

Chapter 39. Quantization

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Topics:

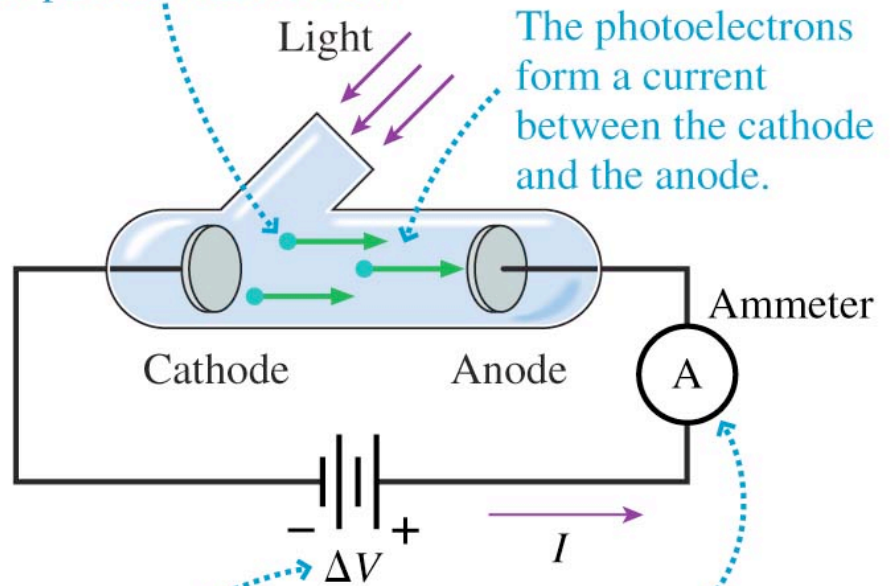
- The Photoelectric Effect
- Einstein's Explanation
- Photons
- Matter Waves and Energy Quantization
- Bohr's Model of Atomic Quantization
- The Bohr Hydrogen Atom
- The Hydrogen Spectrum

The Photoelectric Effect

- In 1886 Hertz noticed, in the course of his investigations, that a negatively charged electroscope could be discharged by shining ultraviolet light on it.
- In 1899, Thomson showed that the emitted charges were electrons.
- The emission of electrons from a substance due to light striking its surface came to be called the **photoelectric effect**.
- The emitted electrons are often called *photoelectrons* to indicate their origin, but they are identical in every respect to all other electrons.

FIGURE 39.1 Lenard's experimental device to study the photoelectric effect.

Ultraviolet light causes the metal cathode to emit electrons. This is the photoelectric effect.



The photoelectrons form a current between the cathode and the anode.

The potential difference can be changed or reversed.

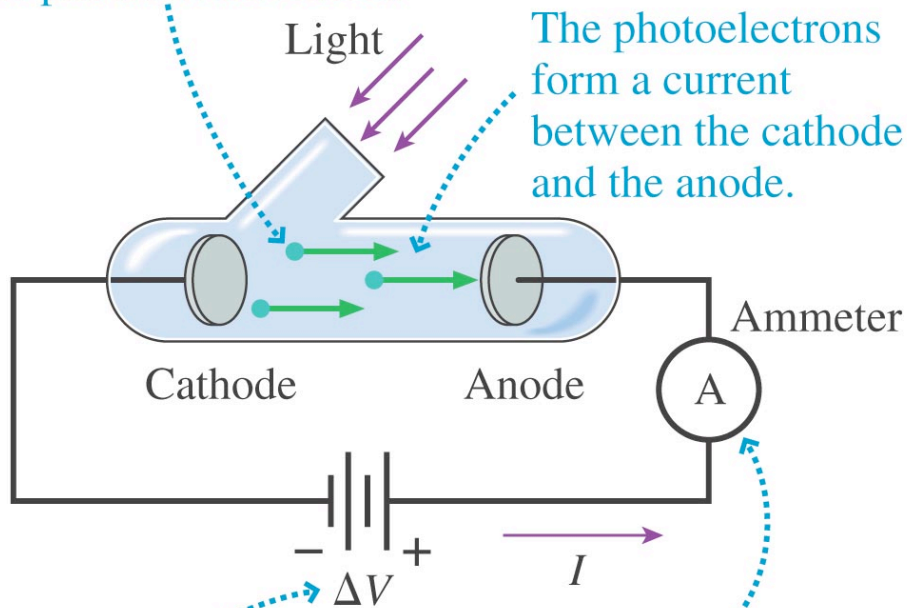
The current can be measured while the potential difference, the light frequency, and the light intensity are varied.

Lenard's Observations

1. Current I is proportional to light intensity
2. Current appears without delay
3. Photo electrons are emitted only if light frequency exceeds threshold f_0 .
4. The value of the threshold frequency depends on the type of metal
5. If potential difference is positive current is independent of potential. If potential is sufficiently negative (V_{stop}) no current is observed independent of light intensity.
6. Stopping potential is independent of intensity

Lenard's Experiment

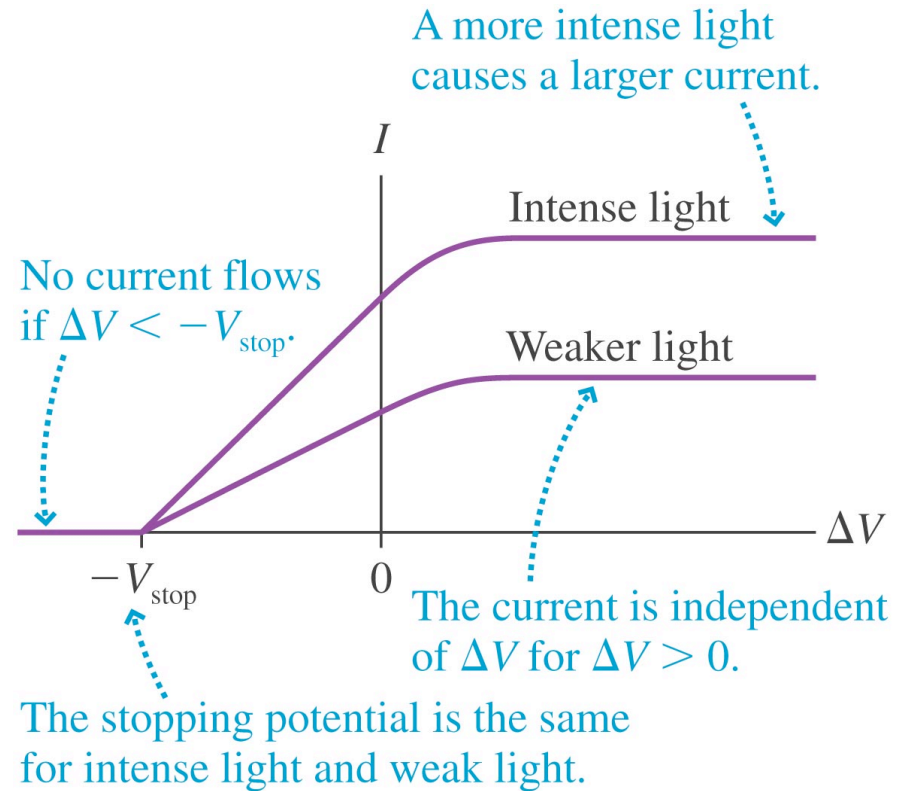
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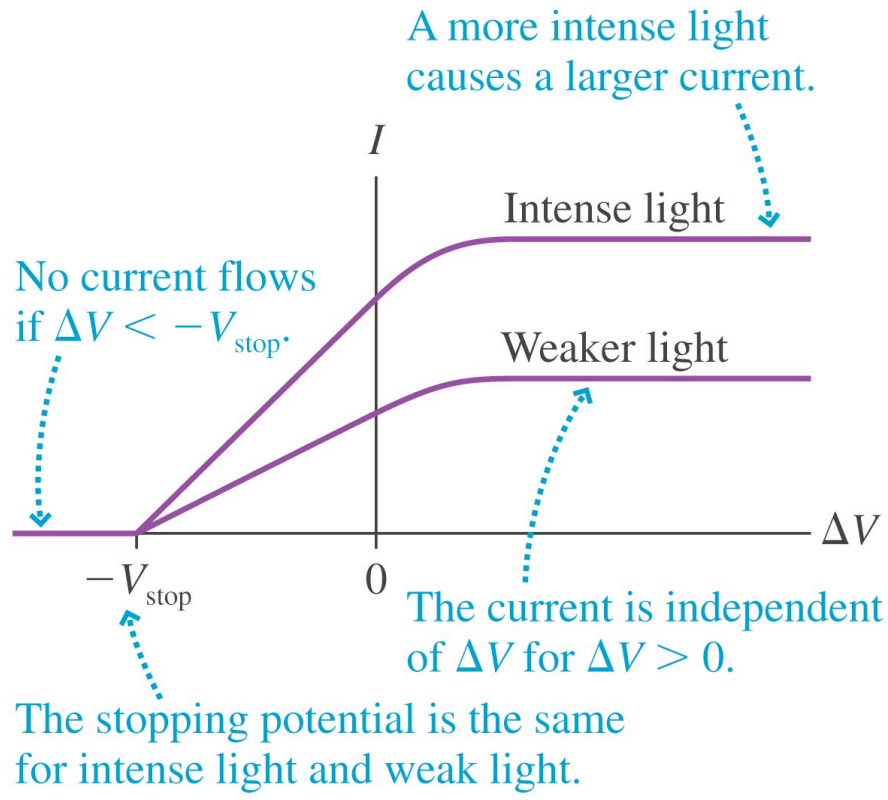


A more intense light causes a larger current.

No current flows if $\Delta V < -V_{stop}$.

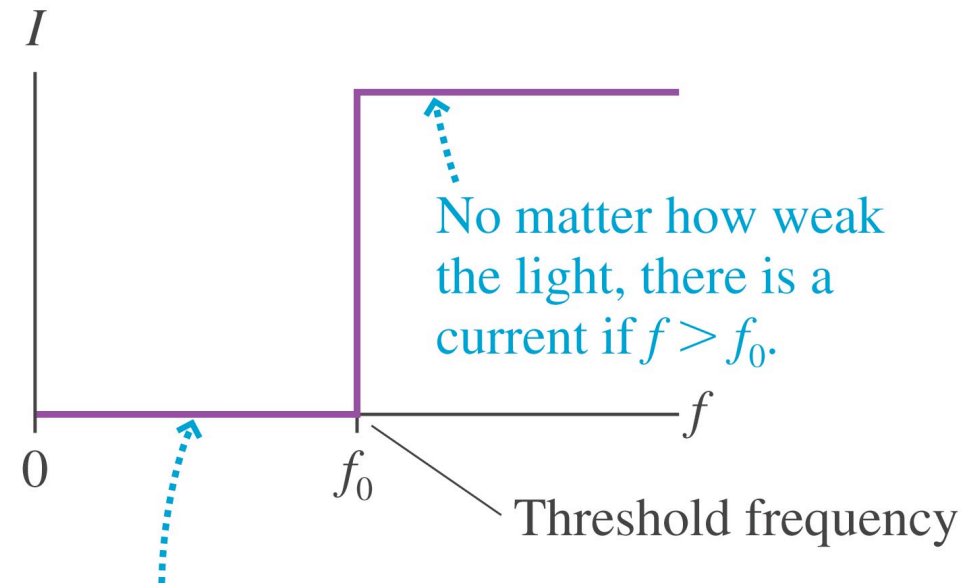
The current is independent of ΔV for $\Delta V > 0$.

The stopping potential is the same for intense light and weak light.



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V_{stop} depends on frequency of light



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Characteristics of the Photoelectric Effect

1. The current I is directly proportional to the light intensity.
2. Photoelectrons are emitted *only* if the light frequency f exceeds a **threshold frequency** f_0 .
3. The value of the threshold frequency f_0 depends on the type of metal from which the cathode is made.
4. If the potential difference ΔV is positive, the current does not change as ΔV is increased. If ΔV is made negative, the current decreases until, at $\Delta V = \Delta V_{\text{stop}}$ the current reaches zero. The value of V_{stop} is called the **stopping potential**.
5. The value of V_{stop} is the same for both weak light and intense light. A more intense light causes a larger current, but in both cases the current ceases when $\Delta V = \Delta V_{\text{stop}}$.

What would make an electron leave a solid conductor?

- A. The electron is repelled by the conductor, so it escapes readily.
- B. The electron is attracted to the conductor, but given enough energy it can escape.
- C. The conductor used a random packing algorithm (RPA) to load the dishwasher resulting in the inefficient use of space. The electron had to continually reload the dishwasher and became fed up.

What would make an electron leave a solid conductor?

- A. The electron is repelled by the conductor, so it escapes easily.
- ✓ **B. The electron is attracted to the conductor, but given enough energy it can escape.**
- C. The conductor used a random packing algorithm (RPA) to load the dishwasher resulting in the inefficient use of space. The electron had to continually reload the dishwasher and became fed up.

What would make an electron leave a solid?

The *minimum* energy to remove a drop of water from the pool is mgh .

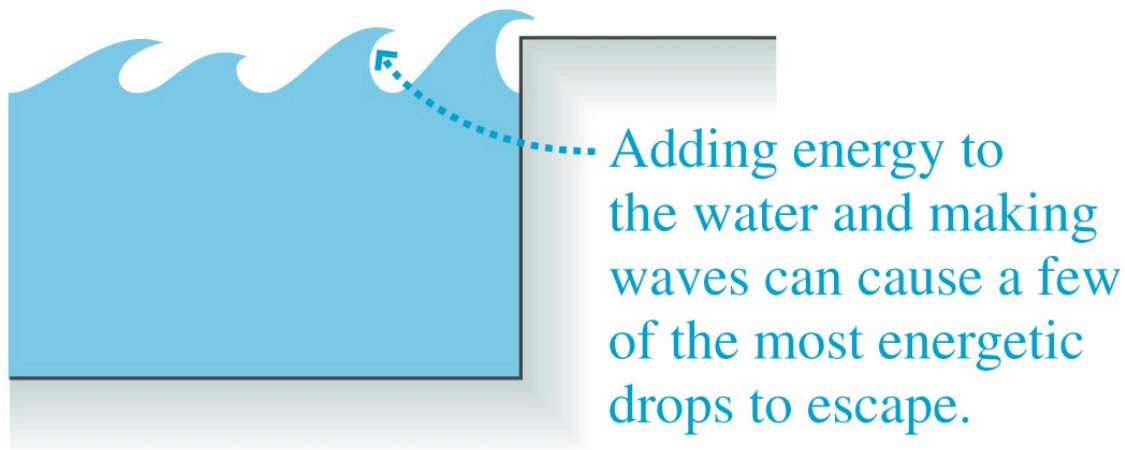
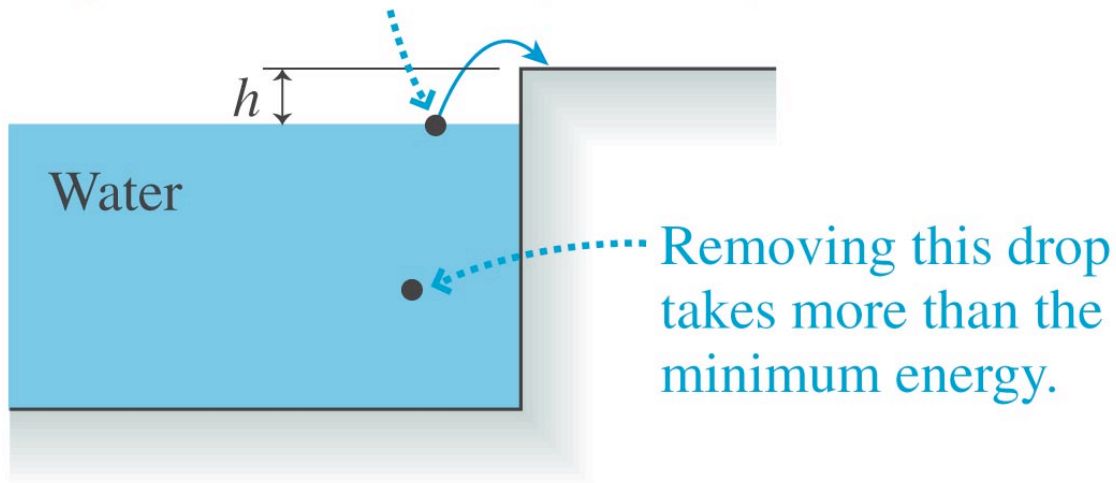


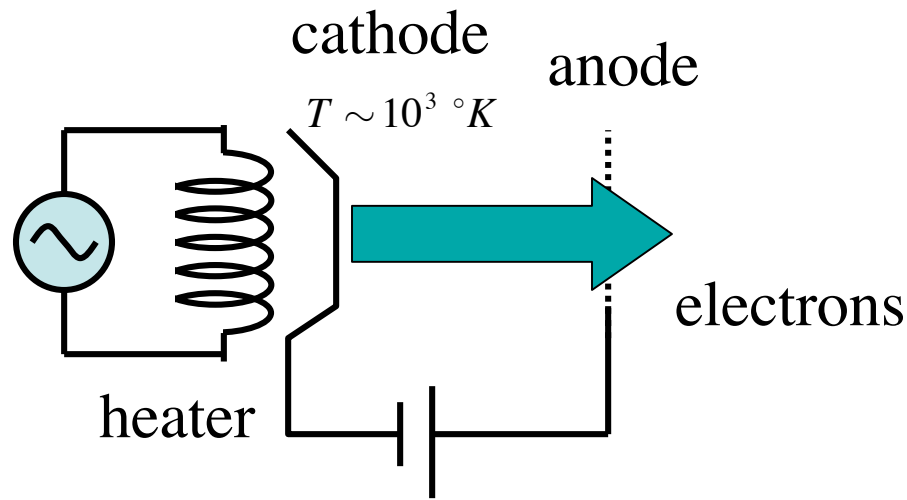
TABLE 39.1 The work function for some of the elements

Element	E_0 (eV)
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

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Compare with ionization energy for H, 13.6 eV

Thermionic Emission Heating electrons to high enough energy will make them leave a solid.



