# Chapter 33. The Magnetic Field

1. Moving charges make a magnetic field

2. The magnetic field exerts a force on moving charges

   Thus moving charges exert forces on each other (Biot Savart Law)

3. A collection of moving charges constitutes and electrical current

4. Currents exert forces and torques on each other

1. Moving charges (or equivalently electrical <u>currents</u>) create a magnetic field,

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}, \qquad *$$

Here $\vec{\mathbf{v}}$ is the velocity of the moving charge.

This is the analog to the expression for the electric field due to a point charge.

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$$

Superposition applies if a number of moving charges are present just as in the case of electric fields. As in the case of electric fields, most of your grief will come from trying to apply the principle of superposition.

* This isn't really true but pretend for the moment that it is.

2. A charge moving through a magnetic field feels a force (called the Lorenz force) given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \,.$$

The total electric and magnetic force on a moving charged particle is the sum of the contributions due to electric and magnetic fields,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) \,.$$

Just to be confusing this is sometimes called the Lorenz force too.

There are a host of consequences of 1. and 2., which will be explored in this chapter. An example is **parallel currents attract** and **antiparallel currents repel**.

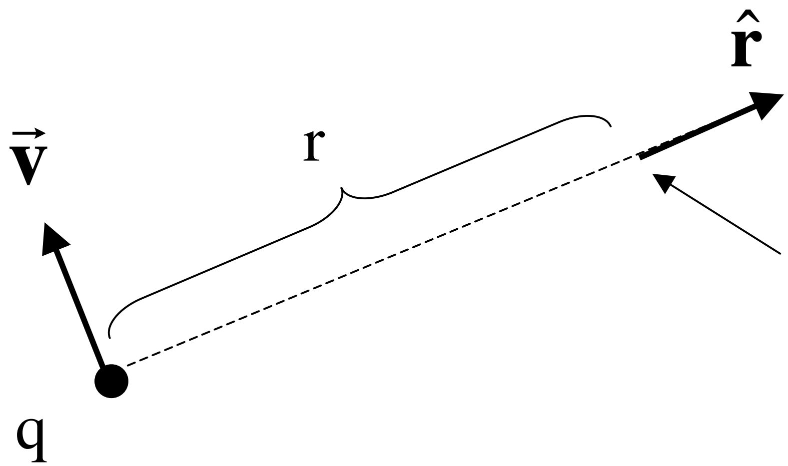
A **big** complication when discussing magnetic fields is the appearance of **vector cross products**. Don't think that you can slide by without learning how to evaluate them.

Electric Field

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2}$$

Magnetic Field

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$




What are the magnitudes and directions of the electric and magnetic fields at this point? Assume q > 0

Comparisons: both go like $r^{-2}$, are proportional to q, have $4\pi$ in the denominator, have funny Greek letters

Differences: E along r, B perpendicular to r and v

# Greek Letters

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{r}}}{r^2} \qquad \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$

$$\varepsilon_0 = 8.8542\times 10^{-12} \quad \textit{Farads / meter} \qquad \mu_0 = 4\pi\times 10^{-7} \quad \textit{Henries / meter}$$

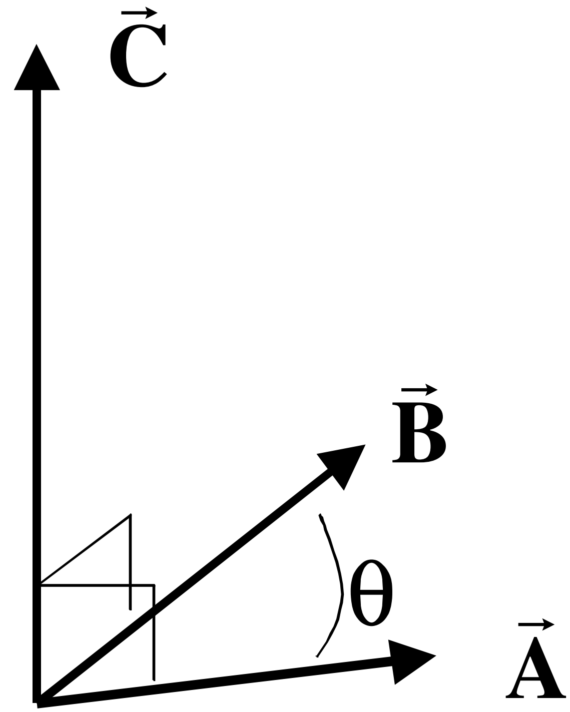
Huh?

