy

$$U = V(u_E + u_B) = AL\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2$$

comes through the area A.

I=Power/Area

$$I = \frac{U}{\Delta t A} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \left|\vec{\mathbf{E}}\right|^2$$

Poynting Vector

The power per unit area flowing in a given direction

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

$$\left|\vec{\mathbf{S}}\right| = I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

What are the units of $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ Ans: Ohms

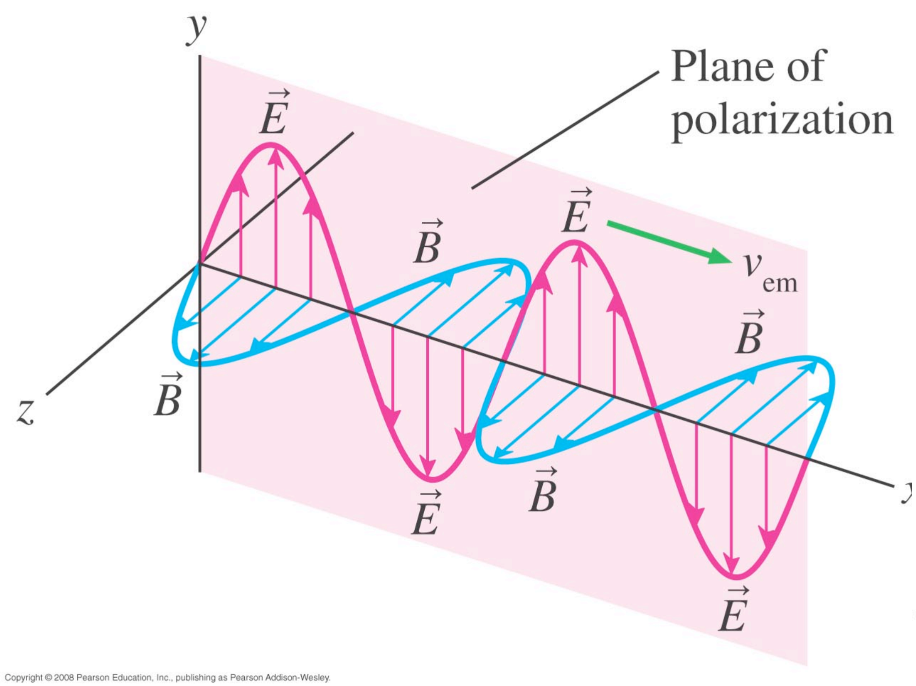
I - W/m^2, E - V/m $\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$

Polarizations

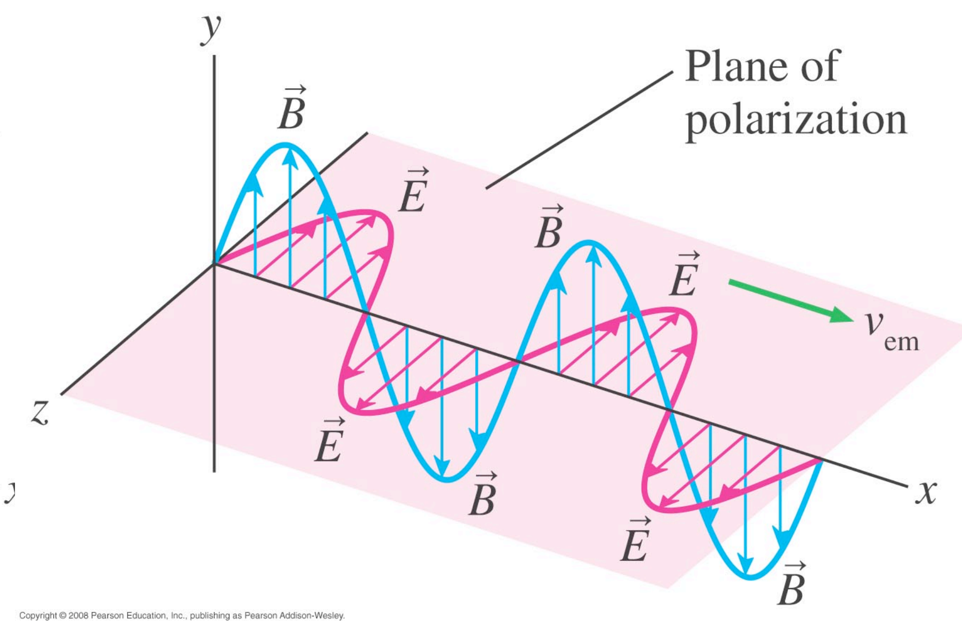
We picked this combination of fields: E_y - B_z

Could have picked this combination of fields: E_z - B_y

(a) Vertical polarization

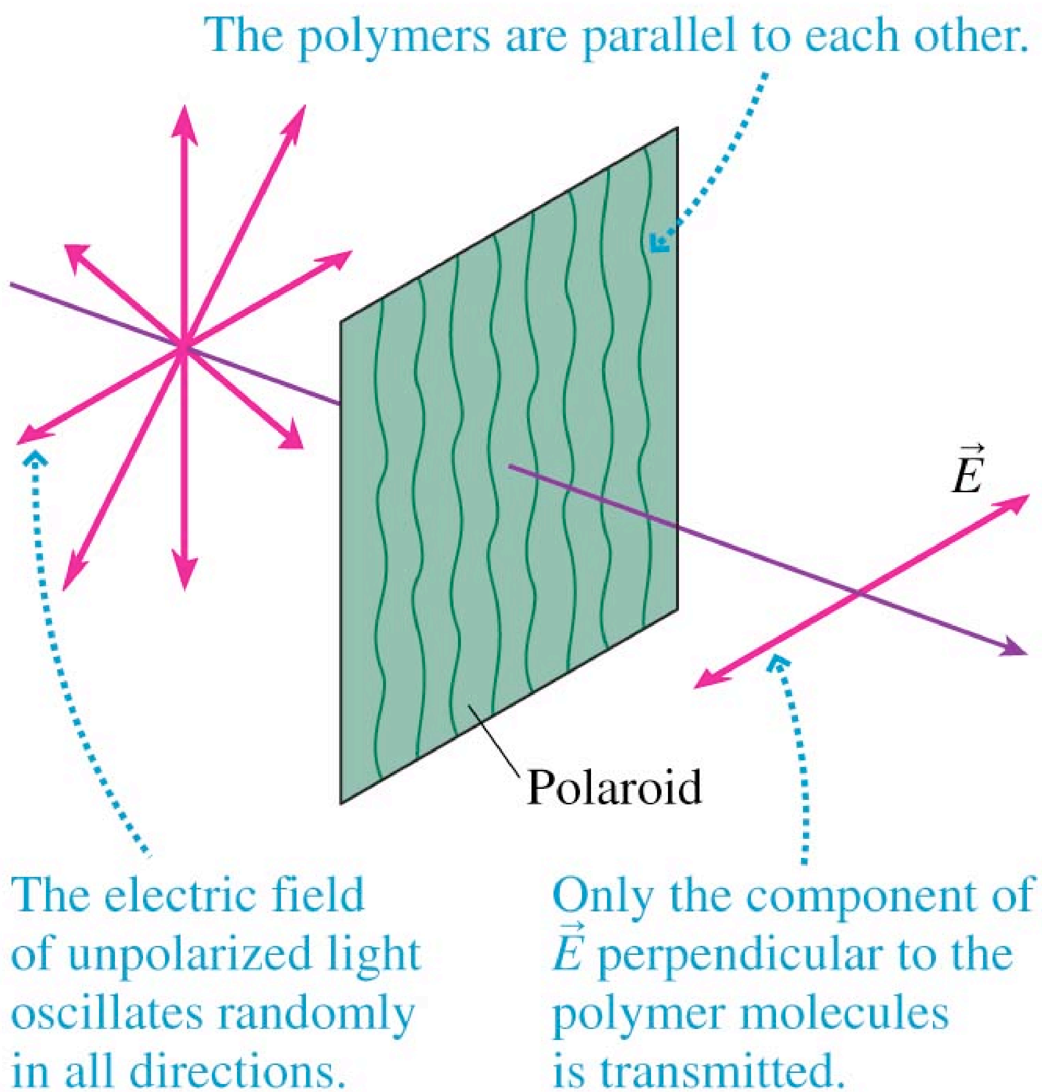


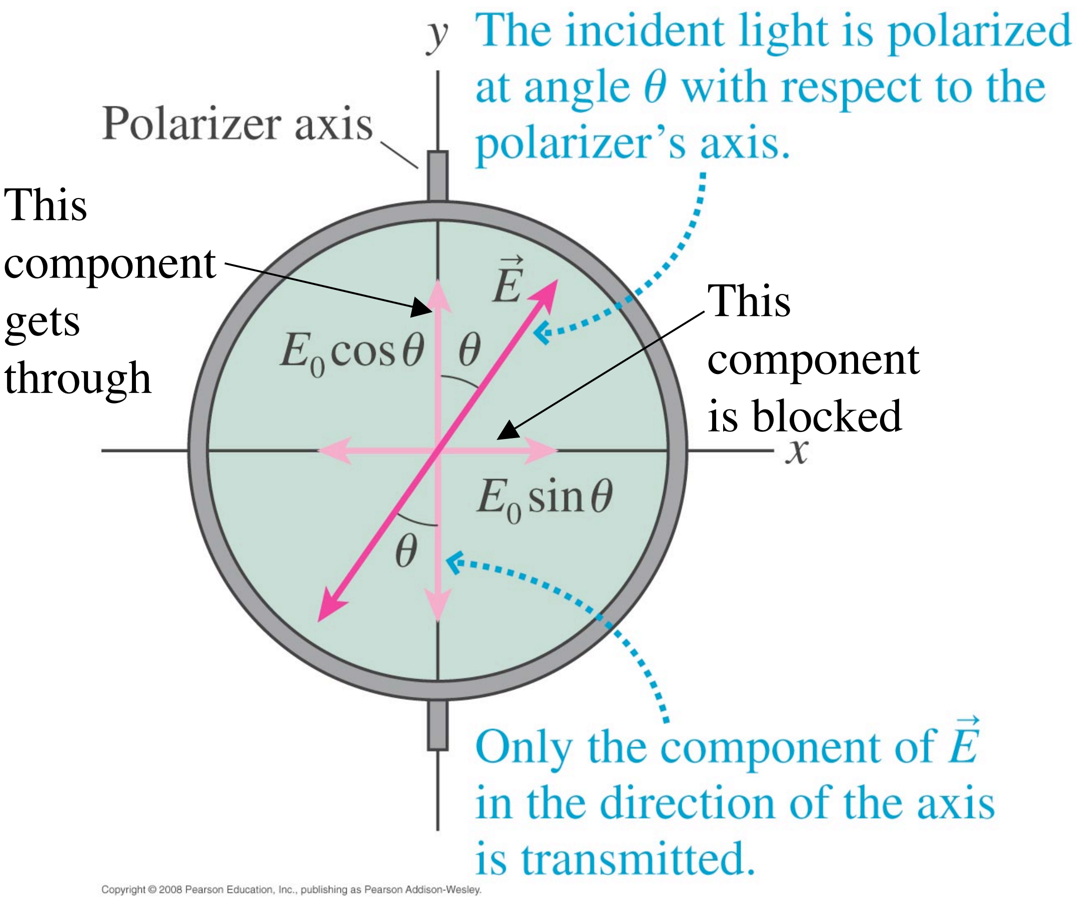
(b) Horizontal polarization



These are called plane polarized. Fields lie in plane

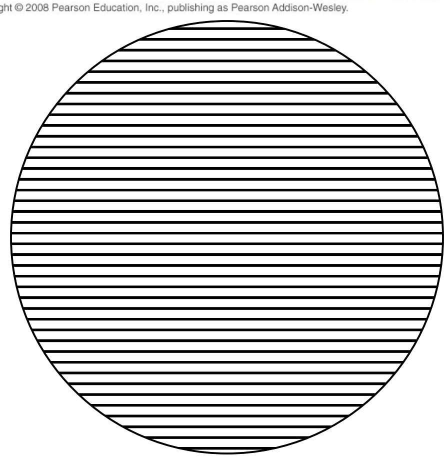
FIGURE 35.28 A polarizing filter.