Heated cathode produces electrons of sufficient energy to overcome work function, E_0 .

Process used in vacuum tubes and cathode ray tubes.

$$T \sim 10^3 \text{ }^\circ\text{K} \sim .1 \text{ eV}$$

Fraction of electrons with high enough energy for tungsten

$$f \sim \exp\left[-\frac{E_0}{kT}\right] = \exp\left[-\frac{E_0}{kT}\right] = \exp\left[-\frac{4.55}{.1}\right] = 1.7 \times 10^{-20}$$

of free electrons/cm³ - 10^{23} + other factors give about 10 A/cm²
 Raise T but don't melt the cathode.

Electrons leave cathode with kinetic energy K_i

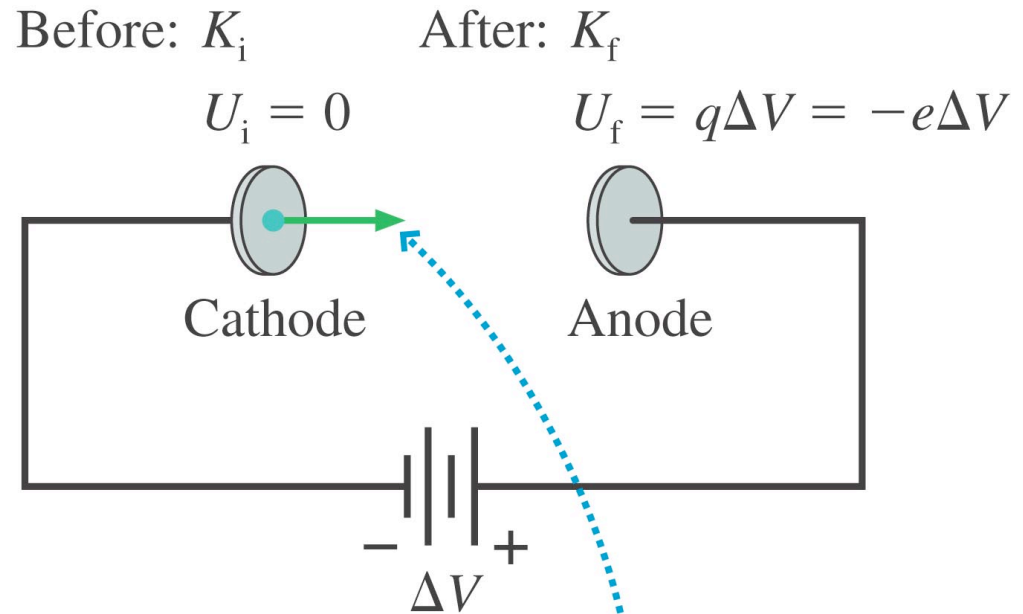
$$K_i = E_{elec} - E_0$$

Energy in metal Work function

Electrons have kinetic energy K_f when they reach the anode.

$$K_f = K_i + e\Delta V$$

$$eV_{stop} = \max[E_{elec} - E_0]$$



Energy is transformed from kinetic to potential as an electron moves from cathode to anode.

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If anode positive, $\Delta V > 0$, all electrons leaving cathode make it to anode.

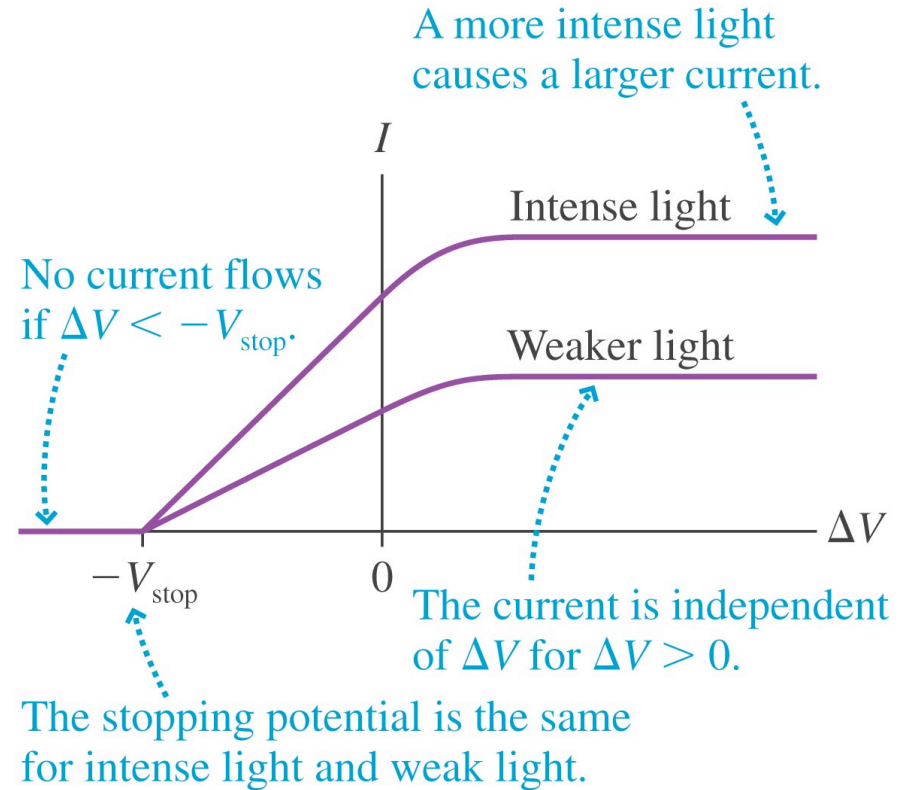
If anode negative, $\Delta V < 0$, some electrons are repelled by anode.

If $\Delta V < -V_{stop}$ no electrons make it to anode

If anode positive, $\Delta V > 0$, all electrons leaving cathode make it to anode.

If anode negative, $\Delta V < 0$, some electrons are repelled by anode.

If $\Delta V < -V_{\text{stop}}$ no electrons make it to anode



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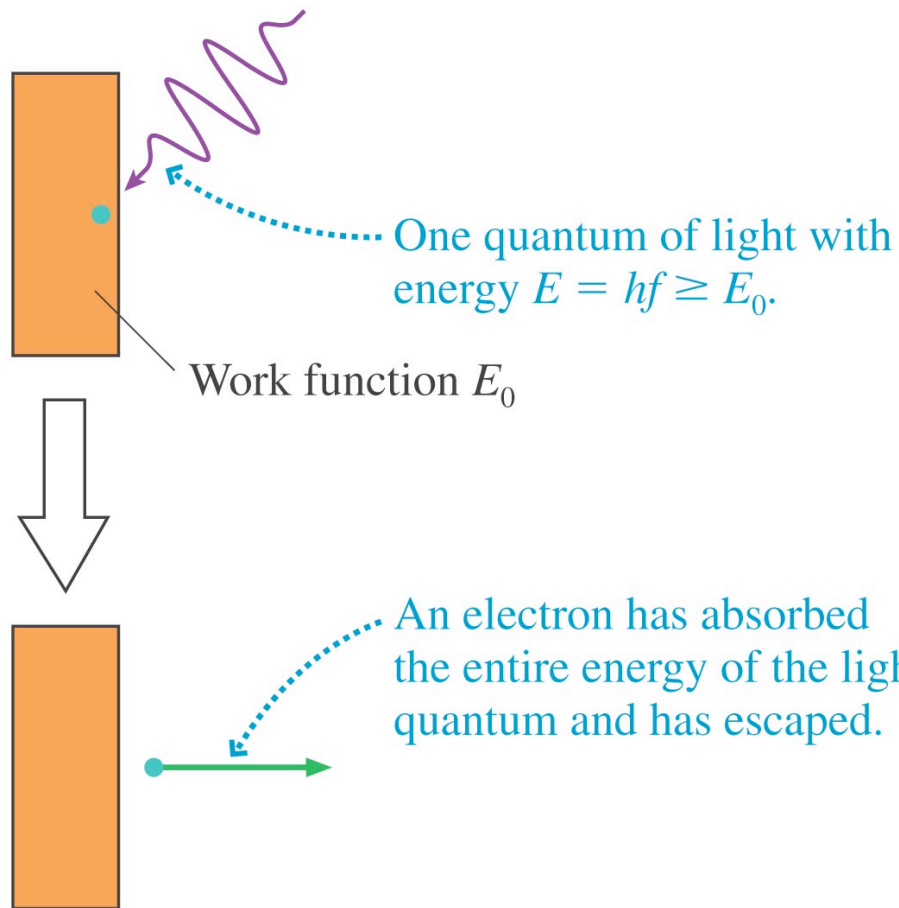
$$eV_{\text{stop}} = \max[E_{\text{elec}} - E_0]$$

Implies that there is a maximum energy for electrons in the metal

Einstein's Postulates

Einstein framed three postulates about light quanta and their interaction with matter:

1. Light of frequency f consists of discrete quanta, each of energy $E = hf$, where h is Planck's constant $h = 6.63 \times 10^{-34}$ J s. Each photon travels at the speed of light $c = 3.00 \times 10^8$ m/s.
2. Light quanta are emitted or absorbed on an all-or-nothing basis. A substance can emit 1 or 2 or 3 quanta, but not 1.5. Similarly, an electron in a metal can absorb only an integer number of quanta.
3. A light quantum, when absorbed by a metal, delivers its entire energy to *one* electron.



$$K_i = E_{elec} - E_0$$

$$K_{\max} = hf - E_0$$

Einstein's Explanation of the Photoelectric Effect

An electron that has just absorbed a quantum of light energy has $E_{\text{elec}} = hf$. (The electron's thermal energy at room temperature is so much less than that we can neglect it.) This electron can escape from the metal, becoming a photoelectron, if

$$E_{\text{elec}} = hf \geq E_0$$

In other words, there is a *threshold frequency*

$$f_0 = \frac{E_0}{h}$$

for the ejection of photoelectrons **because each light quantum delivers all of its energy to one electron.**

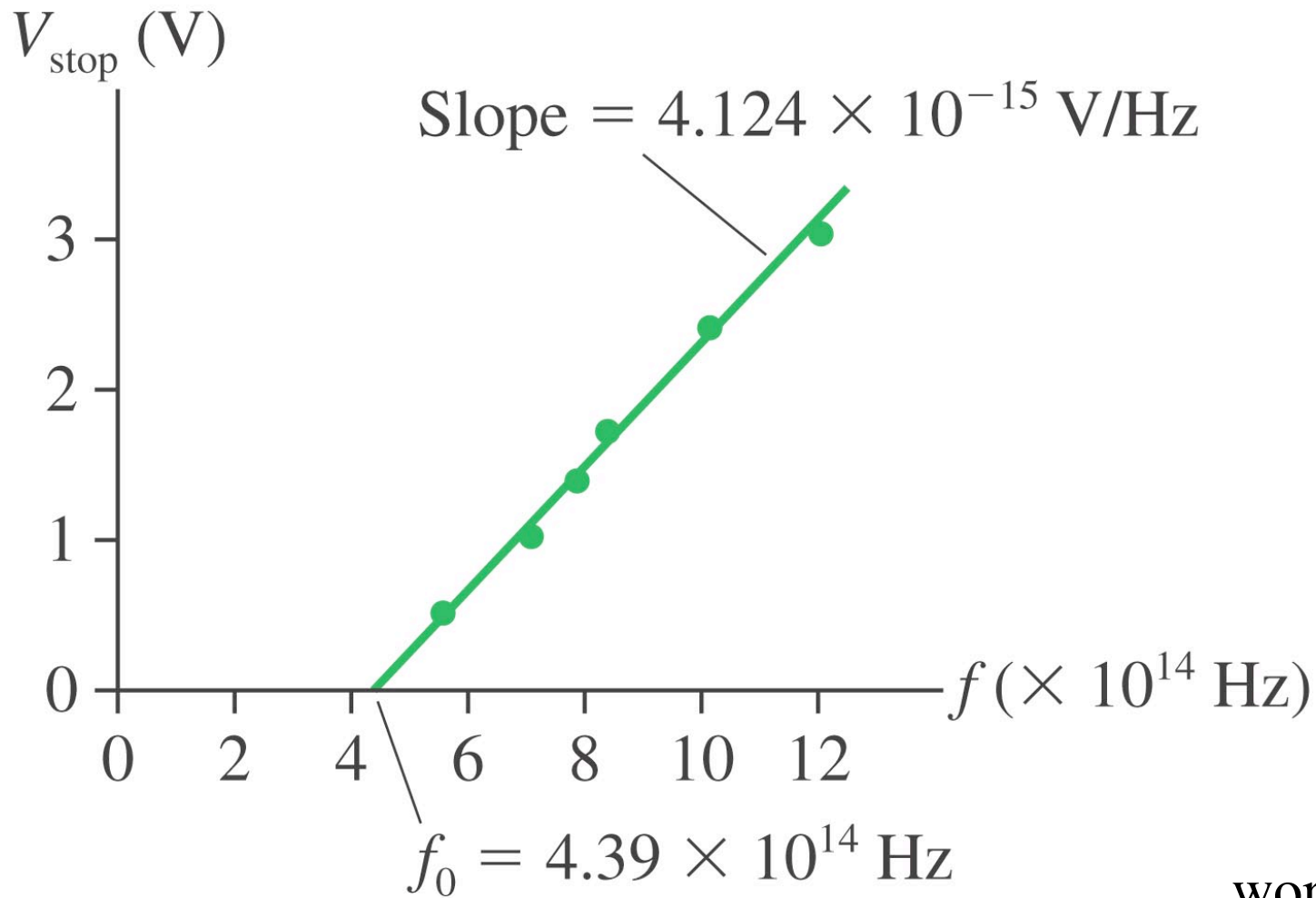
Einstein's Explanation of the Photoelectric Effect

- A more intense light delivers a larger number of light quanta to the surface. These quanta eject a larger number of photoelectrons and cause a larger current.
- There is a distribution of kinetic energies, because different photoelectrons require different amounts of energy to escape, but the *maximum* kinetic energy is

$$K_{\max} = E_{\text{elec}} - E_0 = hf - E_0$$

The stopping potential V_{stop} is directly proportional to K_{\max} . Einstein's theory predicts that the stopping potential is related to the light frequency by

$$V_{\text{stop}} = \frac{K_{\max}}{e} = \frac{hf - E_0}{e}$$



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work function

Millikan's experiment

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{hf - E_0}{e}$$

EXAMPLE 39.3 The photoelectric threshold frequency

QUESTION:

EXAMPLE 39.3 The photoelectric threshold frequency

What are the threshold frequencies and wavelengths for photoemission from sodium and from aluminum?

Sodium: $E_0 = 2.45 \text{ eV}$

Aluminum: $E_0 = 4.28 \text{ eV}$

$$f_0 = E_0/h \quad h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$$

$$\lambda_0 = c/f_0 \quad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

EXAMPLE 39.3 The photoelectric threshold frequency

SOLVE Table 39.1 gives the sodium work function as $E_0 = 2.75$ eV. Aluminum has $E_0 = 4.28$ eV. We can use Equation 39.6, with h in units of eV s, to calculate

$$f_0 = \frac{E_0}{h} = \begin{cases} 6.64 \times 10^{14} \text{ Hz} & \text{sodium} \\ 10.34 \times 10^{14} \text{ Hz} & \text{aluminum} \end{cases}$$

These frequencies are converted to wavelengths with $\lambda = c/f$, giving

$$\lambda = \begin{cases} 452 \text{ nm} & \text{sodium} \\ 290 \text{ nm} & \text{aluminum} \end{cases}$$

EXAMPLE 39.3 The photoelectric threshold frequency

ASSESS The photoelectric effect can be observed with sodium for $\lambda < 452$ nm. This includes blue and violet visible light but not red, orange, yellow, or green. Aluminum, with a larger work function, needs ultraviolet wavelengths $\lambda < 290$ nm.

EXAMPLE 39.4 Maximum photoelectron speed

QUESTION:

EXAMPLE 39.4 Maximum photoelectron speed

What is the maximum photoelectron speed if sodium is illuminated with light of 300 nm?

Maximum kinetic energy: $K_{\max} = hf - E_0$

Maximum speed $mv^2/2 = K_{\max}$

$f = c/\lambda$

EXAMPLE 39.4 Maximum photoelectron speed

SOLVE The light frequency is $f = c/\lambda = 1.00 \times 10^{15}$ Hz, so each light quantum has energy $hf = 4.14$ eV. The maximum kinetic energy of a photoelectron is

$$\begin{aligned}K_{\max} &= hf - E_0 = 4.14 \text{ eV} - 2.75 \text{ eV} = 1.39 \text{ eV} \\ &= 2.22 \times 10^{-19} \text{ J}\end{aligned}$$

Because $K = \frac{1}{2}mv^2$, where m is the electron's mass, not the mass of the sodium atom, the maximum speed of a photoelectron leaving the cathode is

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = 6.99 \times 10^5 \text{ m/s}$$

Note that we had to convert K_{\max} to SI units of J before calculating a speed in m/s.