These funny numbers are a consequence of us having decided to measure charge in Coulombs

Later we will find:

$$1/\sqrt{\varepsilon_0\mu_0} = c \qquad \sqrt{\mu_0/\varepsilon_0} = 377 \quad \textit{Ohms}$$

Aside on cross products: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$




Let $\theta$ be the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, **always chose this to be less than 180°**.

The magnitude of $\vec{\mathbf{C}}$ is given by $\left|\vec{\mathbf{C}}\right| = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta$

The direction of $\vec{\mathbf{C}}$ is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and determined by the "right hand rule"

Right hand rule: put the fingers of your **right hand** in the direction of $\vec{\mathbf{A}}$, then rotate them through the angle $\theta$ (less than 180°) to the direction of $\vec{\mathbf{B}}$. Your thumb now indicates the direction of $\vec{\mathbf{C}}$.

The order of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is important: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}}$

If $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are parallel, then $(\theta=0)$ $\vec{\mathbf{C}} = \mathbf{0}$.

Let's say we are given $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in component form in Cartesian coordinates.

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$
$$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} \, .$$

How do we find the components of $\vec{\mathbf{C}}$?

$$\vec{\mathbf{C}} = C_x\hat{\mathbf{i}} + C_y\hat{\mathbf{j}} + C_z\hat{\mathbf{k}} \, .$$

Answer:

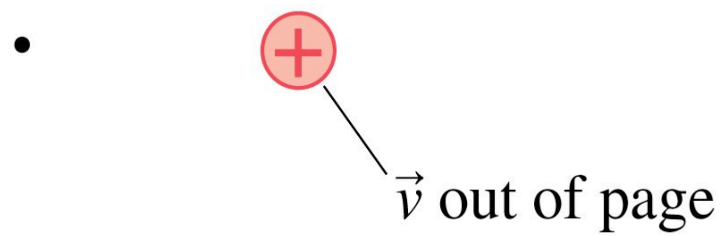
$$C_x = A_yB_z - A_zB_y$$
$$C_y = A_zB_x - A_xB_z \qquad \text{(my favorite)}$$
$$C_z = A_xB_y - A_yB_x$$

Same result as:

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

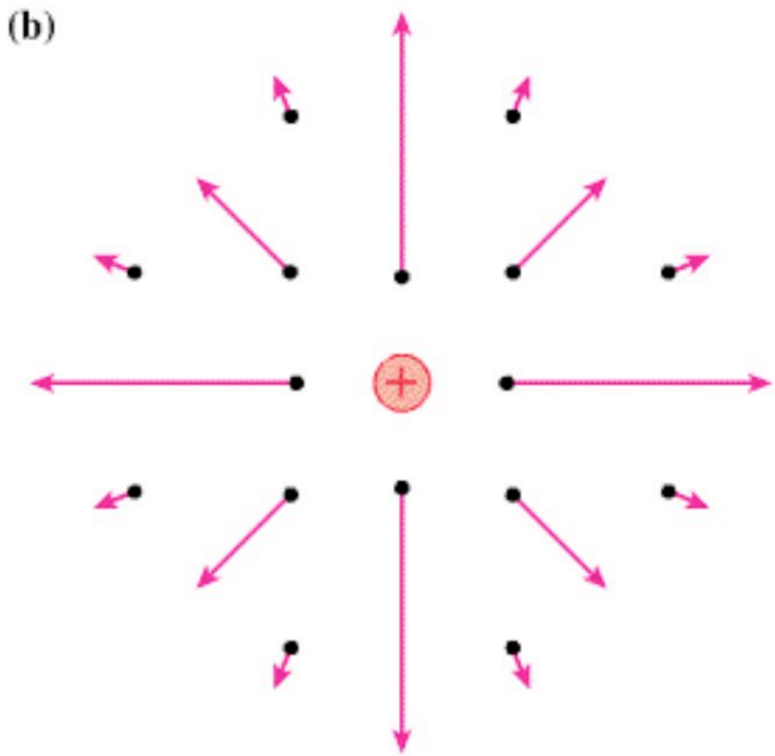
**The positive charge is moving straight out of the page. What is the direction of the magnetic field at the position of the dot?**

