# Chapter 34: Electromagnetic Induction

A time changing magnetic field induces an electric field.

This electric field does not satisfy Coulombs Law

## Faraday's Discovery

A current in a coil is induced if the magnetic field through the coil is changing in time.

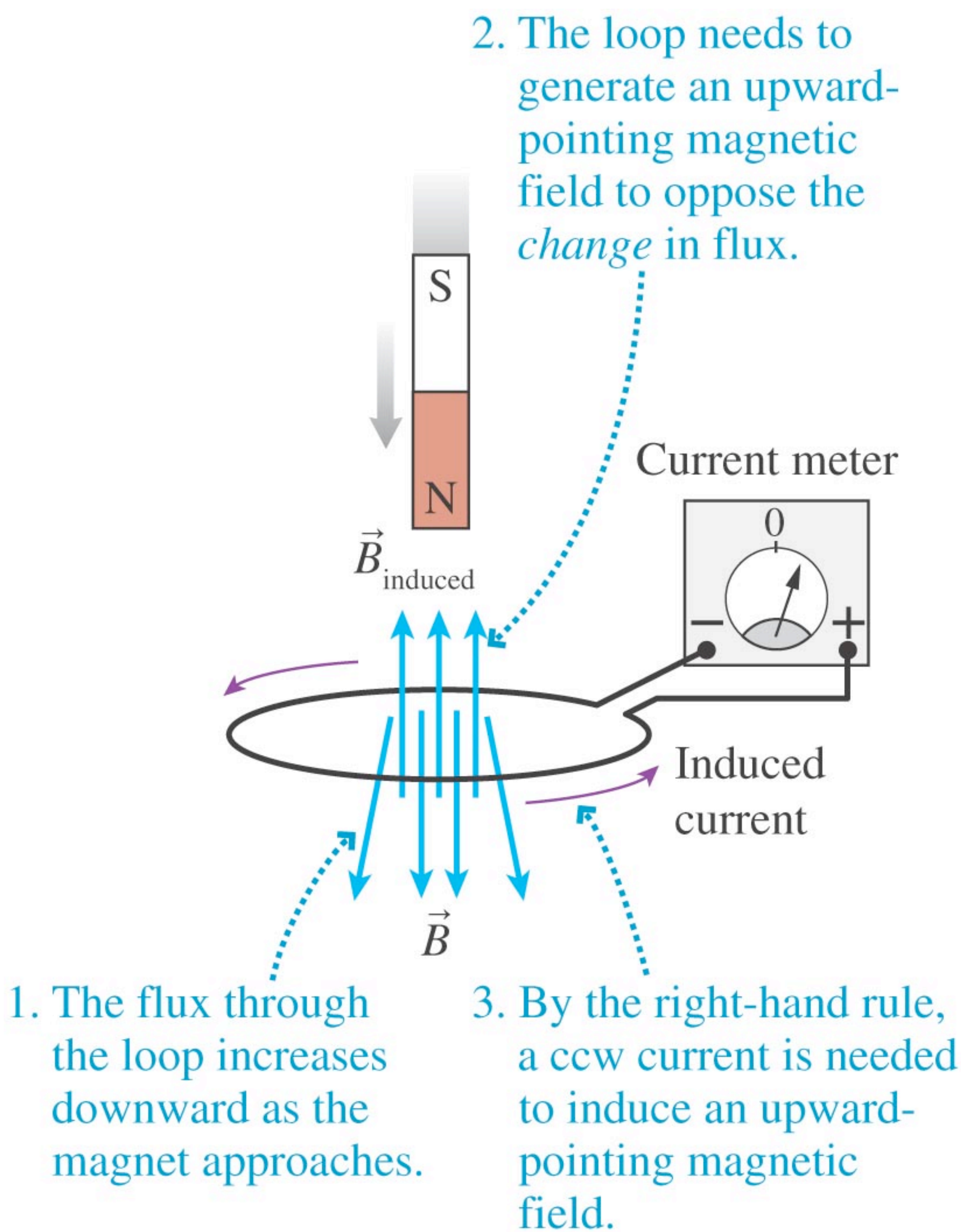
The current can be induced two different ways:

1. By changing the size, orientation or location of the coil in a steady magnetic field.

2. By changing in time the strength of the magnetic field while keeping the coil fixed.

Both cases can be described by the same law: $$EMF = \frac{d}{dt}\Phi$$

The "electromotive force" equals the rate of change of magnetic flux.


2. The loop needs to generate an upward-pointing magnetic field to oppose the *change* in flux.
S
N
Current meter
0
−
+
$\vec{B}_{\text{induced}}$
Induced current
$\vec{B}$
1. The flux through the loop increases downward as the magnet approaches.
3. By the right-hand rule, a ccw current is needed to induce an upward-pointing magnetic field.

The current can be induced two different ways:

1. By changing the size, orientation or location of the coil in a steady magnetic field.

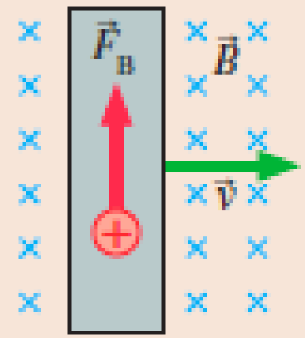
The electromotive force comes from the Lorenz force.
Motional EMF

2. By changing in time the strength of the magnetic field while keeping the coil fixed.

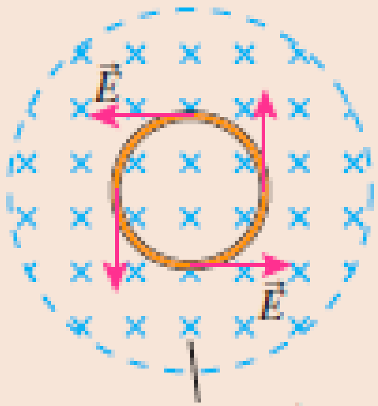
In case #2 the electromotive force comes from an electric field. This requires saying that electric fields can appear that do not satisfy Coulomb's Law!

## Two ways to create an induced current

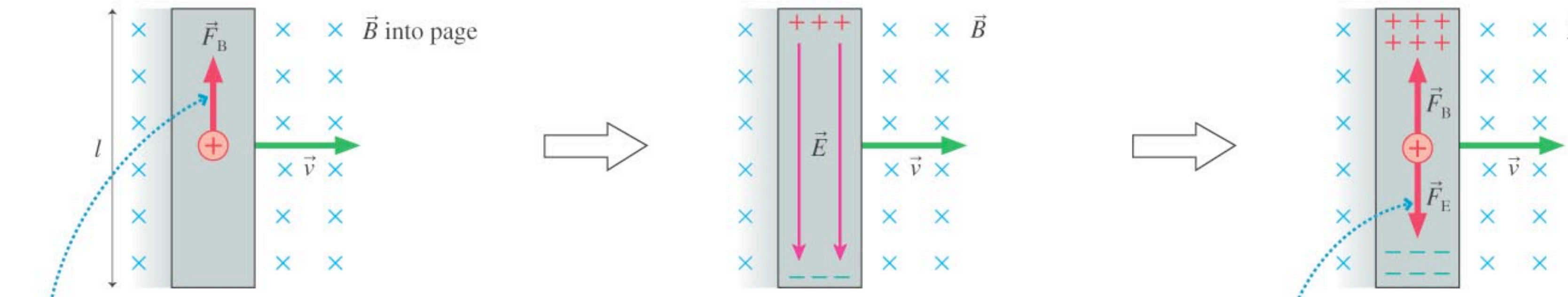
1. **A motional emf** due to magnetic forces on moving charge carriers.



2. An induced electric field due to a changing magnetic field.



# Motional EMF

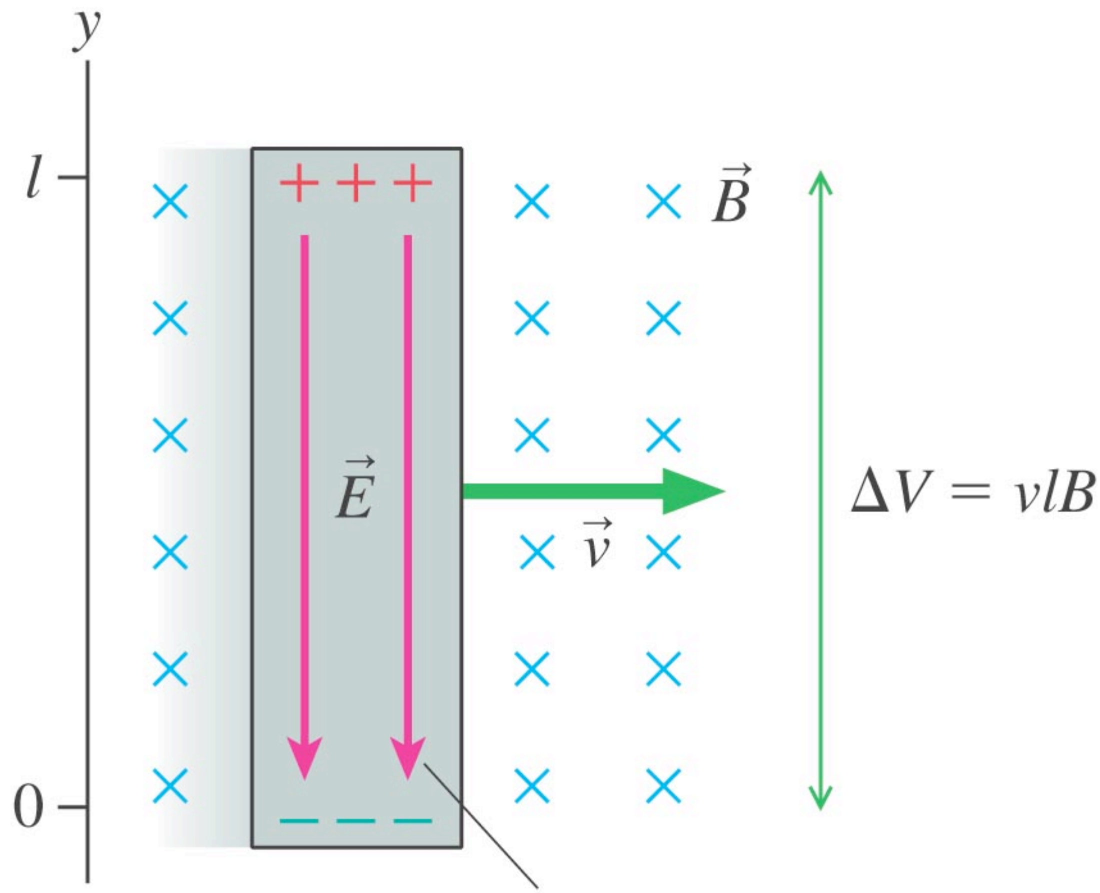



Charge carriers in the wire experience an upward force of magnitude $F_B = qvB$. Being free to move, positive charges flow upward (or, if you prefer, negative charges downward).

The charge separation creates an electric field in the conductor. $\vec{E}$ increases as more charge flows.

The charge flow continues until the downward electric force $\vec{F}_E$ is large enough to balance the upward magnetic force $\vec{F}_B$. Then the net force on a charge is zero and the current ceases.

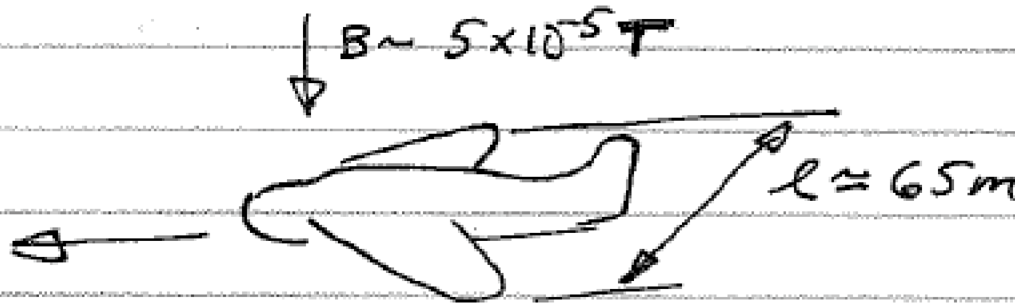
**(a)** Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.



Some numbers

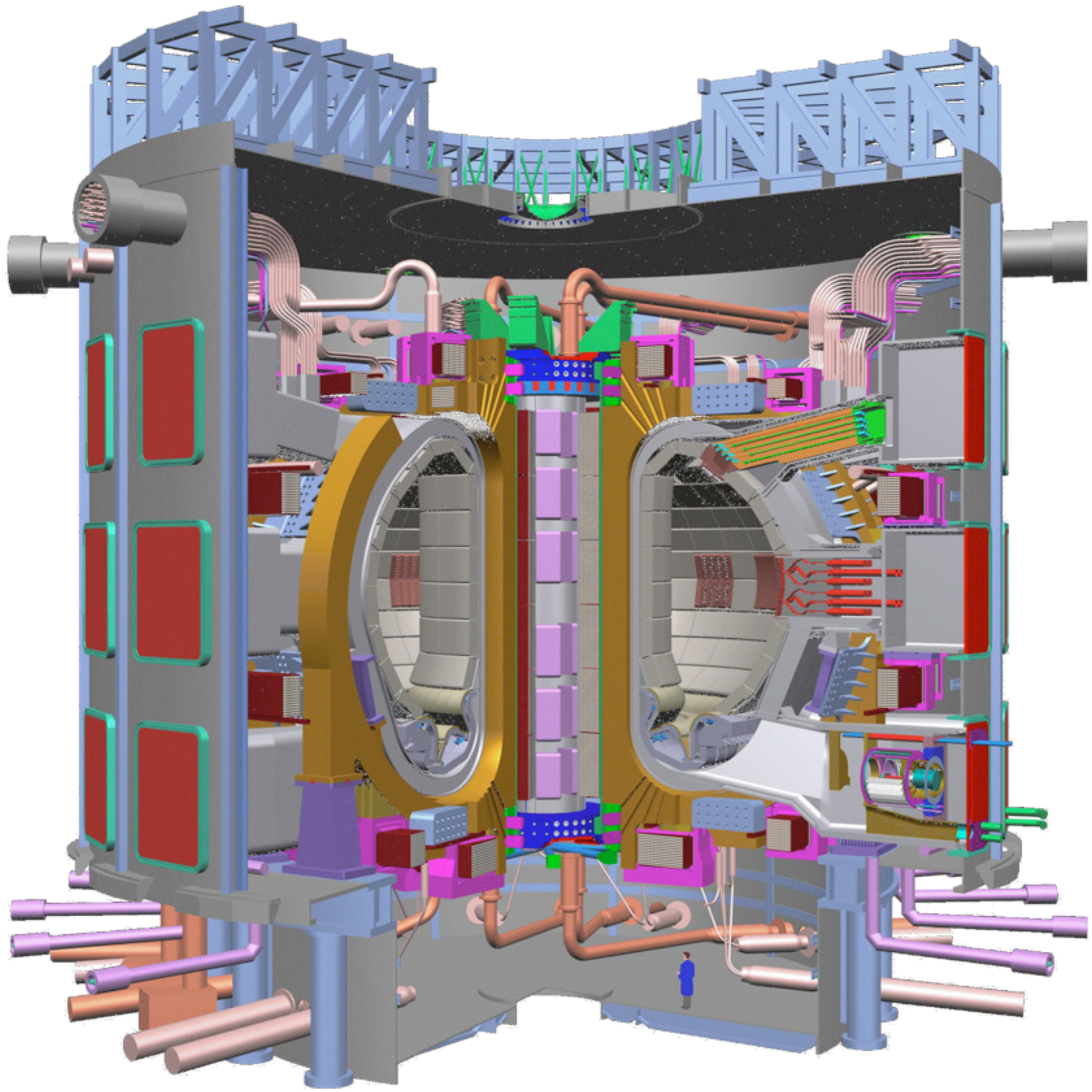
$$\Delta V = V l B$$

What is the potential drop across the wings of an airplane flying through the earth's magnetic field?

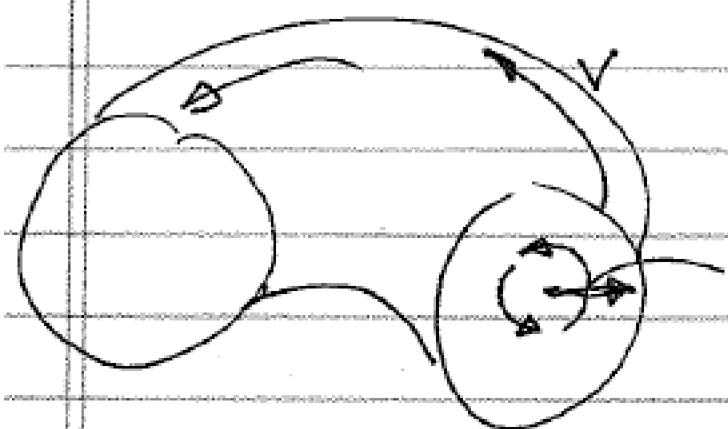



$V = 260$ m/s

$\Delta V = 0.85$ VOLTS



WHAT is the anticipated potential drop from the center of the ITER tokamak to the edge?




Plasma rotates with ~~vel~~ speed ~

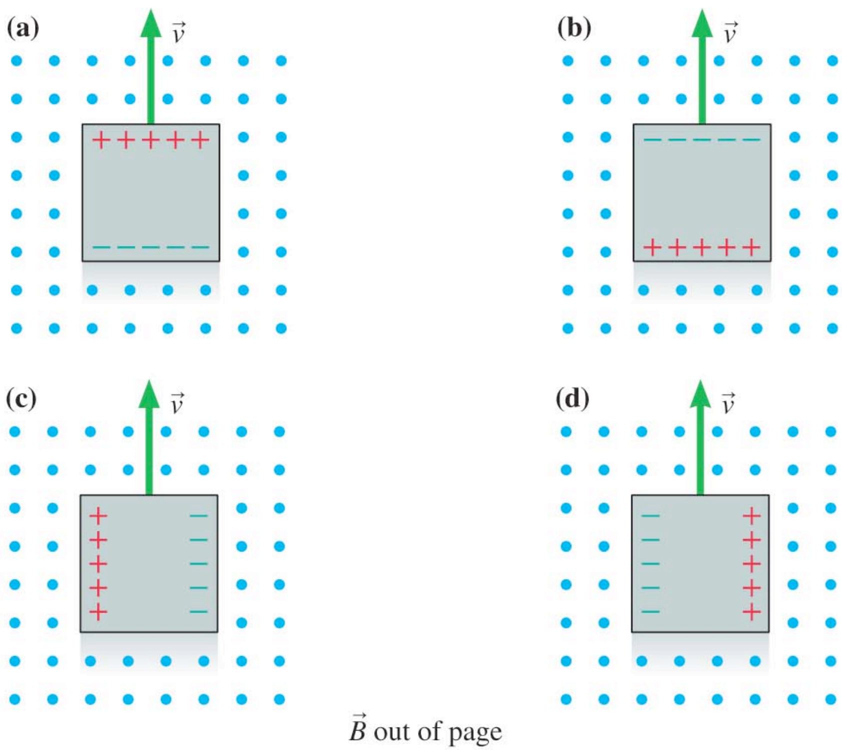
$l \sim 3m$

$B \sim 1T$

$V \sim 9.79 \times 10^{4}$ m/s (62 miles/sec)

$\Delta V = 2.9 \times 10^{5}$ Volts

## A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?



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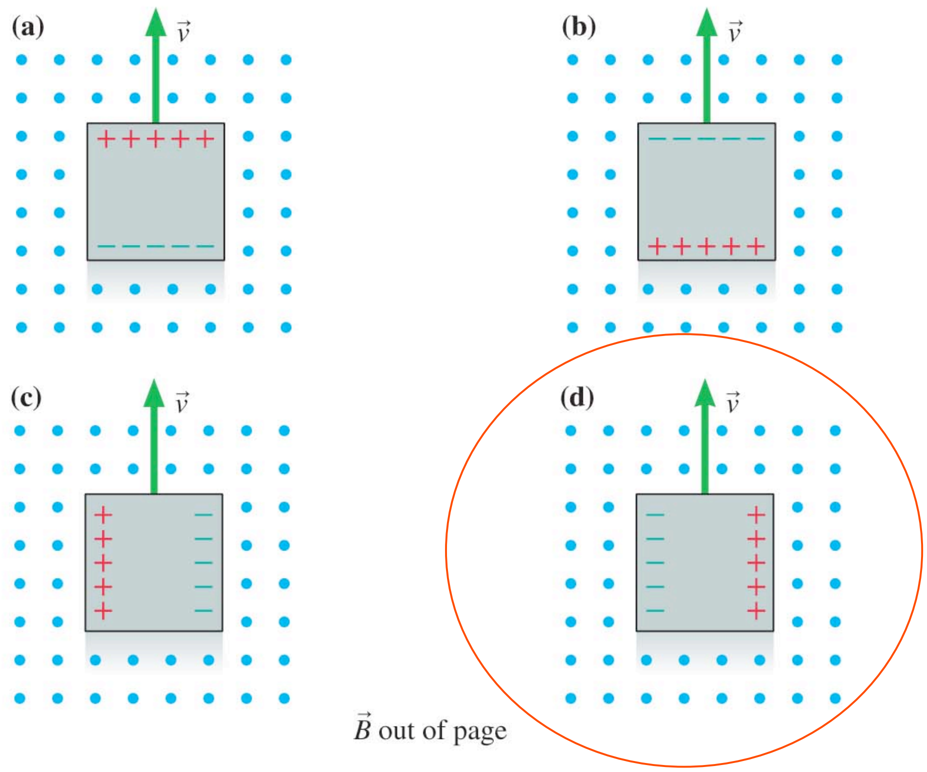
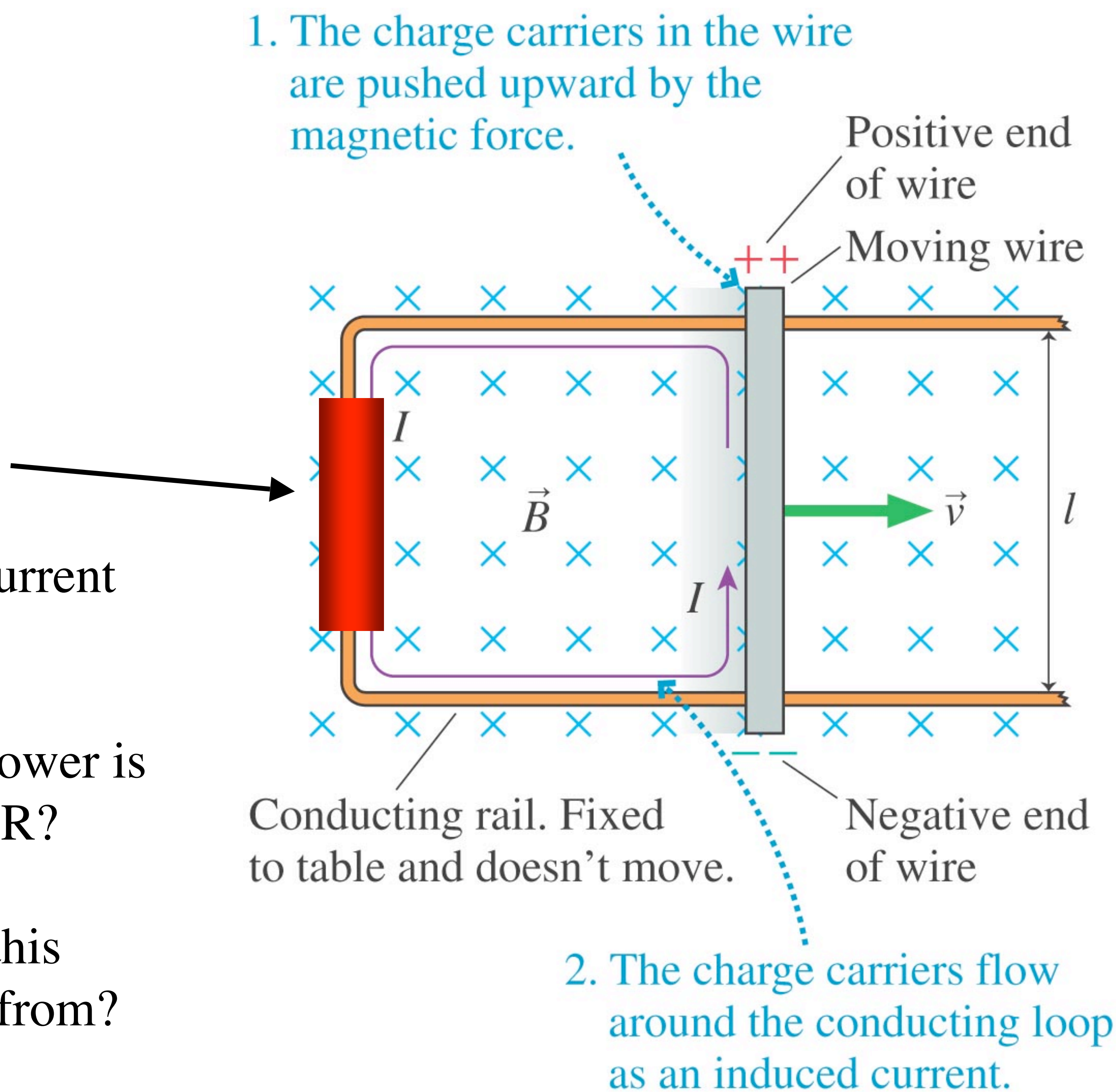


Resistor R

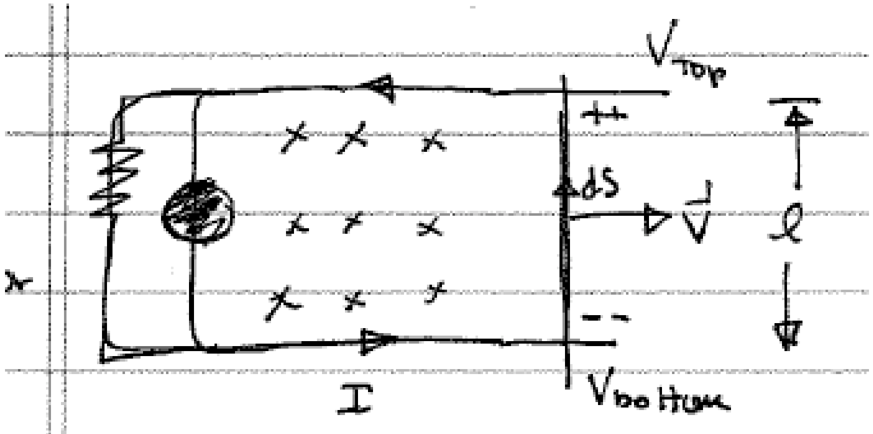
How much current flows?