The wave that passes through the polarizer has an electric field amplitude

$$\left|\vec{\mathbf{E}}\right|_{out} = \left|\cos\theta\right|\left|\vec{\mathbf{E}}\right|_{in}$$



Orientation of polymers

If input light is unpolarized

$$I_{out} = \left\langle \cos^2\theta \right\rangle I_{in} = \frac{1}{2} I_{in}$$

Malus's Law

We now want to expand the picture in the following way:

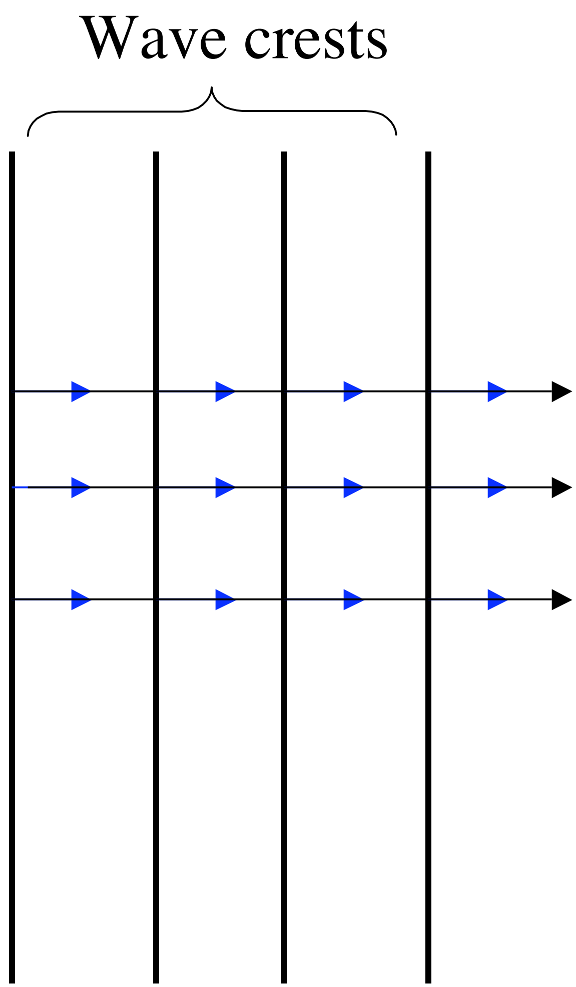
EM waves propagate in 3D not just 1D as we have considered.
- Diffraction - waves coming from a finite source spread out.

EM waves propagate through material and are modified.
- Dispersion - waves are slowed down by media, different frequency waves travel with different speeds

- Reflection - waves encounter boundaries between media. Some energy is reflected.
- Refraction - wave trajectories are bent when crossing from one medium to another.

EM waves can take multiple paths and arrive at the same point.
- Interference - contributions from different paths add or cancel.

When can one consider waves to be like particles following a trajectory?



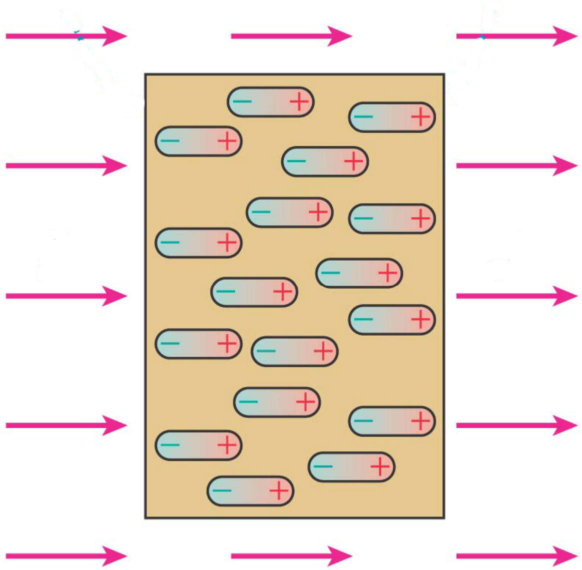
Motion of crests

Direction of power flow

- Wave model: study solution of Maxwell equations. Most complete classical description. Called physical optics.

- Ray model: approximate propagation of light as that of particles following specific paths or “rays”. Called geometric optics.

- Quantum optics: Light actually comes in chunks called photons



$$I \propto \frac{\partial E}{\partial t}$$

Real current given by dielectric constant

Electric flux

$$I_{through} = \varepsilon_0 (\kappa - 1) \frac{d\Phi_e}{dt}$$

κ Dielectric constant

In a dielectric material

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0 (\varepsilon_0 \kappa \frac{d\Phi_e}{dt})$$

κ Dielectric constant

<u>Bottom Line:</u>

Replace ε_0 by $\varepsilon_0 \kappa$

Consequences for EM Plane waves

$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

$$B_z(x,t) = \frac{1}{v_{em}}\left(f_+(x - v_{em}t) - f_-(x + v_{em}t)\right)$$

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0 \kappa} = c/\sqrt{\kappa}$$

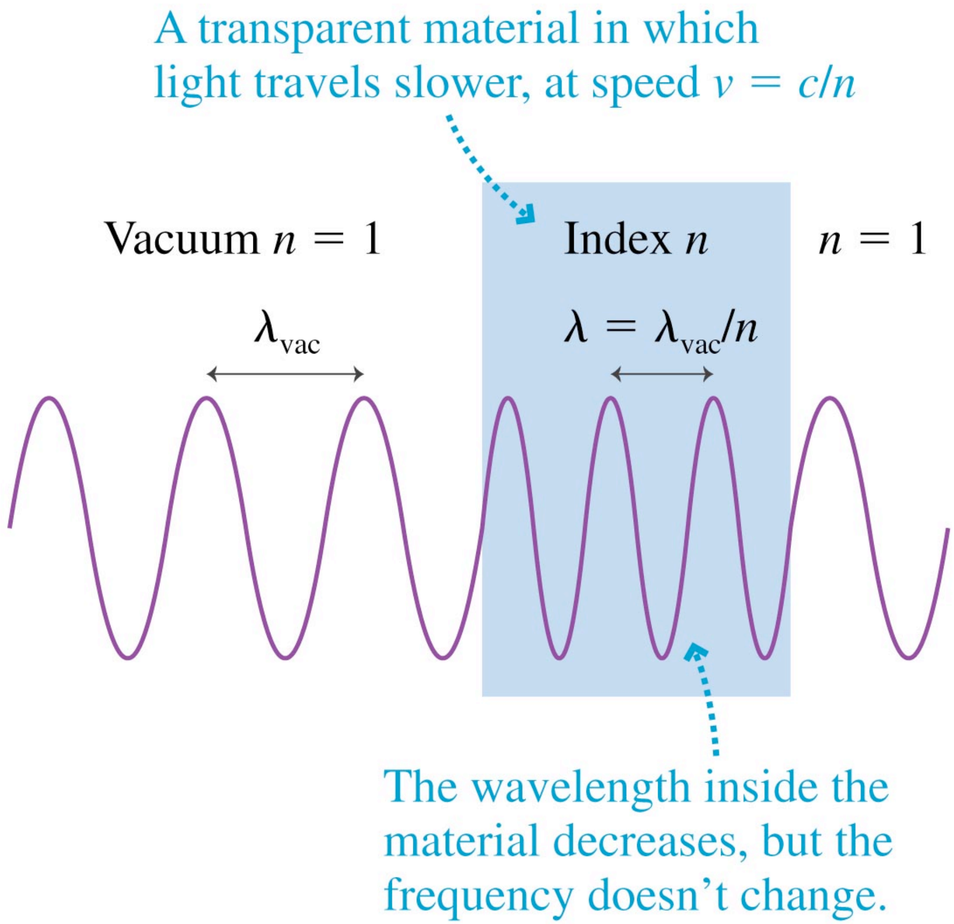
Propagation speed changes
Refraction

Ratio of E to B changes
Reflection

For sinusoidal waves the following is still true

$$f\lambda = v_{em}$$

$$\omega / k = v_{em}$$



For sinusoidal waves the following is still true

$$f\lambda = v_{em}$$

$$\omega / k = v_{em}$$

Frequency is the same in both media

Wavelength changes

Reflection from surface

incident

reflected

transmitted

Region 1

$$v_1 = 1/\sqrt{\mu_0 \varepsilon_0 \kappa_1}$$

Region 2

$$v_2 = 1/\sqrt{\mu_0 \varepsilon_0 \kappa_2}$$

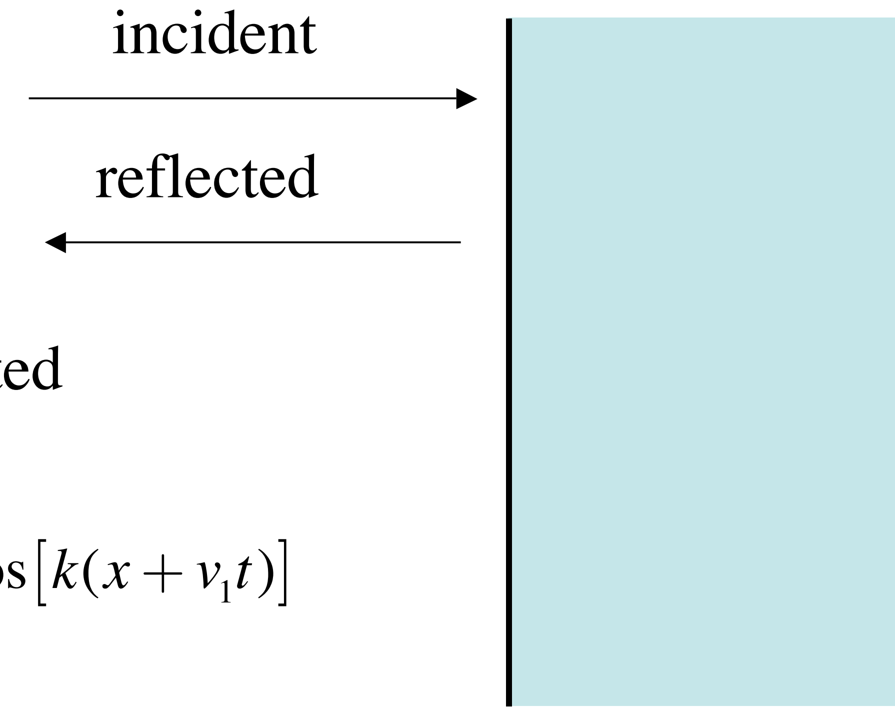
Reflection coefficient $\rho = \dfrac{v_2 - v_1}{v_2 + v_1}$

What if $\kappa_1 = \kappa_2$

"Index matched"