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

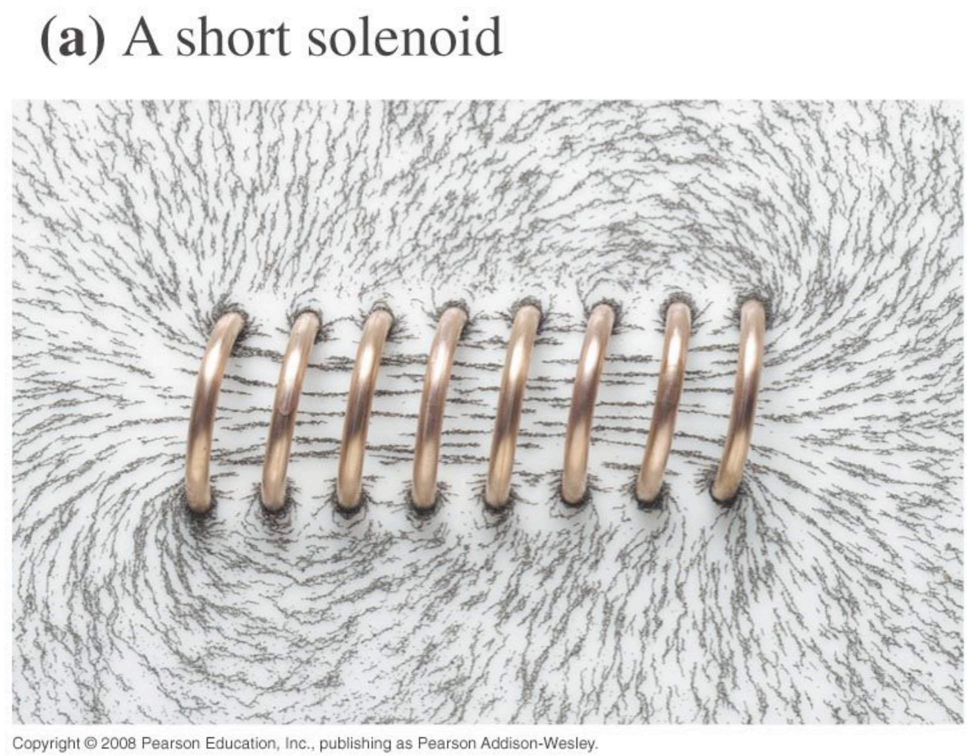



A. Left
B. Right
C. Down
D. Up

Field near a point charge



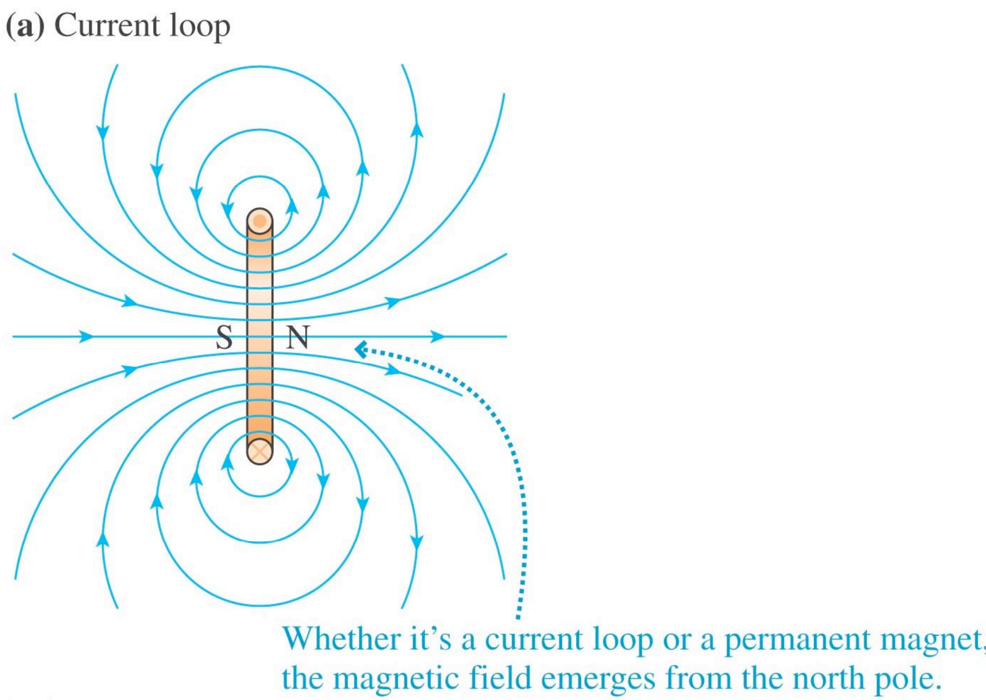

Field lines emerge from charges



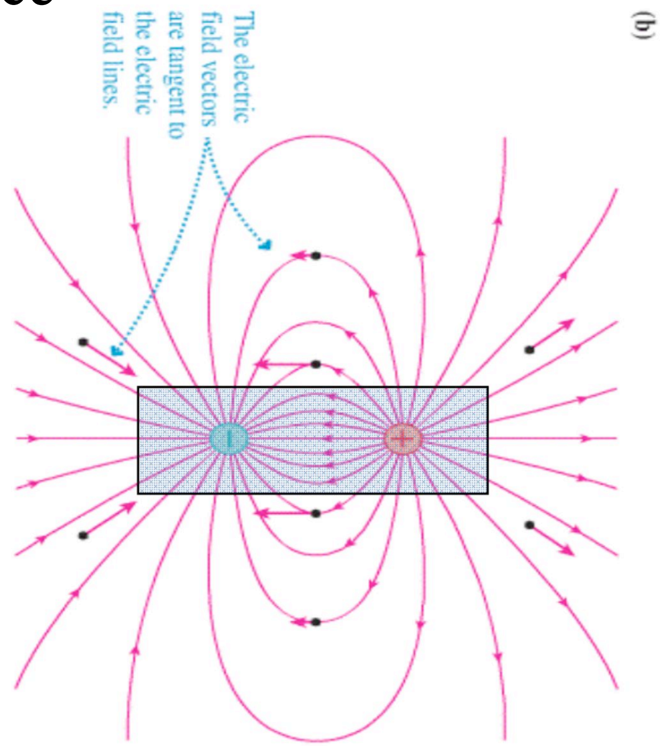

Iron filings align parallel to magnetic field
Field lines do not end