How much power is dissipated in R?

Where does this power come from?



# Potential difference on moving conductor



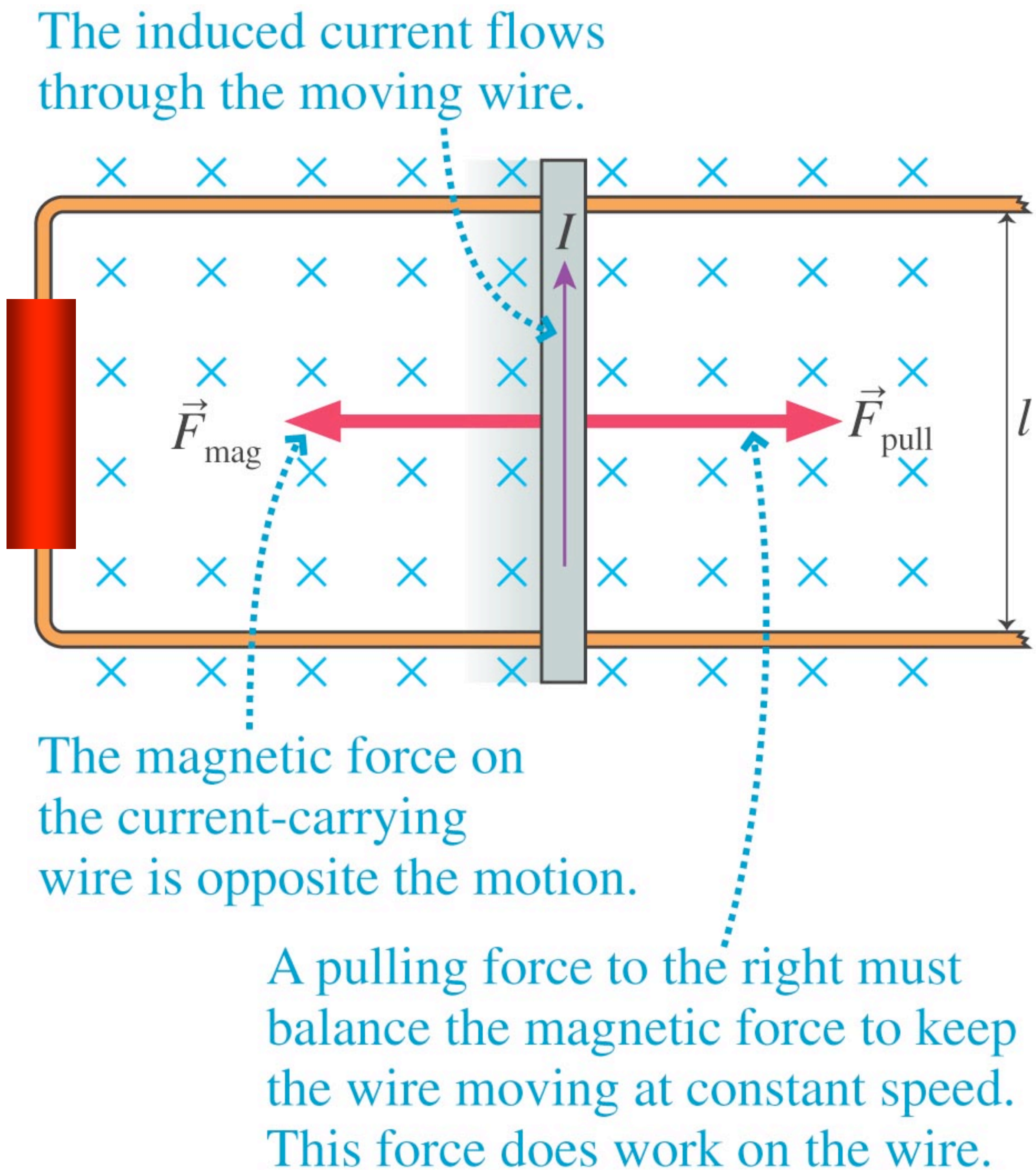

$$\Delta V = V_{top} - V_{bottom} = -\int_{bottom}^{top} \vec{E} \cdot d\vec{s} = vlB$$

Current that flows: $I = \dfrac{\Delta V}{R} = \dfrac{vlB}{R}$ We assume that this current is too small to change B

Power supplied to the resistor $P_{dissipated} = I \Delta V$

Force on moving rail due to B $\vec{F}_{mag} = I\vec{l} \times \vec{B}$

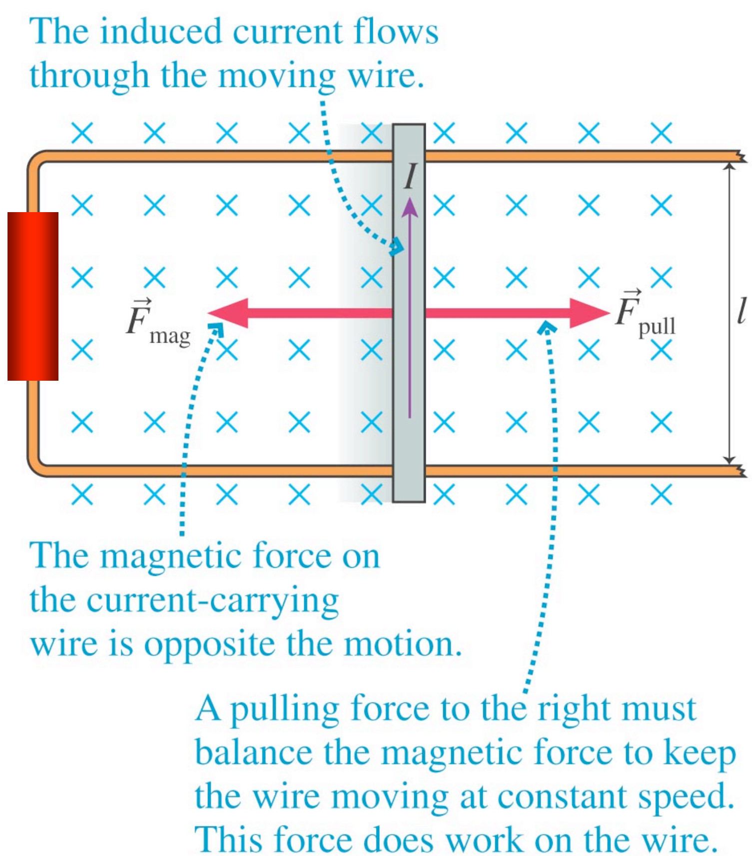
Push needed to keep rail moving $\vec{F}_{pull} = -I\vec{l} \times \vec{B}$



The induced current flows through the moving wire.
I
$\vec{F}_{mag}$
$\vec{F}_{pull}$
l
The magnetic force on the current-carrying wire is opposite the motion.
A pulling force to the right must balance the magnetic force to keep the wire moving at constant speed. This force does work on the wire.

# Power needed to keep rail moving

$$\vec{v}\cdot\vec{F}_{pull} = I\left(lvB\right) = I\Delta V = P_{dissipated}$$



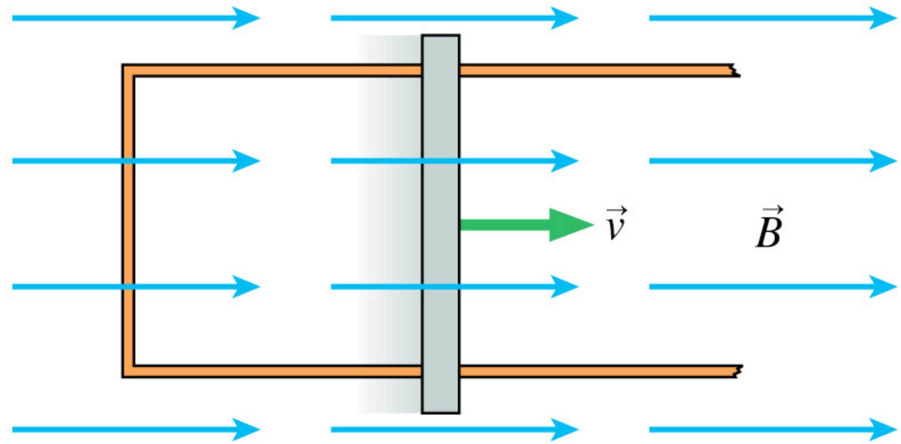



Work done by agent doing the pulling winds up as heat in the resistor

What happens when $R \to 0$?

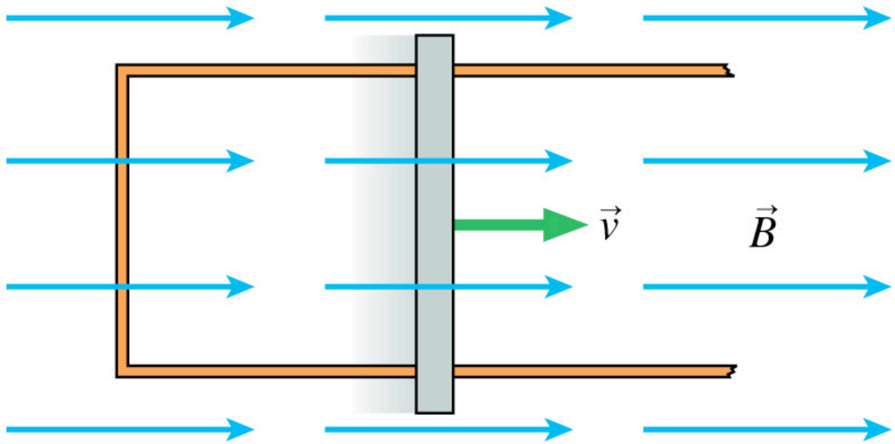
$$I = \frac{\Delta V}{R} = \frac{vlB}{R} \to \infty$$

At some point we can't ignore B due to I.




**Is there an induced current in this circuit? If so, what is its direction?**

A. No
B. Yes, clockwise
C. Yes, counterclockwise




**Is there an induced current in this circuit? If so, what is its direction?**

✔ **A. No**
B. Yes, clockwise
C. Yes, counterclockwise

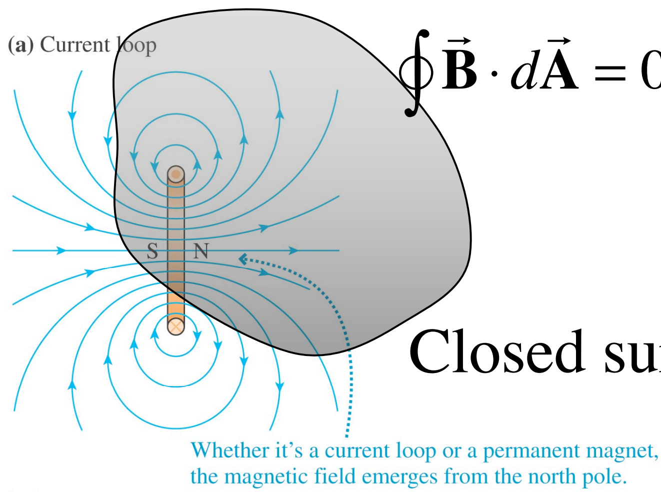
Magnetic Flux

$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

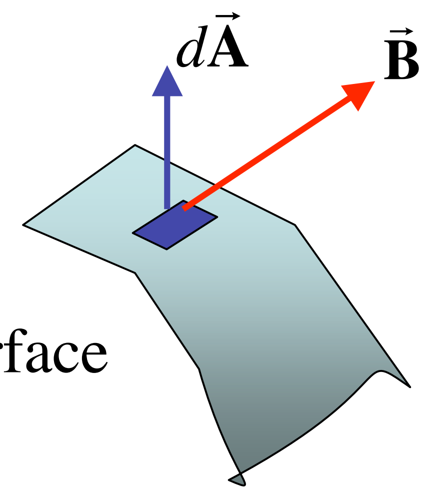
← Some surface

Remember for a closed surface $\Phi = 0$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

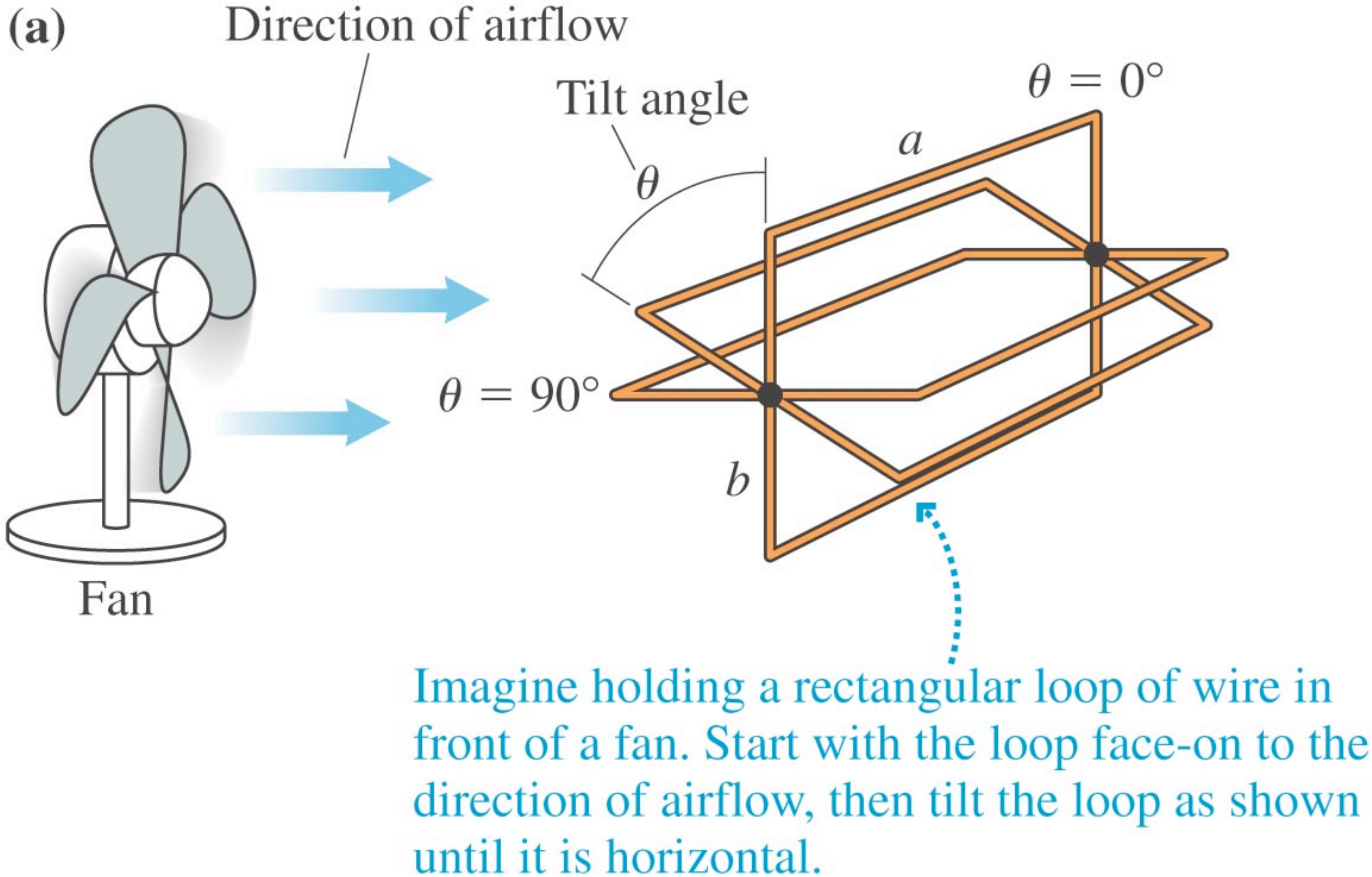


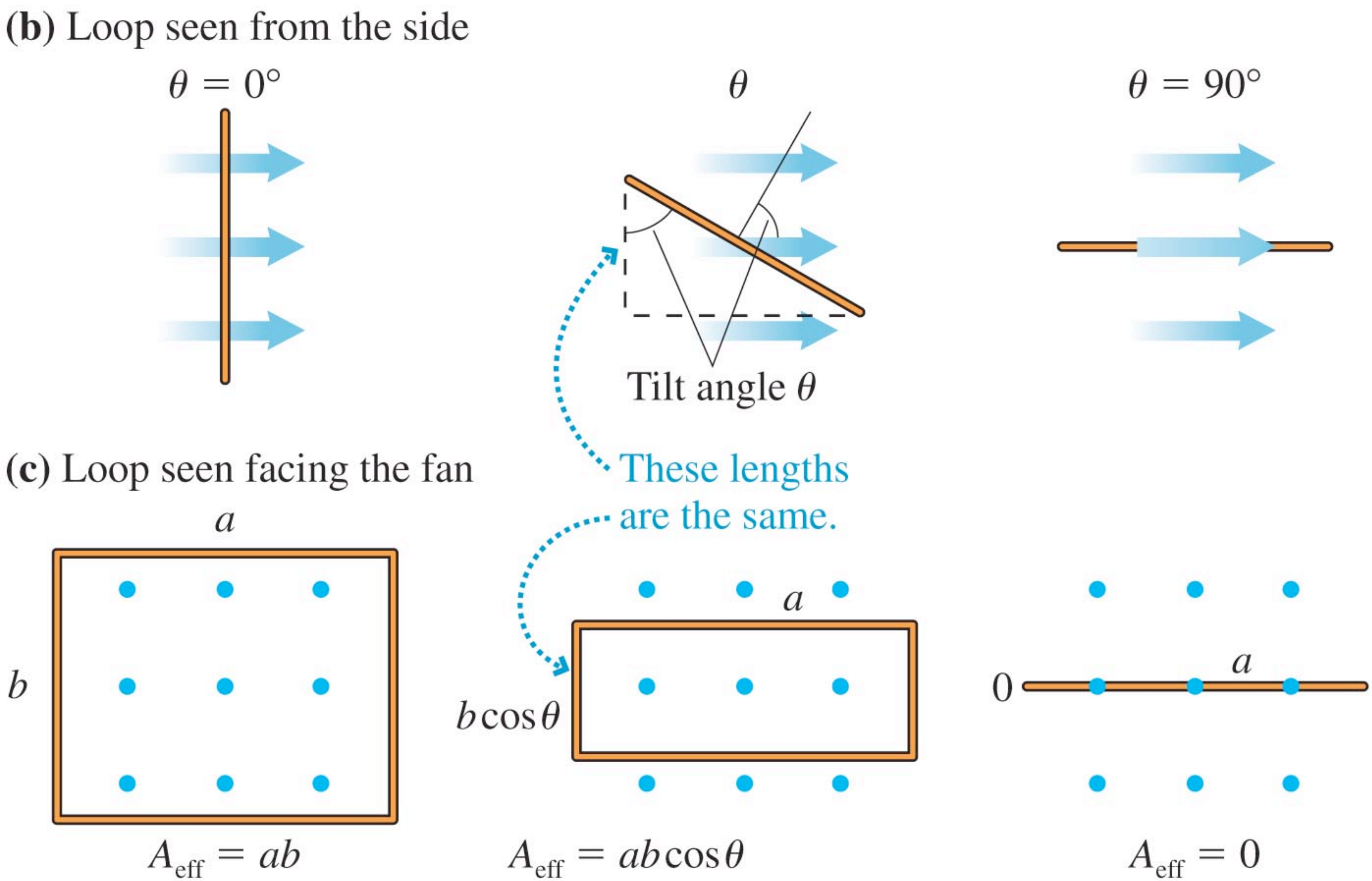

Closed surface




Open surface

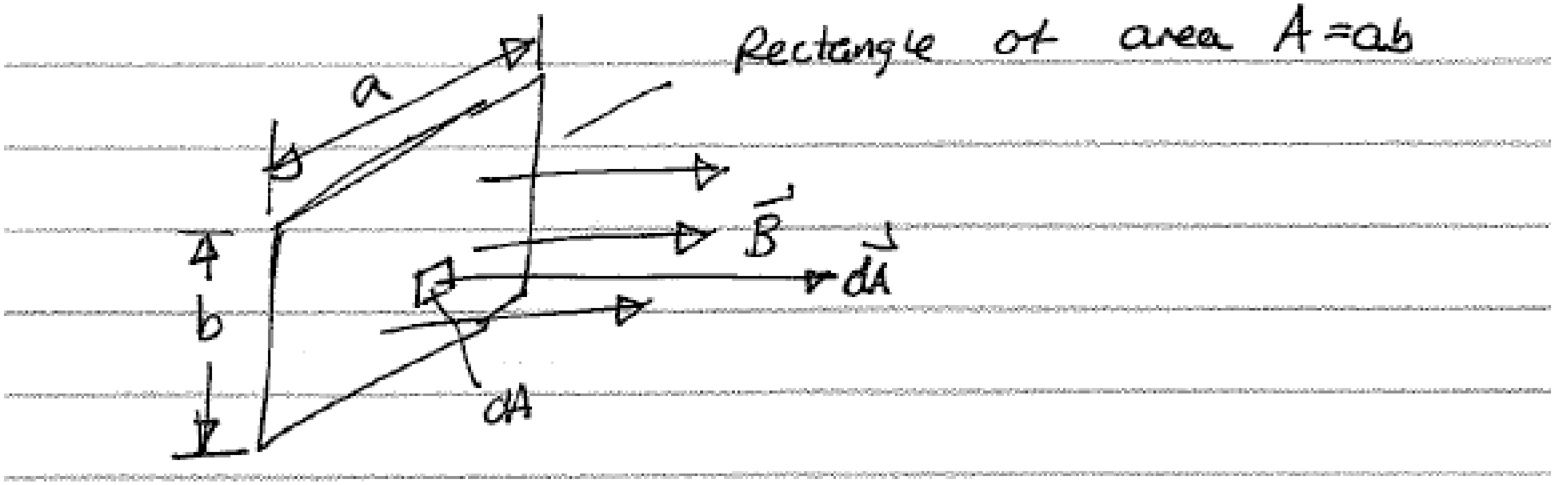
Magnetic flux measures how much magnetic field passes through a given surface


(a)
Direction of airflow
Tilt angle
θ
θ = 0°
a
θ = 90°
b
Fan
Imagine holding a rectangular loop of wire in front of a fan. Start with the loop face-on to the direction of airflow, then tilt the loop as shown until it is horizontal.