Characteristics of the Photoelectric Effect

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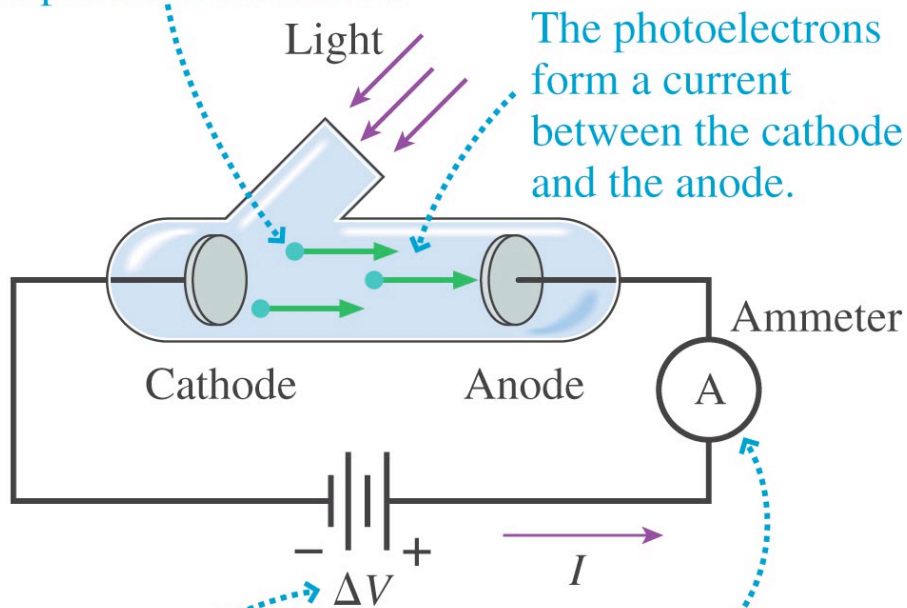
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Lenard's Experiment

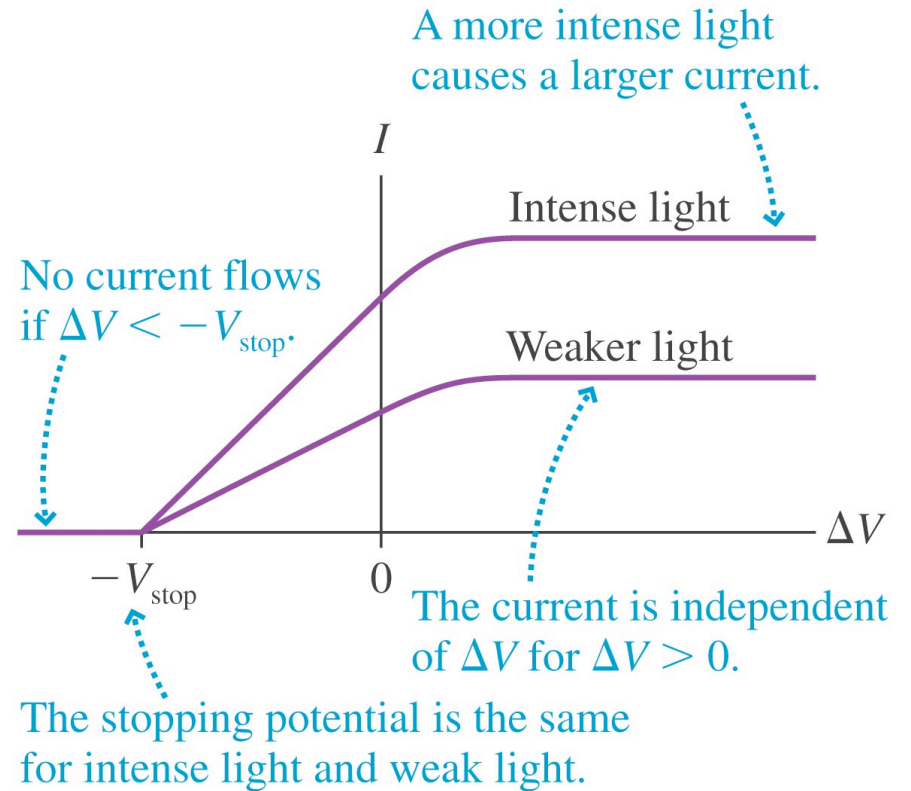
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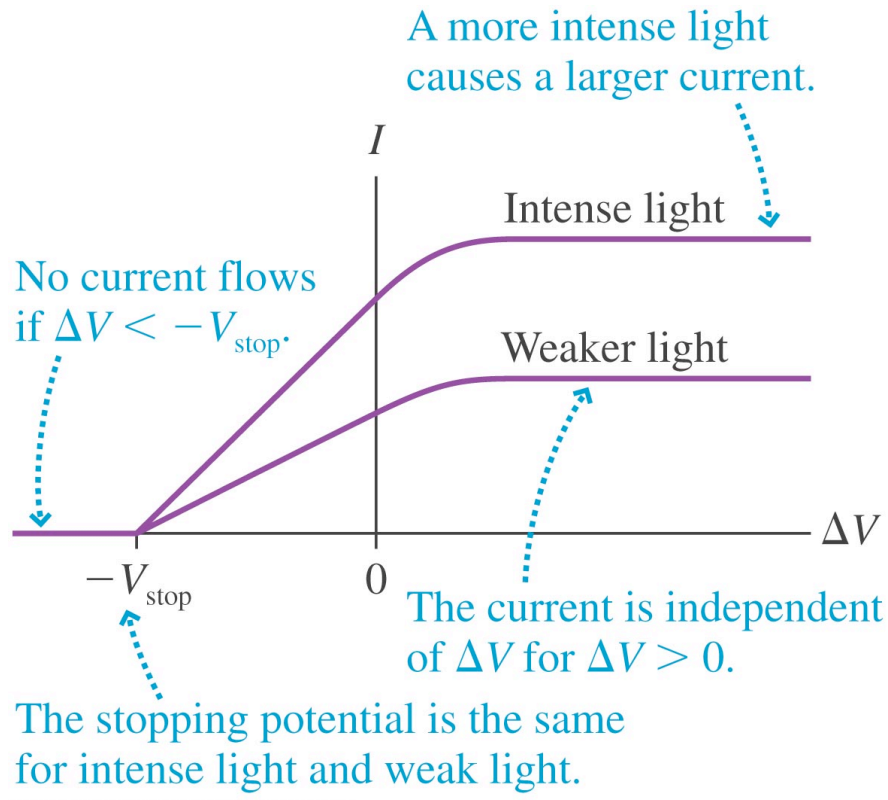


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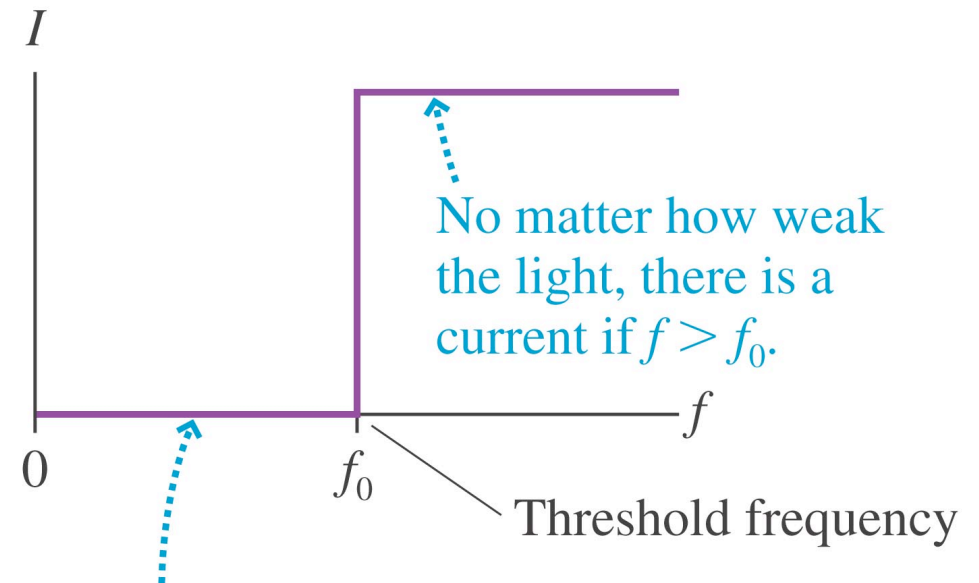
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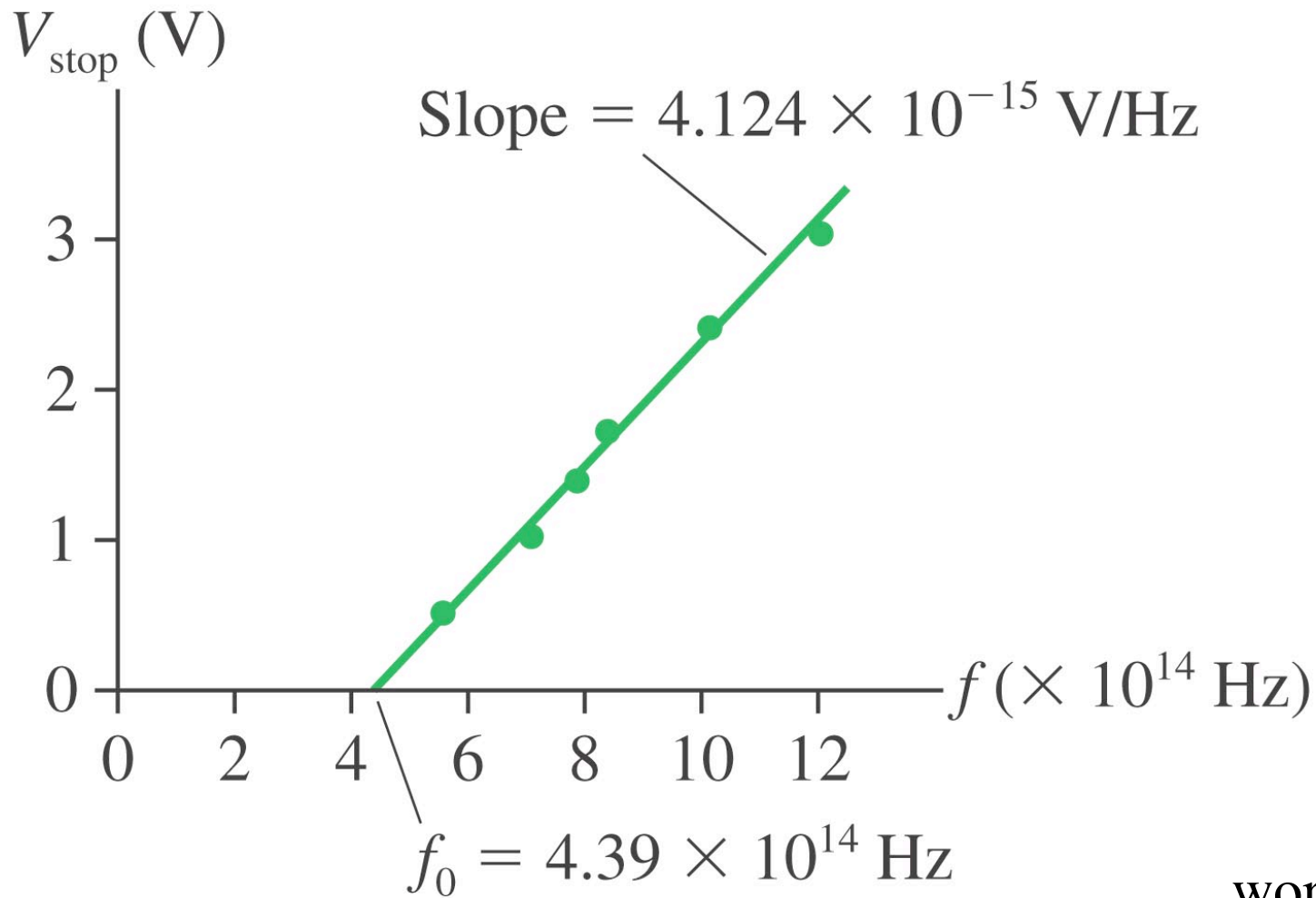
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V_{stop} depends on frequency of light



No matter how intense the light, there is no current if $f < f_0$.

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work function

Millikan's experiment

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{hf - E_0}{e}$$


The work function of metal A is 3.0 eV. Metals B and C have work functions of 4.0 eV and 5.0 eV, respectively. Ultraviolet light shines on all three metals, creating photoelectrons. Rank in order, from largest to smallest, the stopping potential for A, B, and C.

A. $V_C > V_B > V_A$

B. $V_A > V_B > V_C$

C. $V_A = V_B = V_C$


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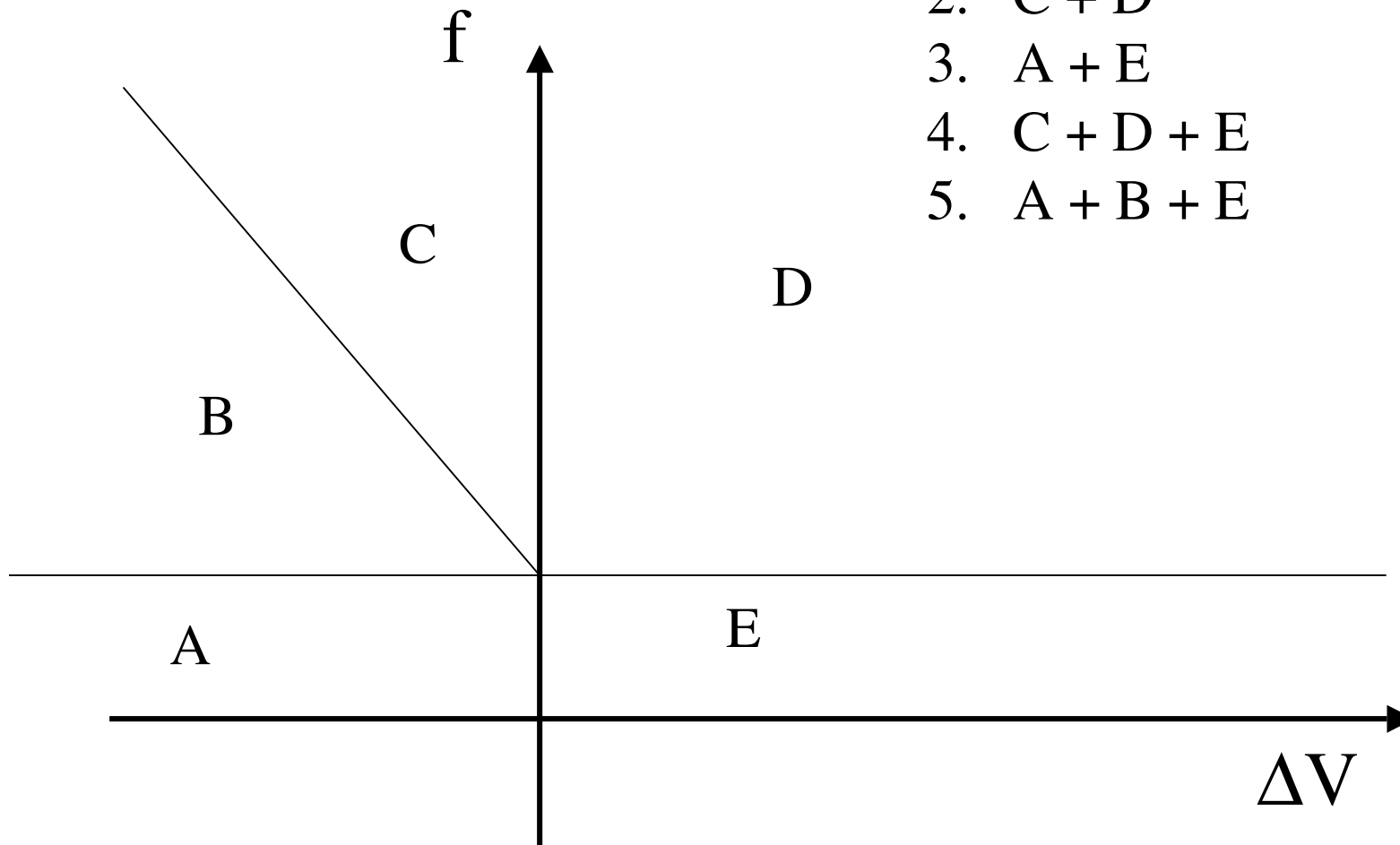
The intensity of a beam of light is increased but the light's frequency is unchanged. Which of the following is true?

- A. The photons are larger.
- B. There are more photons per second.
- C. The photons travel faster.
- D. Each photon has more energy.