Interference in 1 Dimension

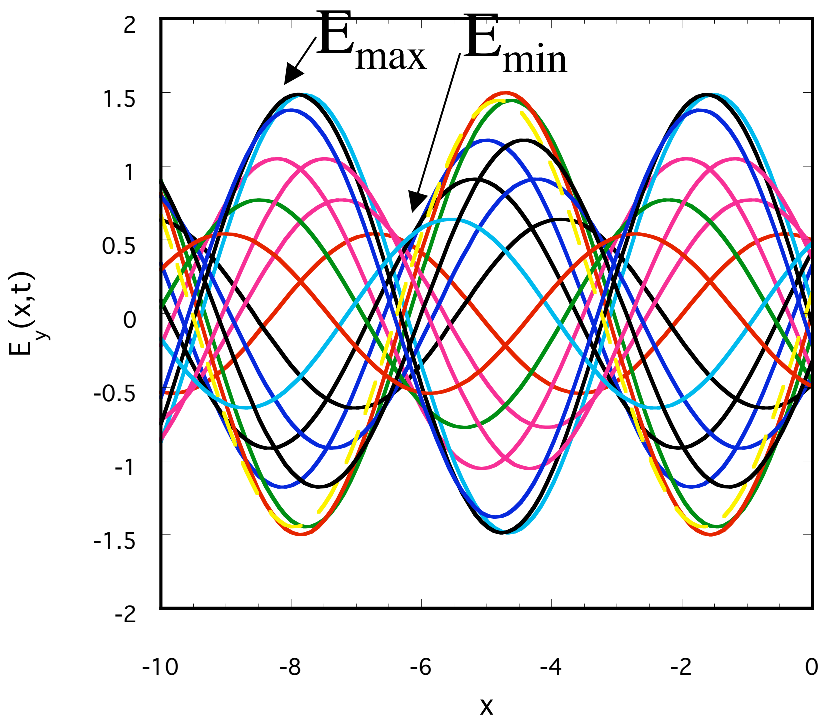
Incident and reflected fields add (superposition)



incident reflected

$$E_y(x,t) = E_0 \cos\left[k(x - v_1 t)\right] + \rho E_0 \cos\left[k(x + v_1 t)\right]$$

Incident and reflected waves will interfere, changing the peak electric field at different points



Case #4: partial reflection ρ= -0.5

E plotted versus x for several values of t

How far apart are the minima?
The Maxima?
What is peak E?

When reflection is not total there are still local maxima and minima.

$$\frac{E_{\max}}{E_{\min}} = \frac{1+|\rho|}{1-|\rho|} \quad = \text{VSWR}$$

Voltage Standing Wave Ratio
Pronounced "vizwarr"

Summary

1. Waves are modified by dielectric constant of medium, κ.
2. All our Maxwell equations are valid provided we replace

$$\varepsilon_0 \rightarrow \varepsilon_0 \kappa$$

3. Speed of waves is lowered. (Index of refraction - *n*)

$$n = \frac{\textit{speed of light in vacuum}}{\textit{speed of light in material}} = \frac{c}{v_{em}} = \sqrt{\kappa}$$

4. Frequency of wave does not change in going from one medium to another. Wavelength does. $\lambda = \lambda_{vac} / n$

5. Waves are reflected at the boundary between two media. (Reflection coefficient)

$$\rho = \frac{v_2 - v_1}{v_2 + v_1} = \frac{n_1 - n_2}{n_1 + n_2}$$

6. Reflected waves interfere with incident waves.

Distance between interference maxima/minima $\lambda / 2$

Ratio of maximum peak field to minimum peak field (Voltage standing wave ratio)

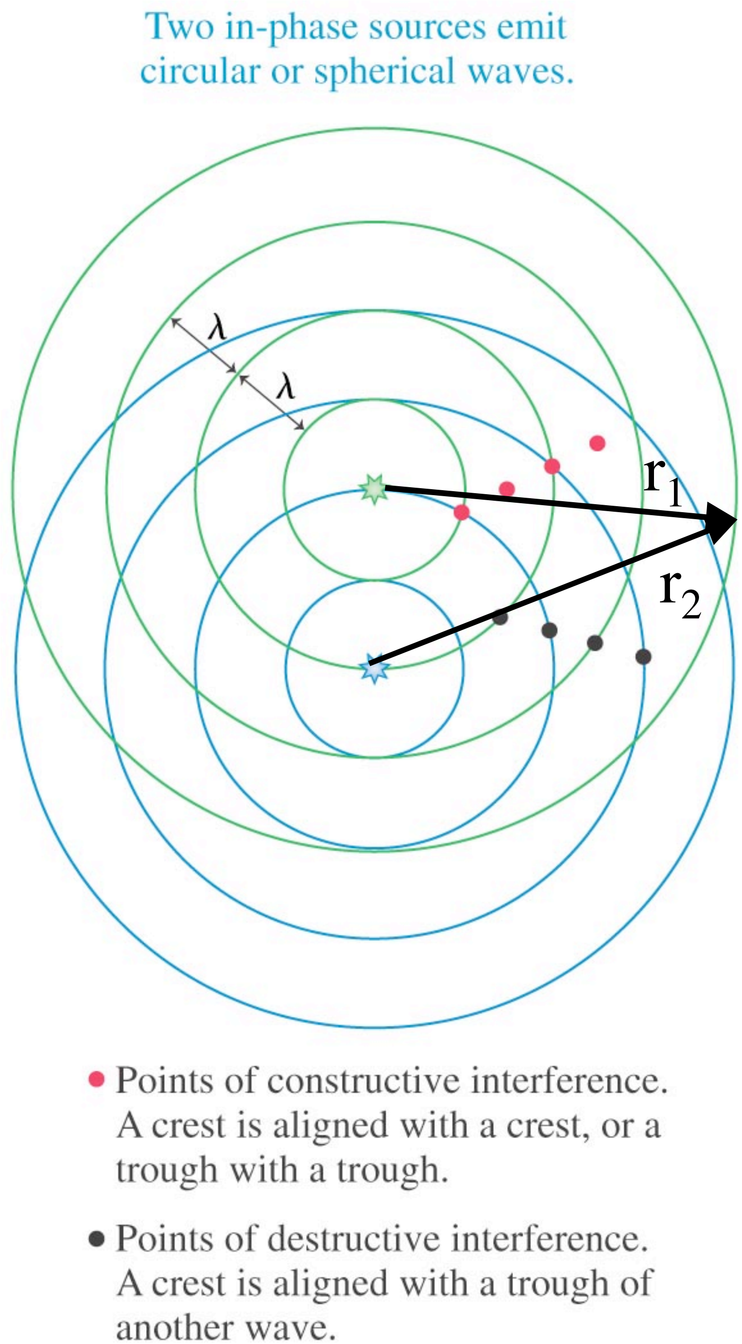
$$\frac{E_{\max}}{E_{\min}} = \frac{1+|\rho|}{1-|\rho|} \quad = \text{VSWR}$$

Chapters 21 & 22

Interference and Wave Optics

Waves that are coherent can add/cancel

Patterns of strong and weak intensity



Two sources that have exactly the same frequency. "Coherent"

$$E(r,t) = A(r_1)\cos(kr_1 - \omega t + \phi_1)$$

$$+ A(r_2)\cos(kr_2 - \omega t + \phi_2)$$

Sources will interfere constructively when

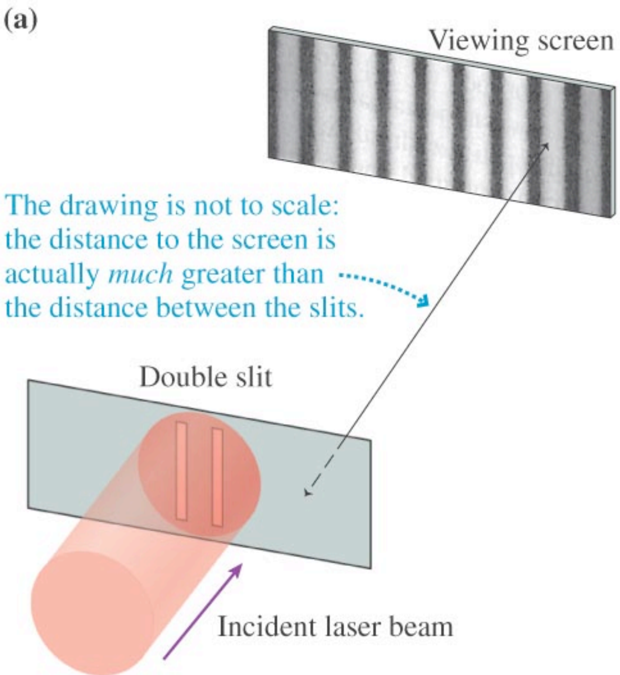
$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = 2\pi m$$
$$m = 0, 1, 2, \ldots$$

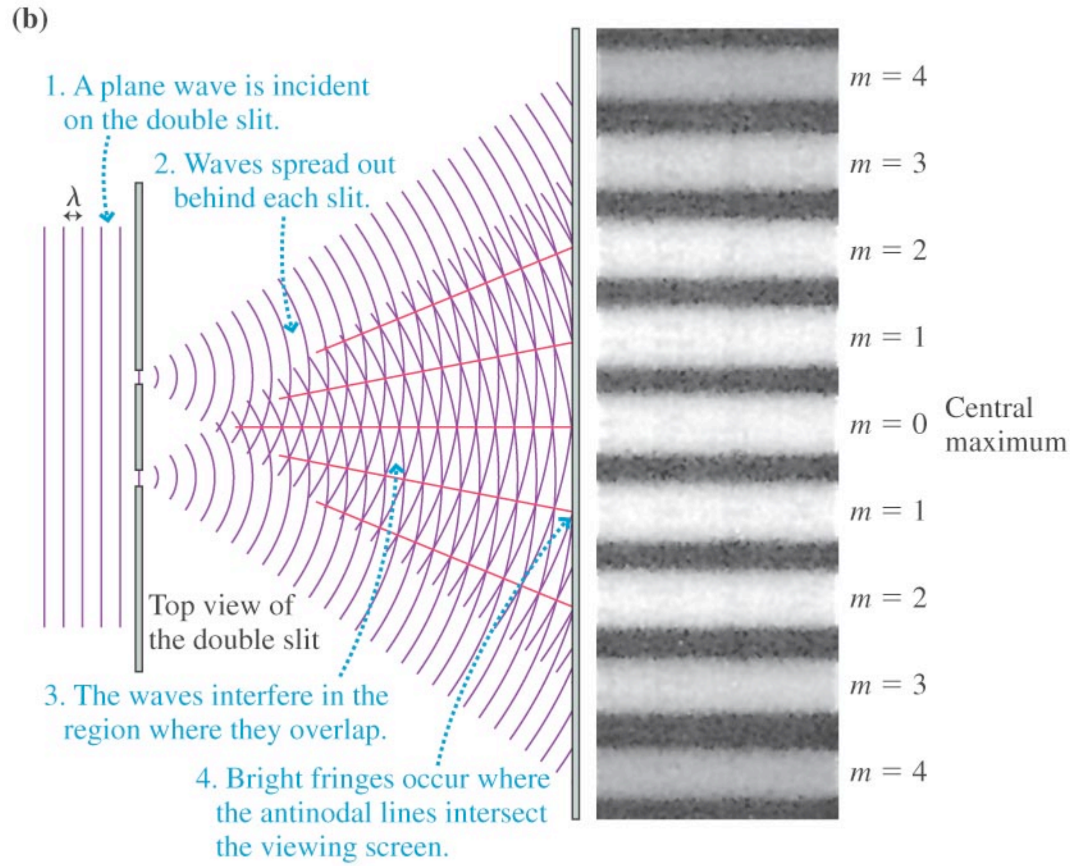
Sources will interfere destructively when