(b) Loop seen from the side
θ = 0°
θ
θ = 90°
Tilt angle θ
(c) Loop seen facing the fan
These lengths
are the same.
a
b
a
b cos θ
a
0
A_eff = ab
A_eff = ab cos θ
A_eff = 0

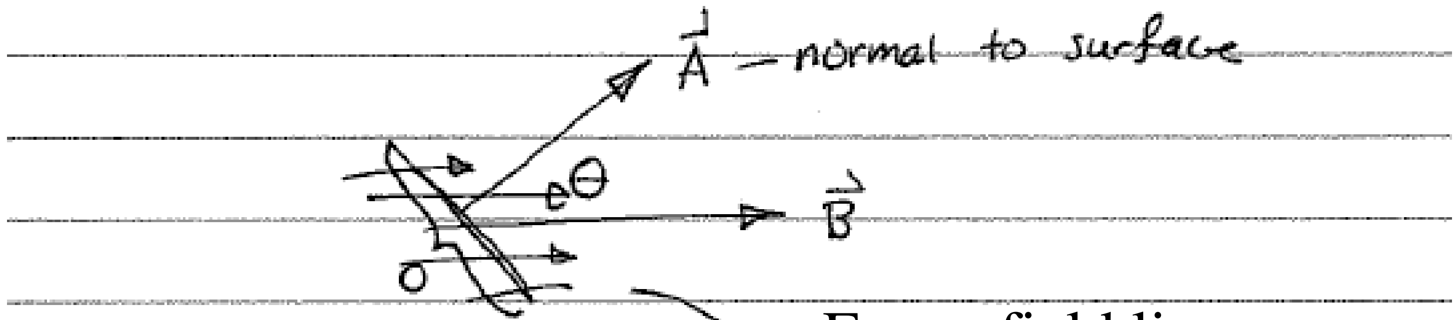
Rectangular surface in a constant magnetic field. Flux depends on orientation of surface relative to direction of B




Suppose the rectangle is oriented do that $\vec{\mathbf{B}}$ and $d\vec{\mathbf{A}}$ are parallel

$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \left|\vec{\mathbf{B}}\right| A = \left|\vec{\mathbf{B}}\right| ab$$

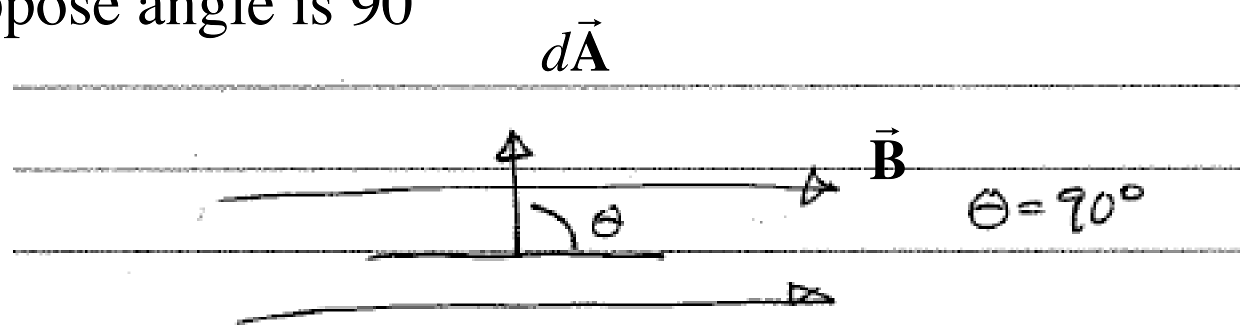
Suppose I tilt the rectangle by an angle θ




Fewer field lines pass through rectangle

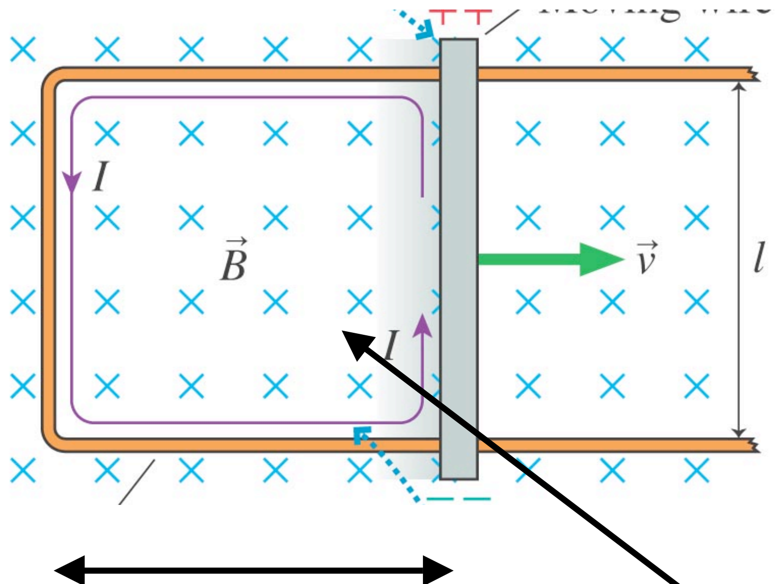
$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \left|\vec{\mathbf{B}}\right| A \cos\theta$$

Suppose angle is 90°




$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \left|\vec{\mathbf{B}}\right| A \cos 90^\circ = 0$$

# A suggestive relation




this length is vt

$$A = lvt$$

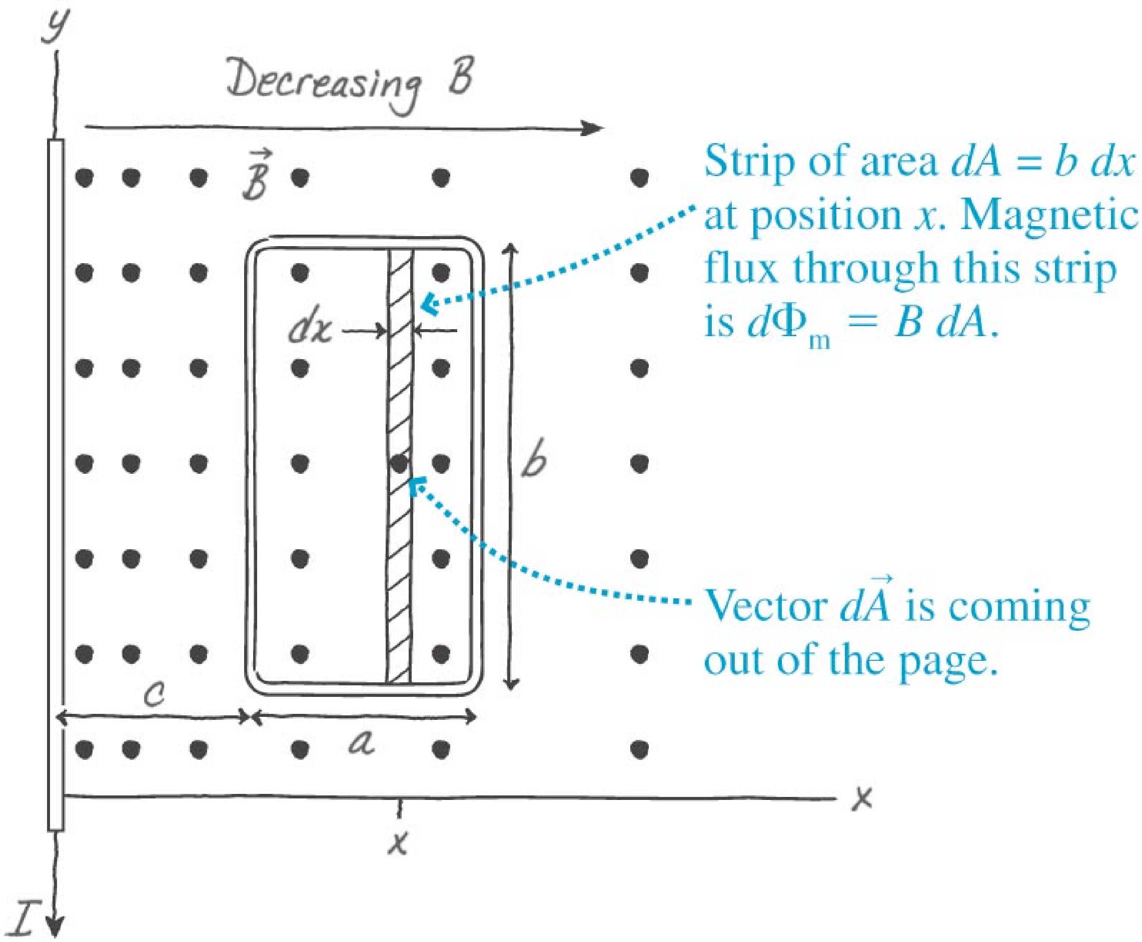
$$\Delta V = V_{top} - V_{bottom} = -\int_{bottom}^{top} \vec{E} \cdot d\vec{s} = vlB$$

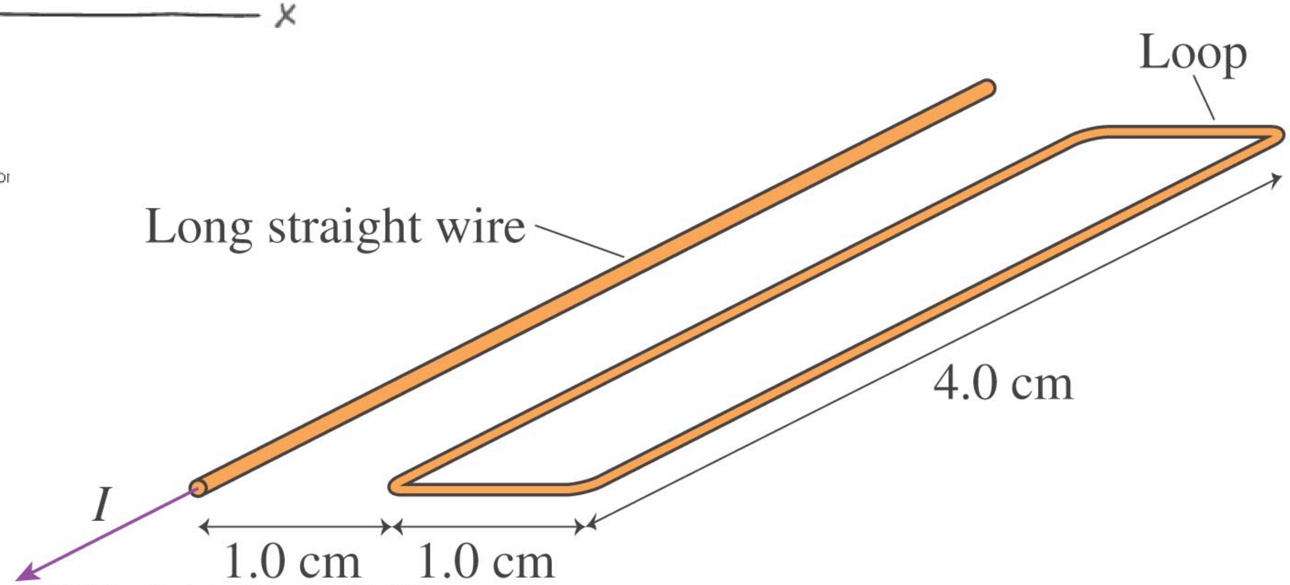
Define A to be out of page,

B is into page

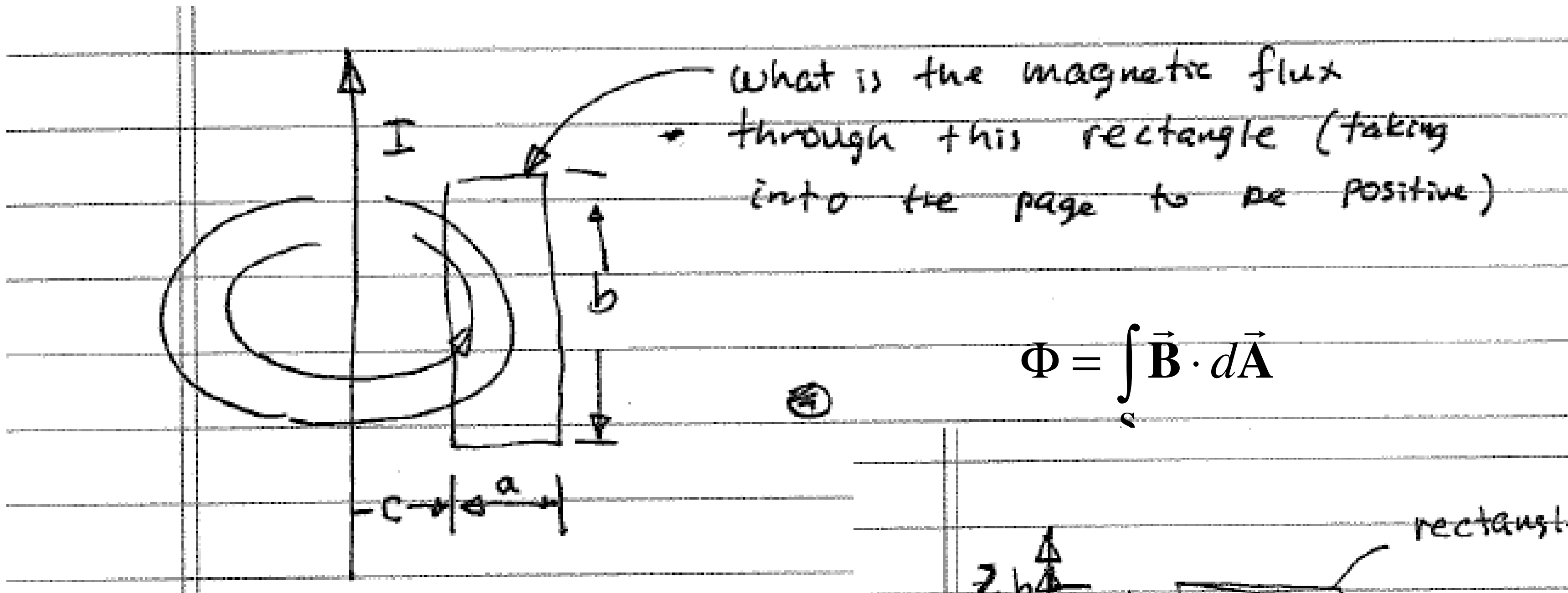
$$\Phi = \int_{S} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\left|\vec{\mathbf{B}}\right| A = -\left|\vec{\mathbf{B}}\right| lvt$$

$$\Delta V = -\frac{d\Phi}{dt} = vlB$$

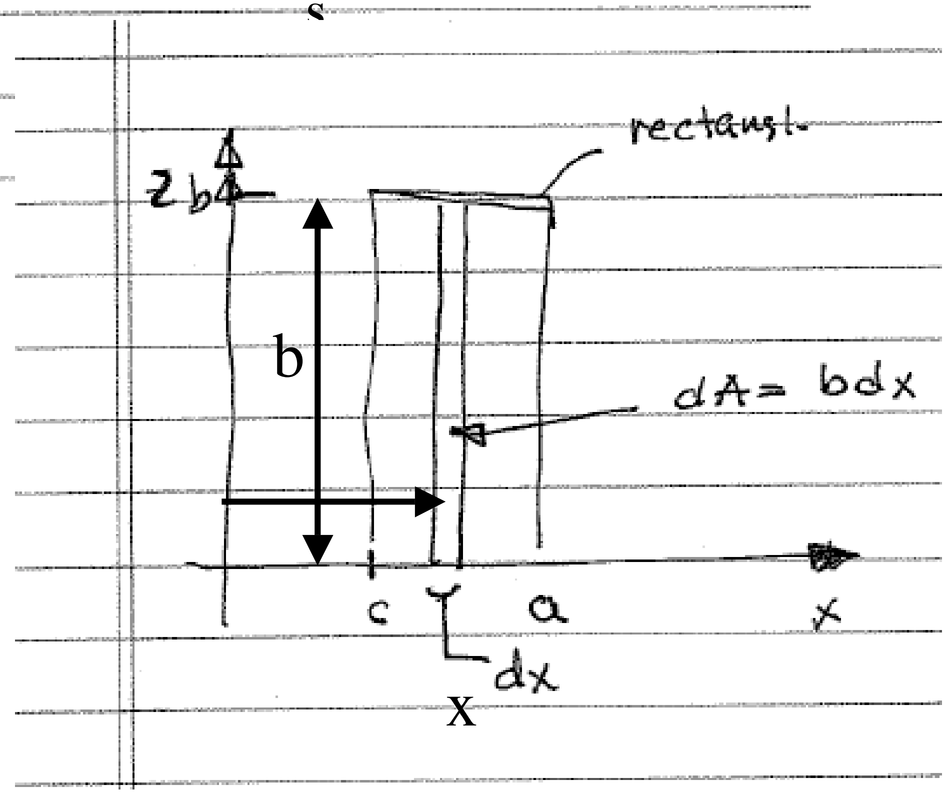

y
Decreasing B
$\vec{B}$
Strip of area $dA = b\,dx$ at position $x$. Magnetic flux through this strip is $d\Phi_m = B\,dA$.
dx
b
Vector $d\vec{A}$ is coming out of the page.
c
a
x
x
I



Loop
Long straight wire
4.0 cm
I
1.0 cm
1.0 cm

# Example of non-uniform B - Flux near a current carrying wire



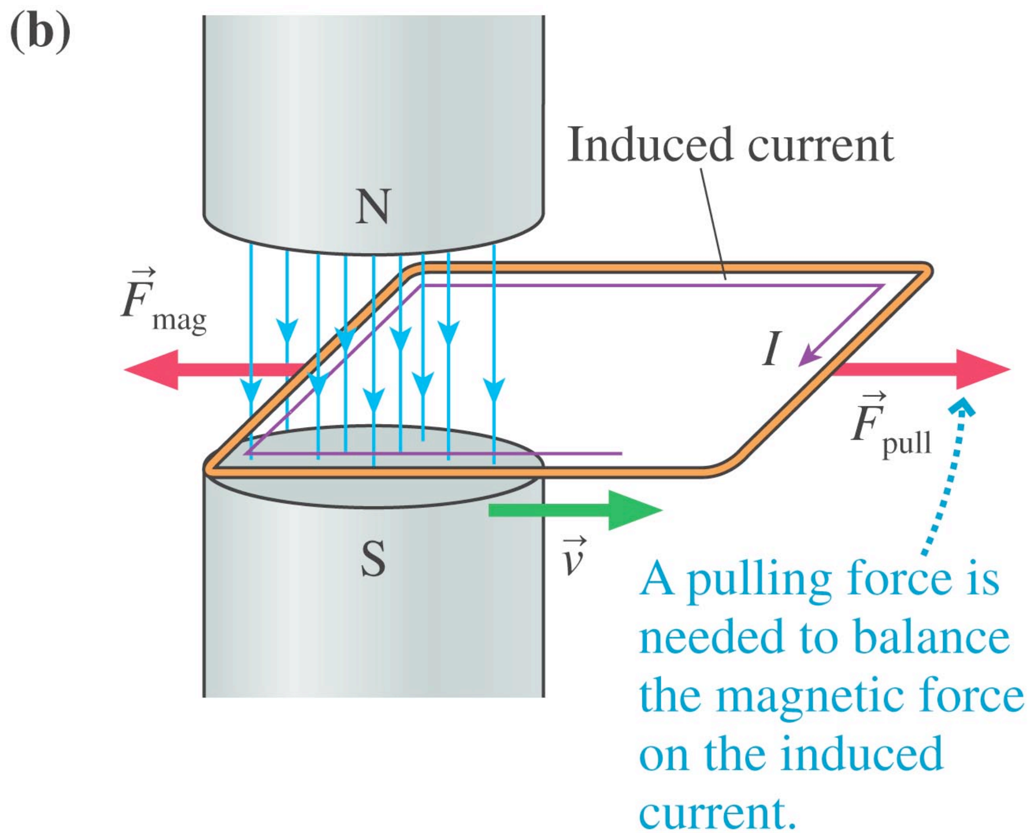

$$\Phi = \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$



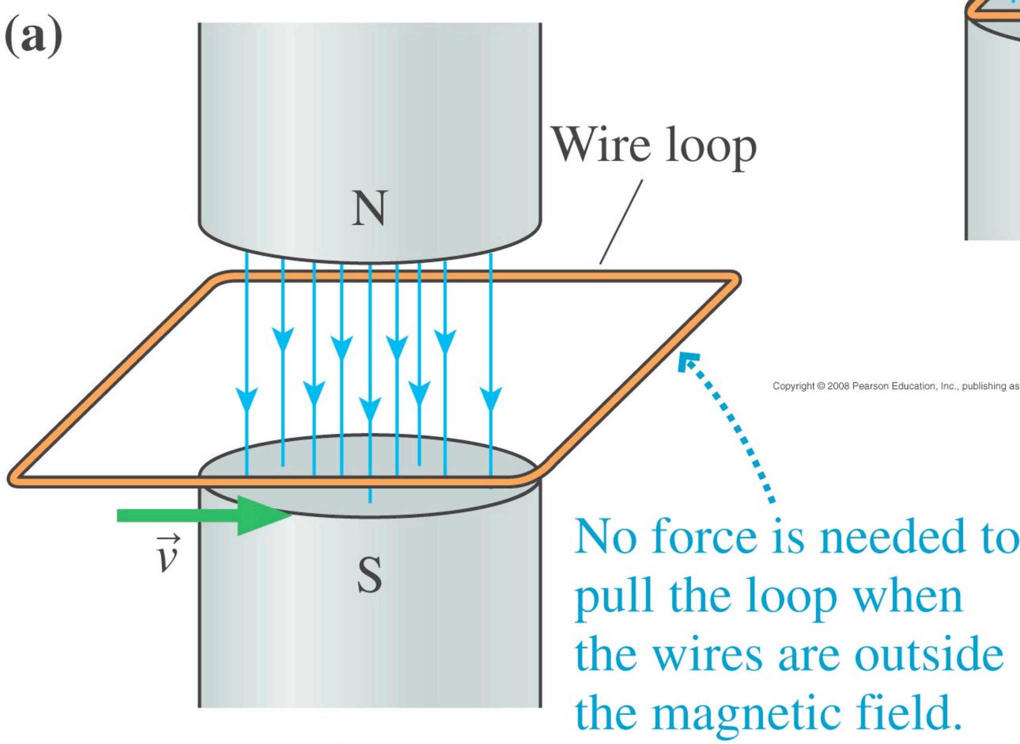

$$\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{2\pi x} \qquad \left|d\vec{\mathbf{A}}\right| = bdx$$

$$= \int_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int_{x=c}^{x=c+a} bdx \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I b}{2\pi} \ln\frac{c+a}{c}$$

