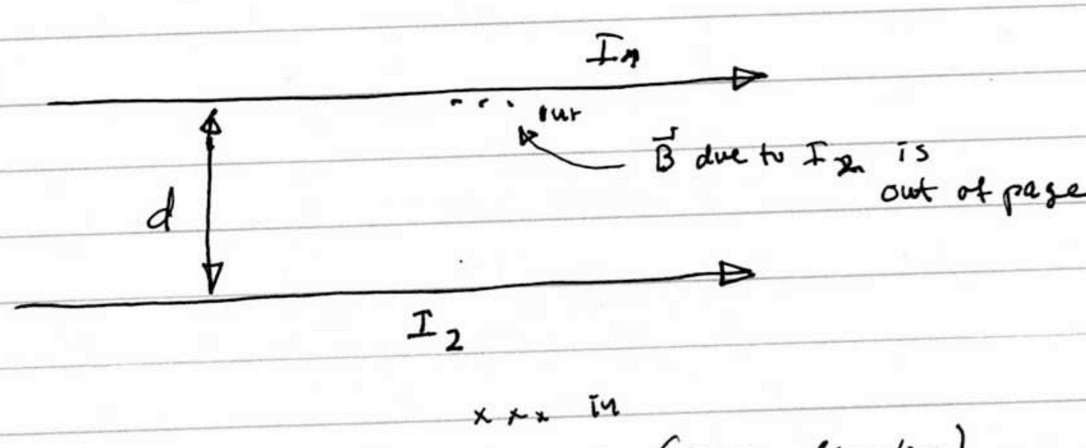
33.8 Force between two Parallel wires



Suppose currents are parallel (same direction)

$\vec{B}$ at I_1 due to I_2 is out of page (RHR)

$|\vec{B}|$ at I_1 due to I_2 is $|\vec{B}| = \frac{\mu_0}{2\pi d} I_2$

~~directi~~

Force due to 2 on 1 $\quad \vec{F} = I_1 \vec{\ell} \times \vec{B}$ $\quad$ (what if antiparallel? F is up)

Direction of $\vec{\ell} \times \vec{B}$ is <u>down</u>

$$|\vec{F}| = \frac{I_1 I_2 \mu_0 L\vec{\ell}|}{2\pi d}$$

$$\left.\frac{\text{Force}}{\text{Length}}\right|_{2 \text{ on } 1} = \frac{I_1 I_2 \mu_0}{2\pi d}$$

Special note: at is at this point we decide how big μ_0 & 1 Ampere should be

$\mu_0 = 4\pi \times 10^{-7}$

UNITS

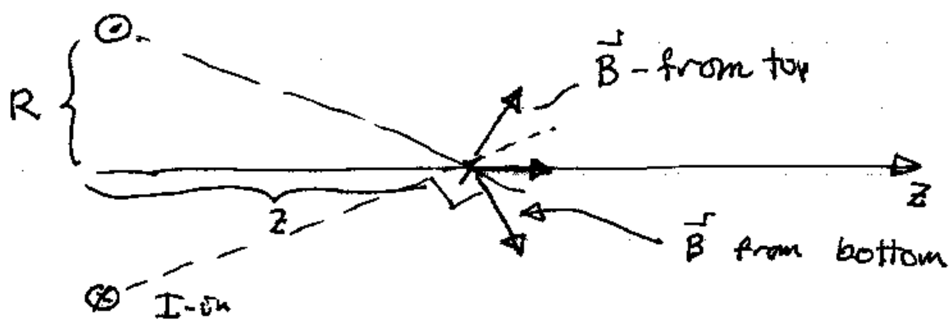
The size of the Ampere is picked so that the constant $\mu_0 = 4\pi \times 10^{-7}$

This then determines the size of a Coulomb . $1C = 1A * 1sec$

This then determines the size of a Volt

$$1V = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$$

33.5 Magnetic Field due to a loop of current



Net $\vec{B}$ (on axis) is parallel to z-axis

See derivation in book

$$|\vec{B}(z)| = \frac{\mu_0}{2\pi} \frac{\pi R^2 I}{(z^2+R^2)^{3/2}}$$

IF $z \gg R$ (Far away from loop)

$$|\vec{B}| \cong \frac{\mu_0}{2\pi} \frac{\pi R^2 I}{|z|^3}$$

Suppose circular loop is replaced by a loop of arbitrary shape (but area A)

$$|\vec{B}| \simeq \frac{\mu_0}{2\pi} \frac{(AI)}{|z|^3}$$

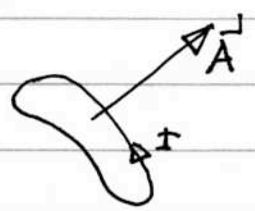
AI = magnetic dipole moment

(also labeled μ)