Magnetic field due to a single loop of current

Electric field due to a dipole - two opposite charges separated by a distance



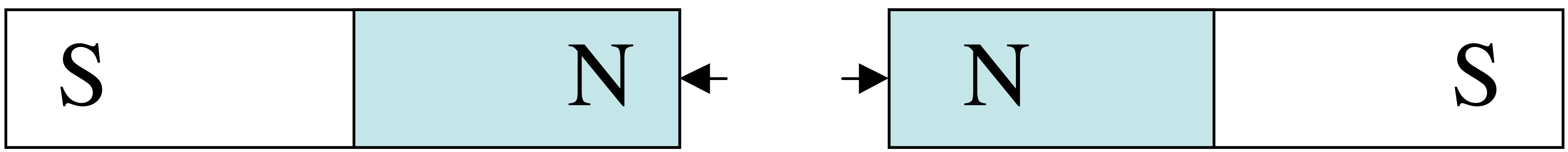


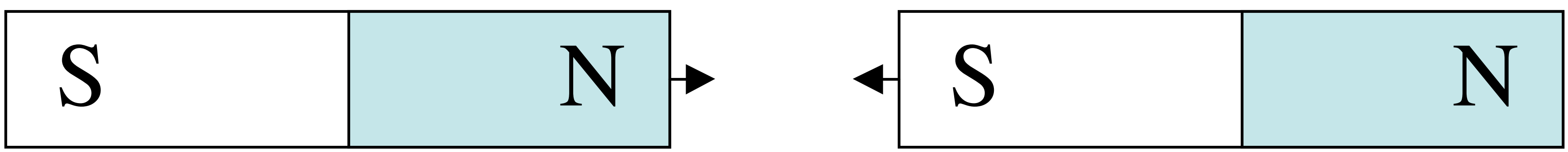
Magnets  are objects that have charges moving inside them
Electrical currents

Magnets have "poles" labeled north and south:

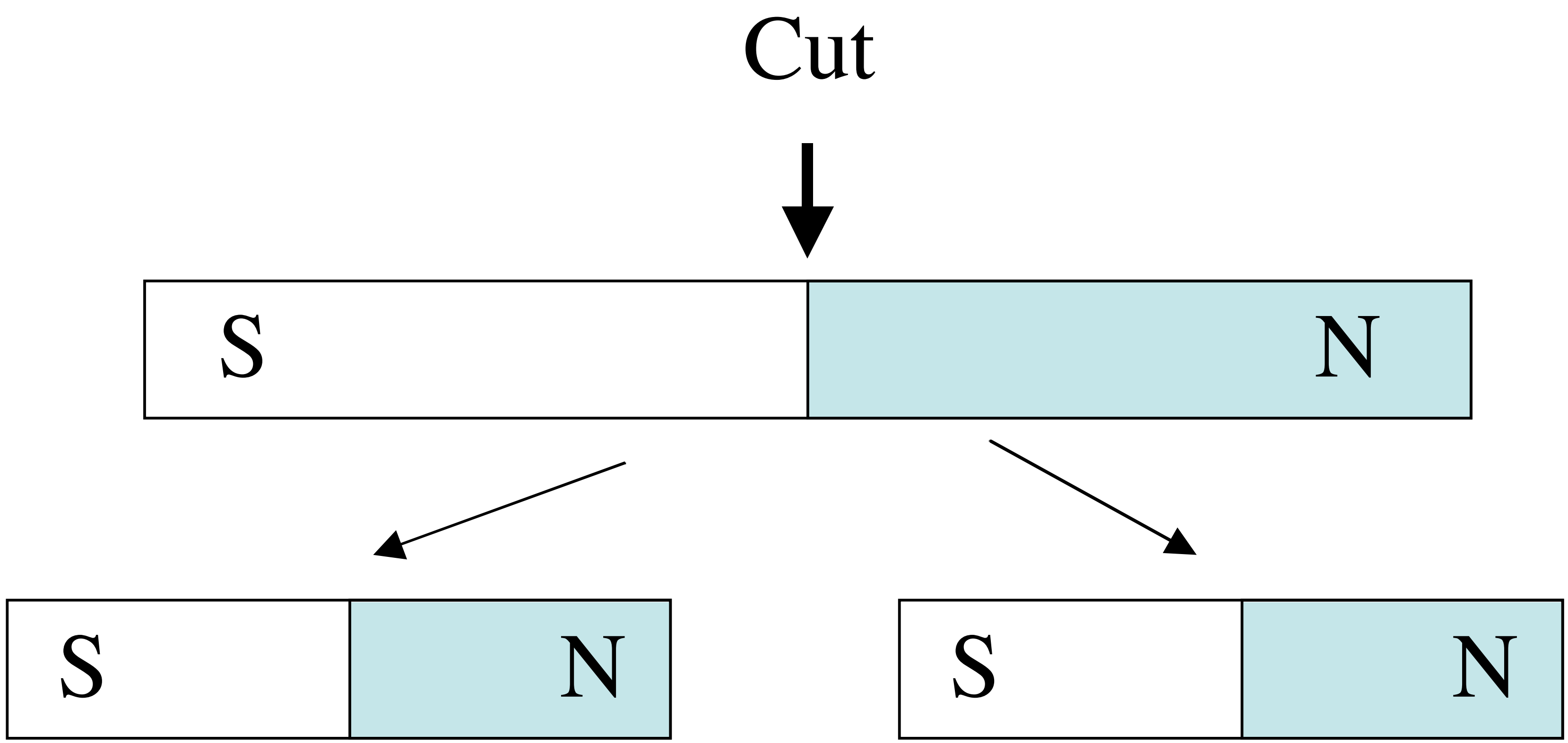
Like poles repel




Unlike poles attract



If you cut a magnet in half, both halves will have a north and south pole.



## 33.7 Force on a moving Charge

For a stationary charge we have

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}(\vec{\mathbf{r}})\,.$$

For a moving charge there is an additional contribution, known as the Lorenz force,

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})\,.$$

Here, $\vec{\mathbf{v}}$ is the velocity of the moving charge.

**Some facts:**

1. For a particle at rest, $\vec{\mathbf{v}} = \mathbf{0}$ the magnetic field exerts no force on a charged particle.
2. The Lorenz force is proportional to the charge, the magnetic field strength and the particle's velocity.