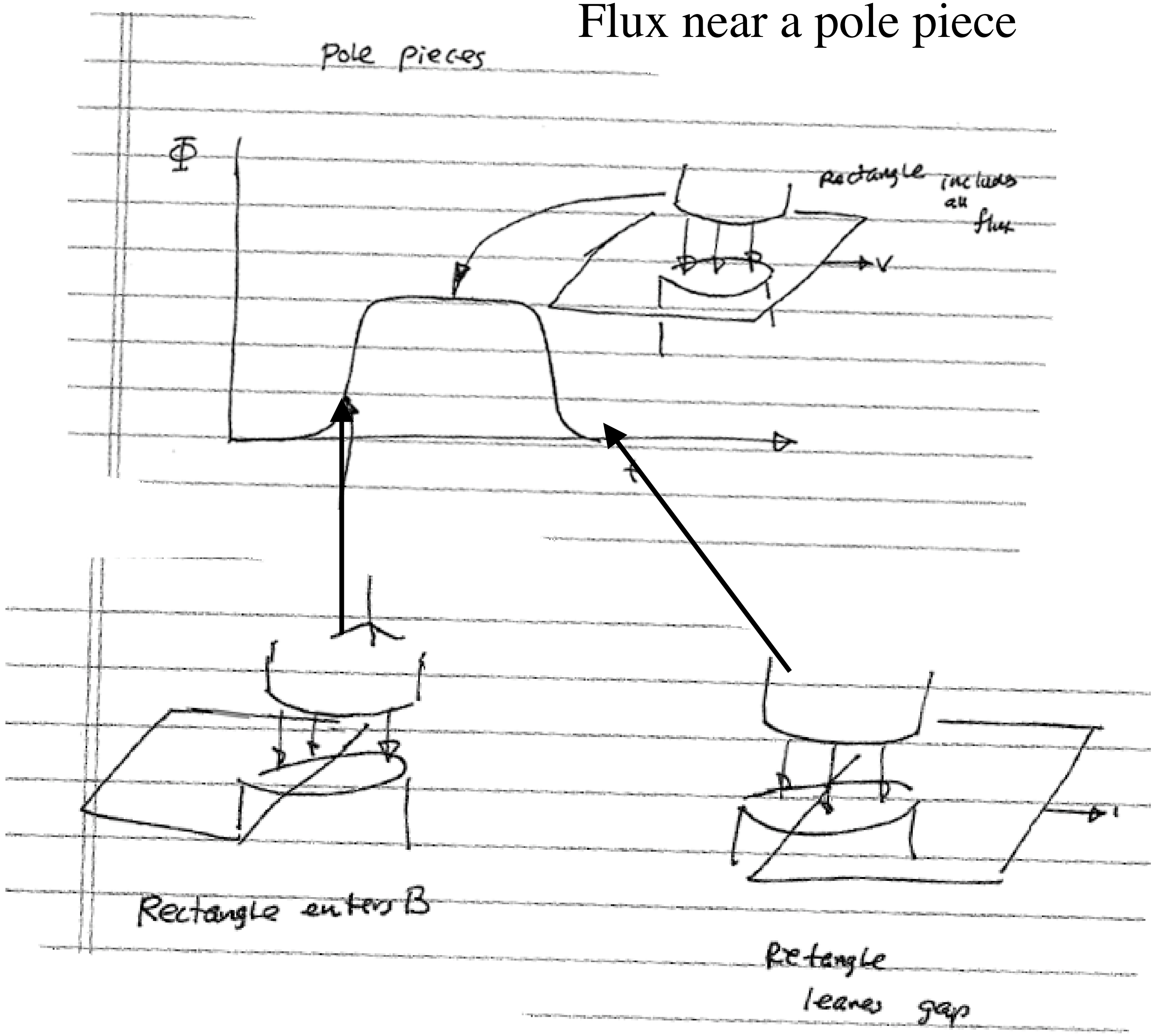
Eddy currents



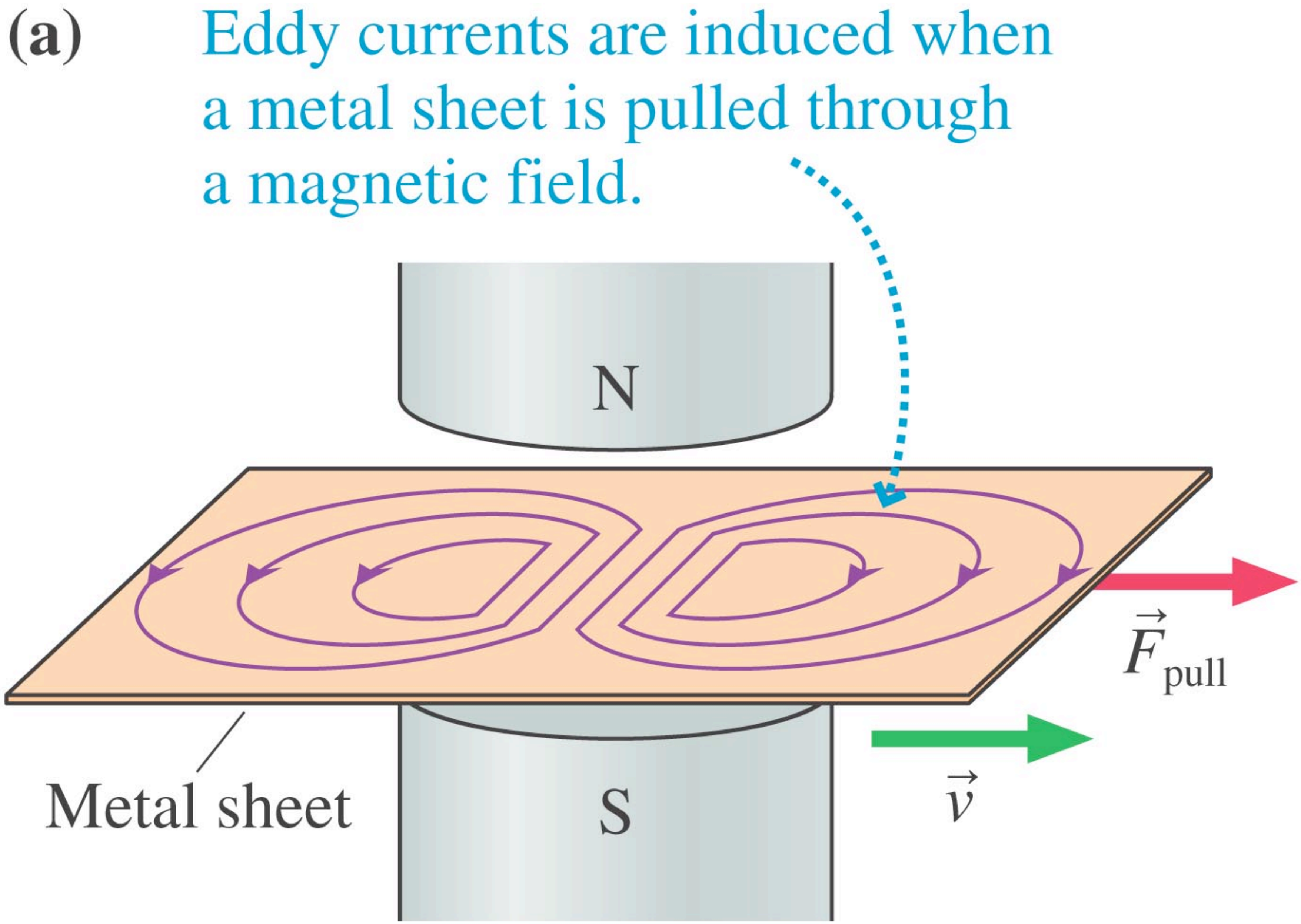


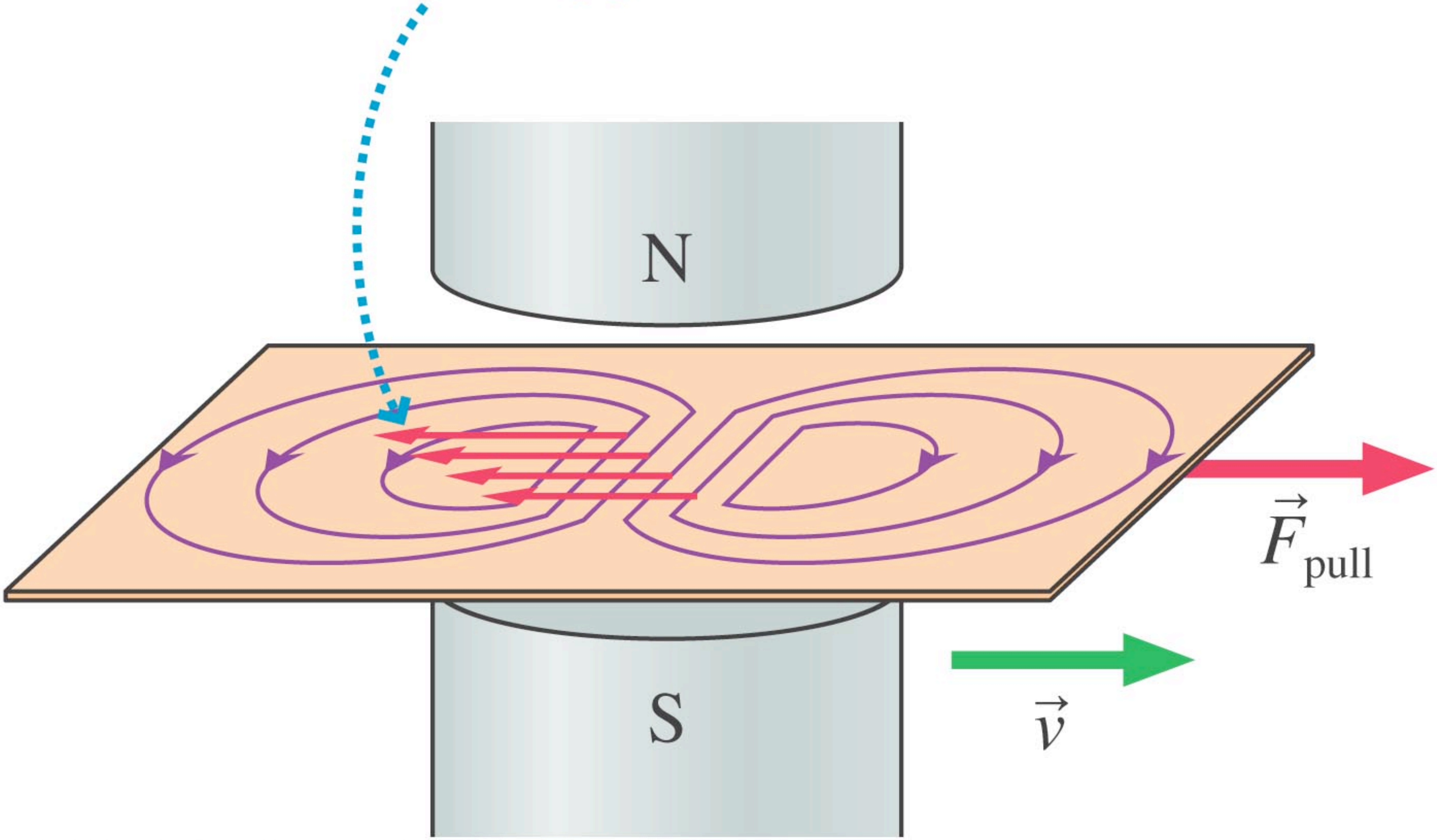
# Flux near a pole piece

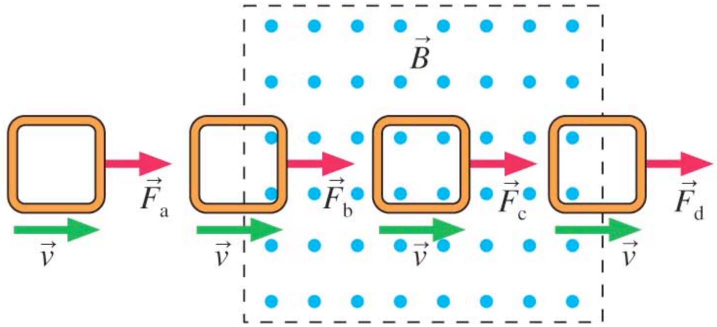


(a)


Eddy currents are induced when a metal sheet is pulled through a magnetic field.
N
S
Metal sheet
$\vec{F}_{pull}$
$\vec{v}$

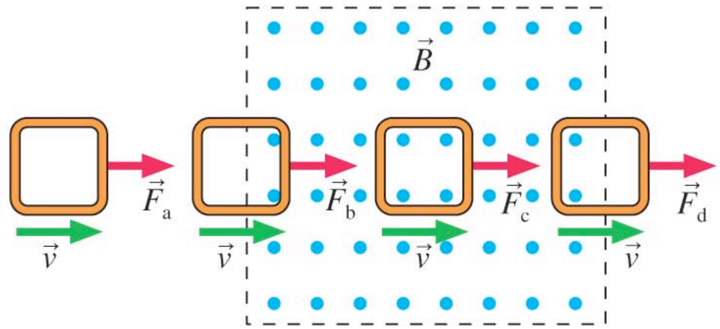
**(b)** The magnetic force on the eddy currents is opposite in direction to $\vec{v}$.






**A square loop of copper wire is pulled through a region of magnetic field. Rank in order, from strongest to weakest, the pulling forces $F_a$, $F_b$, $F_c$ and $F_d$ that must be applied to keep the loop moving at constant speed.**

A. $F_b = F_d > F_a = F_c$
B. $F_c > F_b = F_d > F_a$
C. $F_c > F_d > F_b > F_a$
D. $F_d > F_b > F_a = F_c$
E. $F_d > F_c > F_b > F_a$

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✔ **A.** $\boldsymbol{F_b = F_d > F_a = F_c}$

B. $F_c > F_b = F_d > F_a$

C. $F_c > F_d > F_b > F_a$

D. $F_d > F_b > F_a = F_c$

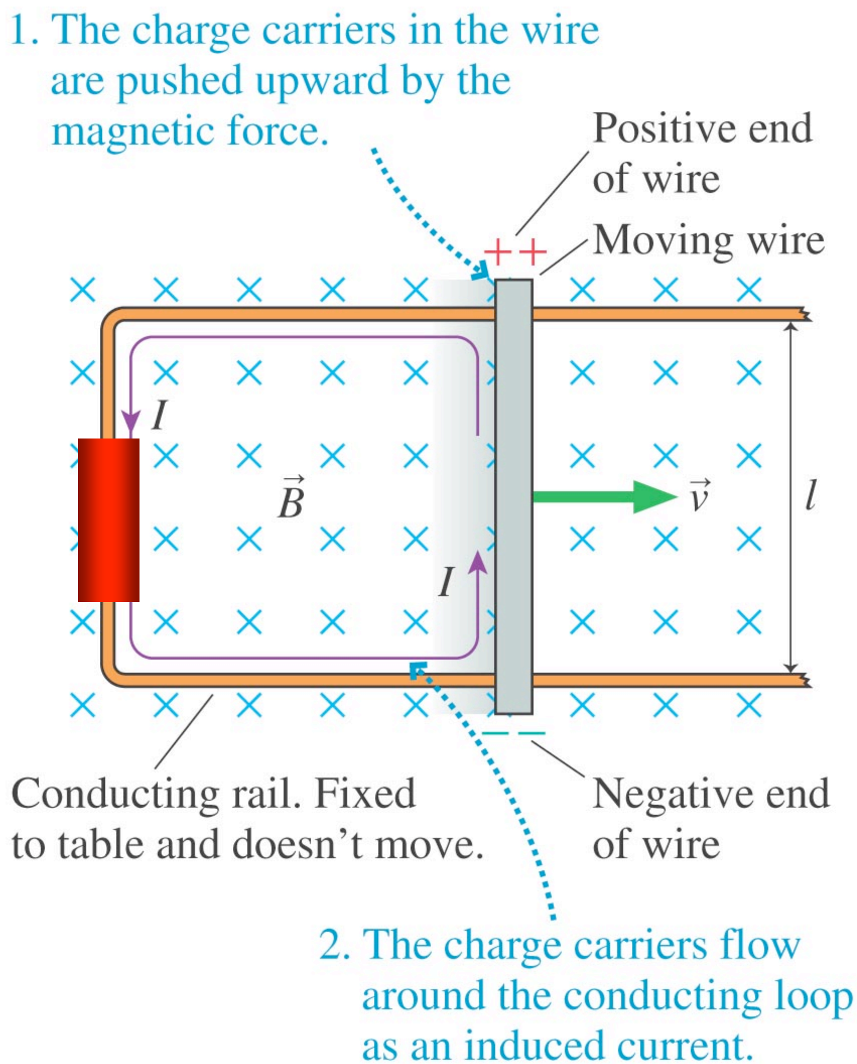
E. $F_d > F_c > F_b > F_a$

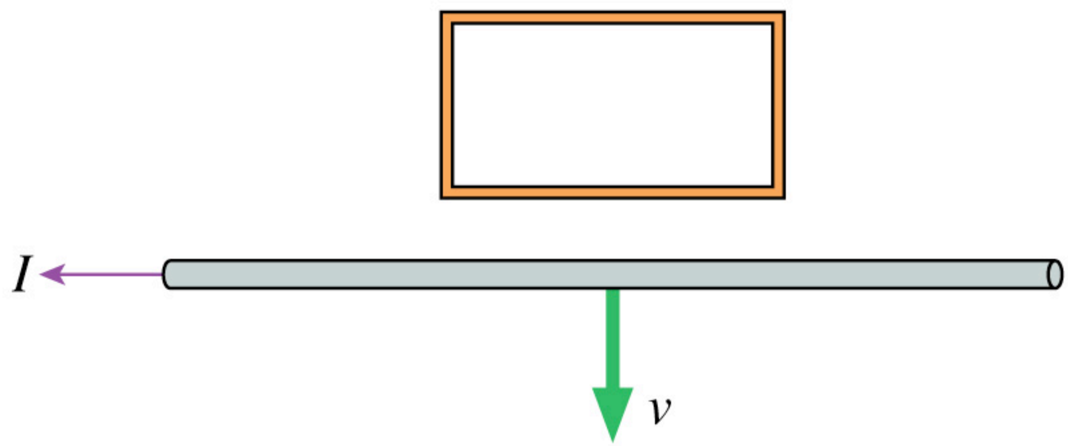
Lenz's Law

In a loop through which there is a change in magnetic flux, and EMF is induced that tends to resist the change in flux

What is the direction of the magnetic field made by the current I?

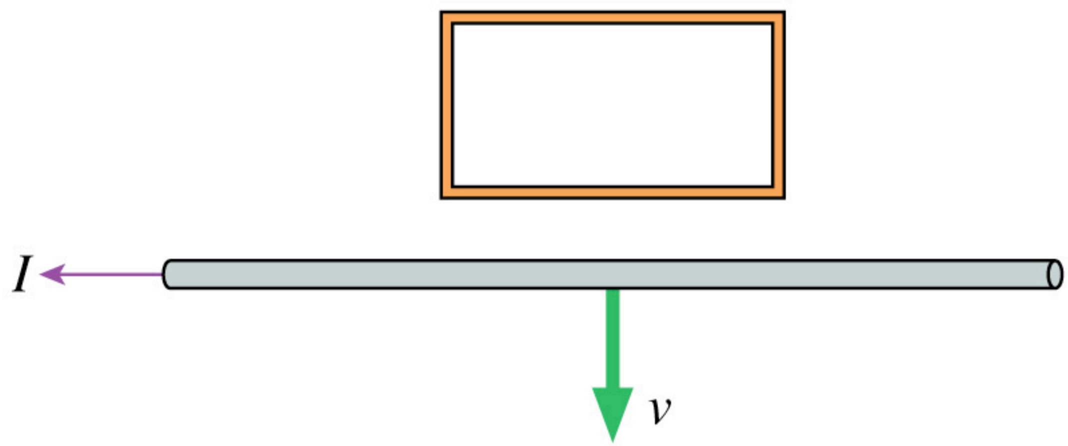
A. Into the page
B. Out of the page






**A current-carrying wire is pulled away from a conducting loop in the direction shown. As the wire is moving, is there a cw current around the loop, a ccw current or no current?**

A. There is no current around the loop.
B. There is a clockwise current around the loop.
C. There is a counterclockwise current around the loop.

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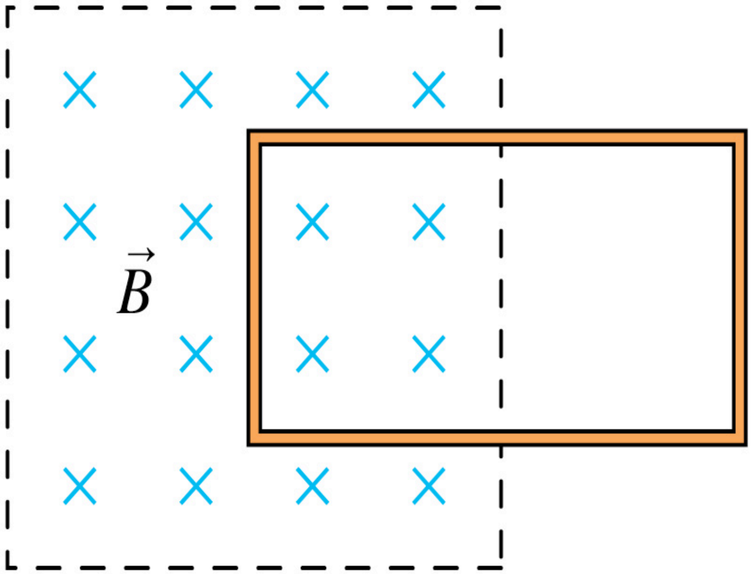
**A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?**




A. The loop is pulled to the left, into the magnetic field.
B. The loop is pushed to the right, out of the magnetic field.
C. The loop is pushed upward, toward the top of the page.
D. The loop is pushed downward, toward the bottom of the page.
E. The tension is the wires increases but the loop does not move.