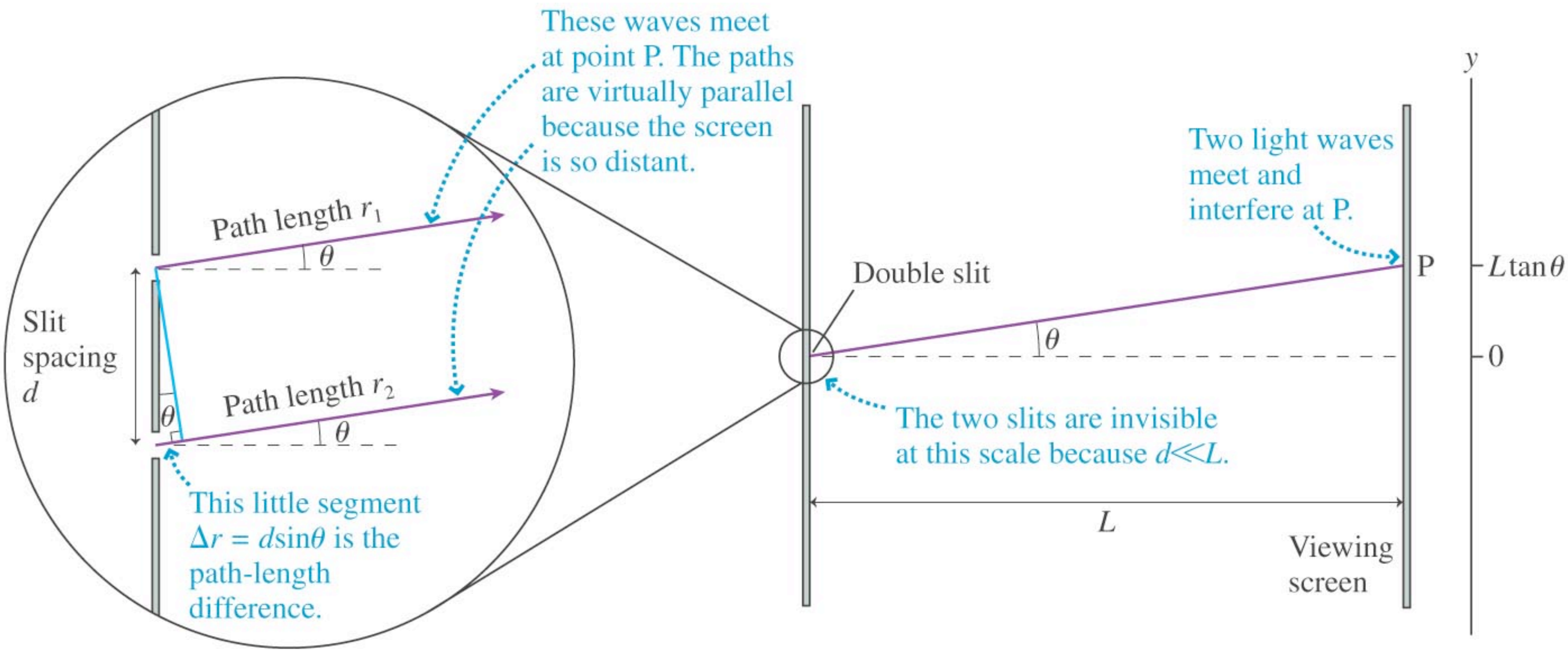
$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = 2\pi\left(m + \frac{1}{2}\right)$$

Interference of light

Coherence because sources are at exactly the same frequency







Sources will interfere <u>constructively</u> when

$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = k\Delta r = 2\pi m$$

Phases same because source comes
from a single incident plane wave

$$k\Delta r = kd\sin\theta = 2\pi m$$

$$\sin\theta_m \approx \theta_m = m\lambda / d$$

$$m = 0, 1, 2, \ldots$$

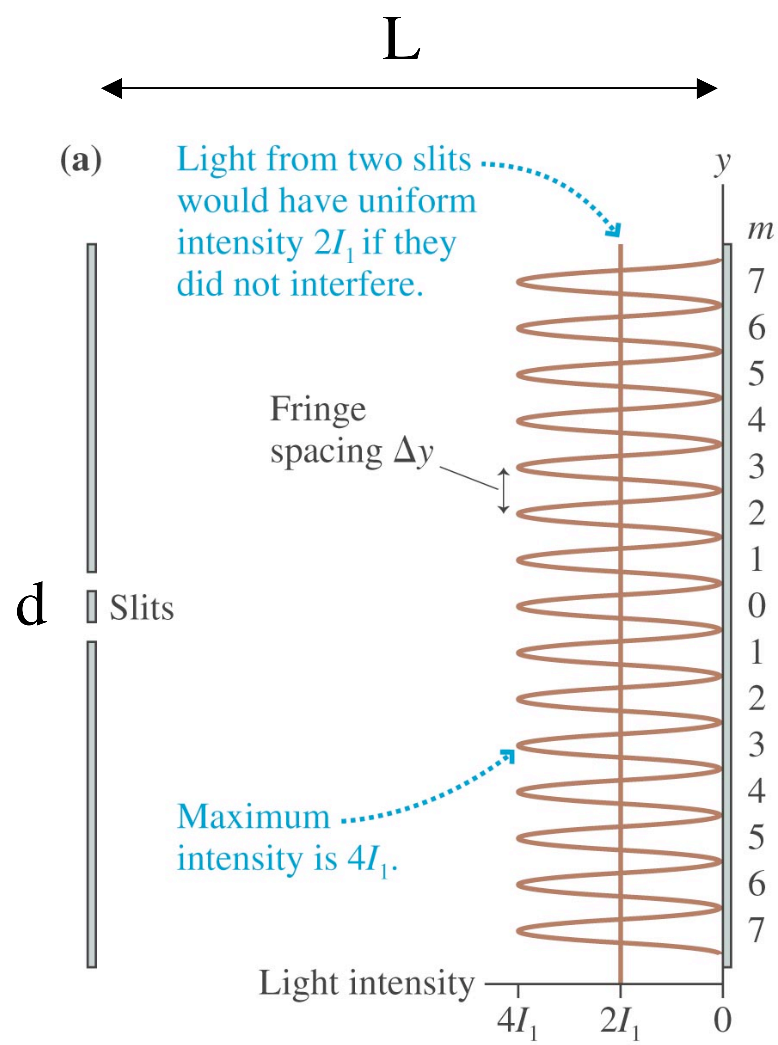
Dark fringes $\sin\theta_m \approx \theta_m = \left(m + \frac{1}{2}\right)\lambda / d$

Intensity on a distant screen

$$I = \sqrt{\frac{\varepsilon_0}{\mu_0}}|\mathbf{E}|^2$$

$$I_{ave} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|2A\cos\left(\frac{k\Delta r}{2}\right)\right|^2$$

$$k\Delta r = kd\sin\theta \simeq kd\,\theta = \frac{2\pi}{\lambda}d\,\frac{y}{L}$$