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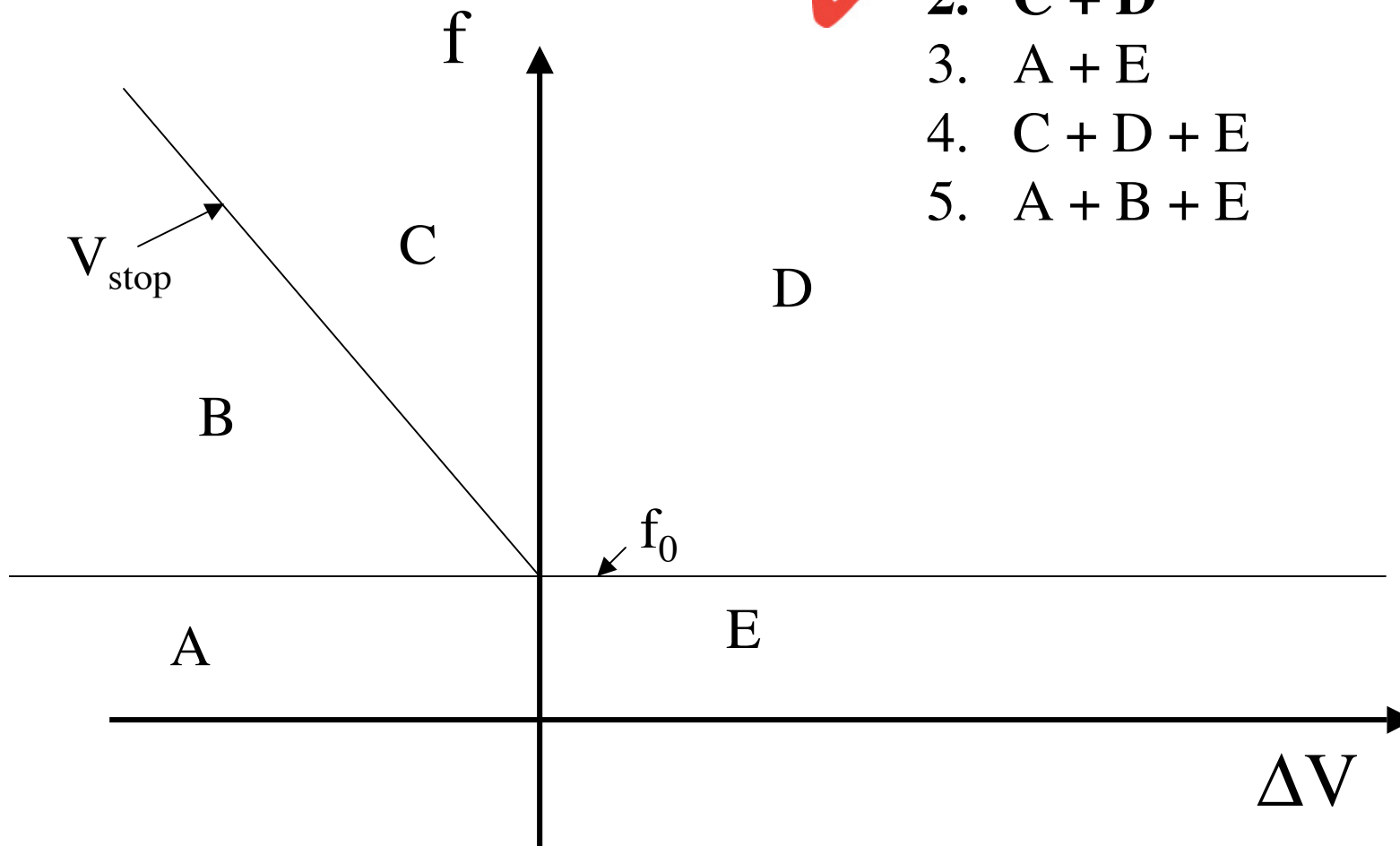
- A. The photons are larger.
-  **B. There are more photons per second.**
- C. The photons travel faster.
- D. Each photon has more energy.

In which regions of parameter space would we collect photoelectrons?

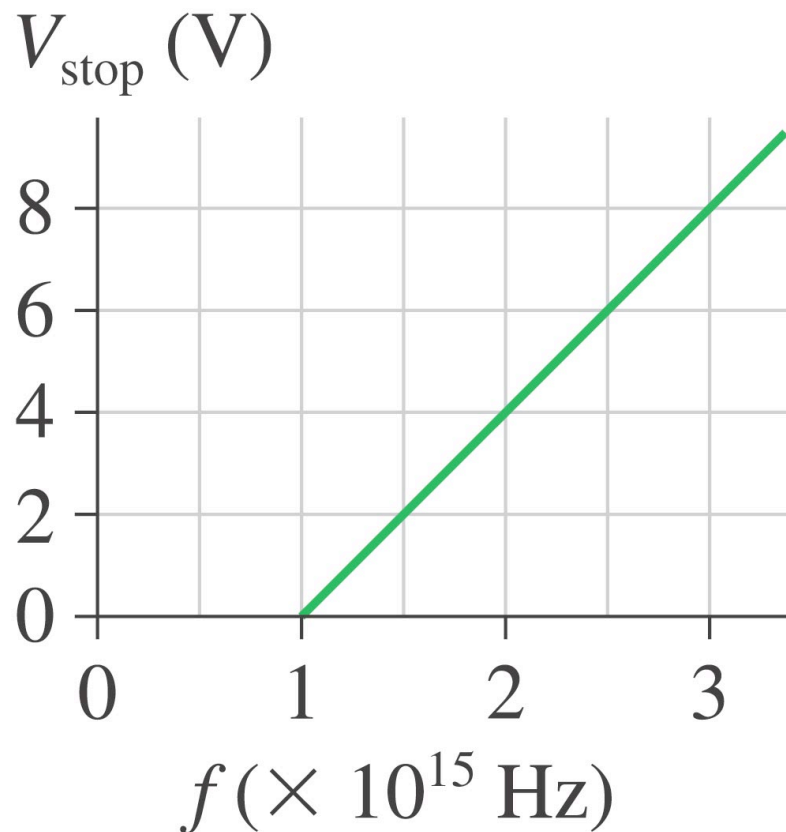


1. $A + B$
2. $C + D$
3. $A + E$
4. $C + D + E$
5. $A + B + E$

In which regions of parameter space would we collect photoelectrons?



1. A + B
2. **C + D**
3. A + E
4. C + D + E
5. A + B + E



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Pictured are the results of an experiment with a cathode of unknown material.

-what is the work function of the cathode?

-what is the experimental value of h ?

$$V_{\text{stop}} = \frac{hf - E_0}{e}$$

$$8V - 0V = \frac{h(3 - 1) \times 10^{15}}{e}$$

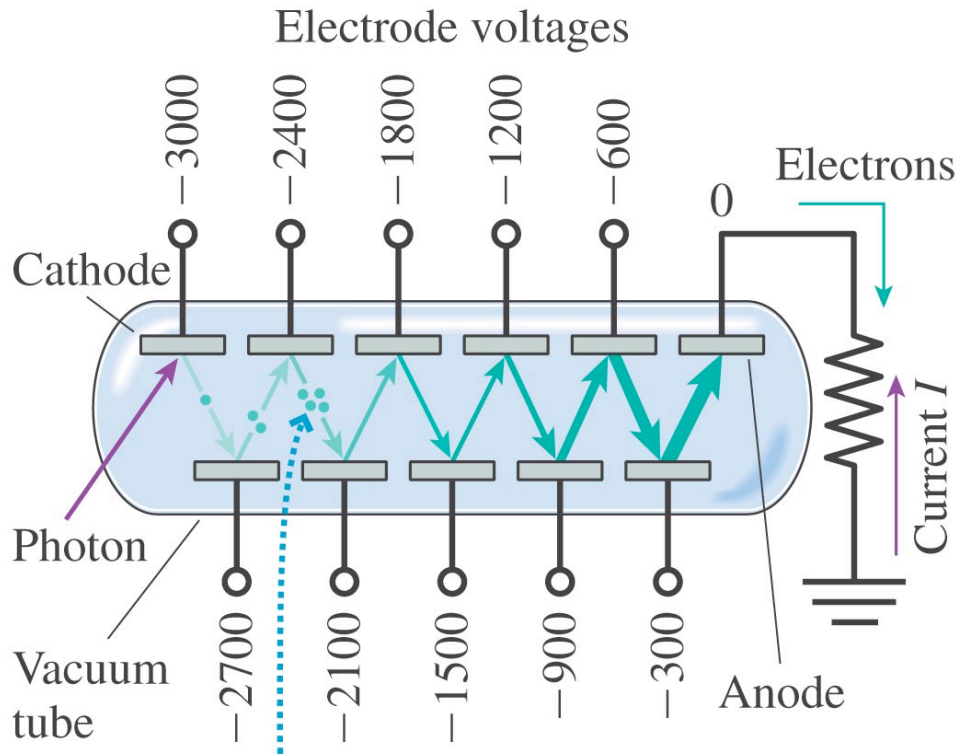
$$h = 4 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$E_0 = hf_0 = h \cdot 1 \times 10^{15}$$

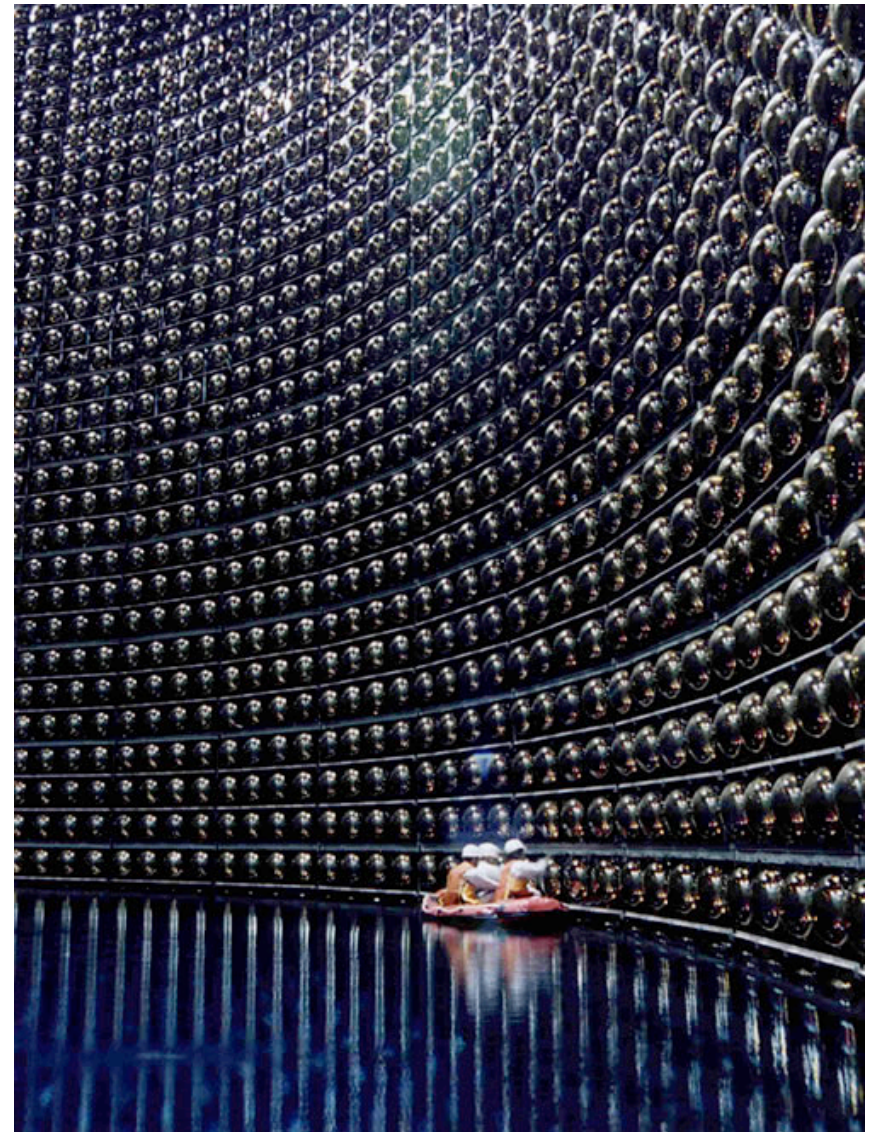
$$E_0 = 4 \text{ eV}$$

Photomultiplier tube measures single photons

(a) A photomultiplier tube



The electron bunch grows after each collision with an electrode.



amazingdata.com/mediadata12/Image/amazing_fun

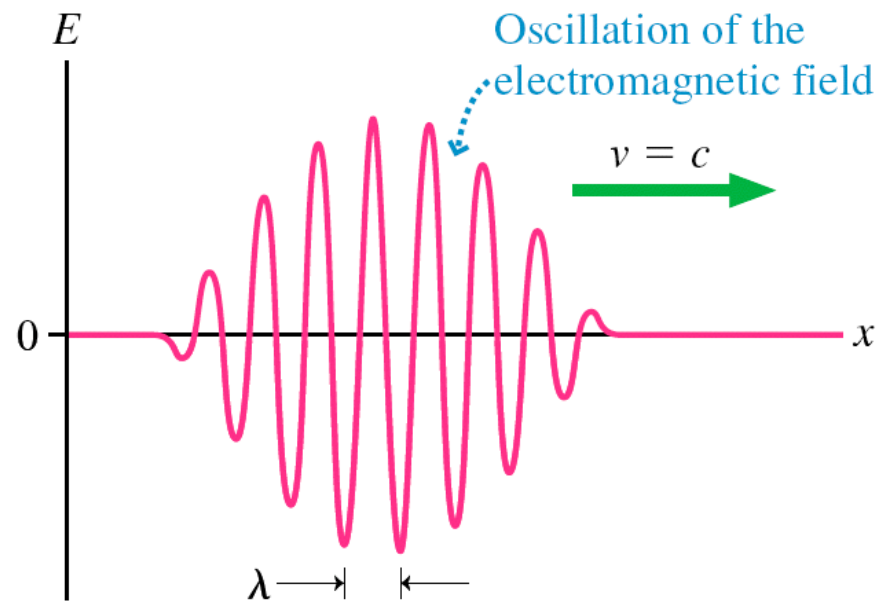
When an electron strikes a metal, multiple electrons can be released by the metal. Secondary emission

Nov. 2001 6k tubes costing \$3k/each imploded in a chain reaction.

Photons

Photons are sometimes visualized as **wave packets**. The electromagnetic wave shown has a wavelength and a frequency, yet it is also discrete and fairly localized.

FIGURE 39.11 A wave packet has wave-like and particle-like properties.



Matter Waves and Energy Quantization

In 1924 de Broglie postulated that *if* a material particle of momentum $p = mv$ has a wave-like nature, then its wavelength must be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad p = h / \lambda$$

where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J s). This is called the **de Broglie wavelength**.

Photons $E = hf$ $E = pc$ $p = h / \lambda$

For light:

$$E = pc$$

$$f = c / \lambda$$

Quantization:

$$E = hf$$

Photon momentum $p = h / \lambda$

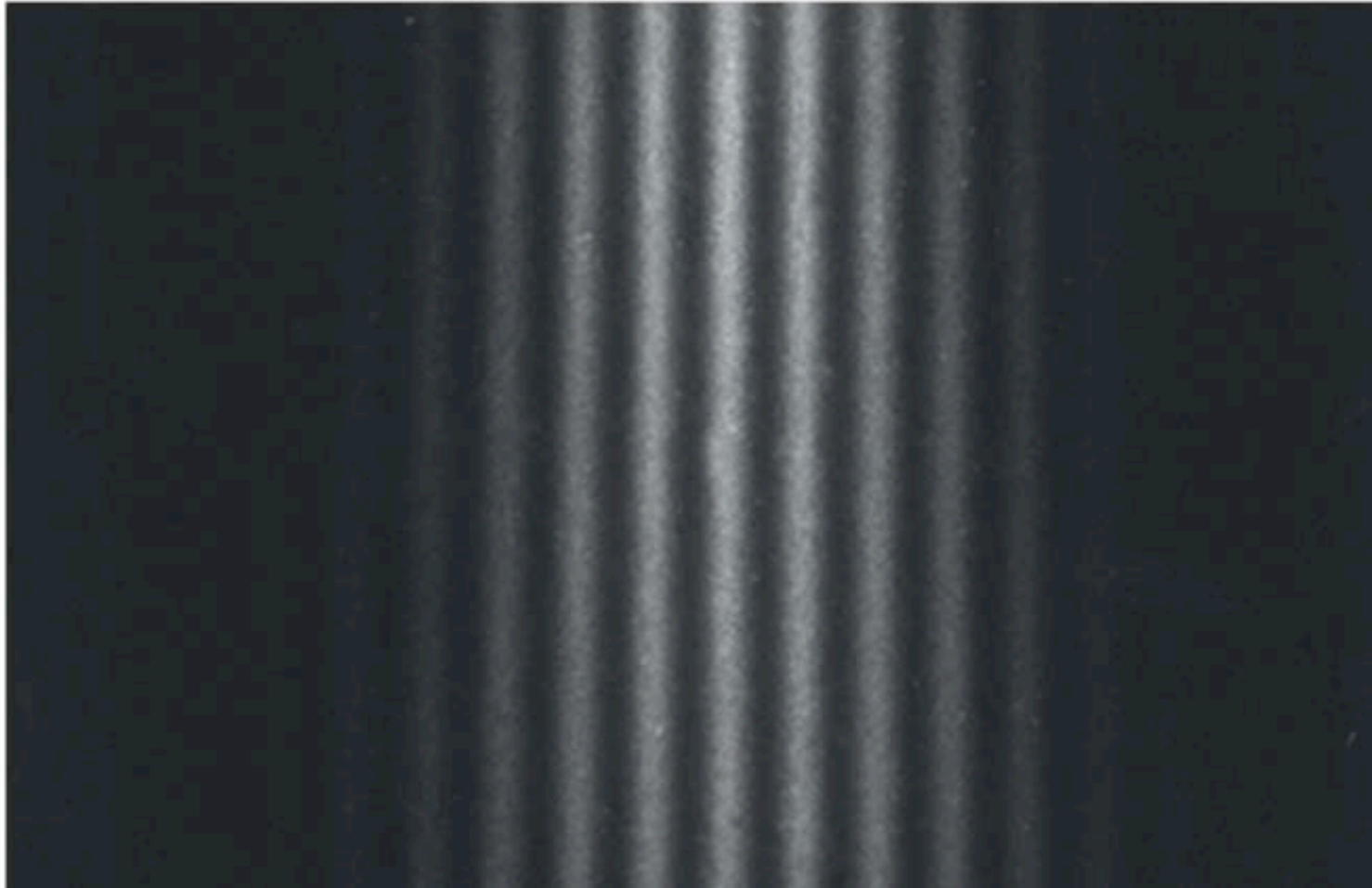
deBroglie assumed that $p = h / \lambda$ is equally fundamental

For nonrelativistic particles $E = \frac{mv^2}{2} = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m}$

What is the particle's frequency?

$$hf = E$$

FIGURE 39.13 A double-slit interference pattern created with electrons.