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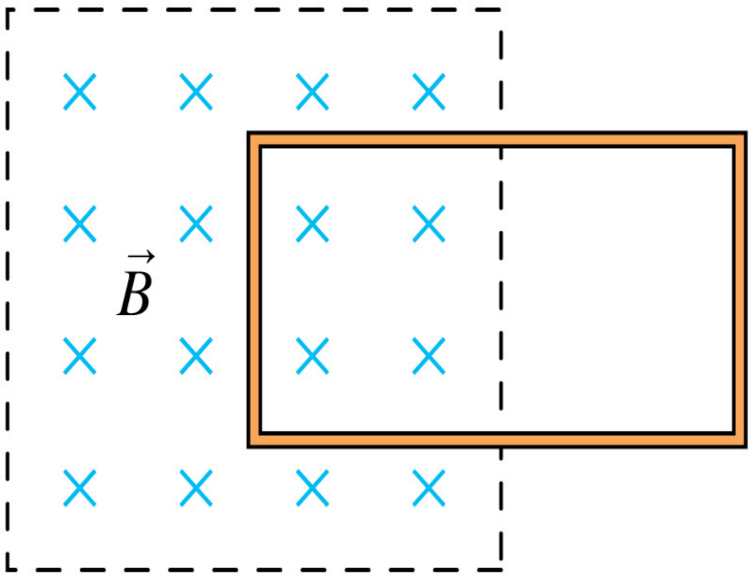


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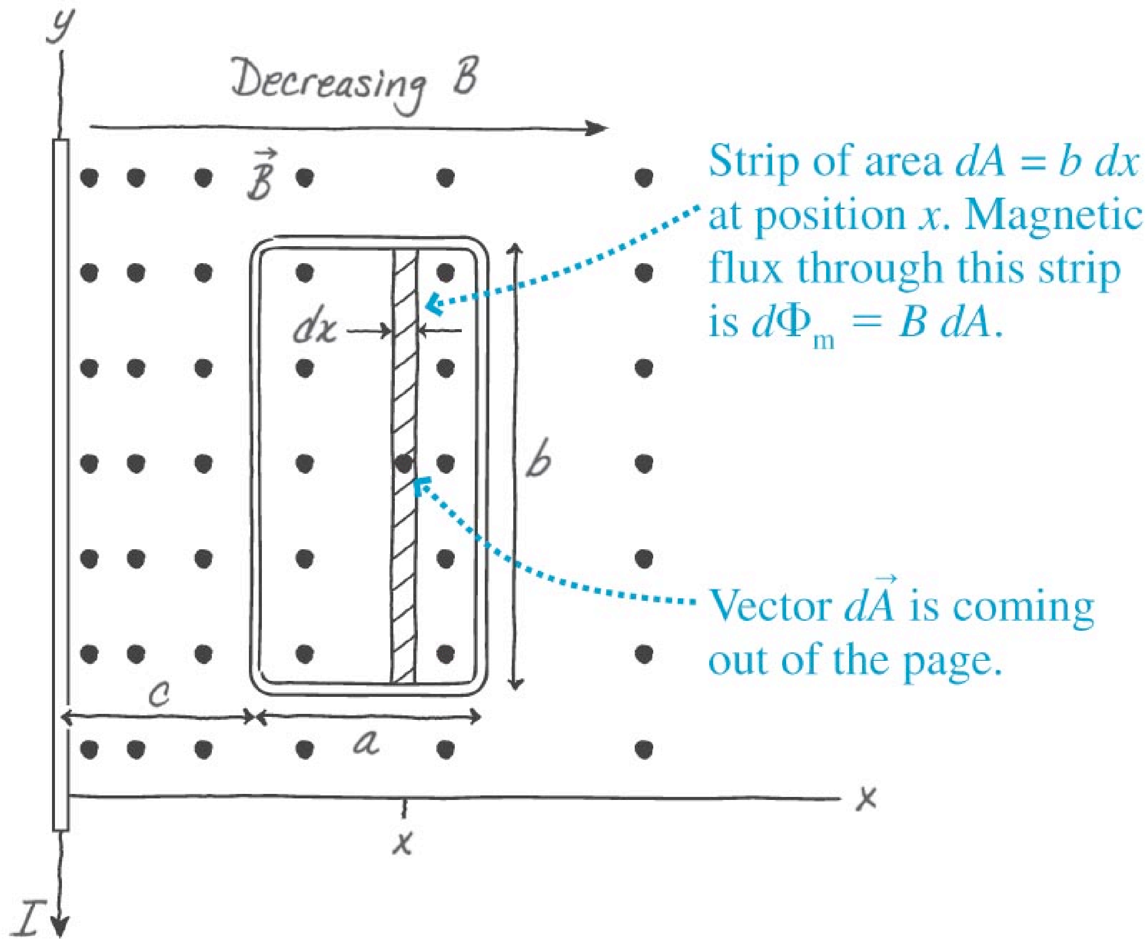
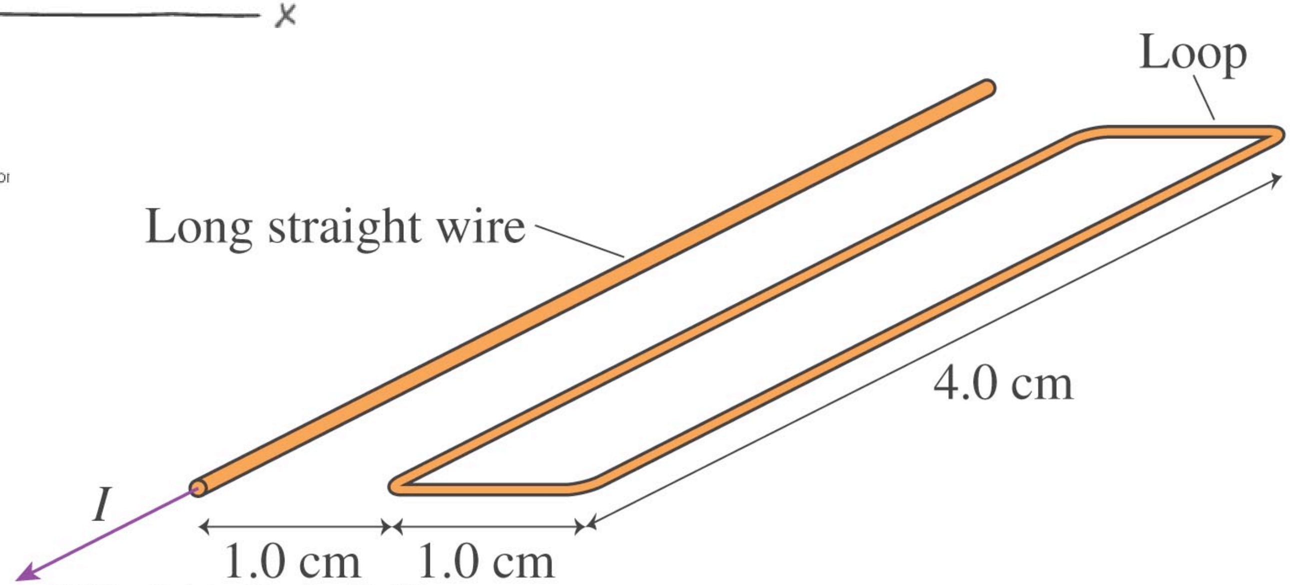
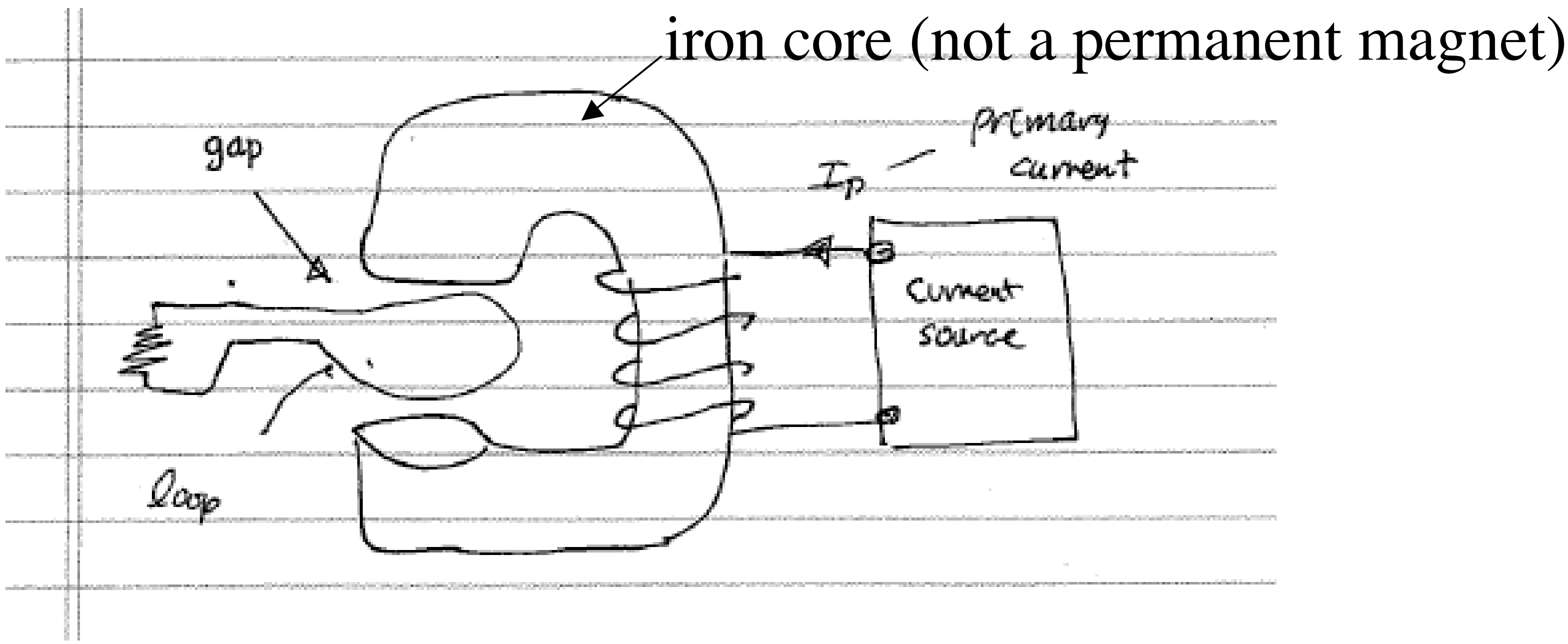

Decreasing B
$\vec{B}$
Strip of area $dA = b\,dx$ at position $x$. Magnetic flux through this strip is $d\Phi_m = B\,dA$.
dx
b
Vector $d\vec{A}$ is coming out of the page.
c
a
x
x
y
I




Loop
Long straight wire
4.0 cm
I
1.0 cm
1.0 cm

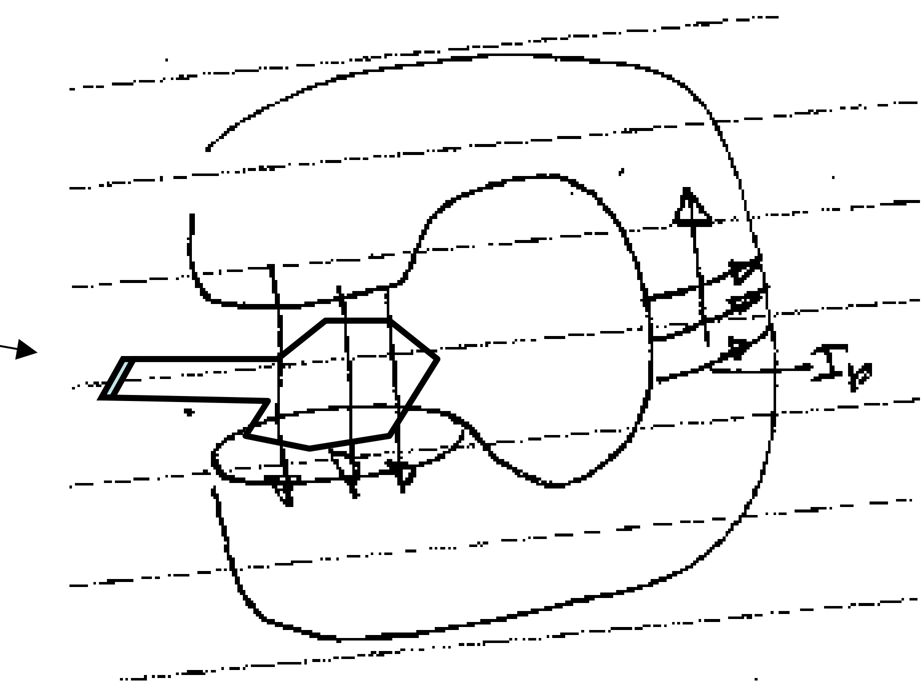
# A Transformer




Questions:

What will be the direction of B in the gap?
If I hold $I_p$ fixed, what will be the current in the loop?
If I increase $I_p$ what will be the direction of current in the loop?
If I decrease $I_p$ what will be the direction of current in the loop?

What will be the direction of B in the gap?



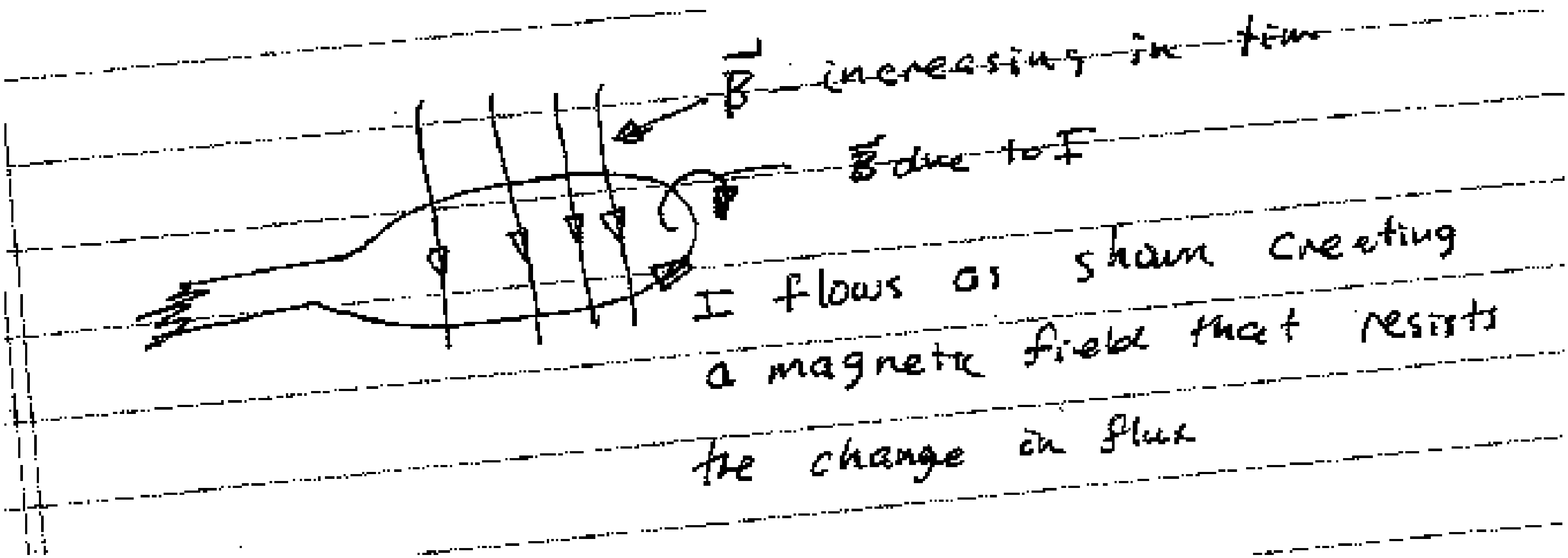

Answer: Down

Primary makes B - up in core, returns through gap.

If I hold Ip fixed what will be the current in the loop?

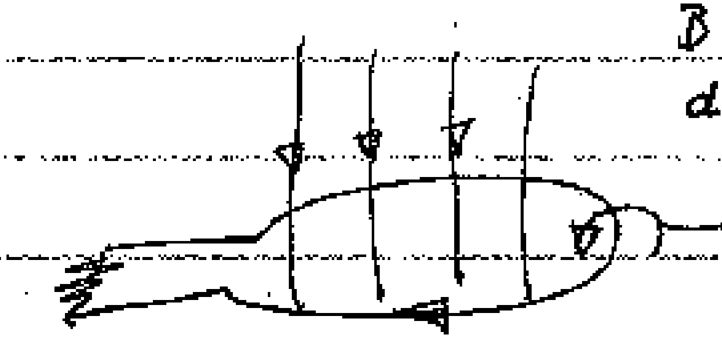
Answer: Zer, flux through loop is not changing

If I increase Ip from one positive value to a larger positive value what will be the direction of the current in the loop?



What if I lower Ip but don't make it negative. What will be the direction of current in the loop?

$\vec{B}$ is still down, but decreasing in time

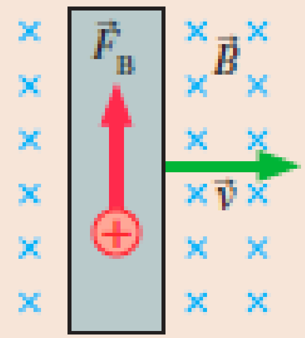


$\vec{B}$ due to I

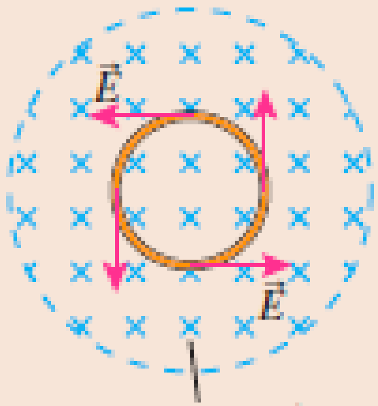
I flows as shown resisting decrease in flux

# Two ways to create an induced current

1. **A motional emf** due to magnetic forces on moving charge carriers.