General Result for $\vec{B}$ far from loop

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{3\vec{r}\,\vec{r}\cdot\vec{\mu} - r^2\vec{\mu}}{|\vec{r}|^5}$$

Where



$\vec{\mu} = I\vec{A}$

$|\vec{A}|$ = area of loop

direction of $\vec{A}$ determined by RHR

Special case $\vec{r}$ is parallel to $\vec{\mu}$

$$3\vec{r}\,\vec{r}\cdot\vec{\mu} - r^2\vec{\mu} = 2r^2\vec{\mu}$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{|\vec{r}|^3}$$

Ampere's Law – Gauss' Law

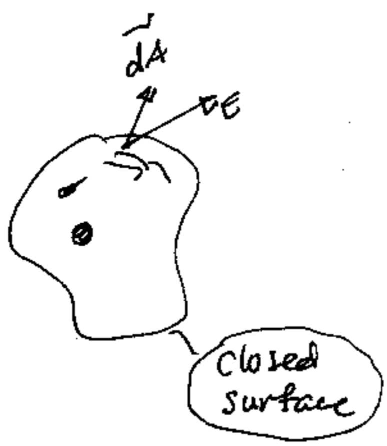
Recall Consequences of Coulomb's Law

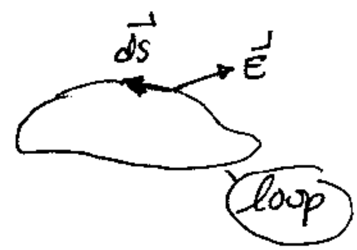
1) Gauss' Law $\oint_{\text{closed surface}} \vec{E}\cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$

Electric flux leaving a ~~close~~ any closed surface Q

2) Line integral $\oint_{\text{closed loop}} \vec{E}\cdot d\vec{s} = 0$

Line integral of work around closed loop 0





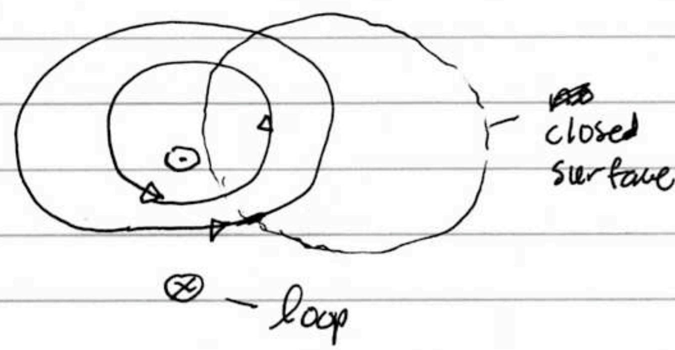
Consequences of Biot-Savart Law

1) Gauss' Law $\oint_{\text{closed surface}} \vec{B}\cdot d\vec{A} = 0$

2) Ampere's Law $\oint_{\text{Loop}} \vec{B}\cdot d\vec{s} = \mu_0 I_{inside}$ (RHR)

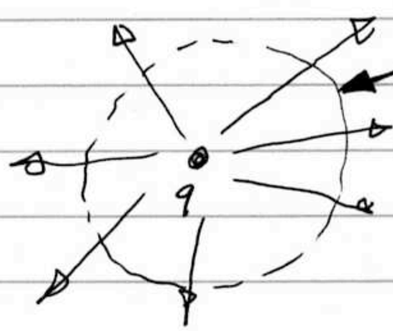
First Consider Gauss' Law

Net magnetic flux leaving any closed surface is always zero



no matter where I draw the surface field lines enter and leave the surface. Net flux is zero.

Not true for electric fields



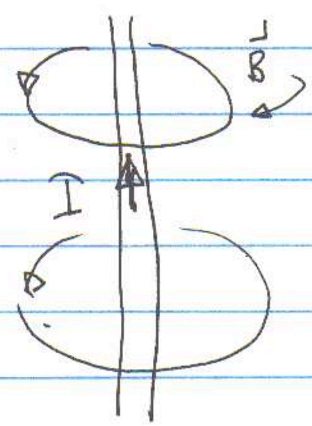
net flux leaving surface is given by charge inside

We say there are no magnetic monopoles

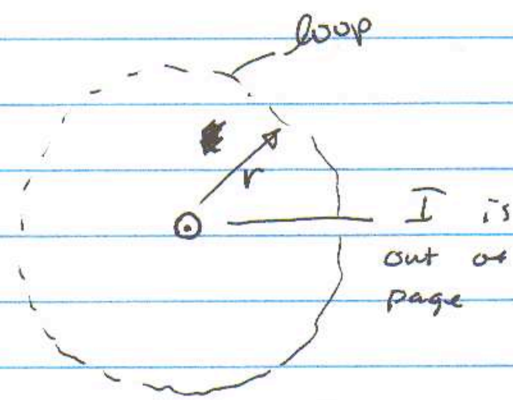
Ampere's Law

$$\oint_{\substack{\text{closed}\\\text{loop}}} \vec{B}\cdot ds = \mu_0 I_{\text{enclosed}}$$