Fringe spacing

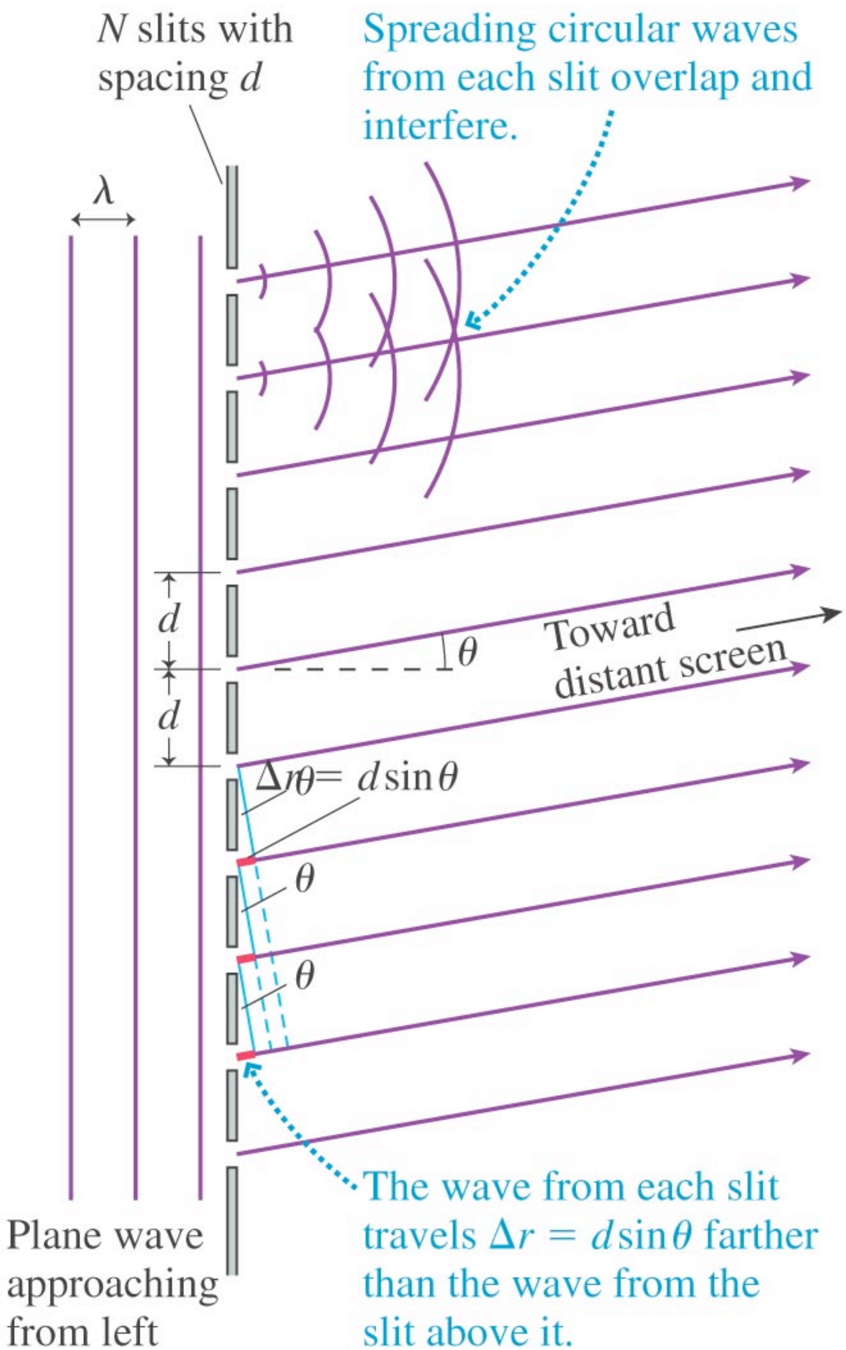
$$\Delta y = \frac{L\lambda}{d}$$

Intensity from a single source

$$I_1 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|A|^2$$

Maximum Intensity at fringe

$$I_{fringe} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|2A|^2 = 2I_1$$



Diffraction Grating
N slits, sharpens bright fringes

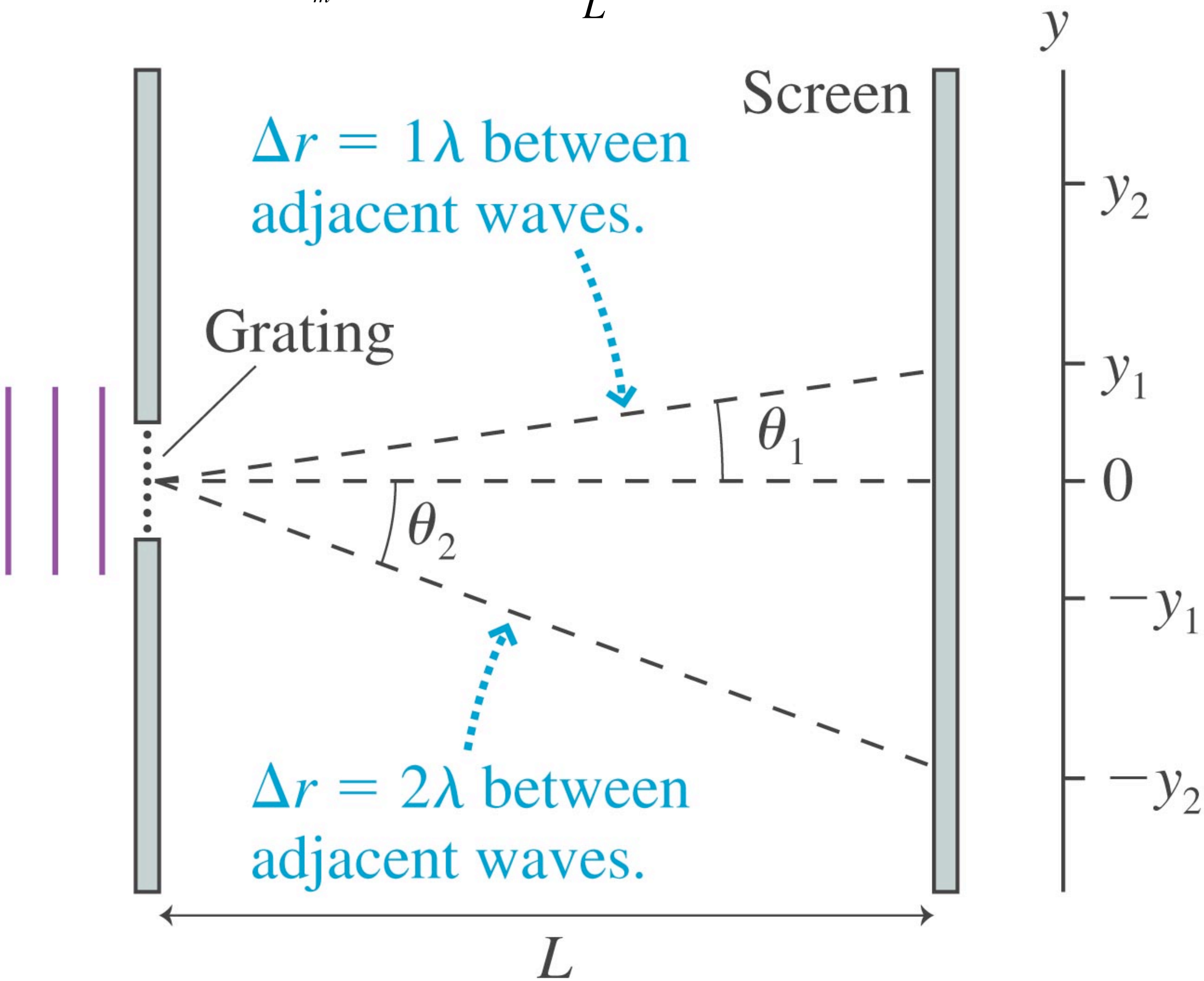
Bright fringes at same
angle as for double slit

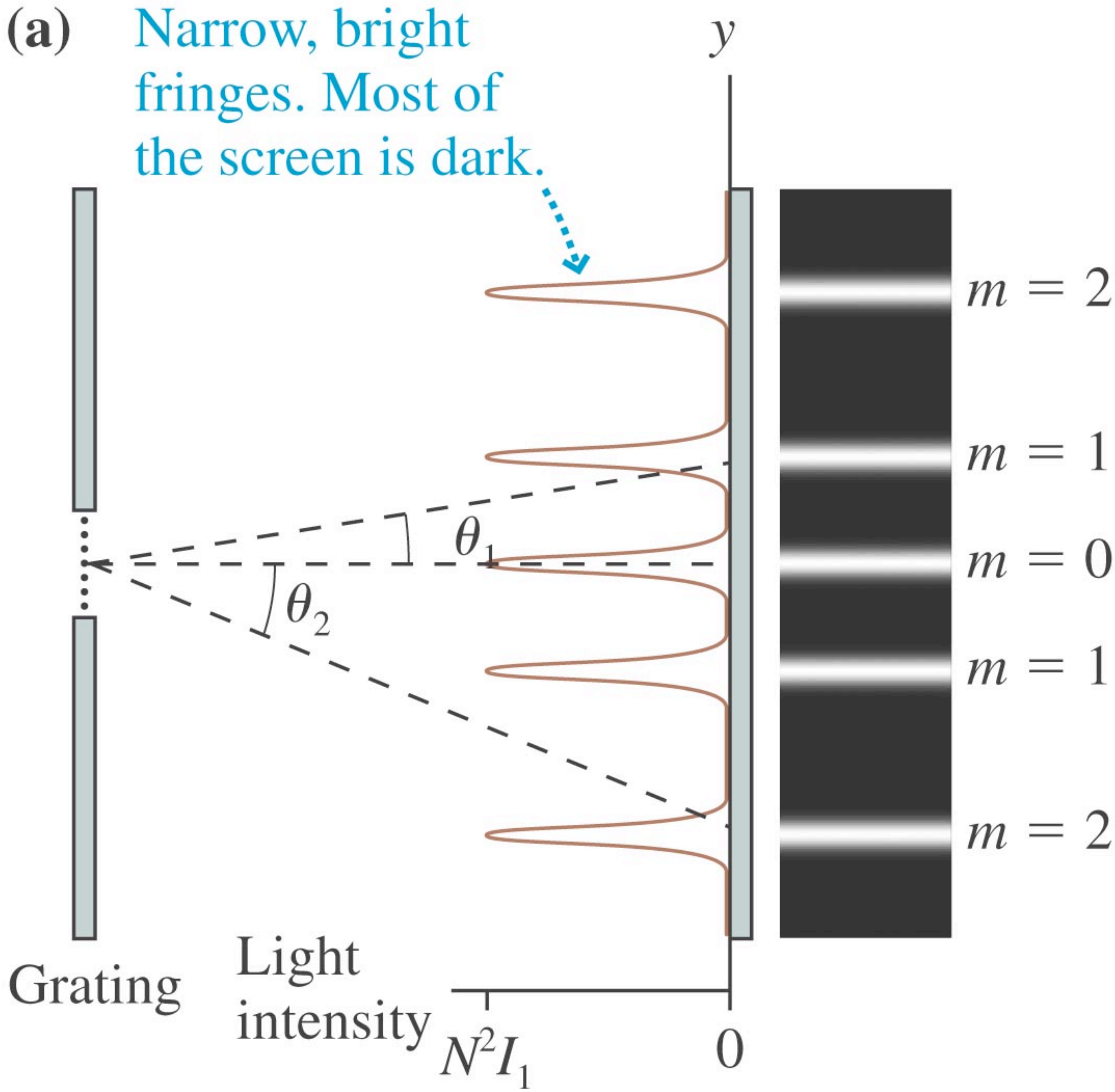
$$\sin\theta_m = m\lambda / d$$

$$m = 0, 1, 2, \ldots$$

Location of Fringes on distant screen

$$\sin\theta_m = m\lambda / d \qquad \frac{y_m}{L} = \tan\theta_m$$





(a)
Narrow, bright fringes. Most of the screen is dark.
y
$m = 2$
$m = 1$
θ_1
$m = 0$
θ_2
$m = 1$
$m = 2$
Grating
Light intensity
$N^2 I_1$
0

Intensity on a distant screen $$I = \sqrt{\frac{\varepsilon_0}{\mu_0}} |\mathbf{E}|^2$$

Average over time $$I_{ave} = \frac{1}{2} I$$

Intensity from a single slit

amplitude from a single slit

$$I_1 = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} |A|^2$$

At the bright fringe N slits interfere constructively

$$I_{fringe} = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}} |NA|^2 = N^2 I_1$$