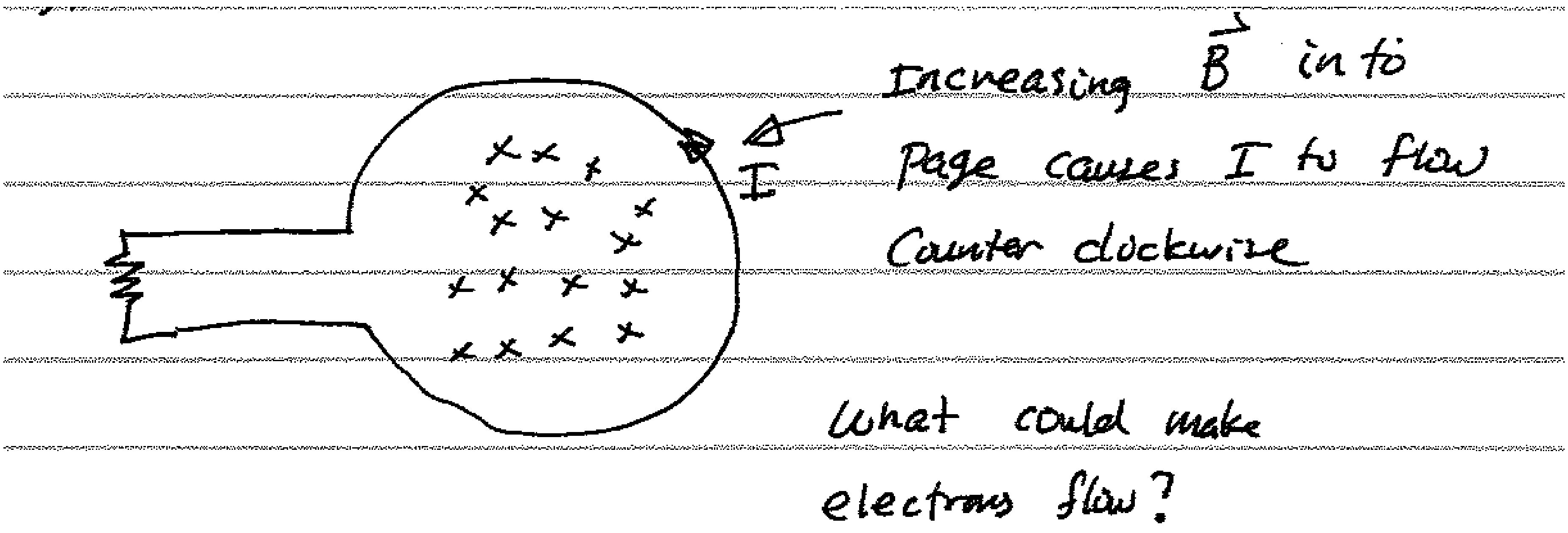



2. An induced electric field due to a changing magnetic field.



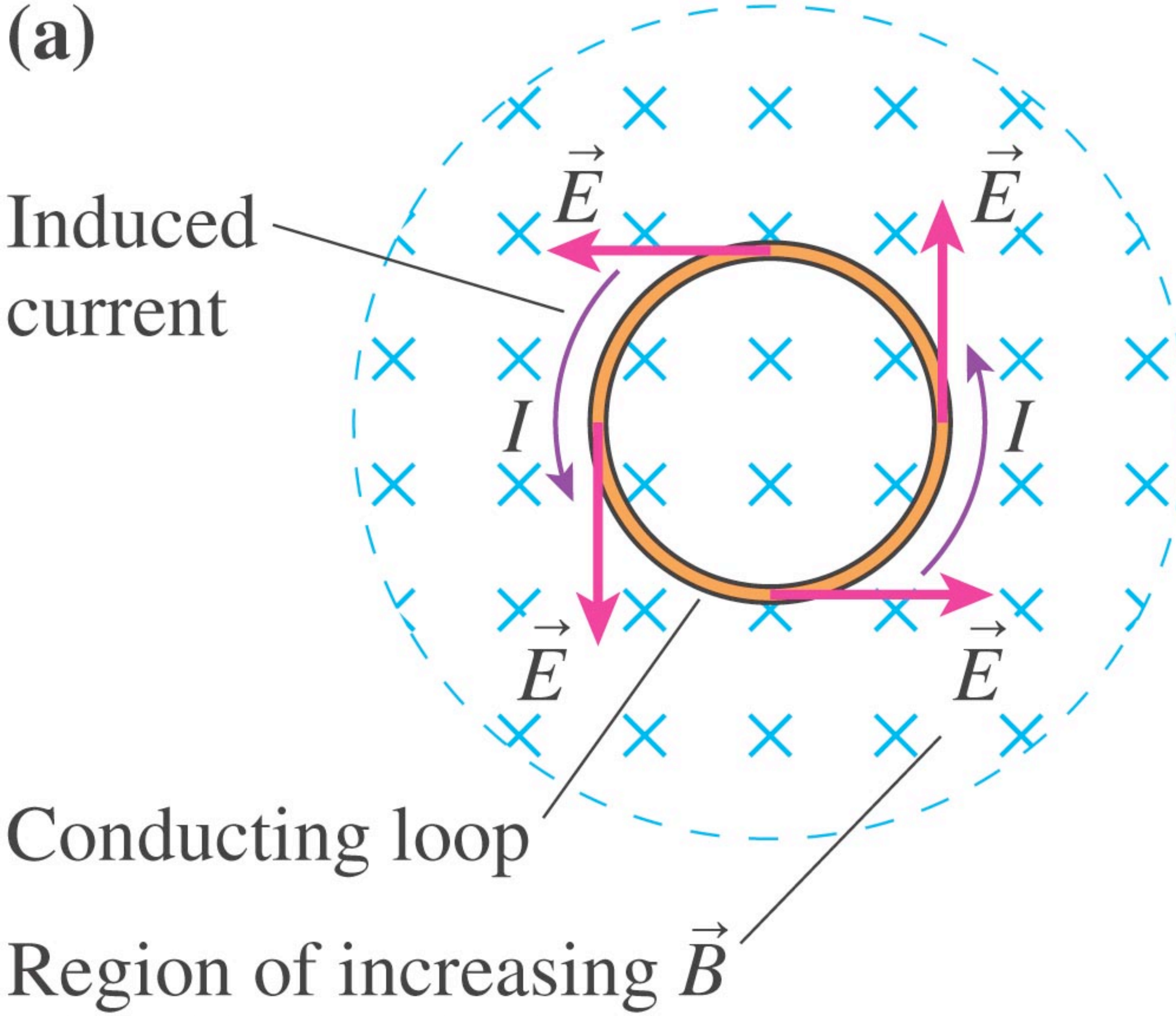
34.6 Induced Electric Field

Time changing magnetic fields induce electric fields

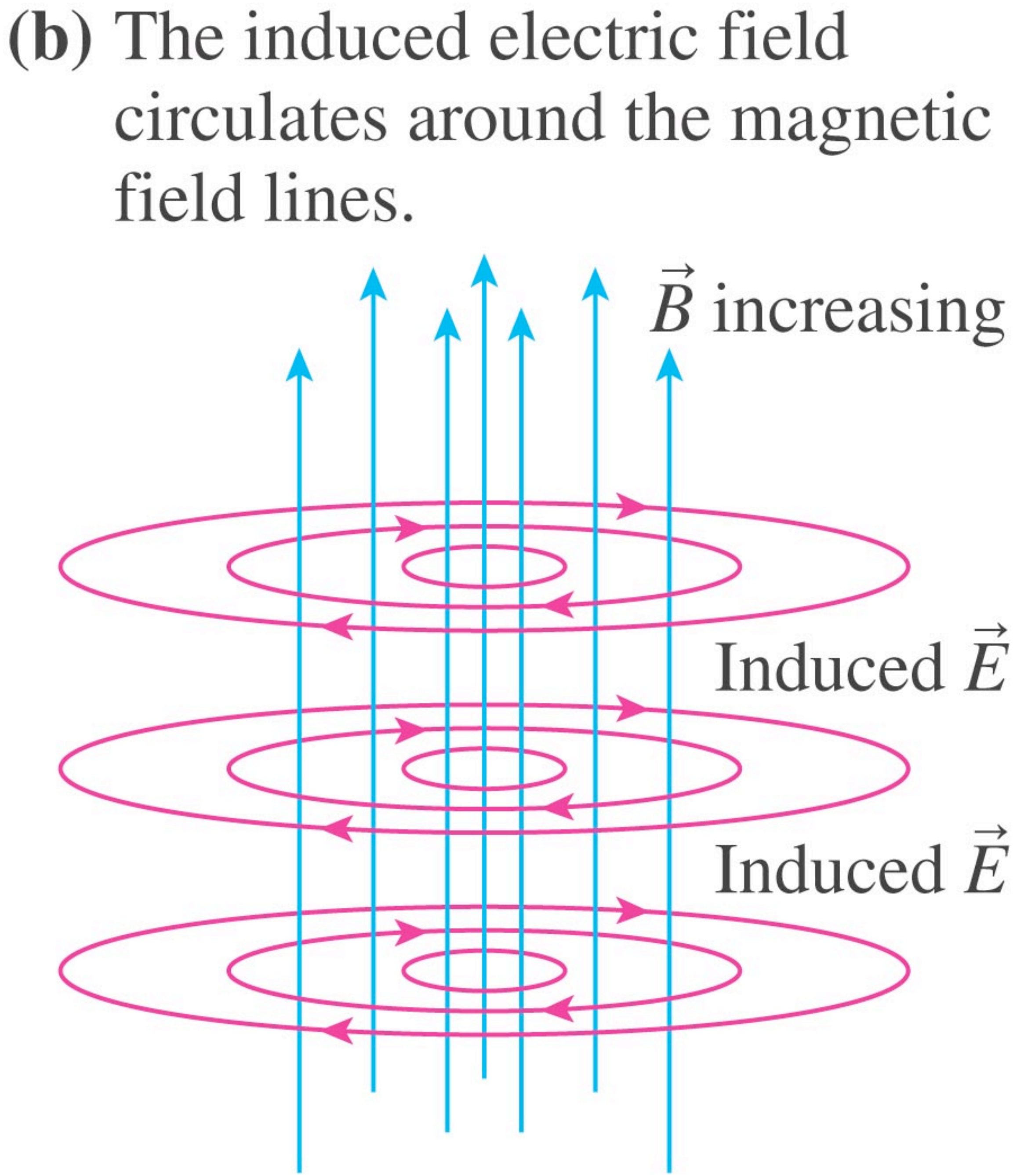



$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

But, the wire is not moving, v=0


(a)
Induced current
$\vec{E}$
$\vec{E}$
$I$
$I$
$\vec{E}$
$\vec{E}$
Conducting loop
Region of increasing $\vec{B}$

There is an electric field even with no wire.

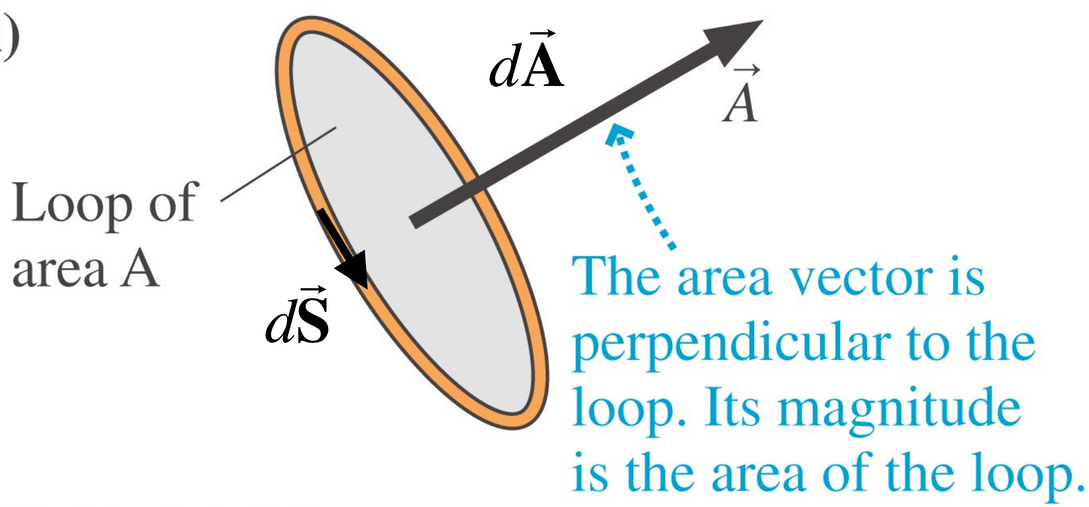


# Faraday's Law for Moving Loops

$$EMF = \oint_{loop}\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) \cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\Phi = -\frac{d}{dt}\int_{Area} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

related by right hand rule

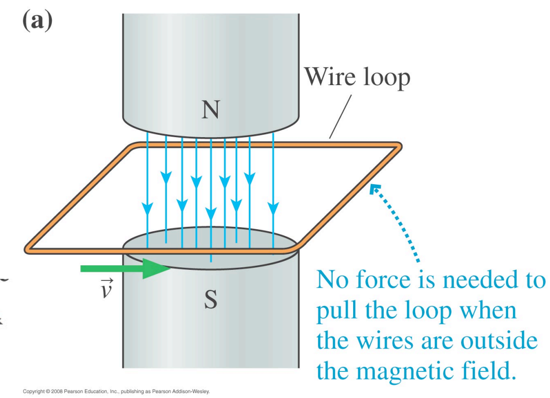
(a)



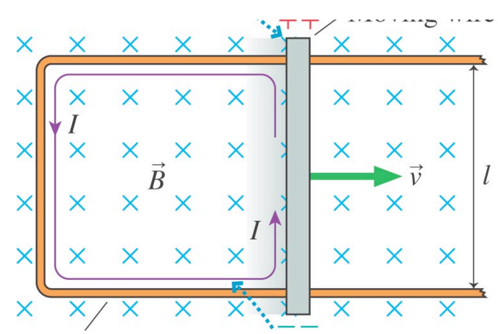
# Reasons Flux Through a Loop Can Change

$$\frac{d}{dt}\Phi = \frac{d}{dt}\int_{Area} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

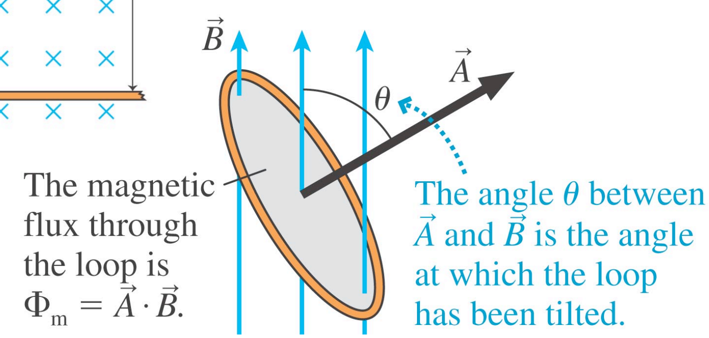
A. Location of loop can change



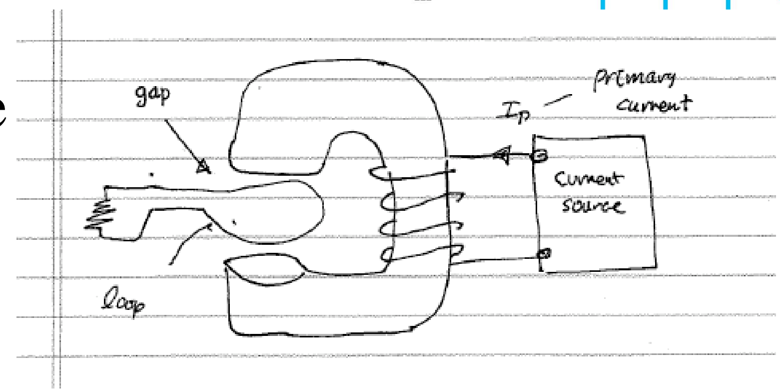

B. Shape of loop can change




C. Orientation of loop can change




D. Magnetic field can change



## Faraday's Law for Moving Loops

$$EMF = \oint_{loop} \left( \vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}} \right) \cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\Phi = -\frac{d}{dt}\int_{Area} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

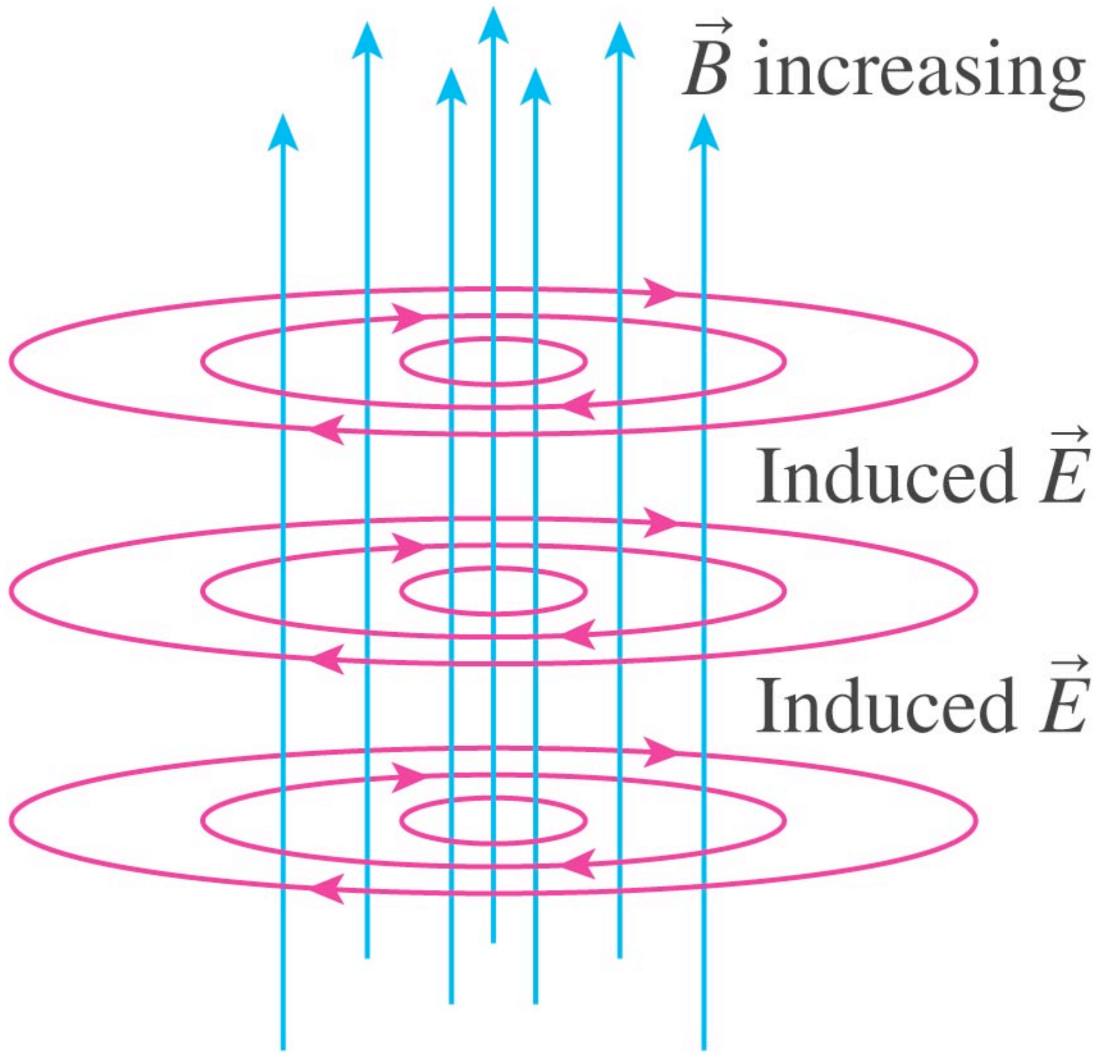
## Faraday's Law for Stationary Loops

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\int_{Area} \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{A}}$$

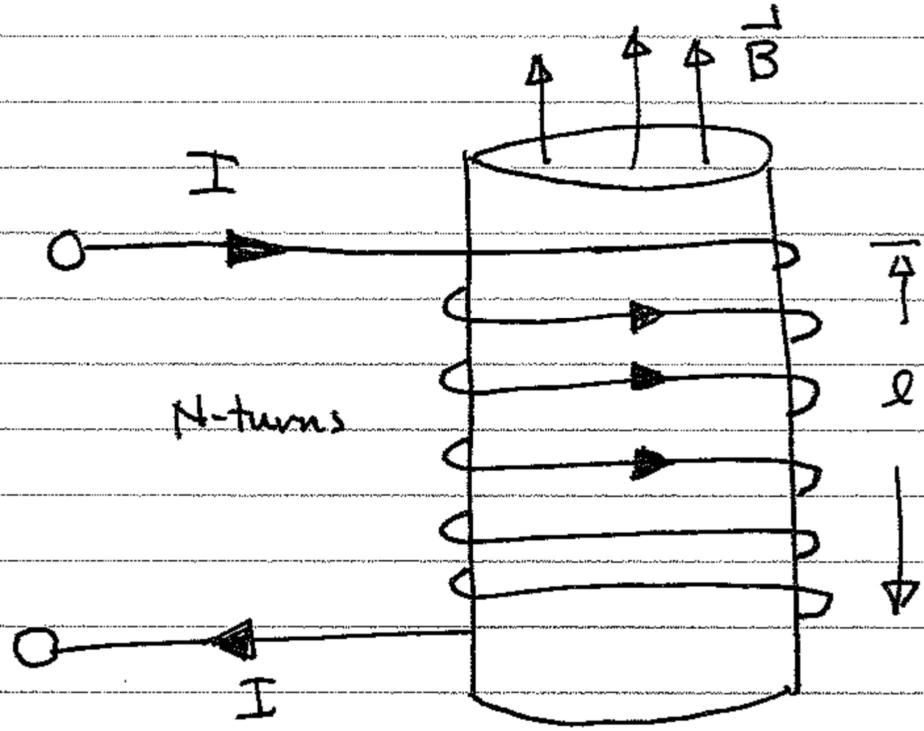
Only time derivative of B enters

There is an electric field even with no wire.

**(b)** The induced electric field circulates around the magnetic field lines.



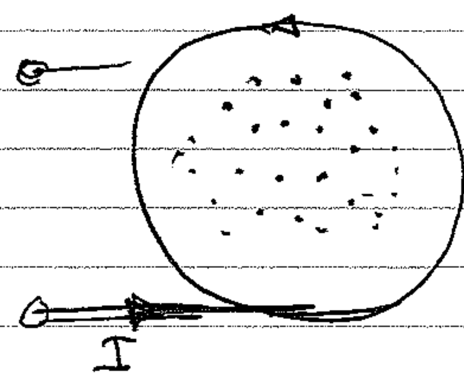
Consider a solenoid with N turns




Put your right thumb in direction of I. Fingers give direction of $\vec{B}$ (up inside)

$$|\vec{B}| = \frac{\mu_0 I N}{\ell}$$

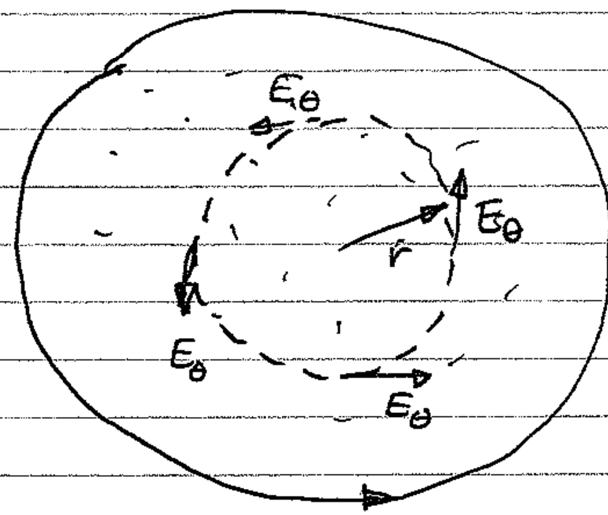
VIEW FROM ABOVE




$\vec{B}$ out of page

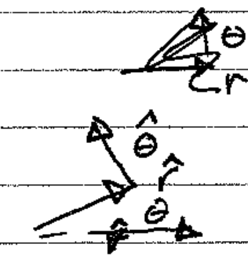
Calculate induced $\vec{E}$-field as a function of r

Consider a loop of radius r




$\vec{B}$ - out of page

$\vec{B} = B_z \hat{k}$




Q: Which direction is $\vec{E}$? $+\hat{\theta}$ or $-\hat{\theta}$

Ans: We don't know, Is B increasing or decreasing?

$$E_\theta = -\frac{r}{2}\frac{\partial B_z}{\partial t}$$

# Faraday's Law for Stationary Loops

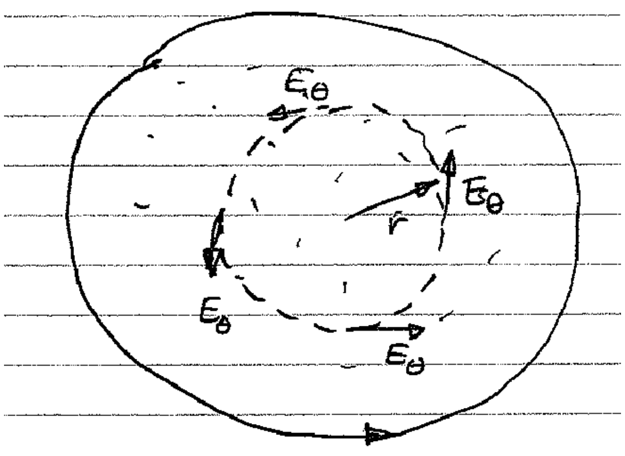
ccw

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\int_{Area} \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{A}}$$

Out of page (+z)

Only time derivative of B enters

Call component of E in θ direction $E_\theta(r,t)$

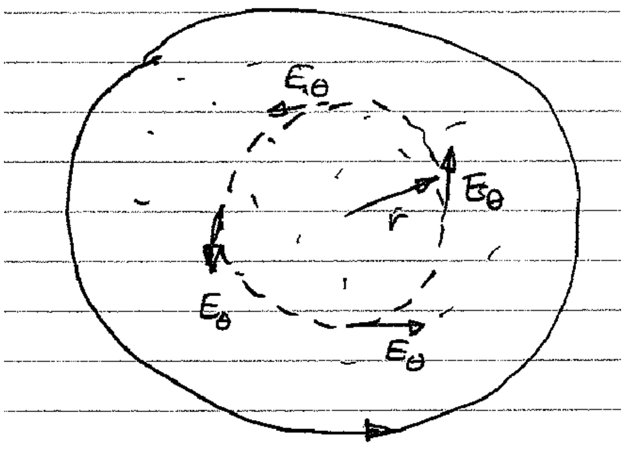



$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = 2\pi r E_\theta(r,t)$$

$$\int_{Area} \frac{\partial \vec{\mathbf{B}}}{\partial t} \cdot d\vec{\mathbf{A}} = \pi r^2 \frac{\partial B_z}{\partial t}$$

Therefore: $$E_\theta(r,t) = -\frac{r}{2}\frac{\partial B_z}{\partial t}$$

Is Lenz's law satisfied ????




$$E_\theta(r,t) = -\frac{r}{2}\frac{\partial B_z}{\partial t}$$

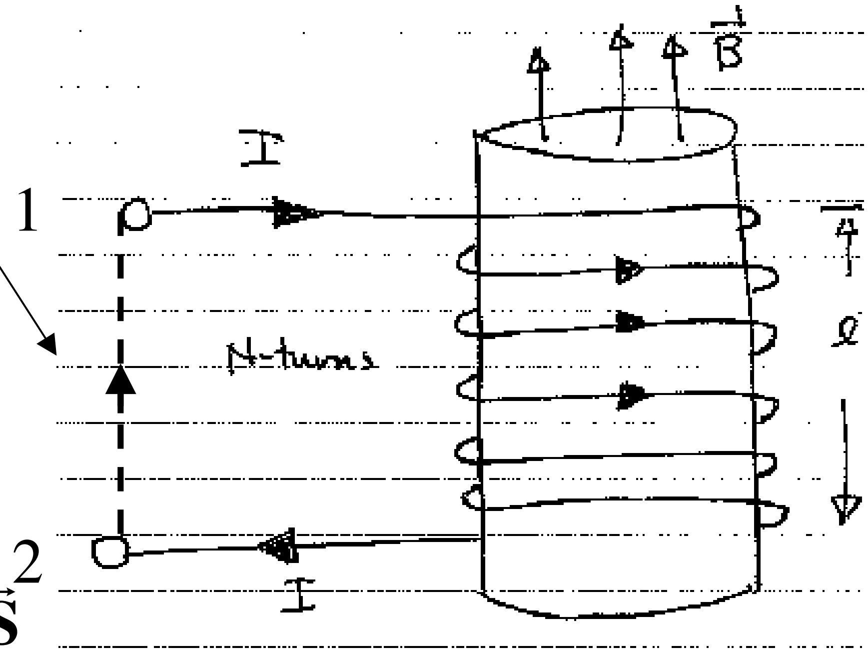
$B_z$ - out of page and increasing

An induced current would flow:

A Clockwise

B Counterclockwise

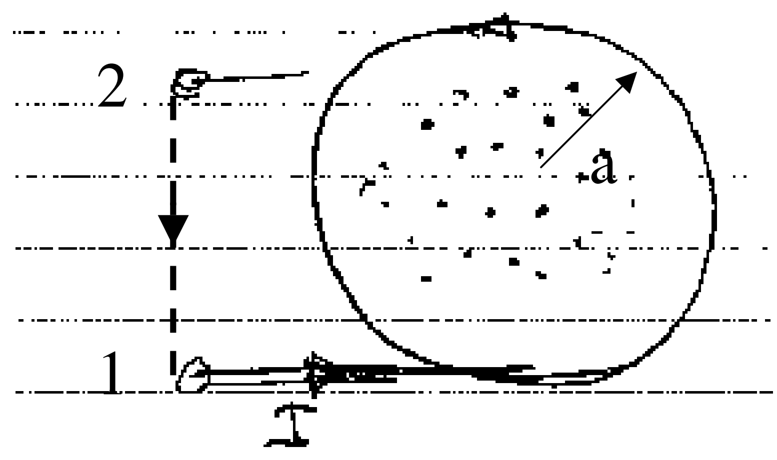
What is $\int_2^1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$