EXAMPLE 39.6 The de Broglie wavelength of an electron

QUESTION:

EXAMPLE 39.6 The de Broglie wavelength of an electron

What is the de Broglie wavelength of a 1.0 eV electron?

$$\begin{array}{ccc} p = h / \lambda & E = \frac{mv^2}{2} & p = mv \\ \downarrow & \swarrow & \\ \lambda = h / p & v = \sqrt{\frac{2E}{m}} & \end{array}$$

EXAMPLE 39.6 The de Broglie wavelength of an electron

SOLVE An electron with $1.0 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ of kinetic energy has speed

$$v = \sqrt{\frac{2K}{m}} = 5.9 \times 10^5 \text{ m/s}$$

Although fast by macroscopic standards, this is a slow electron because it gains this speed by accelerating through a potential difference of a mere 1 V. Its de Broglie wavelength is

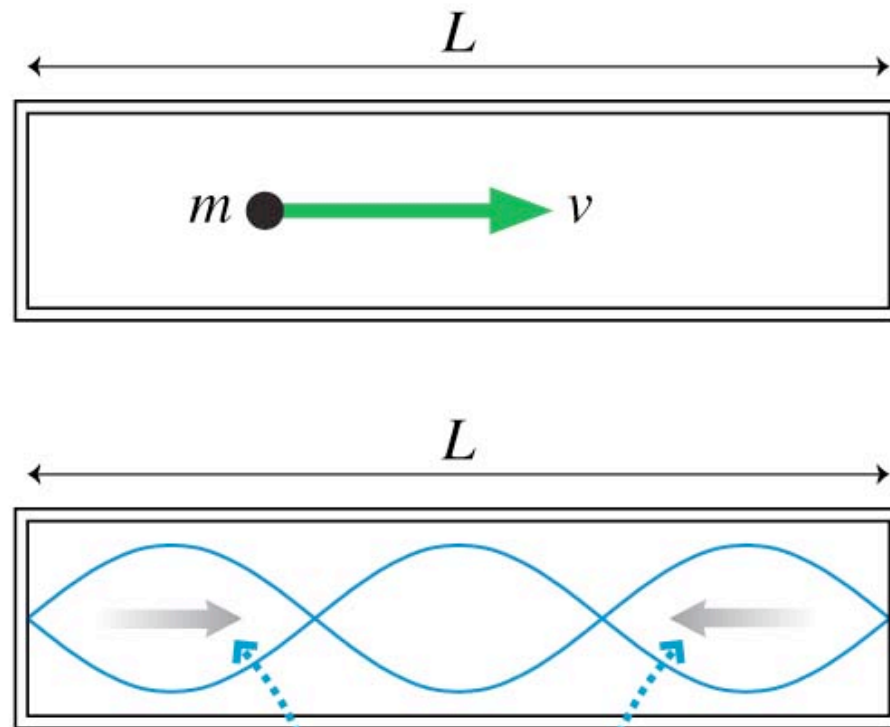
$$\lambda = \frac{h}{mv} = 1.2 \times 10^{-9} \text{ m} = 1.2 \text{ nm}$$

EXAMPLE 39.6 The de Broglie wavelength of an electron

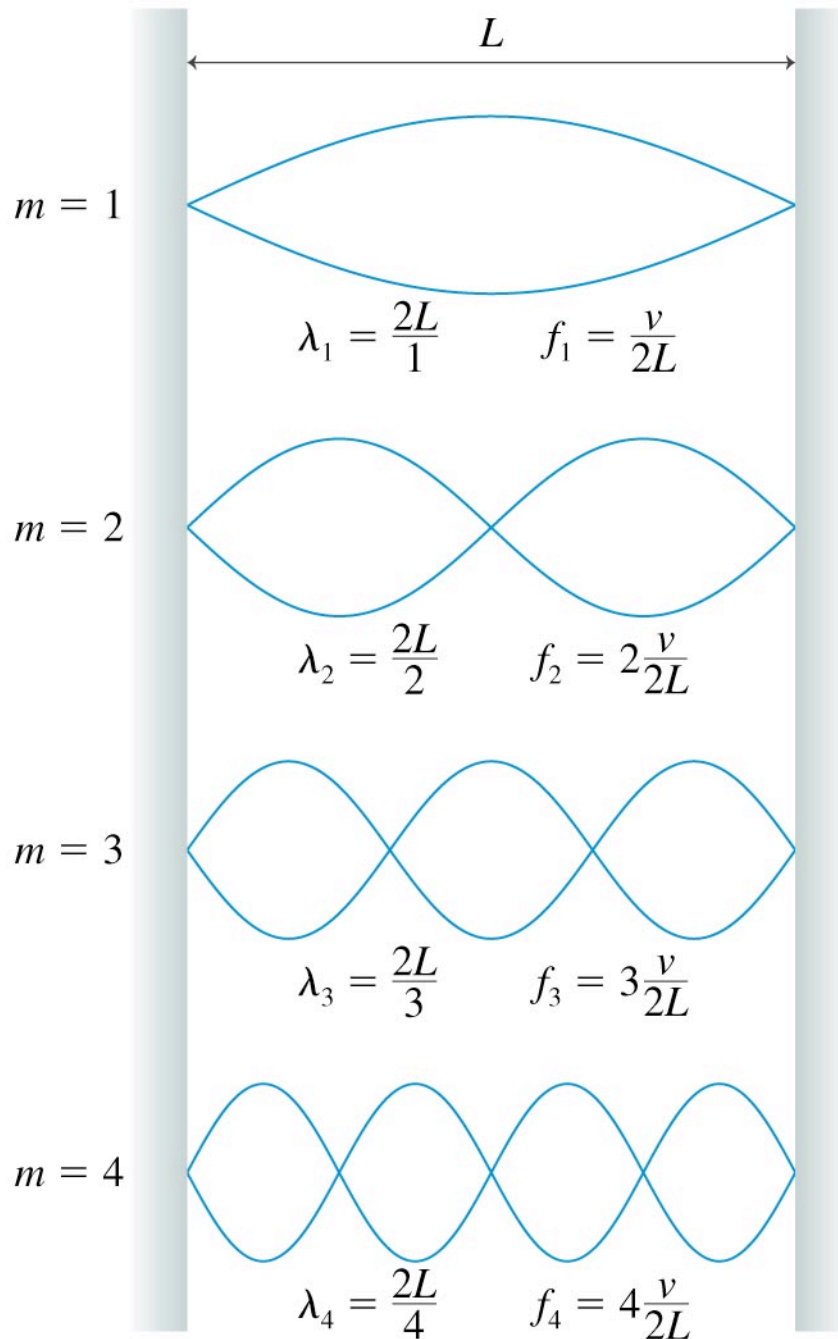
ASSESS The electron's wavelength is small, but it is larger than the wavelengths of x rays and larger than the approximately 10^{-10} m spacing of atoms in a crystal.

Quantization

FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.



Matter waves travel in both directions.



vertical displacement of string

$$\frac{\partial^2 D(x,t)}{\partial t^2} = c_s^2 \frac{\partial^2 D(x,t)}{\partial x^2}$$

$$D(x,t) = D_0 \cos(\omega_m t) \sin\left(\frac{m\pi x}{L}\right)$$

must be zero at $x=0$ and $x=L$

For waves on a string

$$\omega_m = c_s \left(\frac{m\pi}{L}\right)$$

$$f_m = c_s \left(\frac{m}{2L}\right)$$

Quantization of Energy

- Consider a particle of mass m moving in one dimension as it bounces back and forth with speed v between the ends of a box of length L . We'll call this a *one-dimensional box*; its width isn't relevant.
- A wave, if it reflects back and forth between two fixed points, sets up a standing wave.
- A standing wave of length L *must* have a wavelength given by

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4, \dots$$

Quantization of Energy

Using the de Broglie relationship $\lambda = h/mv$, a standing wave with wavelength λ_n forms when the particle has a speed

$$v_n = n \left(\frac{h}{2Lm} \right) \quad n = 1, 2, 3, \dots$$

Thus the particle's energy, which is purely kinetic energy, is

$$E_n = \frac{1}{2}mv_n^2 = n^2 \frac{h^2}{8mL^2} \quad n = 1, 2, 3, \dots$$

De Broglie's hypothesis about the wave-like properties of matter leads us to the remarkable conclusion that **the energy of a confined particle is quantized.**

EXAMPLE 39.8 The energy levels of an electron

QUESTION:

EXAMPLE 39.8 The energy levels of an electron

What are the first three allowed energies for an electron confined in a one-dimensional box of length 0.10 nm, about the size of an atom?

$$E = \frac{mv^2}{2} = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m}$$
$$L = n\lambda / 2$$
$$\downarrow$$
$$E_n = n^2 \frac{h^2}{8mL^2} = n^2 E_1 \quad \longleftarrow \quad \lambda_n = 2L / n$$

EXAMPLE 39.8 The energy levels of an electron

SOLVE We can use Equation 39.16, with $m_{\text{elec}} = 9.11 \times 10^{-31}$ kg and $L = 1.0 \times 10^{-10}$ m to find that the fundamental quantum of energy is $E_1 = 6.0 \times 10^{-18}$ J = 38 eV. Thus the first three allowed energies of an electron in a 0.10 nm box are

$$E_1 = 38 \text{ eV}$$

$$E_2 = 4E_1 = 152 \text{ eV}$$

$$E_3 = 9E_1 = 342 \text{ eV}$$

$$E_n = n^2 E_1 \qquad E_1 = \frac{h^2}{8L^2 m}$$

Electron in a box

$$E_n = n^2 E_1 \quad E_1 = \frac{h^2}{8L^2 m}$$

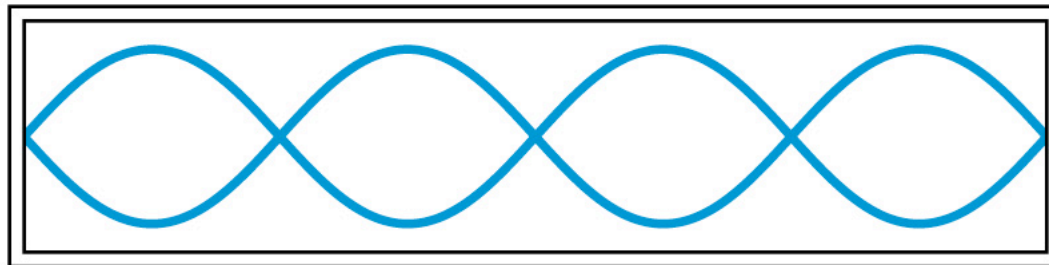
Electron in a hydrogen atom

$$E_n = -\frac{1}{n^2} \frac{e^2}{8\pi\epsilon_0 a_0} = -\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$

Why is one positive and the other negative?

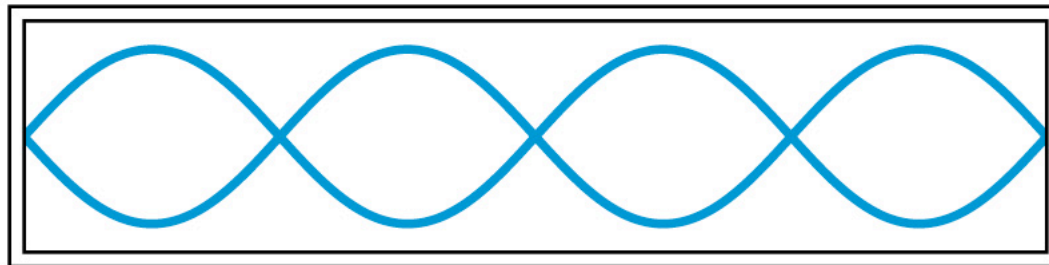
	Atom	Box
Kinetic Energy:	$K = m \frac{v_n^2}{2}$	$K = m \frac{v_n^2}{2}$
Potential Energy:	$U = -\frac{e^2}{4\pi\epsilon_0 r_n}$	$U = 0$

What is the quantum number of this particle confined in a box?



- A. $n = 8$
- B. $n = 6$
- C. $n = 5$
- D. $n = 4$
- E. $n = 3$

What is the quantum number of this particle confined in a box?



- A. $n = 8$
- B. $n = 6$
- C. $n = 5$
- D. $n = 4$
- E. $n = 3$

Electrons confined in identical one dimensional boxes are observed to have energies 12 eV, 27 eV, and 48 eV. What is the length of the boxes?

$$E = \frac{mv^2}{2} = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m}$$

$$L = n\lambda / 2$$

$$\downarrow$$

$$E_n = n^2 \frac{h^2}{8mL^2} = n^2 E_1 \quad \longleftarrow \quad \lambda_n = 2L / n$$

Note:

$$12 = 2^2 3$$

$$27 = 3^2 3$$

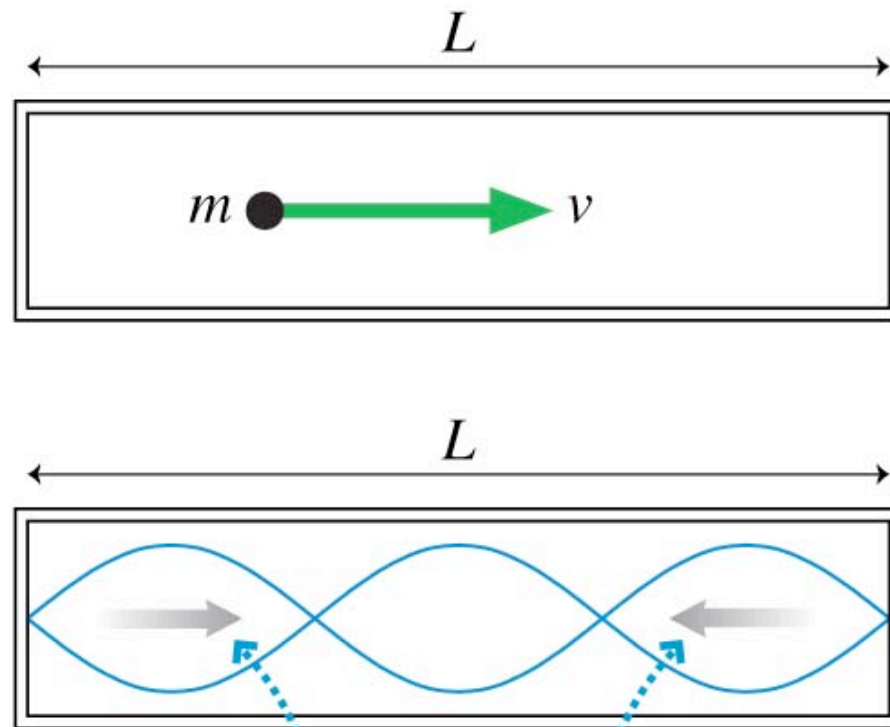
$$48 = 4^2 3$$

$$E_1 = \frac{h^2}{8L^2 m} = 3 \text{ eV}$$

Find L

Quantization

FIGURE 39.15 A particle in a box creates a standing de Broglie wave as it reflects back and forth.



Matter waves travel in both directions.