3. The vector force is the result of the vector cross product of velocity and magnetic field.

Motion of a charged particle in a magnetic field

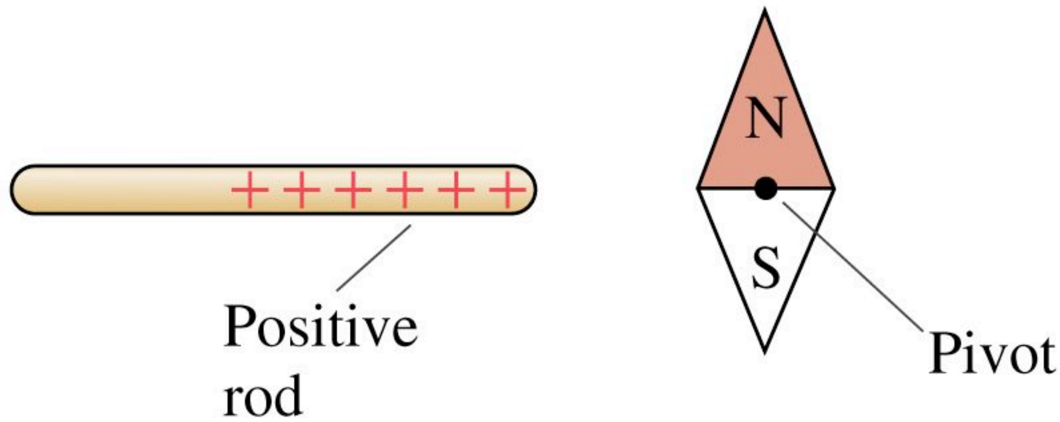
Newton's law: $m\vec{\mathbf{a}} = \vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Note: Lorentz force is always perpendicular to velocity. There fore the magnitude of velocity will not change. This implies the kinetic energy of the particle is constant,

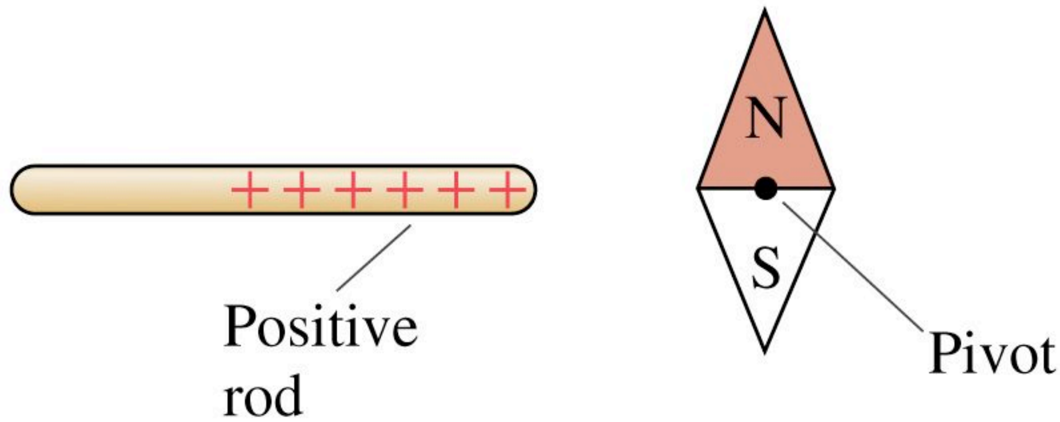
$$\frac{d}{dt}KE = \vec{\mathbf{v}} \cdot \vec{\mathbf{F}} = \mathbf{0}\,, \qquad KE = \frac{m}{2}\left|\vec{\mathbf{v}}\right|^2$$

Note also, if velocity is parallel to magnetic field the force is zero and vector velocity is constant.



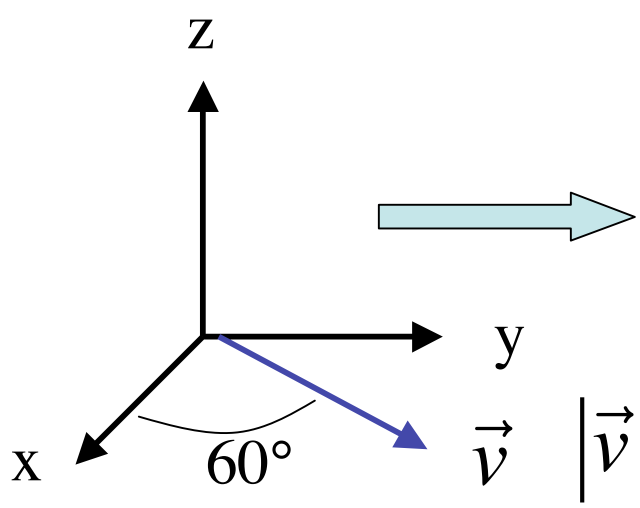

**Does the compass needle rotate clockwise (cw), counterclockwise (ccw) or not at all?**

A. Clockwise
B. Counterclockwise
C. Not at all




**Does the compass needle feel a force from the charged rod?**

A. Attracted
B. Repelled
C. No force
D. Attracted but very weakly It’s complicated




B = 1T in the y direction

$\vec{v}$ $\left|\vec{v}\right| = 10^6$ m/s in x-y plane

What is the force on a moving electron?