N turns

$$\int_2^1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} + \int_{1,\ \text{wire}}^{2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$$

$$= \oint_{Loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -N\pi a^2 \frac{\partial B_z}{\partial t}$$



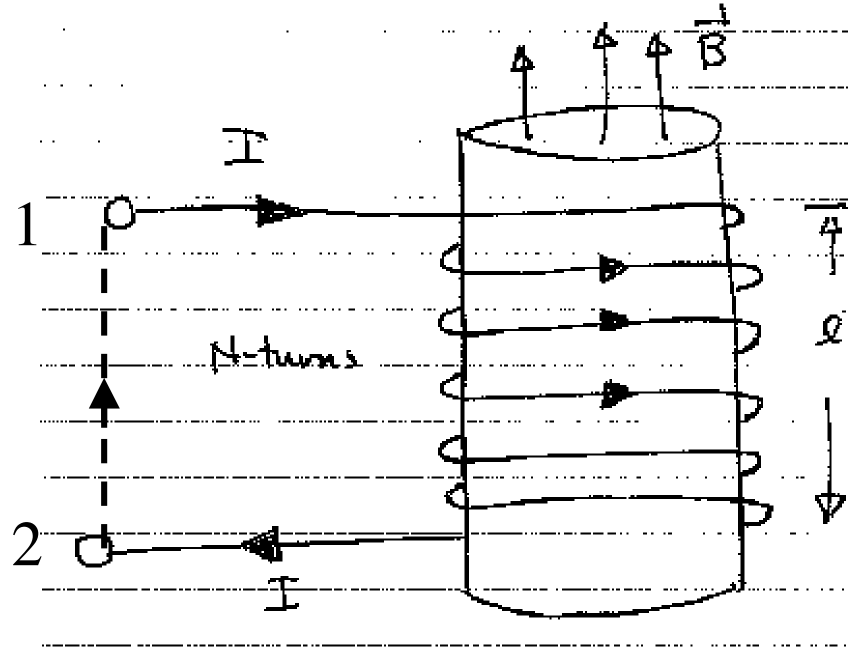

Top view

Question: $\int_{1,\ \text{wire}}^{2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = ?$

A. 0 B. $-\int_2^1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}}$

# Inductance

$$\int_{2}^{1} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -N\pi a^2 \frac{\partial B_z}{\partial t}$$




$$B_z = \frac{\mu_o N I}{l}$$

$$V_1 - V_2 = -\int_{2}^{1} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{\mu_0 N^2 \pi a^2}{l} \frac{dI}{dt} = L \frac{dI}{dt}$$

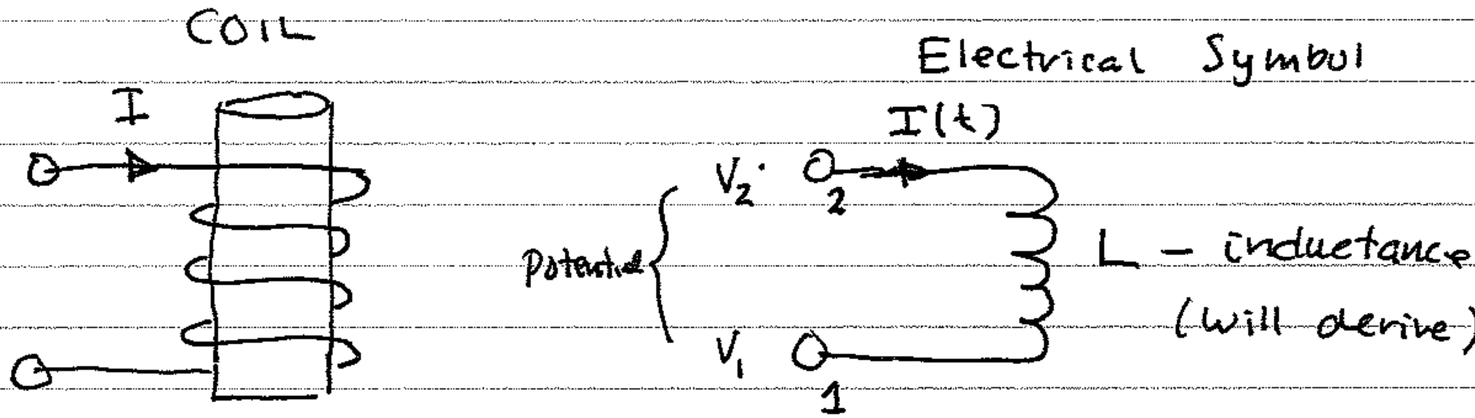
$$L = \frac{\mu_0 N^2 \pi a^2}{l}$$

Depends in geometry of coil, not I

# Inductors

An inductor is a coil of wire

Any length of wire has inductance: but it's usually negligible




Engineering sign convention for labeling voltage and current

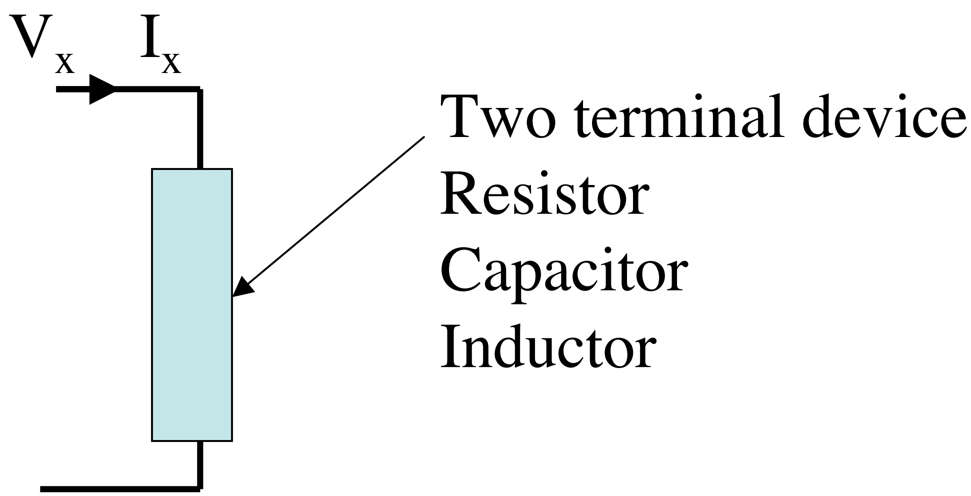
$$V_L = V(2) - V(1) = L\, dI/dt$$

# Engineering Convention for Labeling Voltages and Currents

1. Pick one terminal and draw an arrow going in.

2. Label the current $I_x$.

3. Label the Voltage at that terminal $V_x$. This is the potential at that terminal relative to the other terminal.

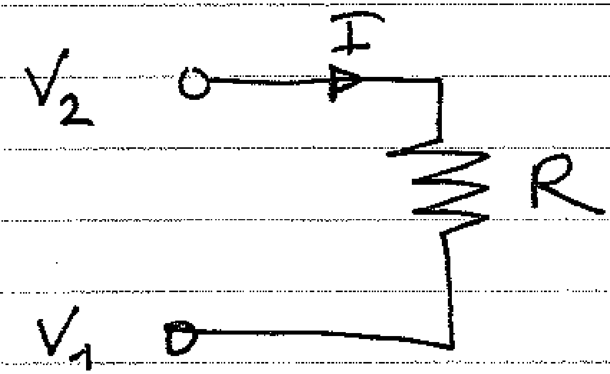



No f-ing minus signs

$$V_R = RI_R$$

$$V_L = L\, dI_L / dt$$

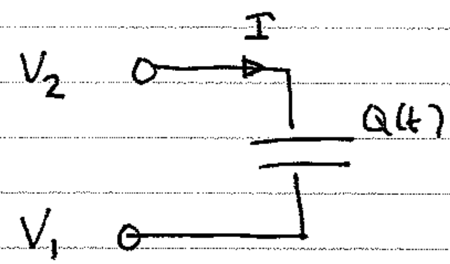
$$I_C = C\, dV_C / dt$$




$$V_R = (V_2 - V_1) = IR$$

**OHM'S LAW**

$$V_R = IR$$



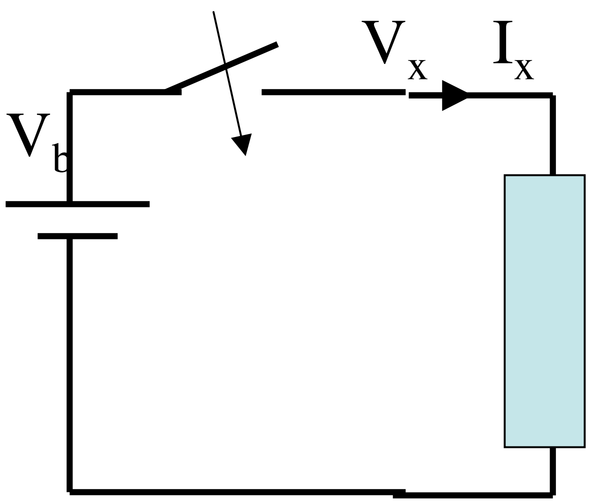

$$Q(t) = C\,(V_2 - V_1)$$

$$= C\,V_c$$

$$\frac{dQ(t)}{dt} = I(t) = C\,\frac{dV_c}{dt}$$

$$I(t) = C\,\frac{dV_c}{dt}$$

Power and Energy to a two terminal device




At t=0 the switch is closed

Then $V_x = V_b$

Current $I_x$ flows.

The power delivered to the device is

$$P = I_x V_x$$

If device is a resistor

$$V_x = I_x R$$

$$P = I_x^2 R > 0$$

If device is an inductor

$$V_x = L \frac{dI_x}{dt}$$

$$P = I_x L \frac{dI_x}{dt} = \frac{d}{dt}\left(\frac{L I_x^2}{2}\right)$$

Energy stored in Inductor

$$P = IL\frac{dI}{dt} = \frac{d}{dt}\left(\frac{LI^2}{2}\right)$$

$$U = \int_0^t dt' P(t') = \int_0^t dt' \frac{d}{dt'}\left(\frac{LI^2}{2}\right) = \left(\frac{LI^2}{2}\right)$$

Where is the energy?

$$B_z = \frac{\mu_o NI}{l} \qquad L = \frac{\mu_0 N^2 \pi a^2}{l}$$

Consider a solenoid

$$\left(\frac{LI^2}{2}\right) = \left(\pi a^2 l\right)\frac{B_z^2}{2\mu_0}$$ = Volume x Energy Density

Energy is stored in the magnetic field

# How much energy is stored in the magnetic field of an MRI machine?

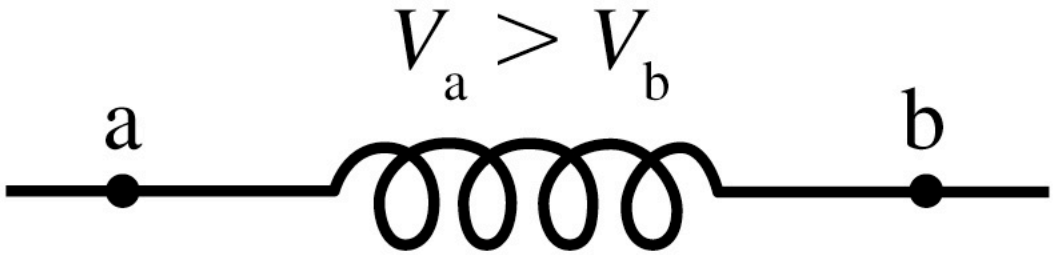

Volume = $\pi a^2 l = \pi (.5)^2 \cdot 2 = 1.57\ m^3$

$\mu_0 = 4\pi \times 10^{-7}$

$U = \frac{1.57}{2} \frac{1^2}{(4\pi \times 10^{-7})} = 6.25 \times 10^5\ J$

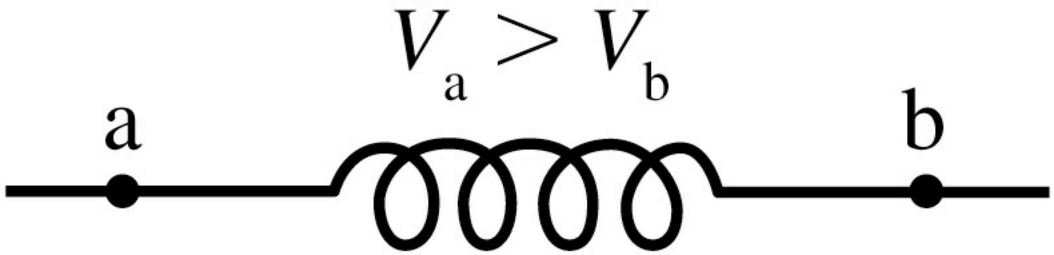
= 1 hair dryer * 625 sec

= 1 hairdryer * 10 minutes




**The potential at a is higher than the potential at b. Which of the following statements about the inductor current *I* could be true?**

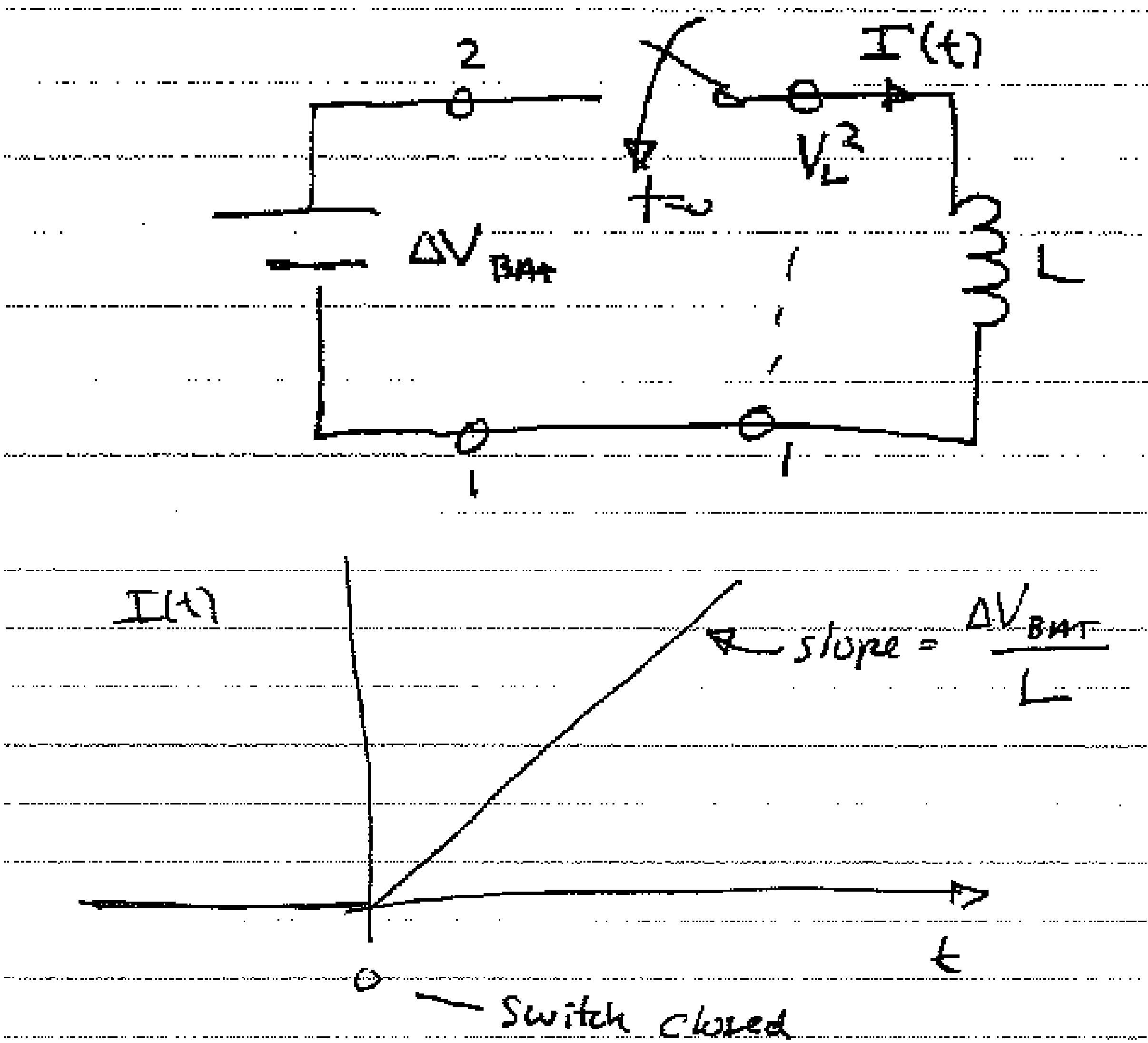
A. I is from b to a and is steady.
B. I is from b to a and is increasing.
C. *I* is from a to b and is steady.
D. *I* is from a to b and is increasing.
E. *I* is from a to b and is decreasing.




**The potential at a is higher than the potential at b. Which of the following statements about the inductor current *I* could be true?**

A. I is from b to a and is steady.
B. I is from b to a and is increasing.
C. *I* is from a to b and is steady.
✔ **D. *I* is from a to b and is increasing.**
E. *I* is from a to b and is decreasing.

What happens to the current in the inductor after I close the switch?




$V_L = L\frac{dI}{dt}$

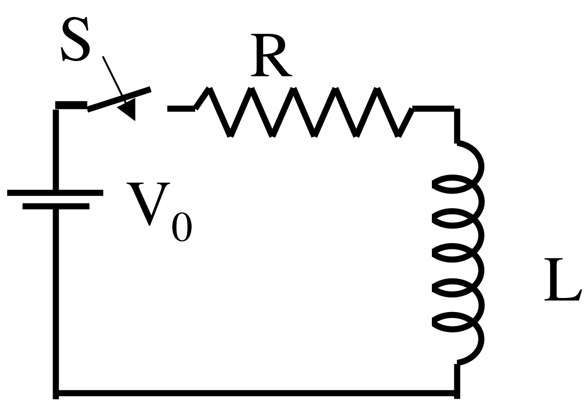
$V_L = \Delta V_{bat}$

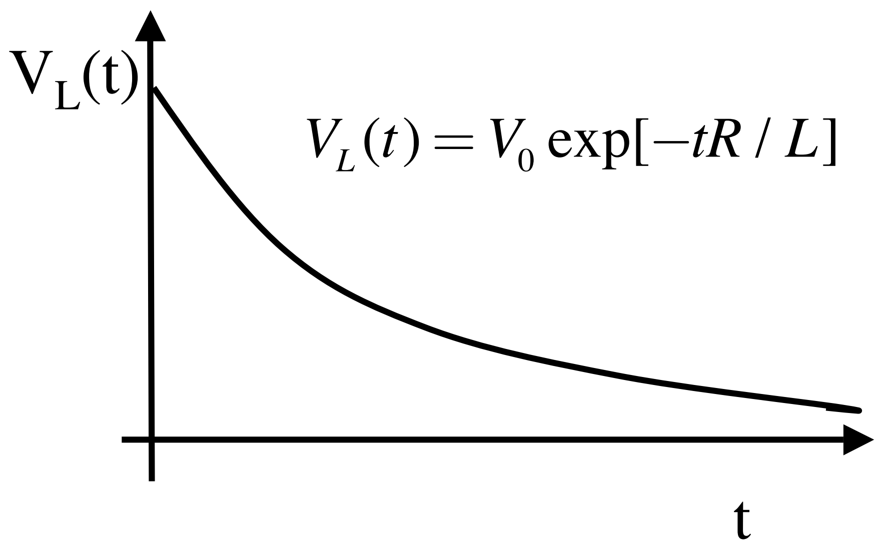
What happens if I open the switch?

Three categories of time behavior

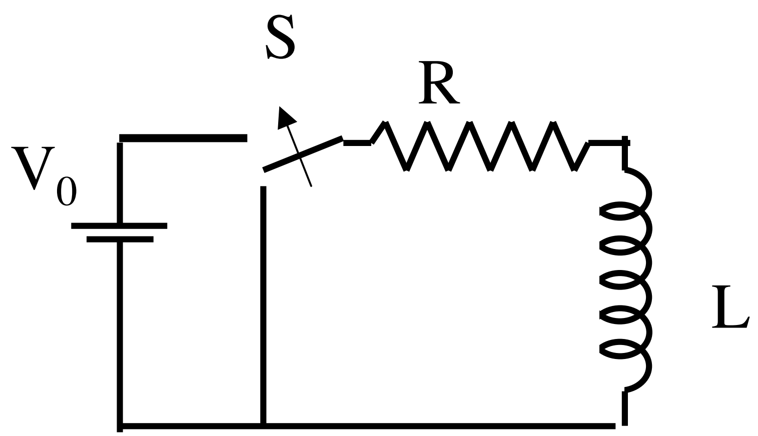
1. Direct Current (DC) Voltages and currents are constants in time. Example: batteries - circuits driven by batteries

2. Transients  Voltages and currents change in time after a switch is opened or closed.  Changes diminish in time and stop if you wait long enough.

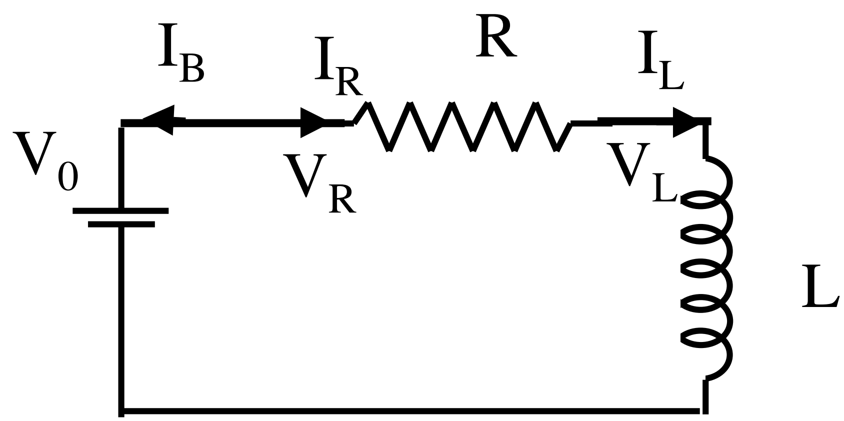



$$V_L(t) = V_0 \exp[-tR / L]$$

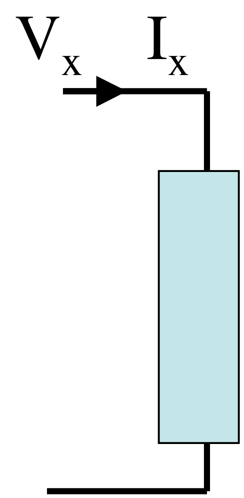



Consider the series connection of an inductor, a resistor and a battery. Initially no current flows through inductor and resistor. At t=0 switch is closed.What happens to current?




Notice, I've gone overboard and labeled every circuit element voltage and current according to the engineering convention.

# A Word about Voltage and Current




Voltage is “across”.

Current is “through”.

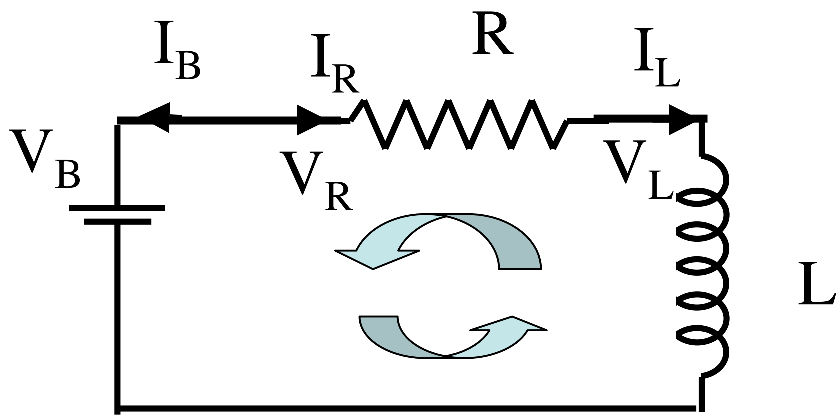
Voltage is the potential difference between the two terminals.