An electron confined in a one-dimensional box emits a 200 nm photon in a transition from the $n=2$ to $n=1$ states. What is the length of the box?

$$E = \frac{mv^2}{2} = \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m} \qquad L = n\lambda / 2$$

$$E_n = n^2 \frac{h^2}{8mL^2} = n^2 E_1 \qquad \longleftarrow \qquad \lambda_n = 2L / n$$

Photon $E_{\text{photon}} = hf = c / \lambda_{\text{photon}}$

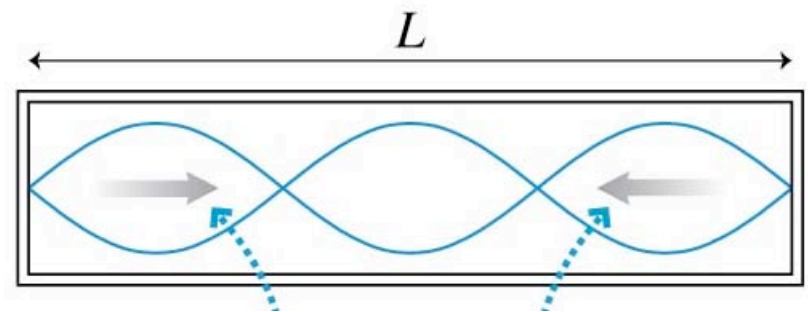
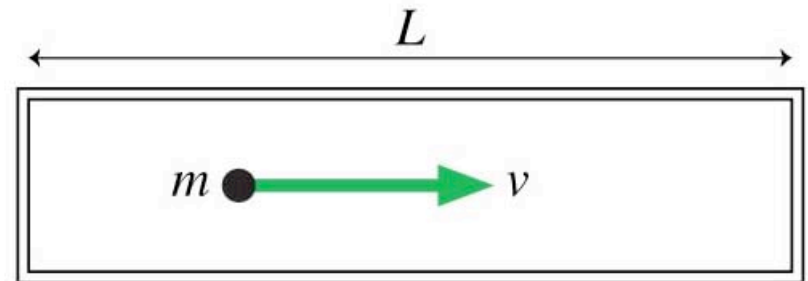
Transition: $E_{\text{photon}} = E_2 - E_1 = (2^2 - 1^2) \frac{h^2}{8L^2 m}$

Find L

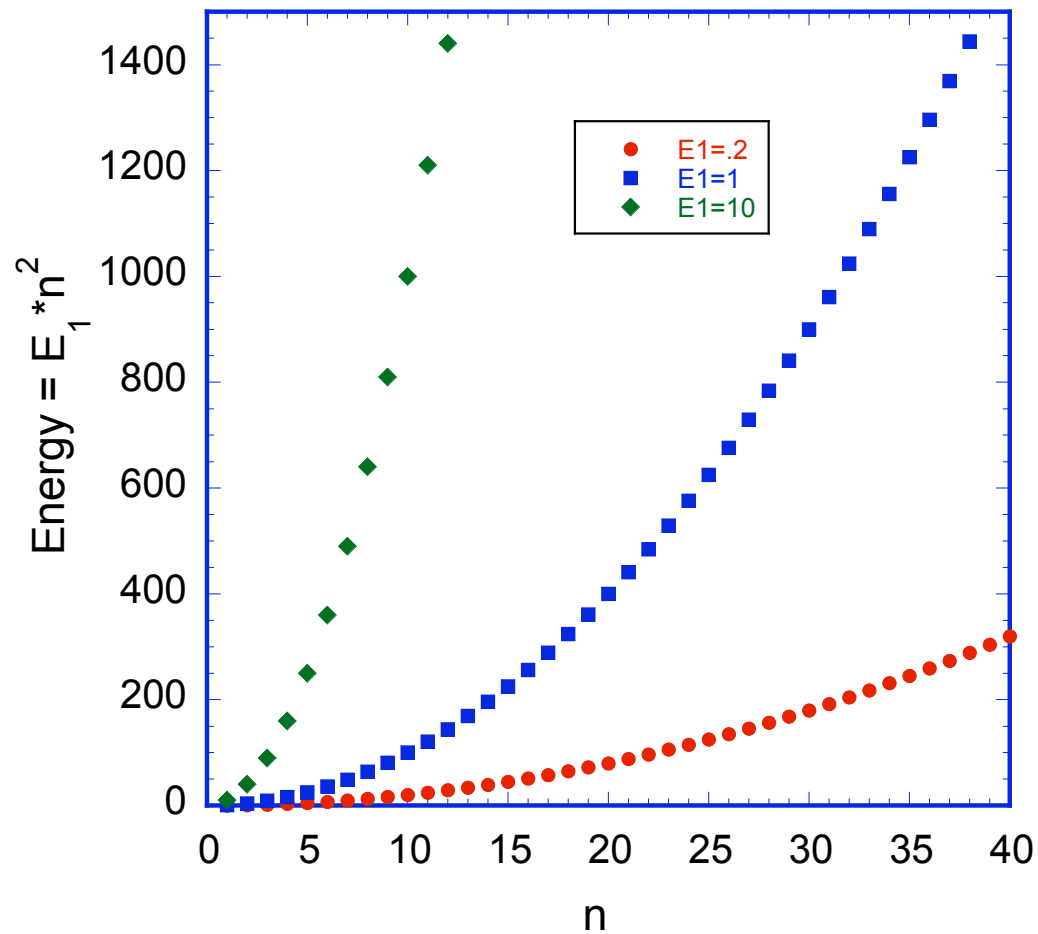
What happens when $L \rightarrow \infty$

$$E_n = n^2 \frac{h^2}{8mL^2} = n^2 E_1$$

$$E_1 = \frac{h^2}{8mL^2} \rightarrow 0$$



Energy becomes a continuous variable.

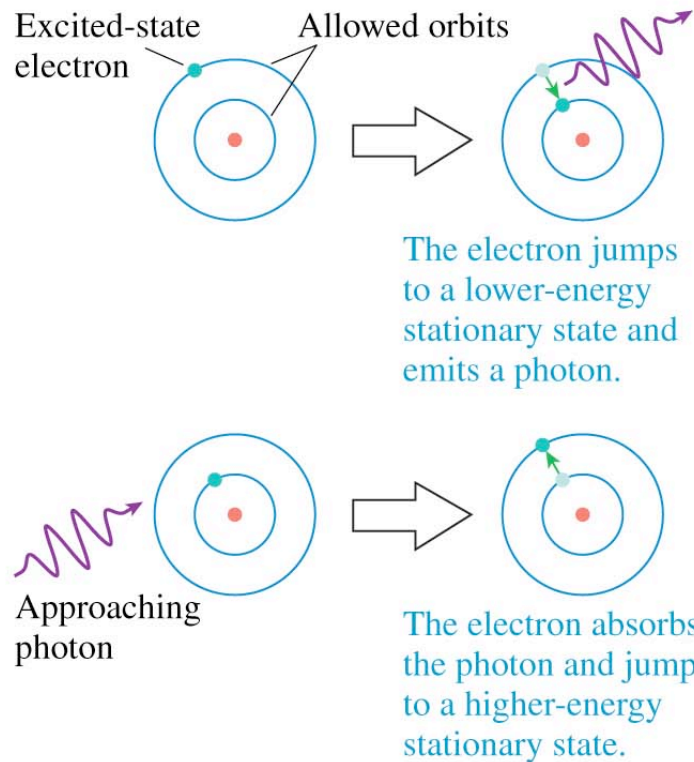


A free particle can have any energy

$$E_1 = \frac{h^2}{8mL^2}$$

FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(a) Emission and absorption of light



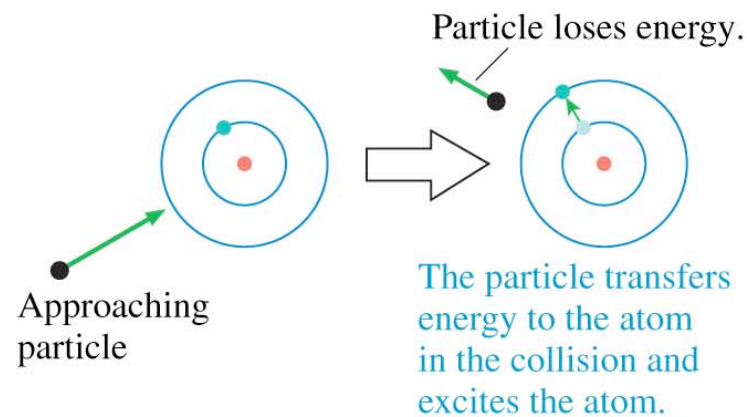
Bohr's Model of Atomic Quantization

6. An atom can move from a lower energy state to a higher energy state by absorbing energy $\Delta E_{\text{atom}} = E_f - E_i$ in an inelastic collision with an electron or another atom.

This process, called **collisional excitation**, is shown.

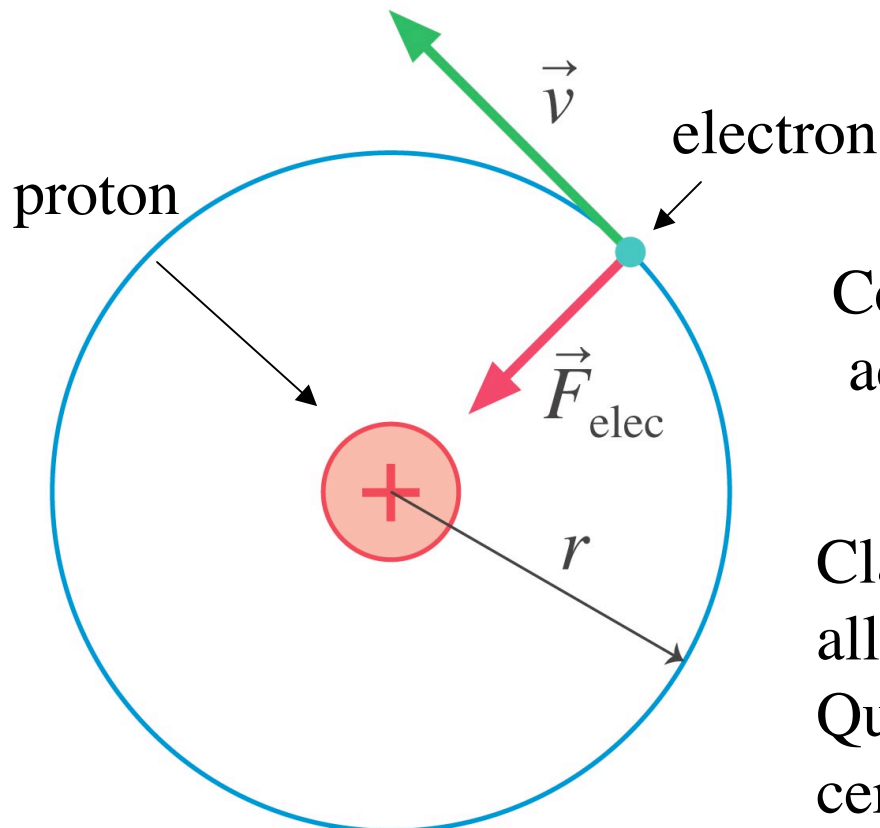
FIGURE 39.17 An atom can change stationary states by emitting or absorbing a photon or by undergoing a collision.

(b) Collisional excitation



39.6 Bohr Model of the Hydrogen Atom (Approximate QM treatment)

Classical Picture



$$m\vec{a} = \vec{F}$$

$$m \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

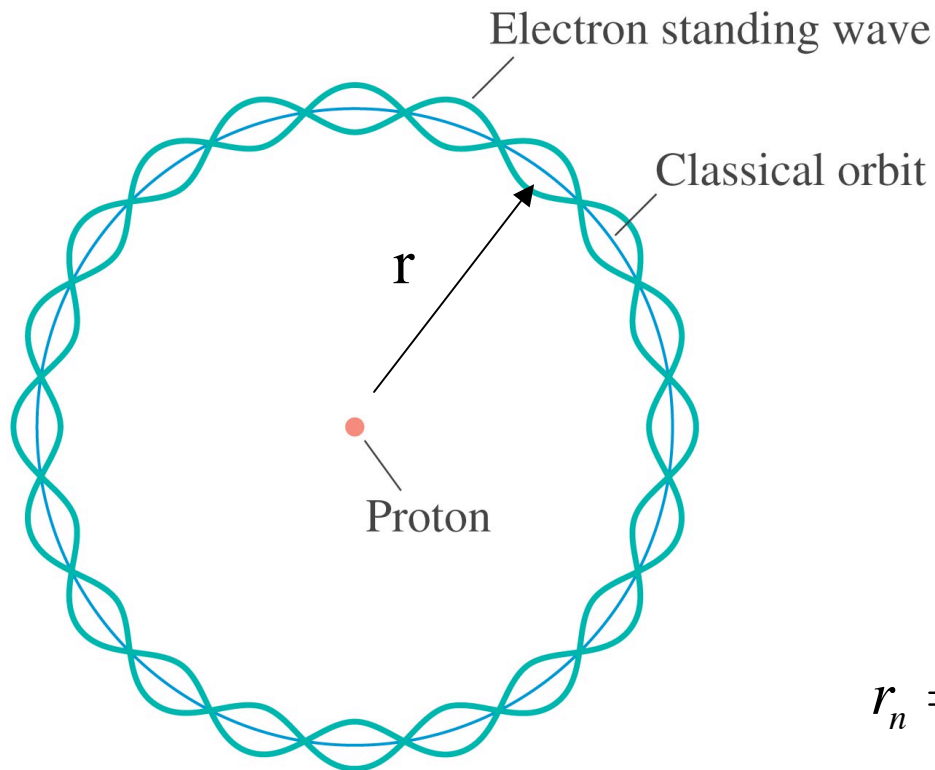
Centripetal
acceleration

Coulomb
force

Classically, any value of v and r are allowed so long as $F=ma$ above. Quantum mechanics says only certain values of r and v are allowed.

Quantum mechanics: Orbit must be an integer # of de Broglie wavelengths

$$2\pi r = n\lambda$$



$$mv = h / \lambda = \frac{hn}{2\pi r}$$

Plug in and solve for r.

$$m \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

Only certain r's are allowed.

$$r_n = n^2 a_0 \quad a_0 = \frac{\epsilon_0 h^2}{\pi m e^2}$$

Bohr radius $a_0 = 5.3 \times 10^{-11} \text{ m}$

What are the total energies (Kinetic + Potential) of these states?

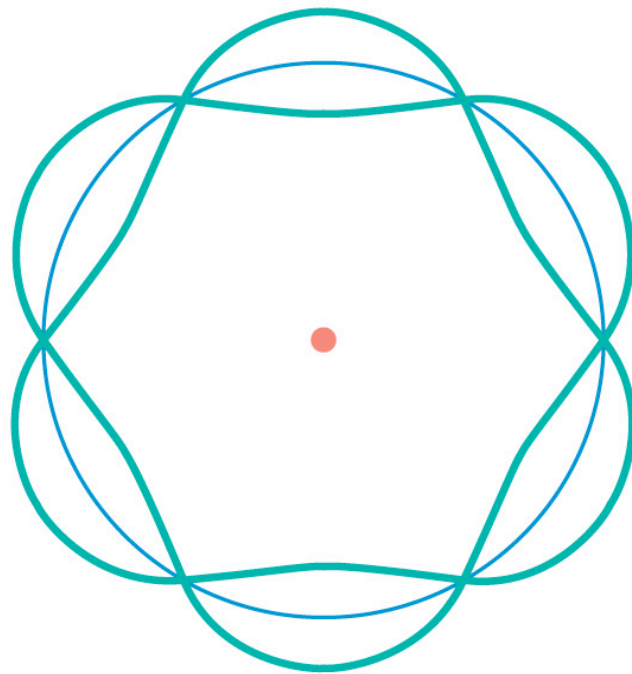
Kinetic Energy: $K = m \frac{v_n^2}{2}$ use
 $mv_n = h / \lambda_n = \frac{hn}{2\pi r_n}$

Potential Energy: $U = -\frac{e^2}{4\pi\epsilon_0 r_n}$

Combining, $E=K+U$ $E_n = -\frac{1}{n^2} \frac{e^2}{8\pi\epsilon_0 a_0} = -\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$

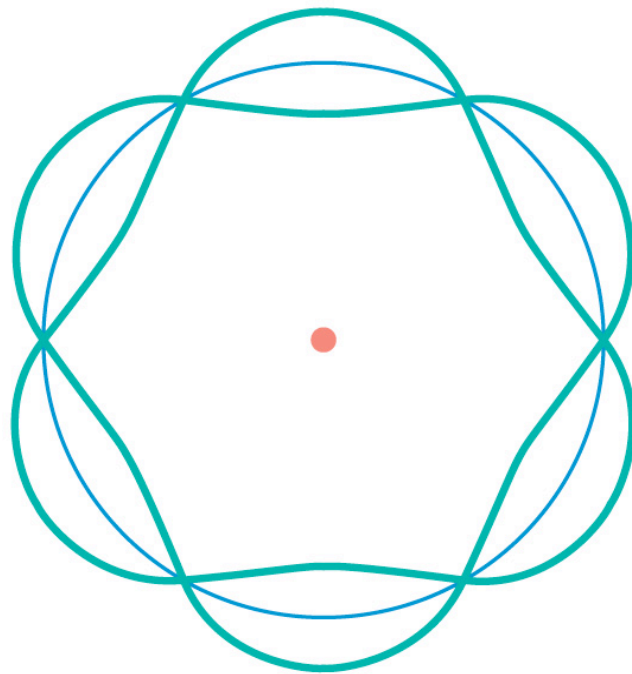
Gives Balmer spectrum

What is the quantum number of this hydrogen atom?



- A. $n = 5$
- B. $n = 4$
- C. $n = 3$
- D. $n = 2$
- E. $n = 1$

What is the quantum number of this hydrogen atom?