Spatial average of intensity must correspond to sum of N slits

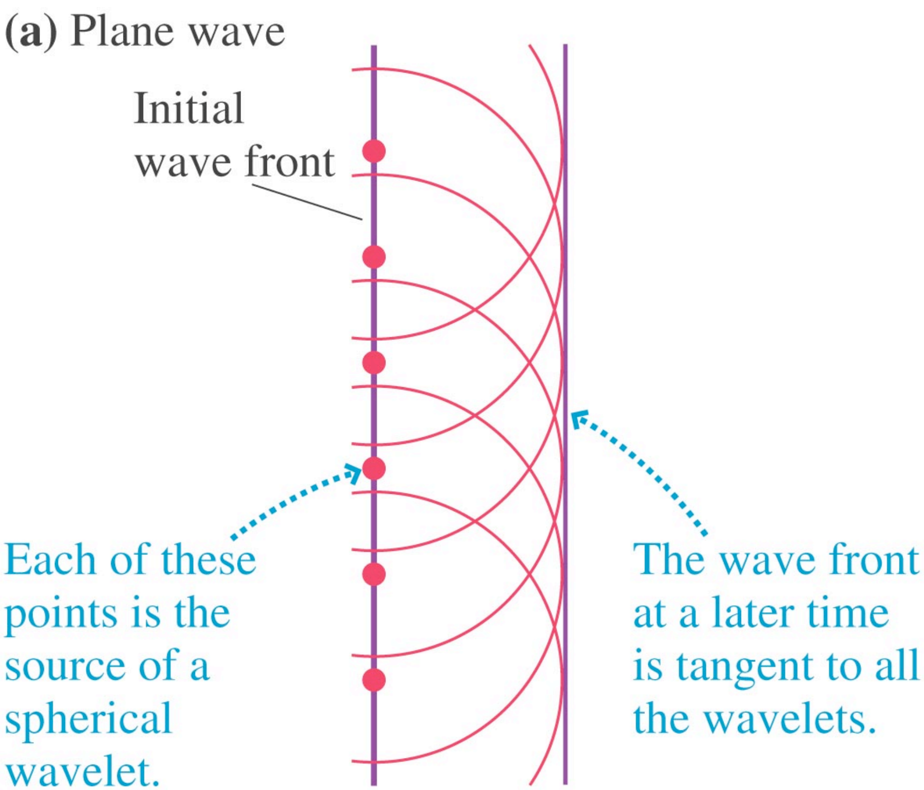
$$I_{SA} = N I_1$$

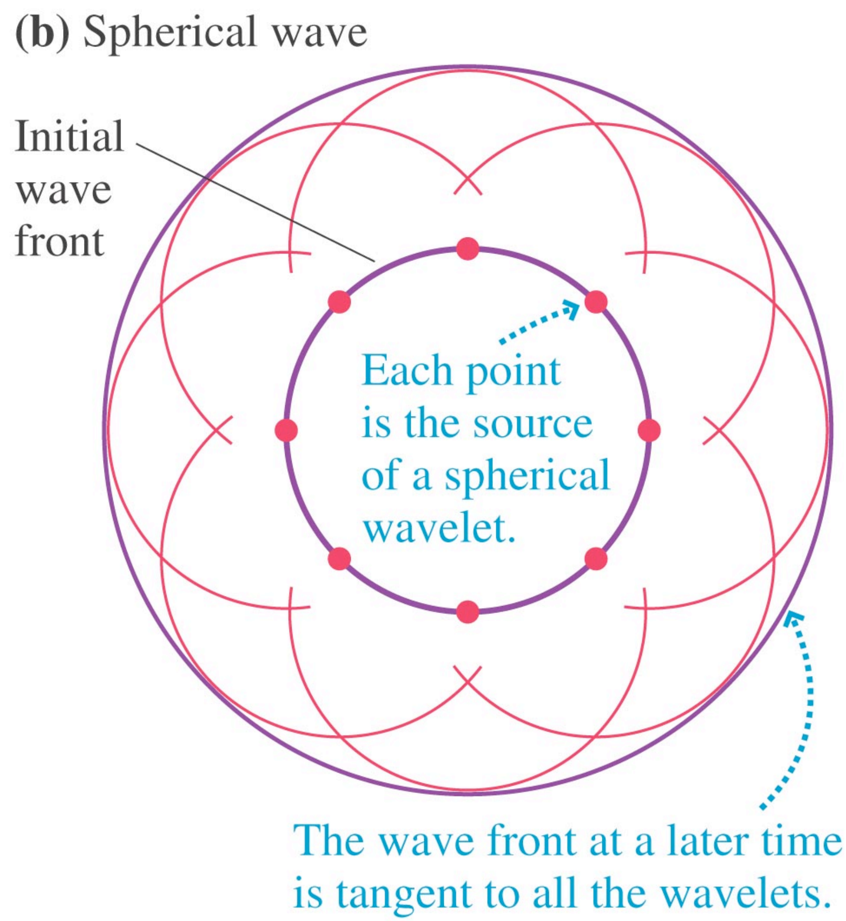
Huygen’s Principle

1. Each point on a wave front is the source of a spherical wavelet that spreads out at the wave speed.

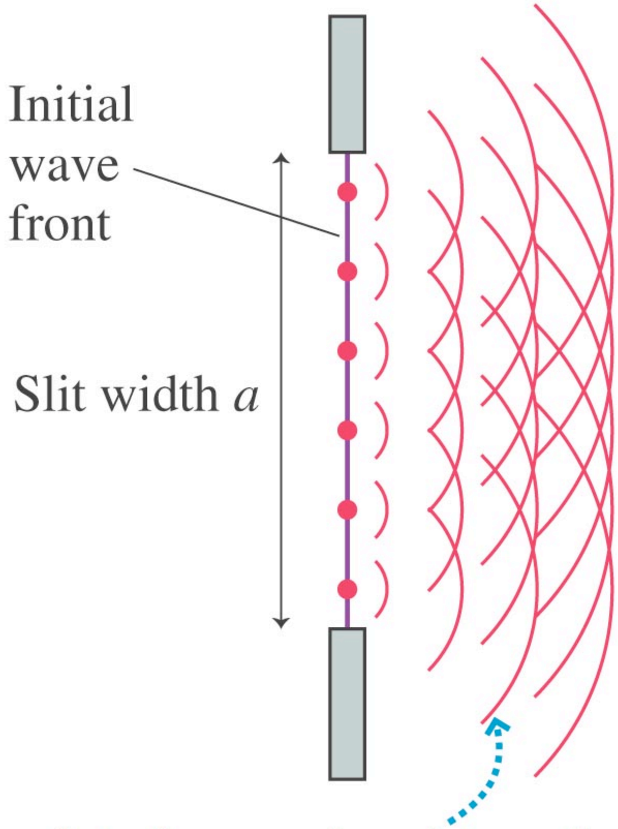
2. At a later time, the shape of the wave front is the line tangent to all the wavelets.

Huygens Principle:

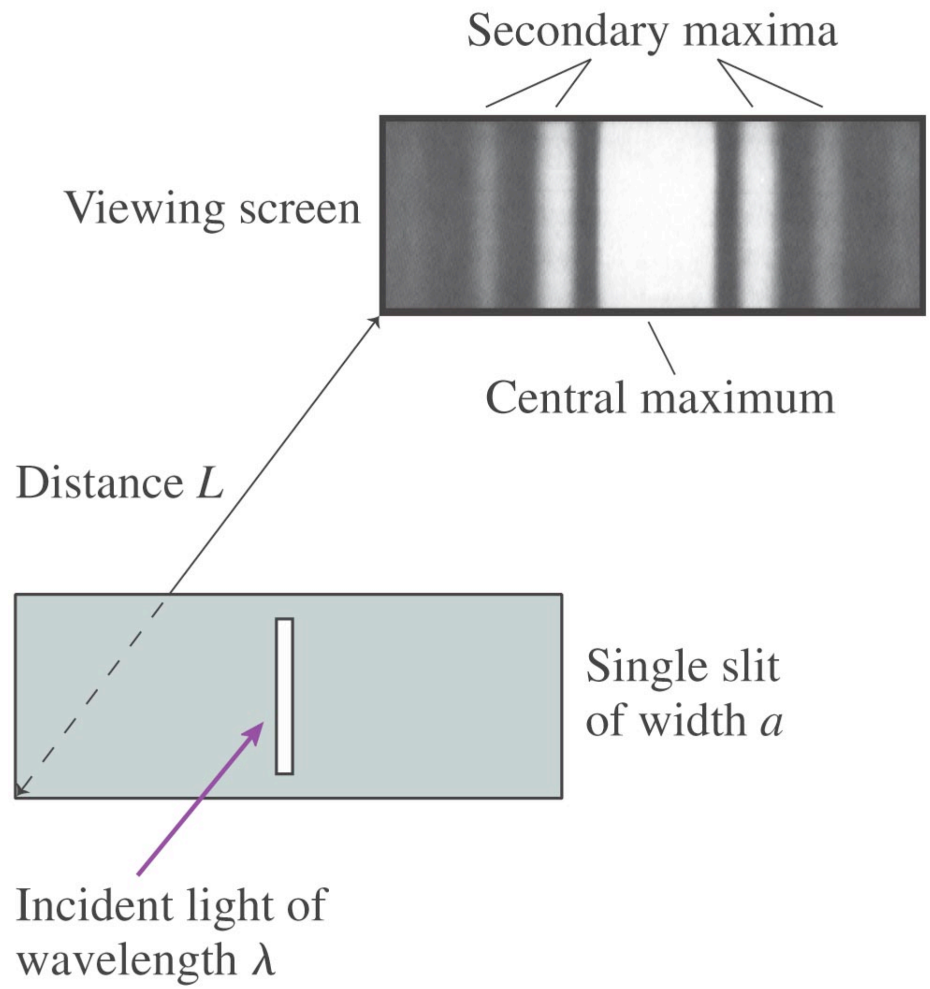




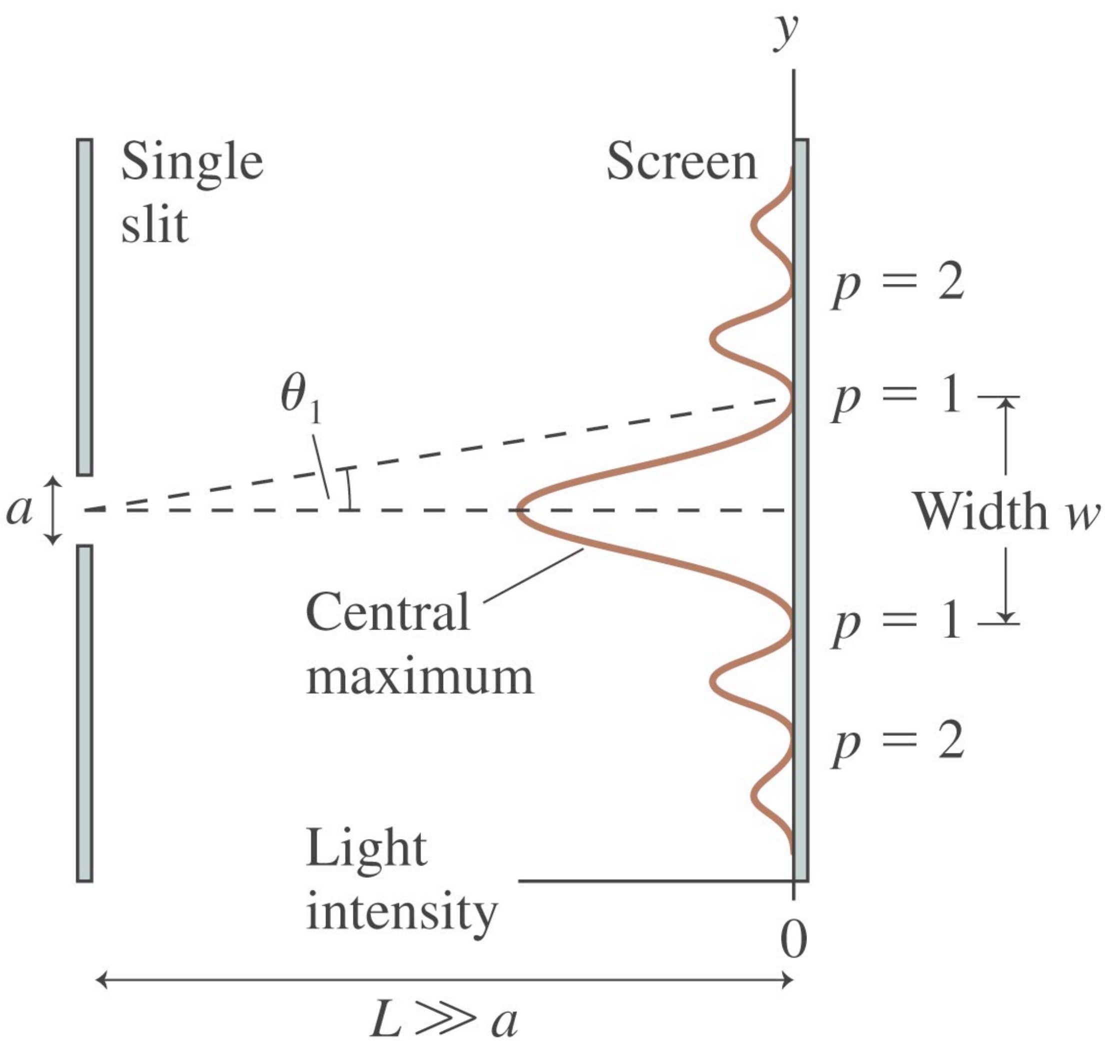
(a) Greatly magnified view of slit



The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.



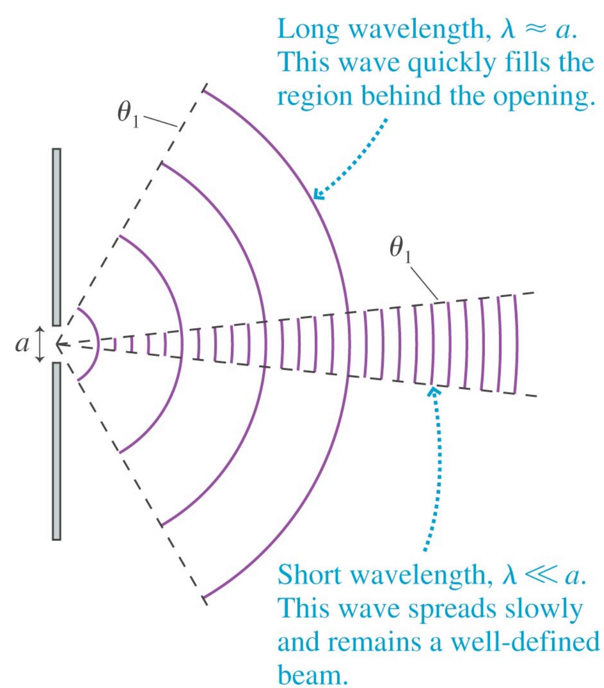
Width of Central Maximum



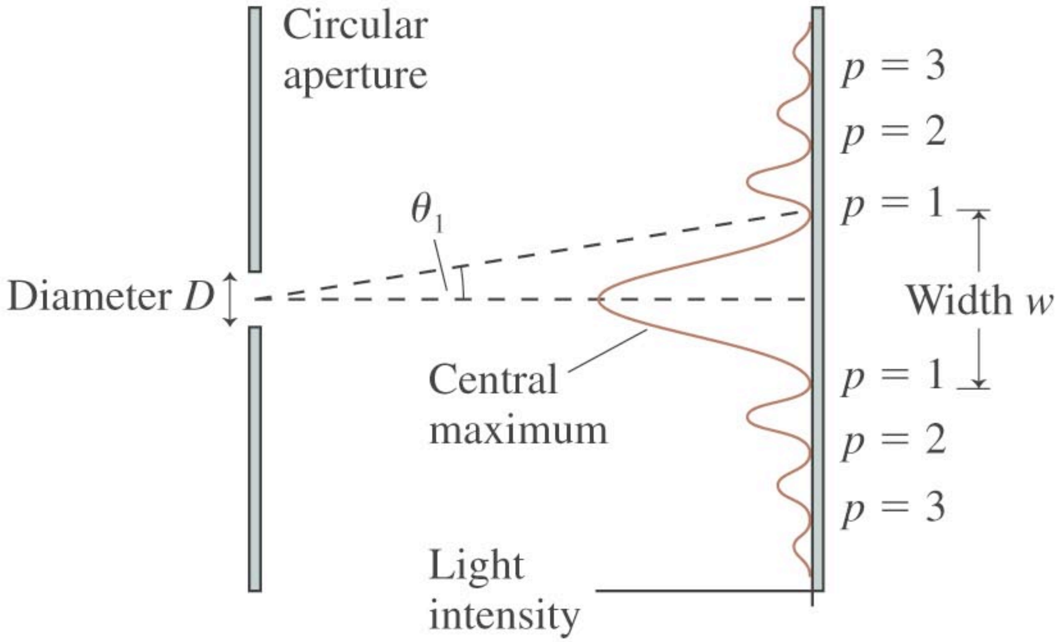
$$\frac{w}{\bar{r}} = \frac{2\lambda}{a}$$