Example:



TOP VIEW



Pick r to be radius of loop

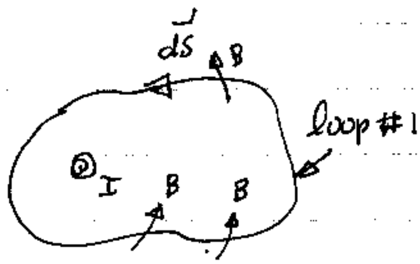
$\vec{dS}$ is tangent to loop and parallel to $\vec{B}$

$$\oint \vec{B}\cdot\vec{dS} = |\vec{B}| \text{ circumference} = |\vec{B}|\, 2\pi r$$

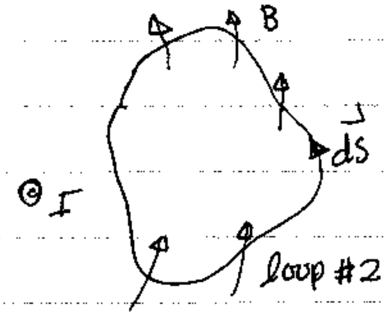
But: we know $|\vec{B}| = \dfrac{\mu_0 I}{2\pi r}$

so $\oint \vec{B}\cdot d\vec{S} = \mu_0 I$

The "miracle" is that even if the loop is not a circle centered on I it's still true



$\oint \vec{B} \cdot \vec{ds} = \mu_0 I$



$\oint \vec{B} \cdot \vec{ds} = 0$

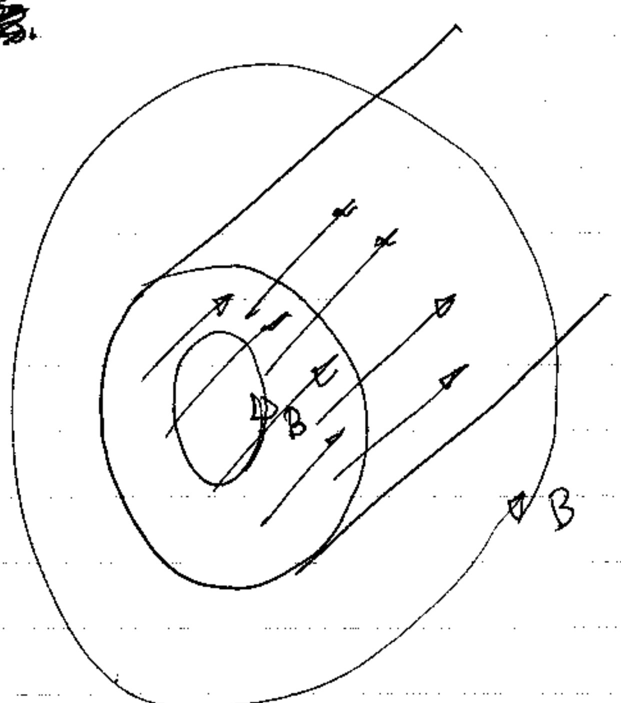
(no current enclosed by loop)

Two examples where Ampere's Law can be applied to find $\vec{B}$

ii) Example 33.8 magnetic field in a current carryin wire

iii) Magnetic field of a solenoid

3.

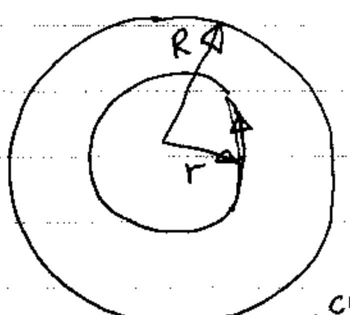


Different strengths of magnetic field at different radii

Suppose current I flows uniformly throughout a wire of radius R

Magnetic field strength will vary with distance from wire $\vec{B}(r)$ r = distance from center of wire