- A. $n = 5$
- B. $n = 4$
- C. $n = 3$
- D. $n = 2$
- E. $n = 1$

Quantization of angular momentum

Circumference is integer wavelengths

$$2\pi r = n\lambda$$

Momentum given by deBroglie

$$mv = h / \lambda = \frac{hn}{2\pi r}$$

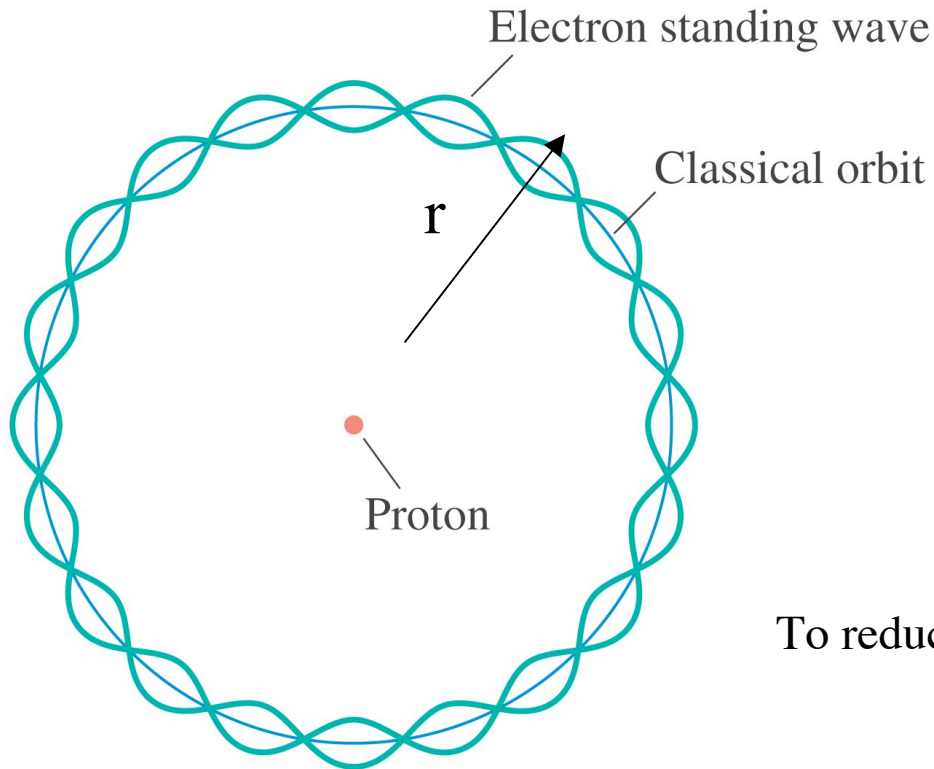
What is angular momentum?

$$l = rmv = r \frac{hn}{2\pi r} = n \frac{h}{2\pi} = n\hbar$$

Integer number of Planck constants

To reduce the number of times we have to write 2π

$$\hbar = \frac{h}{2\pi}$$



Classical wave: $\cos[kx - \omega t]$

Wave number $k = 2\pi / \lambda$

Momentum $p = h / \lambda = \frac{hk}{2\pi} = \hbar k$

Frequency $\omega = 2\pi f$

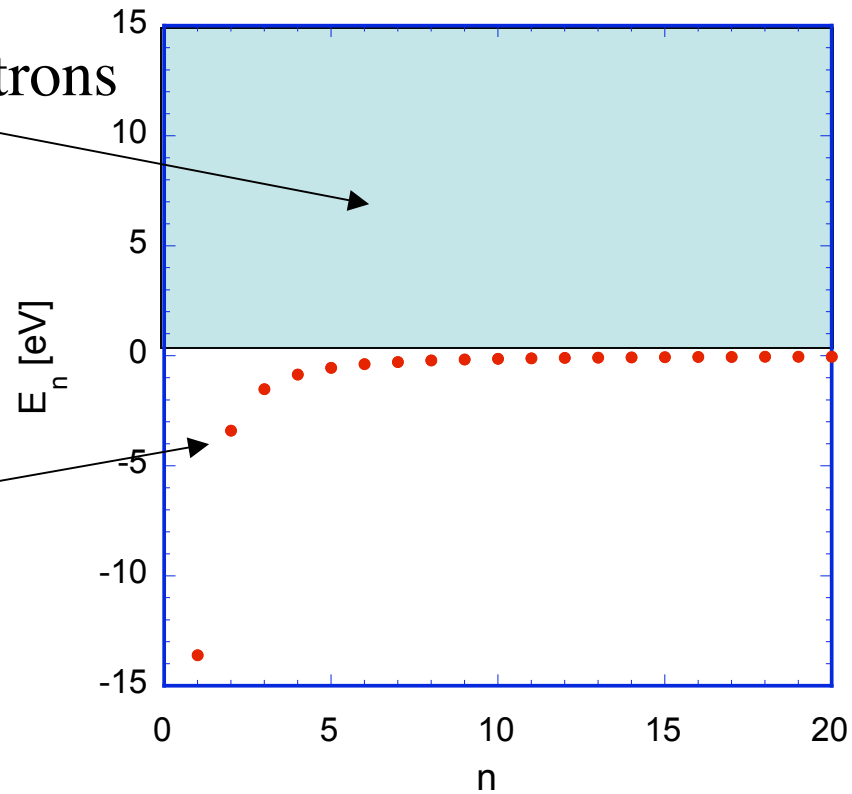
Energy $E = hf = \frac{h\omega}{2\pi} = \hbar\omega$

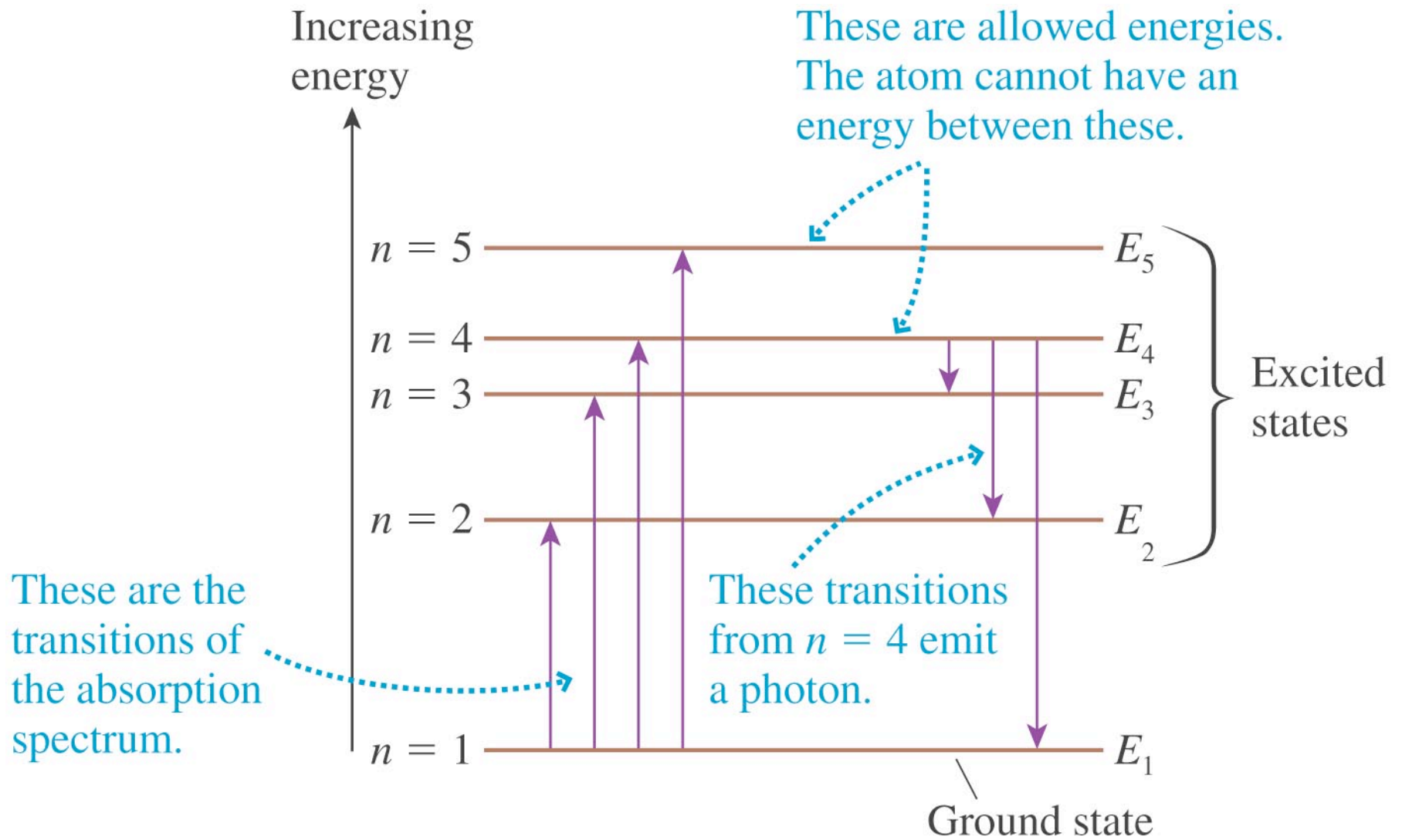
Bound electron energy

$$E_n = -\frac{1}{n^2} \frac{e^2}{8\pi\epsilon_0 a_0} = -\frac{2.18 \times 10^{-18} \text{ J}}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

Here there be free electrons

Bound electrons





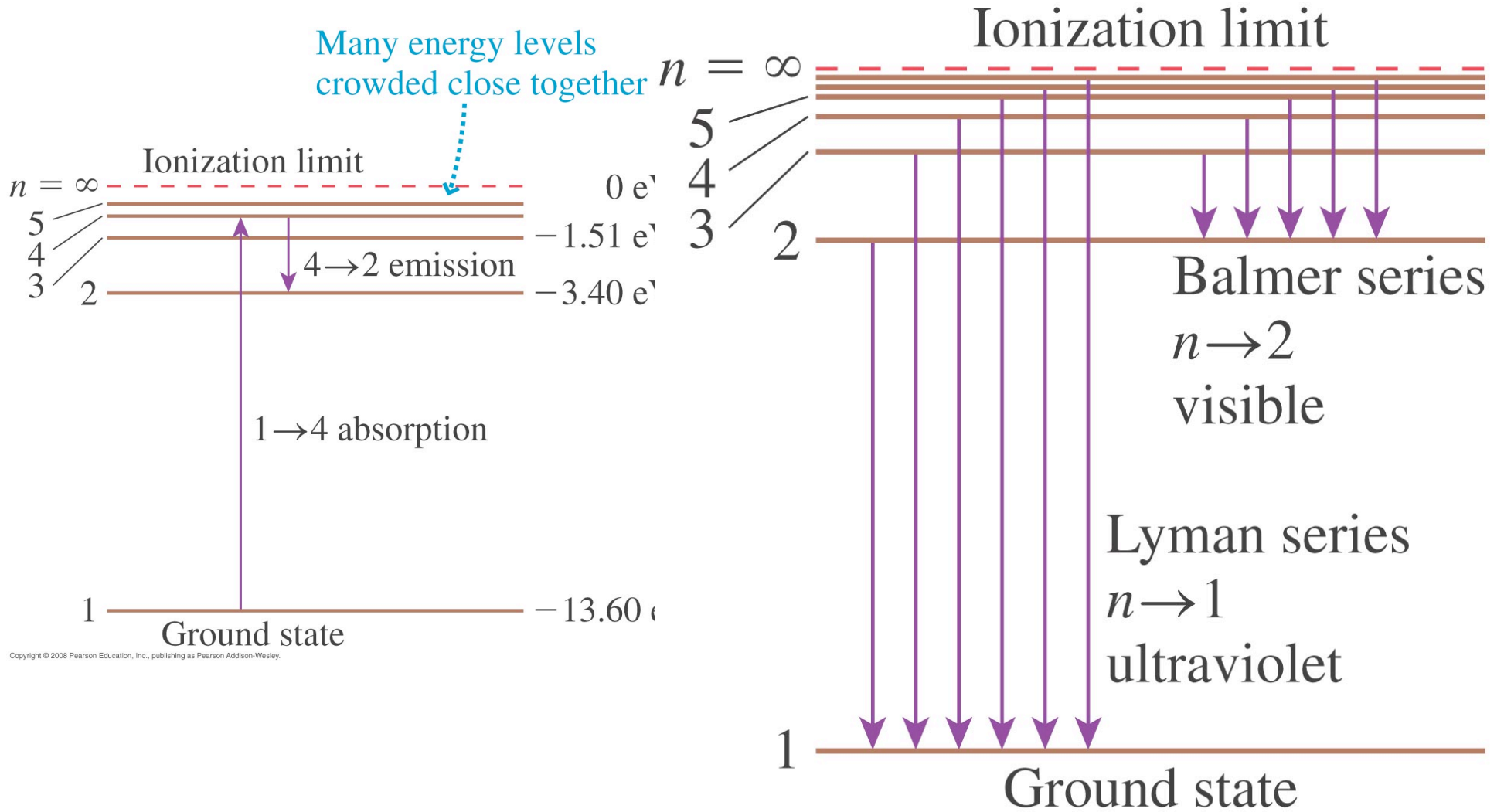
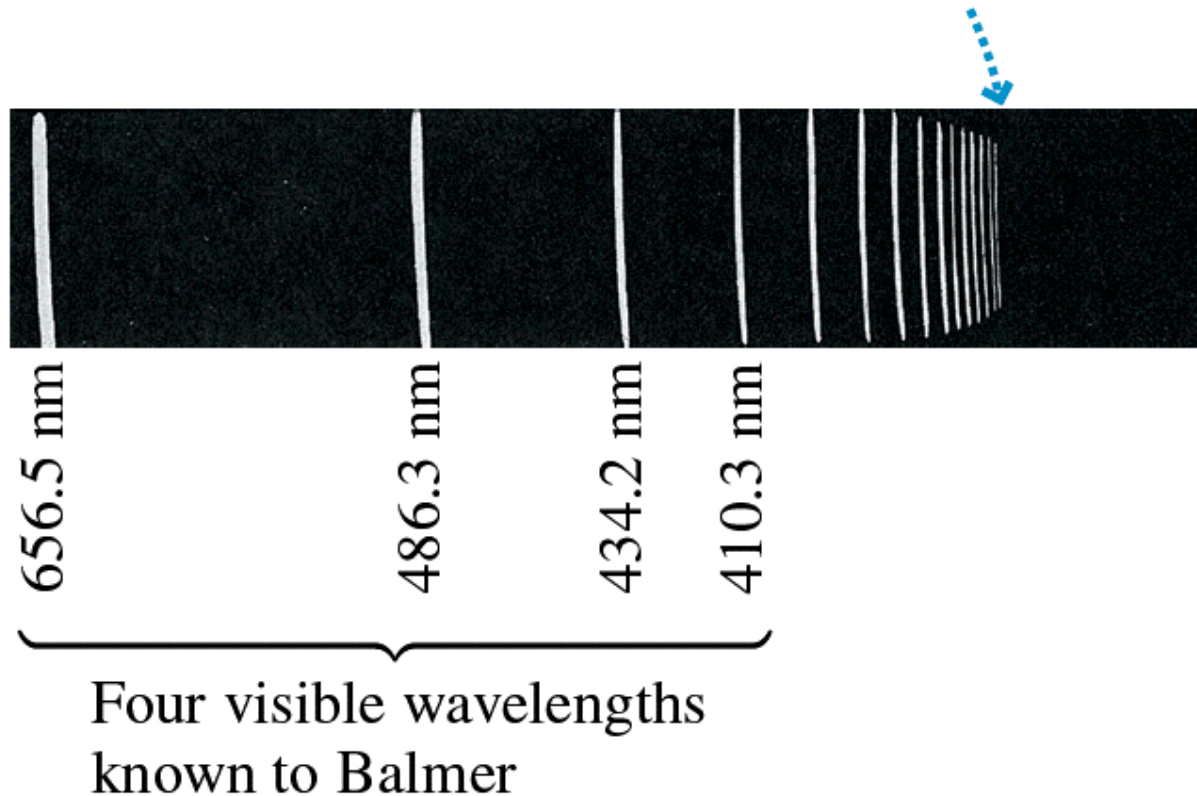
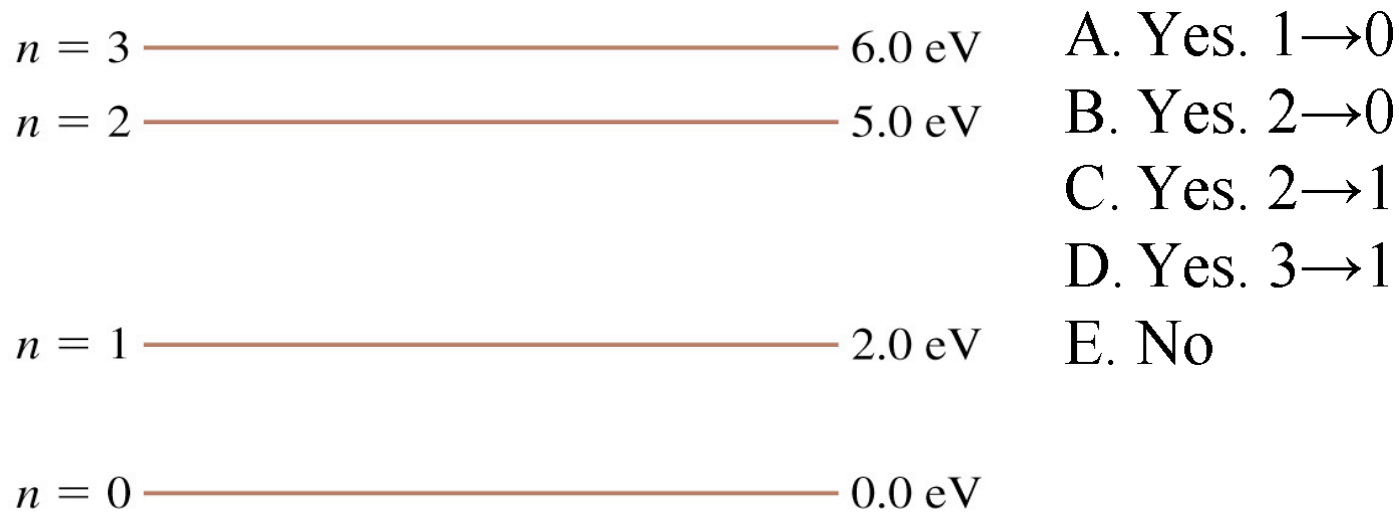


FIGURE 25.3 The Balmer series of hydrogen as seen on the photographic plate of a spectrometer.

The spectral lines extend to the series limit at 364.7 nm.