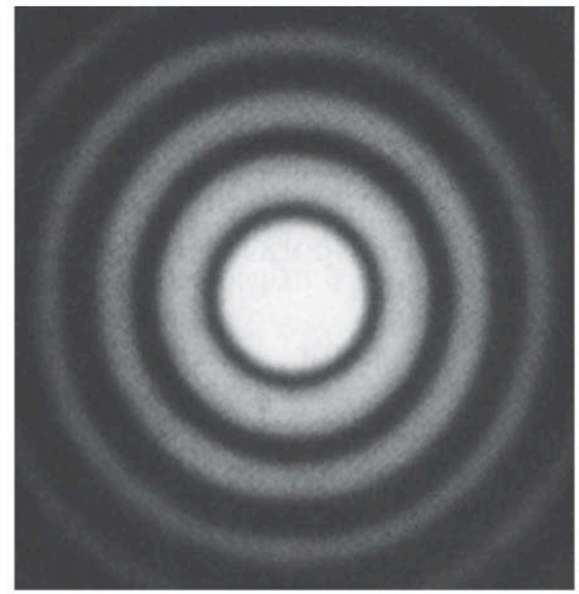
What increases w?

1. Increase distance from slit.
2. Increase wavelength
3. Decrease size of slit



Circular aperture diffraction



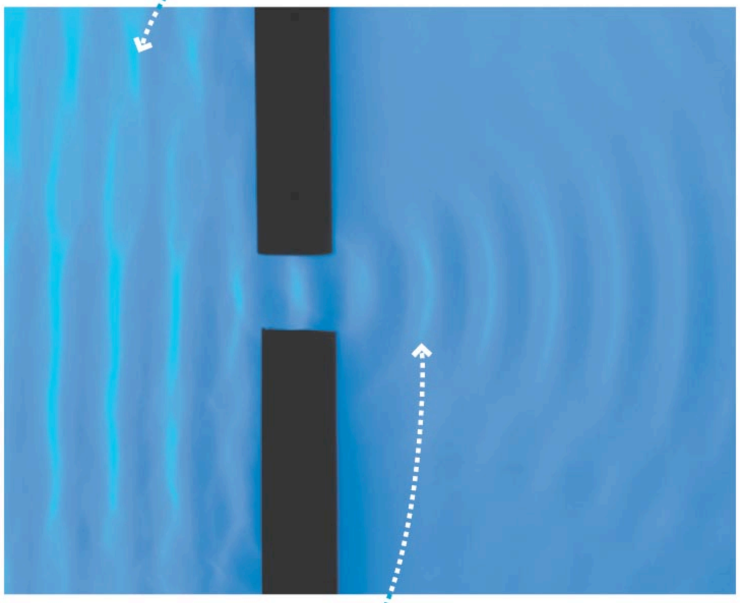


Width of central maximum

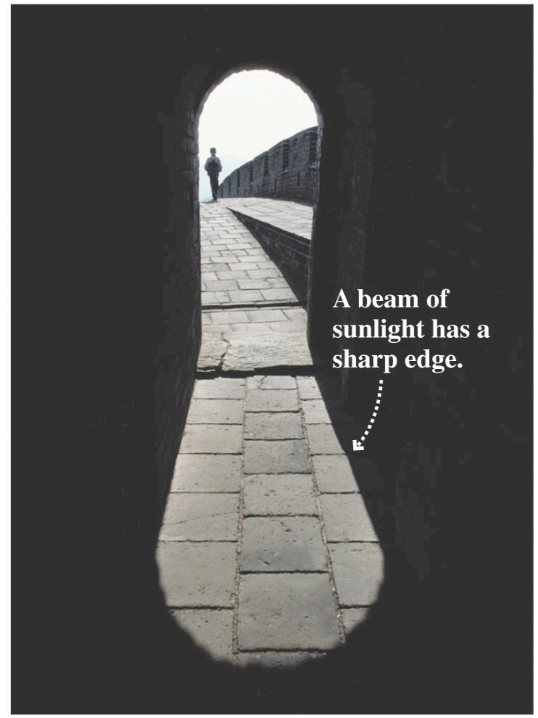
$$\frac{w}{L} = \frac{2.44\lambda}{D}$$

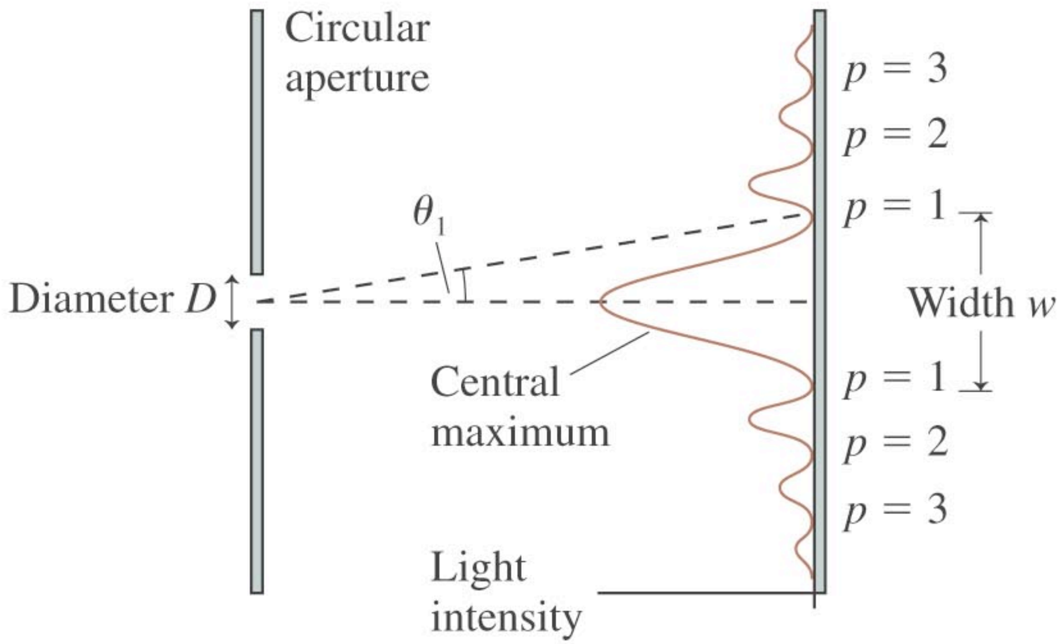
Wave Picture vs Ray Picture

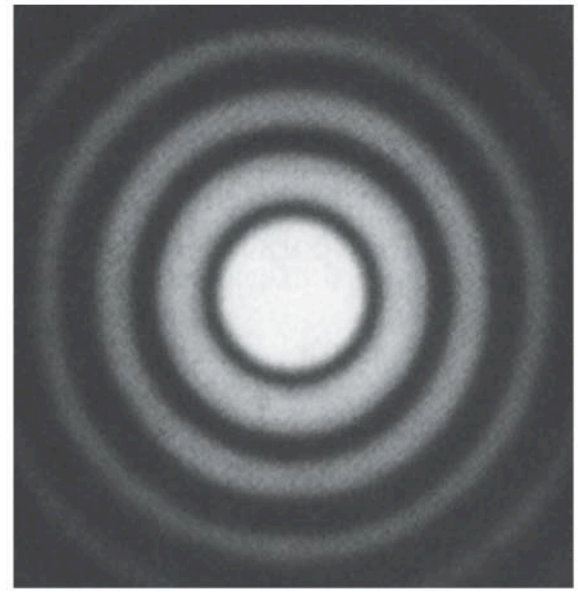
(a)



(b)







If D >> w, ray picture is OK

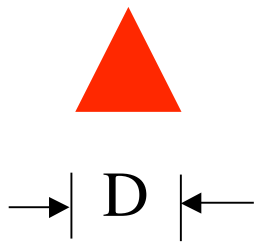
If D <= w, wave picture is needed

$$\frac{w}{L} = \frac{2.44\lambda}{D}$$

Critical size: $D_c = w \Rightarrow D_c = \sqrt{2.44\lambda L}$

If product of wave length and distance to big, wave picture necessary.

Distant object

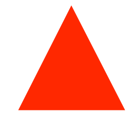


Example suppose object is on surface of sun

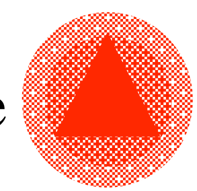
$$L = 1.5 \times 10^{11} m$$

$$\lambda = 500nm = 5 \times 10^{-7} m$$

$$D_c = \sqrt{2.44 \lambda L} = 427 m$$

When will you see  ?

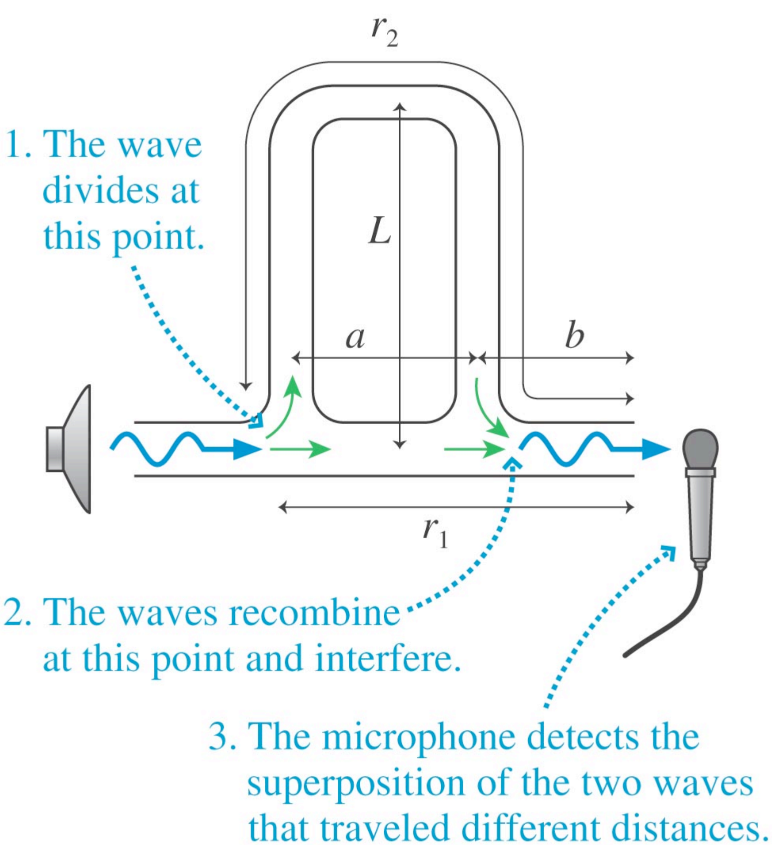
$$D > D_c = \sqrt{2.44 \lambda L}$$

When will you see  ?