<u>Case # 1</u>
draw loop with $r<R$



$$|\vec{B}| = \frac{1}{2\pi r}\mu_0 \frac{\pi r^2}{\pi R^2} I$$

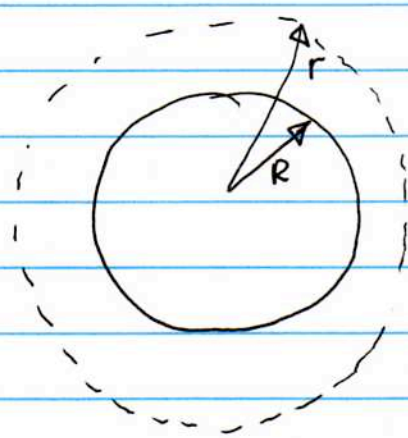
$$\boxed{|\vec{B}| = \frac{\mu_0}{2\pi} I \frac{r}{R^2}}$$

$$\oint \vec{B}\cdot d\vec{s} = \underbrace{2\pi r}_{\text{circumference}} |\vec{B}| = \mu_0 I_{through}$$

$$I_{through} = \frac{\pi r^2}{\pi R^2} I$$

Case #2

draw loop $r > R$

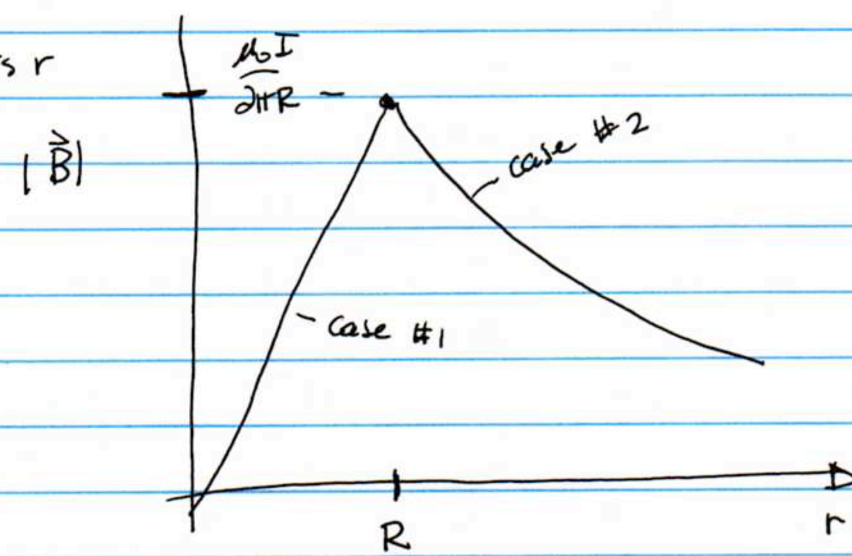


$$\int \vec{B} \cdot d\vec{s} = 2\pi r |\vec{B}| = \mu_0 I_{through}$$

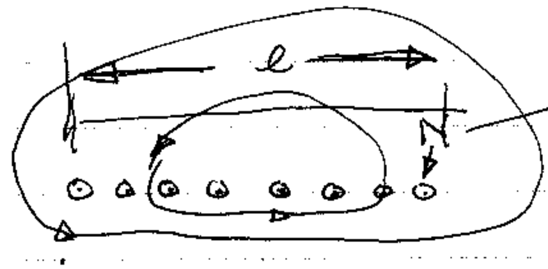
But now $I_{through} = I$
independent of r

$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$ same as the thin wire

Plot $|\vec{B}|$ vs r

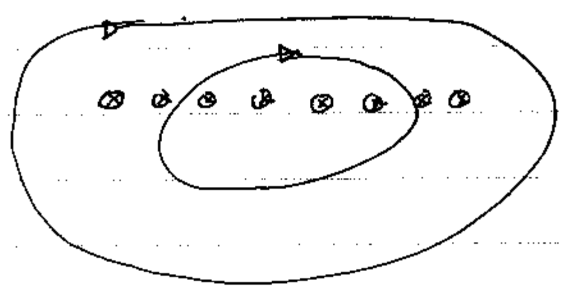


Example #2



N loops of curr
Spaced over distance ℓ

Show picture



How to find $\vec{B}$ on axis?

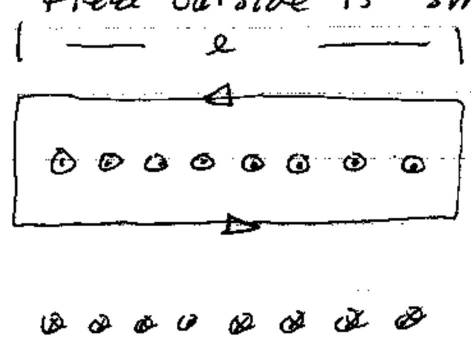
Method #1 sum up contributions from N Loops

$$B_{loop} = \frac{\mu_0}{2\pi} \frac{\pi R^2 I}{(z^2+R^2)^{3/2}}$$

Method #2 assume solenoid is long $\ell >> R$

Then field inside is uniform points in $+z$ direction

Field outside is small



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 N I$$

$$\oint \vec{B} \cdot ds \approx \cancel{B_{inside} - B_{outside}}$$

$$\ell B_{inside}$$

$$\boxed{B_{inside} = \frac{\mu_0 N I}{\ell}}$$