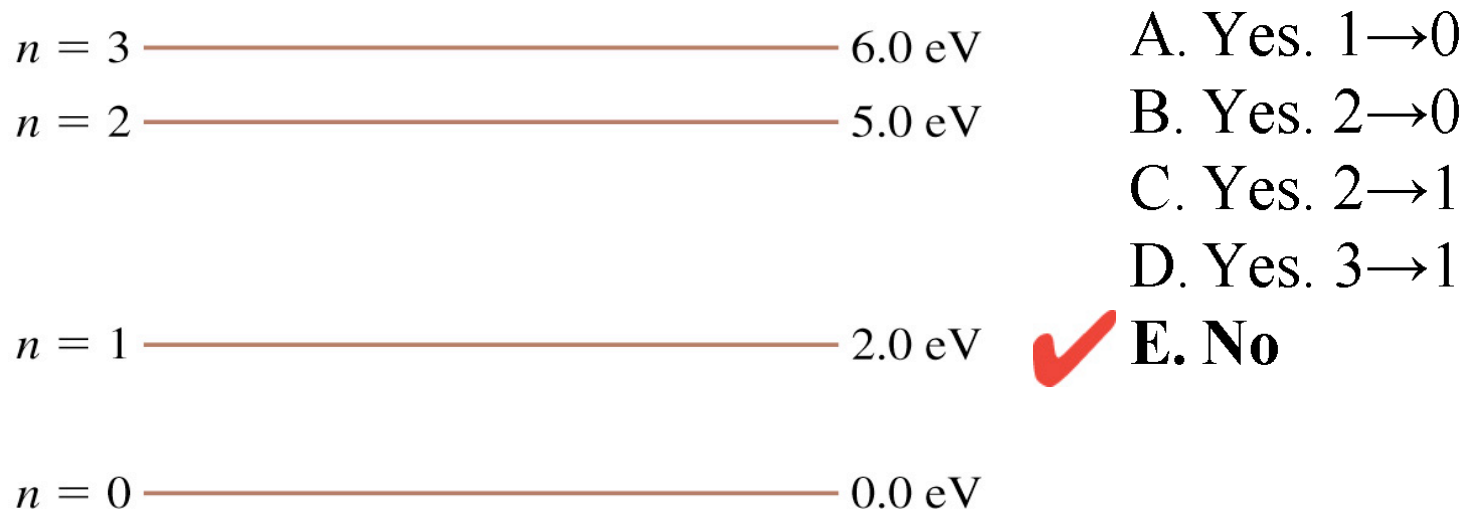


A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the absorption spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will absorb it?



- A. Yes. $1 \rightarrow 0$
- B. Yes. $2 \rightarrow 0$
- C. Yes. $2 \rightarrow 1$
- D. Yes. $3 \rightarrow 1$
- E. No

A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the absorption spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will absorb it?



A. Yes. $1 \rightarrow 0$

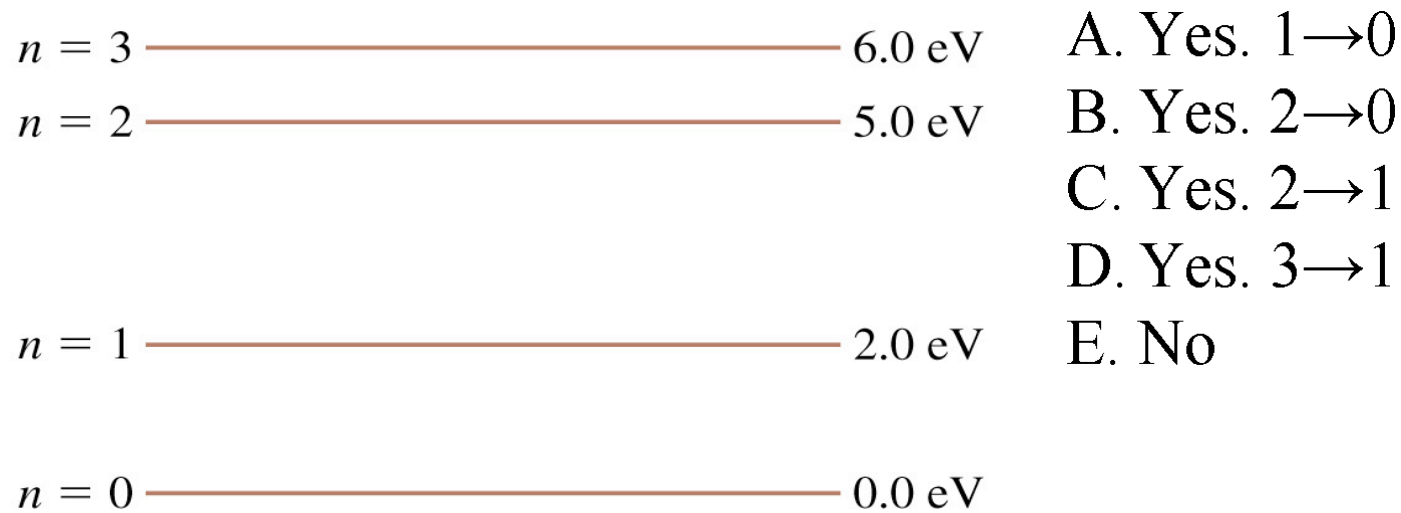
B. Yes. $2 \rightarrow 0$

C. Yes. $2 \rightarrow 1$

D. Yes. $3 \rightarrow 1$

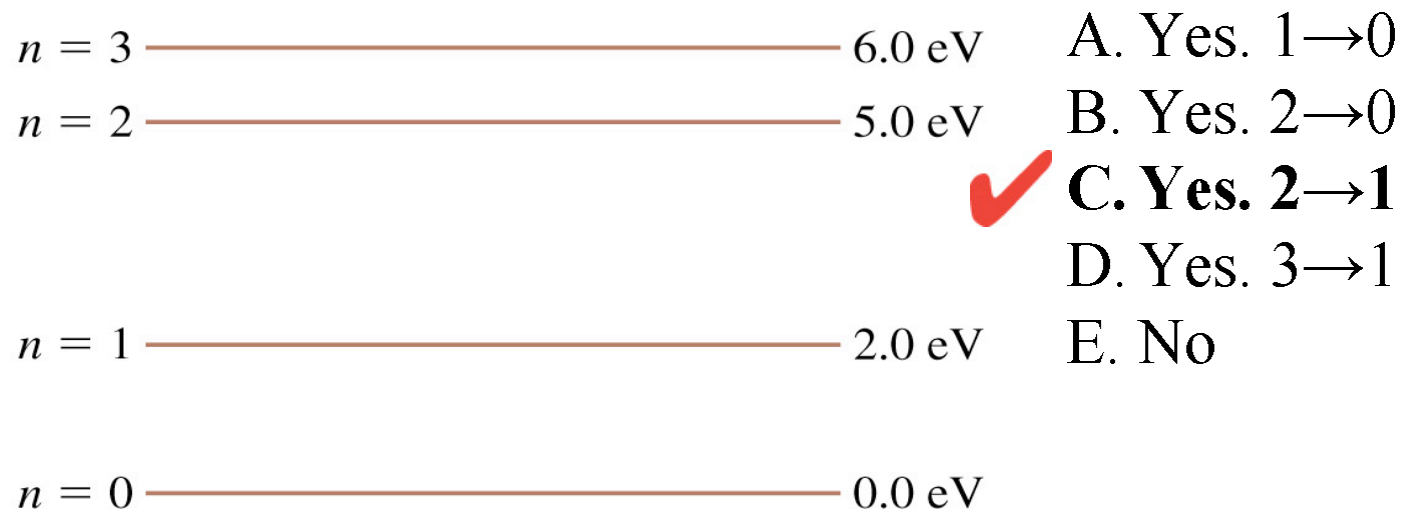
 **E. No**

A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?

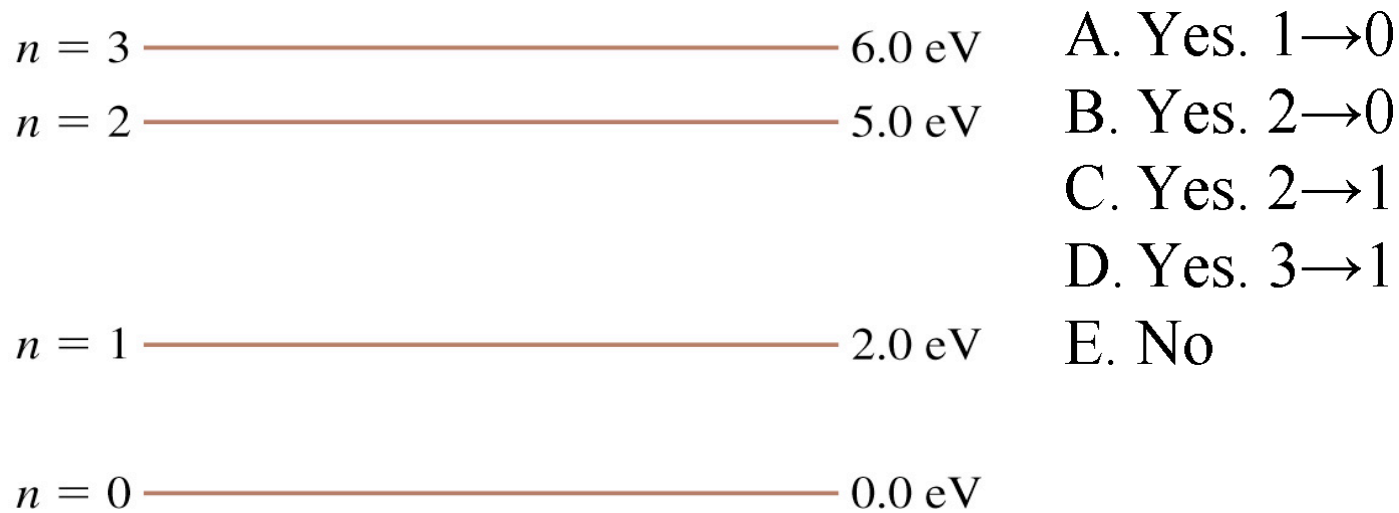


- A. Yes. $1 \rightarrow 0$
- B. Yes. $2 \rightarrow 0$
- C. Yes. $2 \rightarrow 1$
- D. Yes. $3 \rightarrow 1$
- E. No

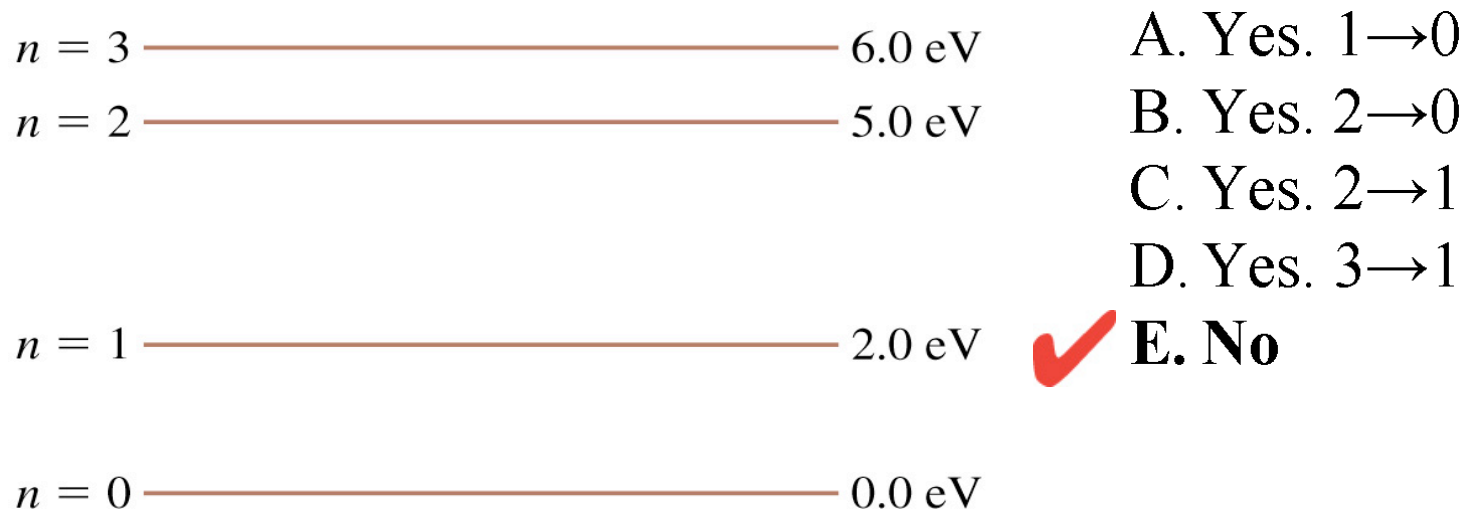
A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0$ eV. Do you expect to see a spectral line with $\lambda = 414$ nm in the emission spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will emit it?



A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the absorption spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will absorb it?



A photon with a wavelength of 414 nm has energy $E_{\text{photon}} = 3.0 \text{ eV}$. Do you expect to see a spectral line with $\lambda = 414 \text{ nm}$ in the absorption spectrum of the atom represented by this energy-level diagram? If so, what transition or transitions will absorb it?



A. Yes. $1 \rightarrow 0$

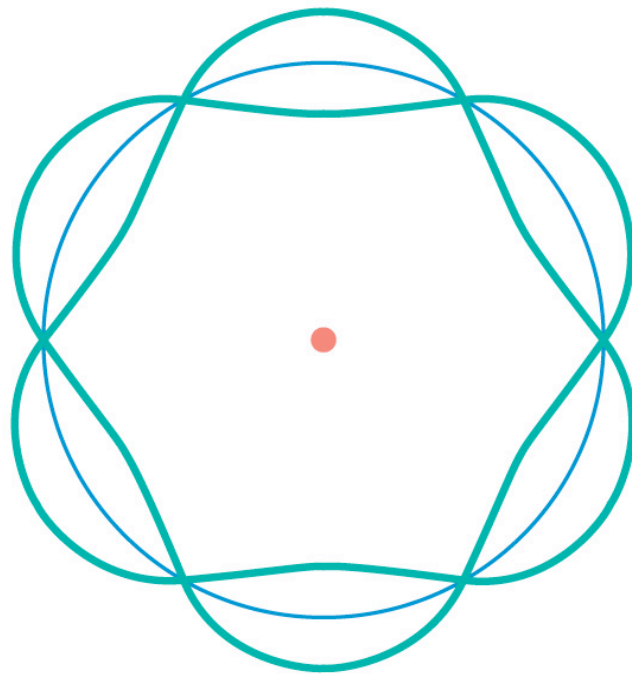
B. Yes. $2 \rightarrow 0$

C. Yes. $2 \rightarrow 1$

D. Yes. $3 \rightarrow 1$

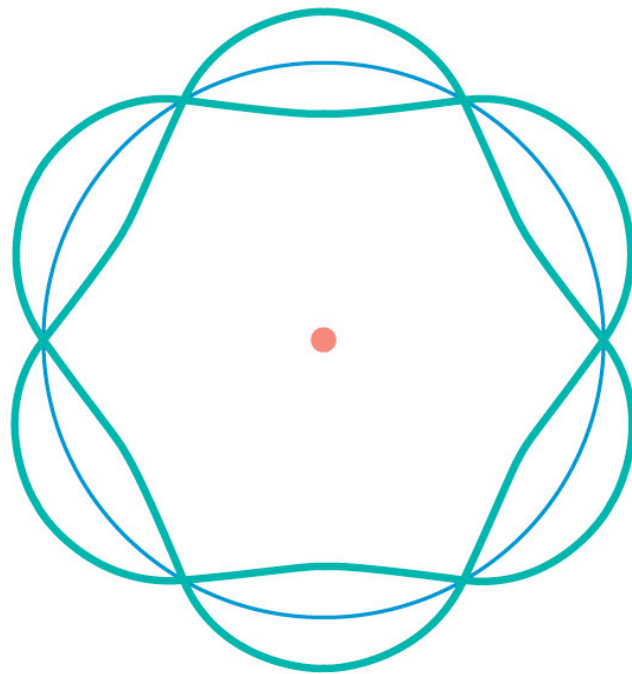
 E. No

What is the quantum number of this hydrogen atom?



- A. $n = 5$
- B. $n = 4$
- C. $n = 3$
- D. $n = 2$
- E. $n = 1$

What is the quantum number of this hydrogen atom?



- A. $n = 5$
- B. $n = 4$
- C. $n = 3$
- D. $n = 2$
- E. $n = 1$

Bohr's Model of Atomic Quantization

1. An atom consists of negative electrons orbiting a very small positive nucleus.
2. Atoms can exist only in certain **stationary states**. Each stationary state corresponds to a particular set of electron orbits around the nucleus. These states can be numbered 1, 2, 3, 4, . . . , where n is the *quantum number*.
3. Each stationary state has an energy E_n . The stationary states of an atom are numbered in order of increasing energy: $E_1 < E_2 < E_3 < \dots$
4. The lowest energy state of the atom E_1 is *stable* and can persist indefinitely. It is called the **ground state** of the atom. Other stationary states with energies E_2, E_3, E_4, \dots are called **excited states** of the atom.

Bohr's Model of Atomic Quantization

5. An atom can “jump” from one stationary state to another by emitting or absorbing a photon of frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{atom}}}{h}$$

where h is Planck's constant and $\Delta E_{\text{atom}} = |E_f - E_i|$.

E_f and E_i are the energies of the initial and final states. Such a jump is called a **transition** or, sometimes, a **quantum jump**.

FIGURE 39.18 An energy-level diagram.