Diffraction blurs image

$$D \leq D_c = \sqrt{2.44 \lambda L}$$

Interferometer



Sources will interfere <u>constructively</u> when

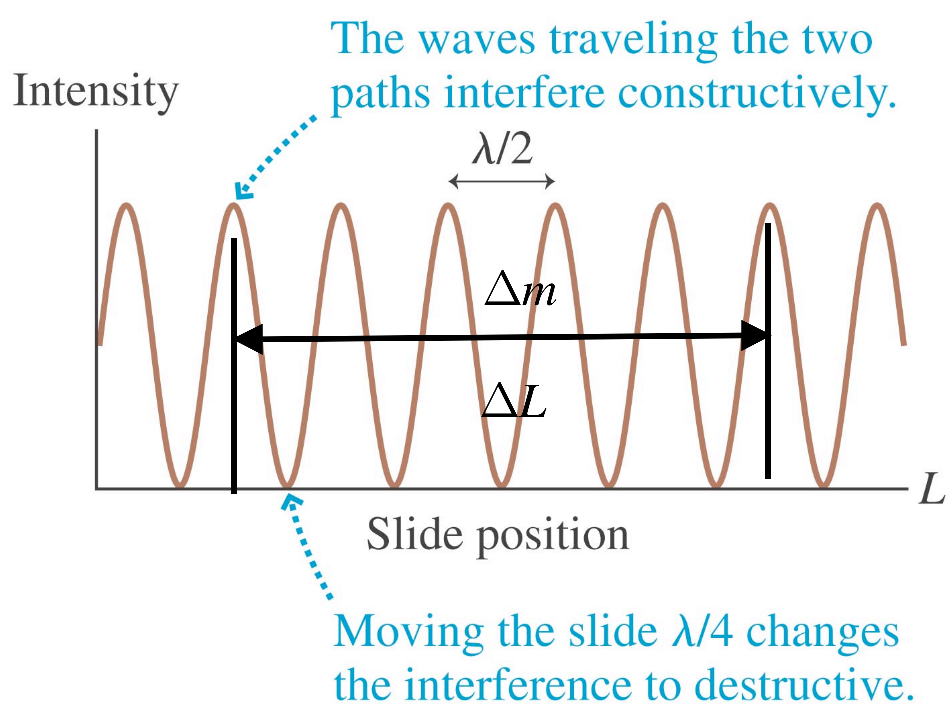
$$\Delta r = 2L = m\lambda \qquad m = 0, 1, 2, \ldots$$

Sources will interfere <u>destructively</u> when

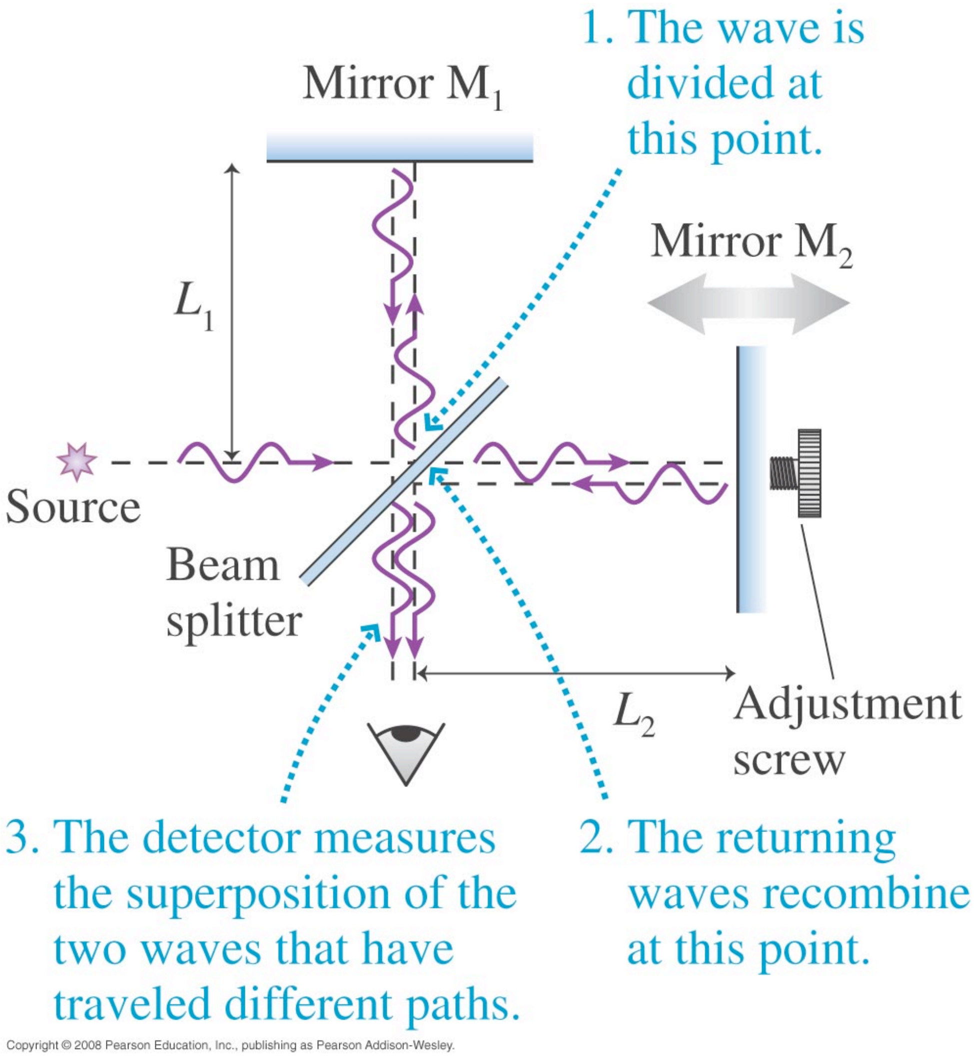
$$\Delta r = 2L = \left(m + \frac{1}{2}\right)\lambda$$

If I vary L

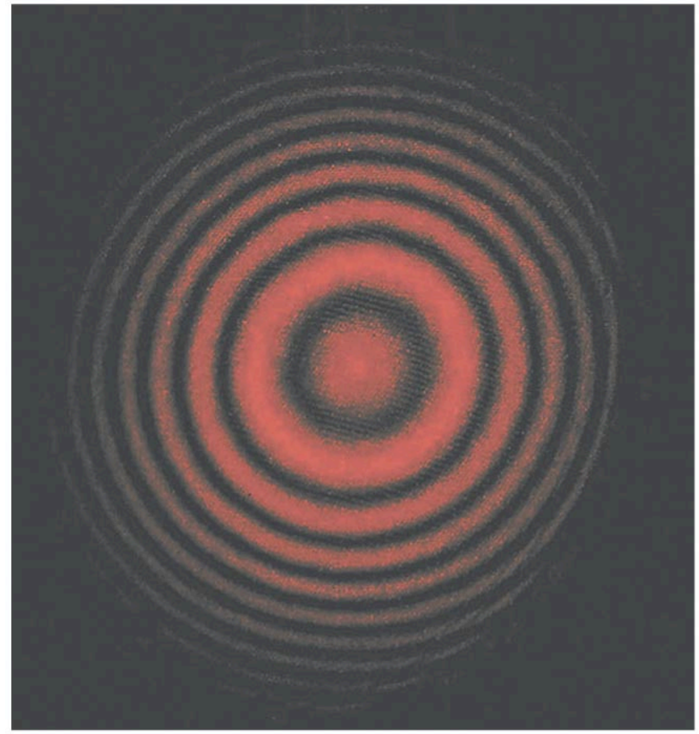
$$\Delta m = \frac{\Delta L}{\lambda / 2}$$



Michelson Interferometer



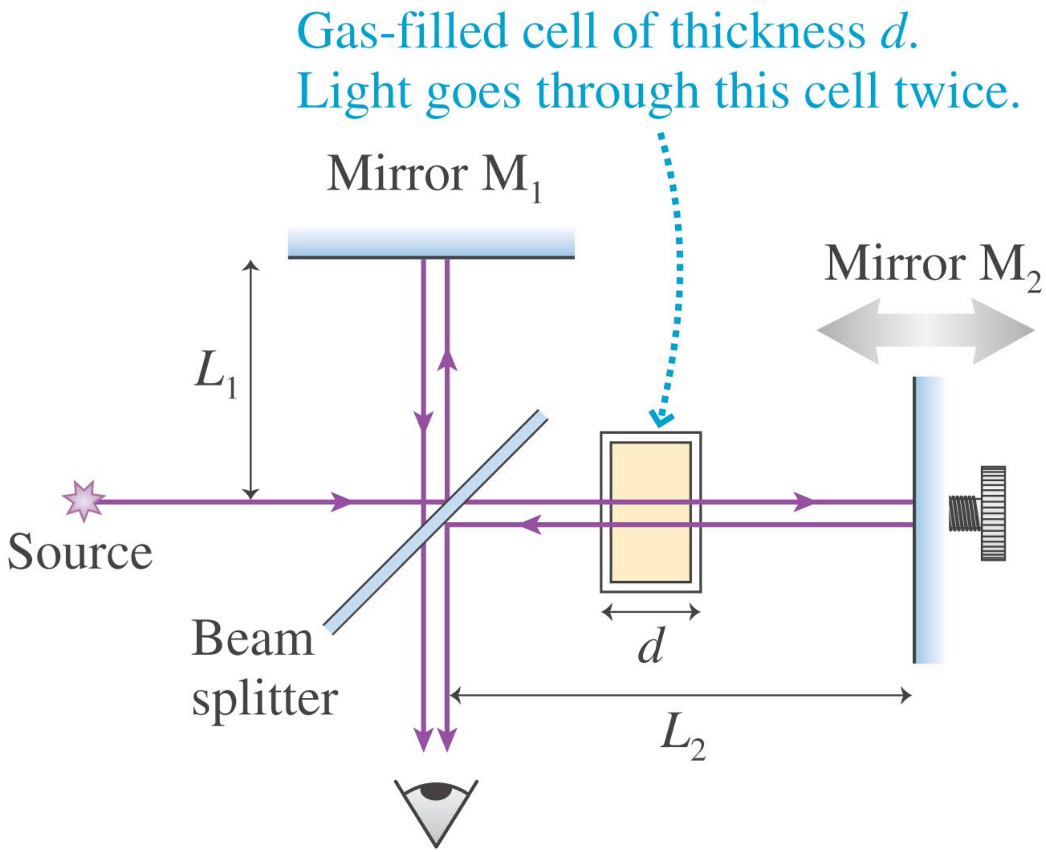
What is seen



As L2 is varied, central spot changes from dark to light, etc. Count changes = Δm

If I vary L_2 $\quad \Delta m = \dfrac{\Delta L_2}{\lambda / 2}$

Measuring Index of refraction



Number of wavelengths in cell when empty

$$m_1 = \frac{2d}{\lambda_{vac}}$$

Number of wavelengths in cell when full

$$m_2 = \frac{2d}{\lambda_{gas}} = \frac{2d}{\lambda_{vac} / n_{gas}}$$

Number of fringe shifts as cell fills up

$$\Delta m = m_2 - m_1 = \left(n_{gas} - 1\right)\frac{2d}{\lambda_{vac}}$$