Current is the amount of charge per unit time flowing through the device.

If you catch yourself saying:

“Voltage through..”.
or
“Current across…”.

You are probably confused.




Kirchhoff's voltage and current laws.

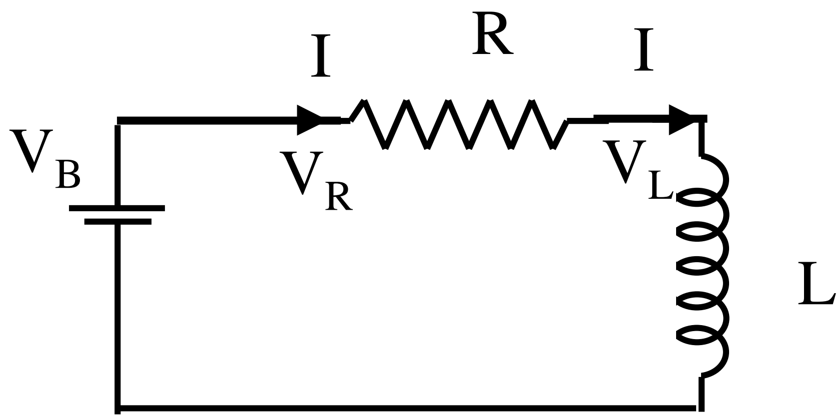
1. The sum of the currents entering any node is zero. (KCL)
2. The sum of the voltages around any loop is zero. (KVL)

#1(KCL) tells us

A. $I_B+I_R+I_L = 0$
B. $I_B=I_R=I_L$
C. $I_B=-I_R, I_R=I_L$

#2(KVL) tells us

A. $V_B+V_R+V_L = 0$
B. $V_L+V_R-V_B=0$
C. $V_B=V_R=V_L$




Now I have cleaned things up making use of
$I_B$=-$I_R$, $I_R$=$I_L$=I.

Now use device laws:
$V_R = RI$
$V_L = L\,dI/dt$

KVL: $V_L + V_R - V_B = 0$

$$L\frac{dI}{dt} + RI - V_B = 0$$

This is a differential equation that determines I(t). Need an initial condition I(0)=0

$$L\frac{dI(t)}{dt} + RI(t) - V_B = 0, \quad I(0) = 0$$

This is a linear, ordinary, differential equation with constant coefficients.

Linear: only first power of unknown dependent variable and its derivatives appears. No $I^2$, $I^3$ etc.
Ordinary: only derivatives with respect to a single independent variable - in this case t.
Constant coefficients: L and R are not functions of time.

Consequence: We can solve it!

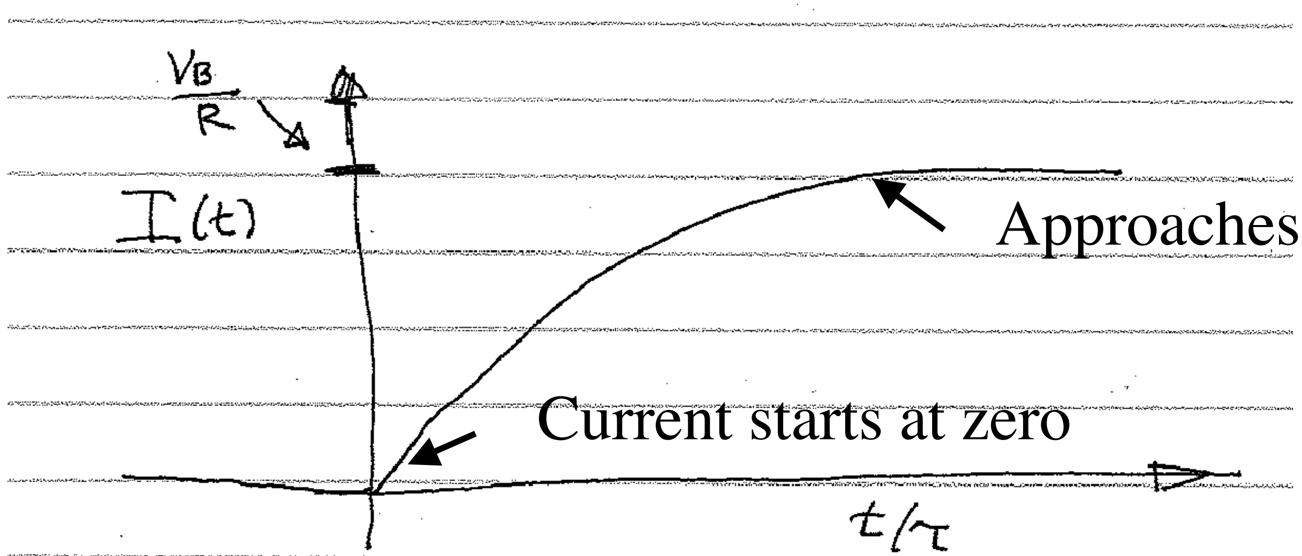
$$L\frac{dI(t)}{dt} + RI(t) - V_B = 0, \quad I(0) = 0$$

Solution:

$$I(t) = \frac{V_B}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = (L/R)$$

This is called the “L over R” time.



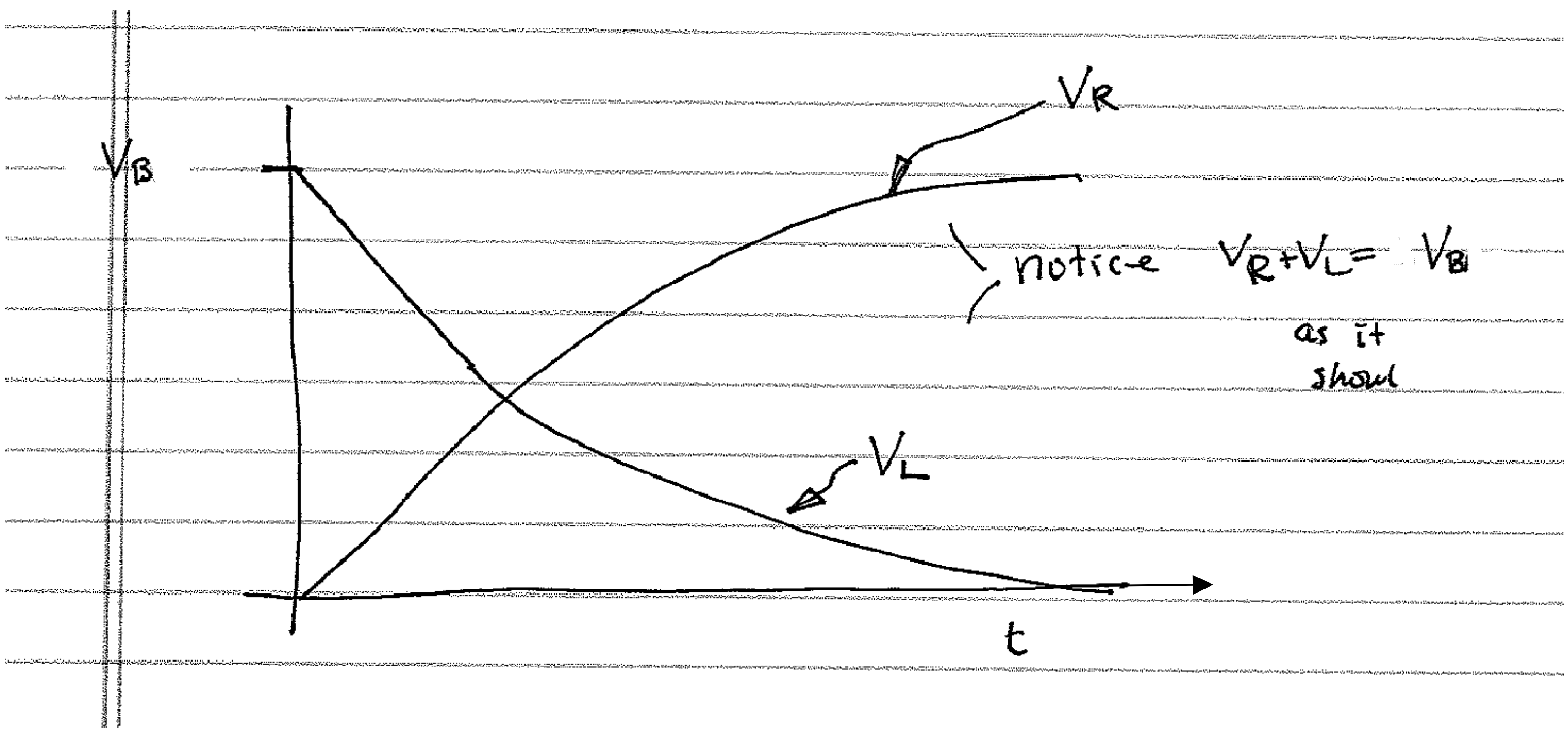

Let’s verify

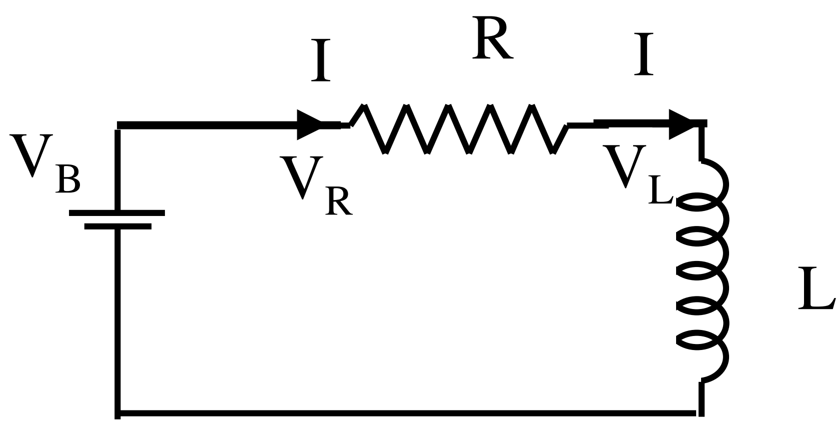
What is the voltage across the resistor and the inductor?

$$I(t) = \frac{V_B}{R}\left(1 - e^{-t/\tau}\right)$$

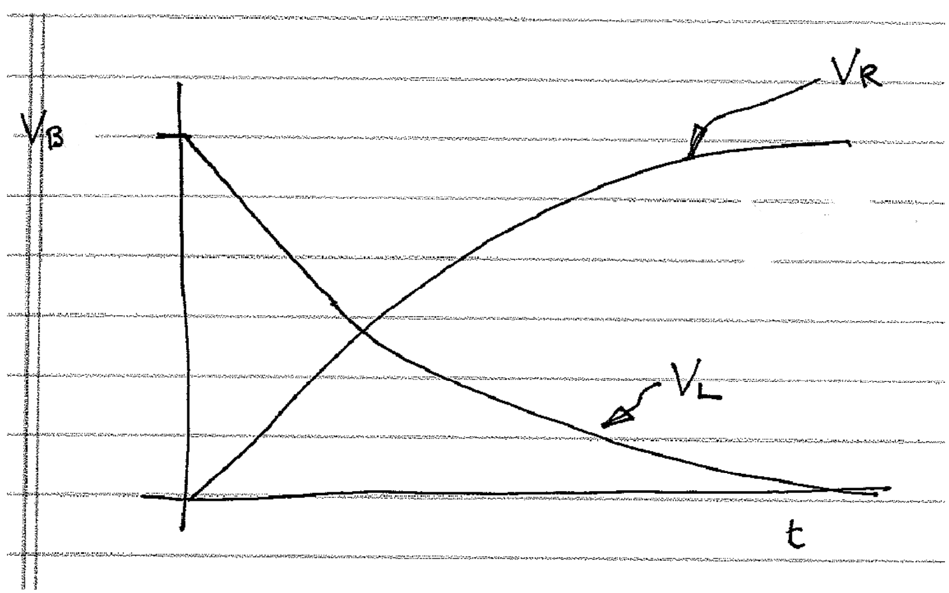
$$V_R = RI(t) = V_B\left(1 - e^{-t/\tau}\right)$$

$$V_L = L\frac{dI}{dt} = V_B e^{-t/\tau}$$






Initially I is small and $V_R$ is small.
All of $V_B$ falls across the inductor, $V_L=V_B$.
Inductor acts like an open circuit.

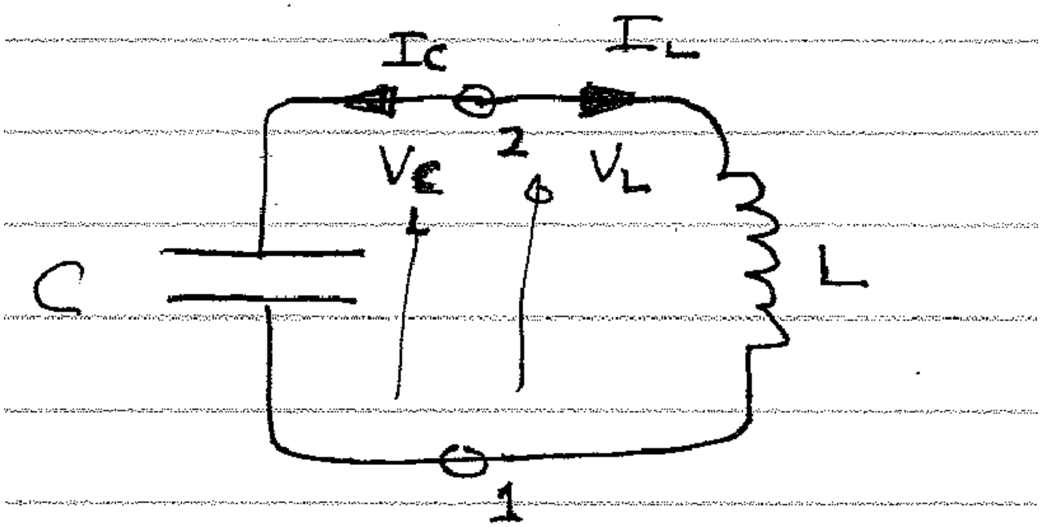



Time asymptotically I stops changing and $V_L$ is small.
All of $V_B$ falls across the resistor, $V_R=V_B$. $I=V_B/R$
Inductor acts like an short circuit.

Now for a Mathematical Interlude

How to solve a linear, ordinary differential equation with constant coefficients

# The L-C circuit




$$V_C = V_L = V$$

$$I_C + I_L = 0$$

$$V(t) = L\frac{dI_L}{dt}$$

$$I_C = C\frac{dV(t)}{dt}$$

KCL says:

A. $I_C=I_L$
B. $I_C+I_L=0$
C. $V_L=L\, dI_L/dt$

KVL says:

A. $V_C=V_L$
B. $V_C+V_L=0$
C. $I_C=C\, dV_L/dt$

What about initial conditions?

Must specify: $I_L(0)$ and $V_C(0)$

$$V_C = V_L = V$$

$$I_C + I_L = 0$$

$$V(t) = L\frac{dI_L}{dt}$$

$$I_C = C\frac{dV(t)}{dt}$$

What about initial conditions?

Must specify: $I_L(0)$ and $V_C(0)$

---

$$\frac{dI_C}{dt} + \frac{dI_L}{dt} = 0$$

$$\frac{dI_C}{dt} = C\frac{d^2V(t)}{dt^2}$$

$$\frac{dI_L}{dt} = \frac{V(t)}{L}$$

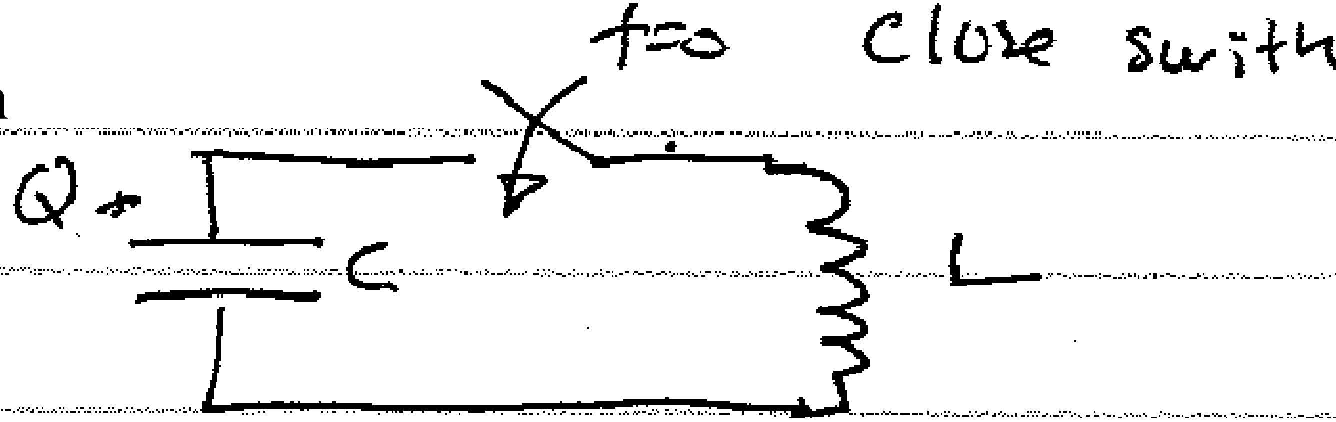
$$\frac{d^2V(t)}{dt^2} + \frac{V(t)}{LC} = 0$$

$$V(0) = V_C(0)$$

$$I_L(0) = -I_C(0) = -C\left.\frac{dV}{dt}\right|_{t=0}$$

Let's take a special case of no current initially flowing through the inductor

Initial charge on capacitor




$$\frac{d^2V(t)}{dt^2} + \frac{V(t)}{LC} = 0$$

$$V(0) = V_C(0)$$

$$I_L(0) = 0 = -C\frac{dV}{dt}\bigg|_{t=0}$$

Solution

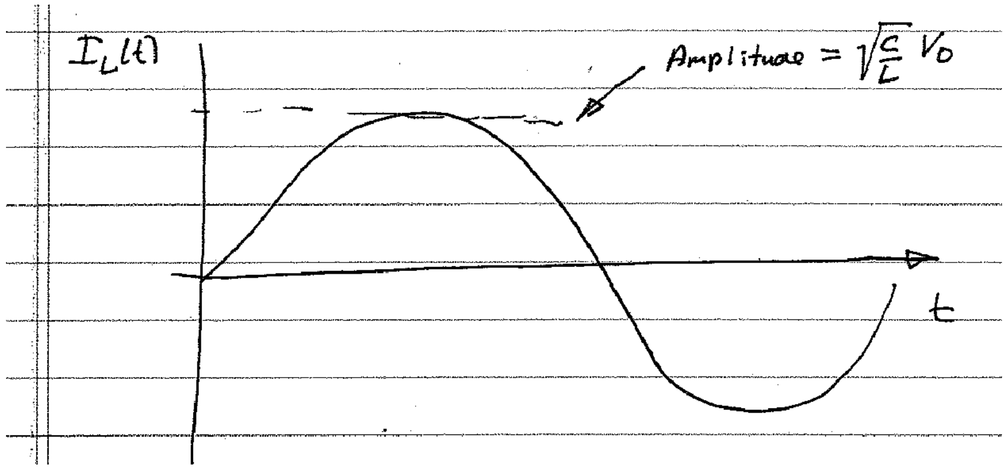
A: $V(t) = V_C(0)\cos(\omega t)$

$\omega = 1/\sqrt{LC}$

B: $V(t) = V_C(0)\sin(\omega t)$

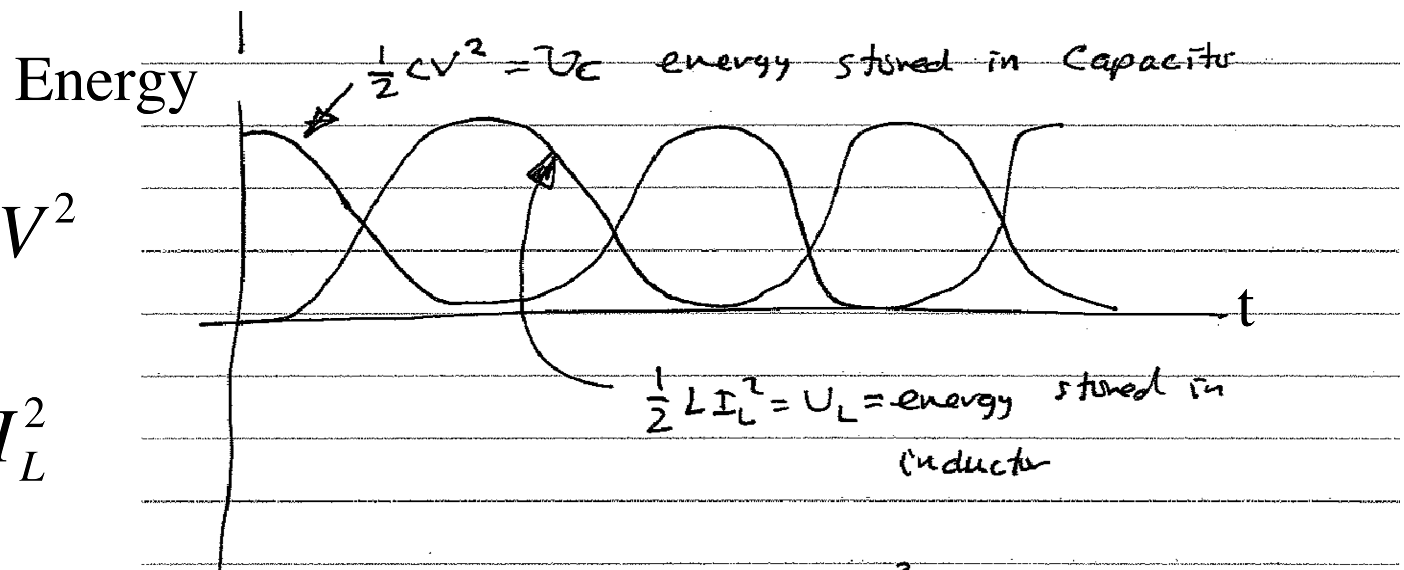
# Current through Inductor and Energy Stored

$$I_L(t) = -C\frac{dV}{dt} = C\omega V_0 \sin\omega t = \sqrt{\frac{C}{L}}\, V_0 \sin\omega t$$




$$U_C = \frac{1}{2}CV^2$$

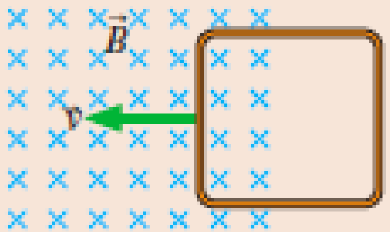
$$U_L = \frac{1}{2}LI_L^2$$



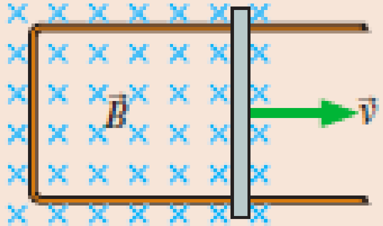

$$U_C + U_L = \text{const} = \frac{1}{2}CV(0)^2$$

## Three ways to change the flux

1. A loop moves into or out of a magnetic field.




2. The loop changes area or rotates.




3. The magnetic field through the loop increases or decreases.