$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \left|\vec{\mathbf{v}}\right|\left|\vec{\mathbf{B}}\right| \sin\theta$$

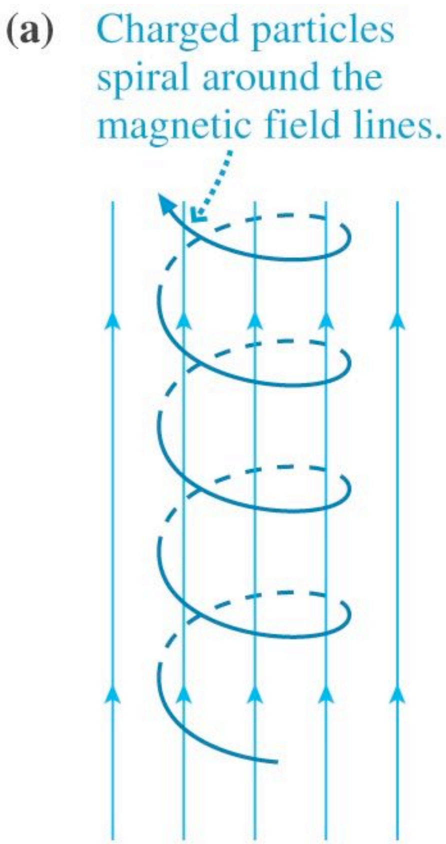
What is θ in this problem?

A. 60°
B. 30°
C. 72 ° and sunny

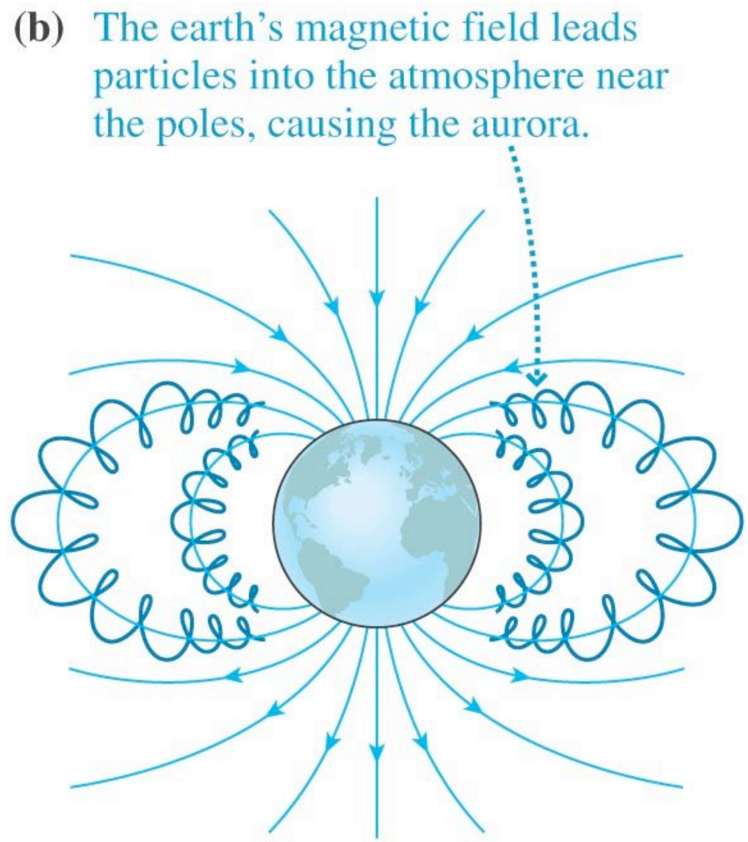
$$\vec{\mathbf{F}} = -1.6 \times 10^{-19}(C) * 10^{6}(m/s) * 1(T) * \frac{1}{2}\widehat{\mathbf{k}}$$

$$\vec{\mathbf{F}} = -0.8 \times 10^{-13}\widehat{\mathbf{k}} \quad \text{Newtons}$$

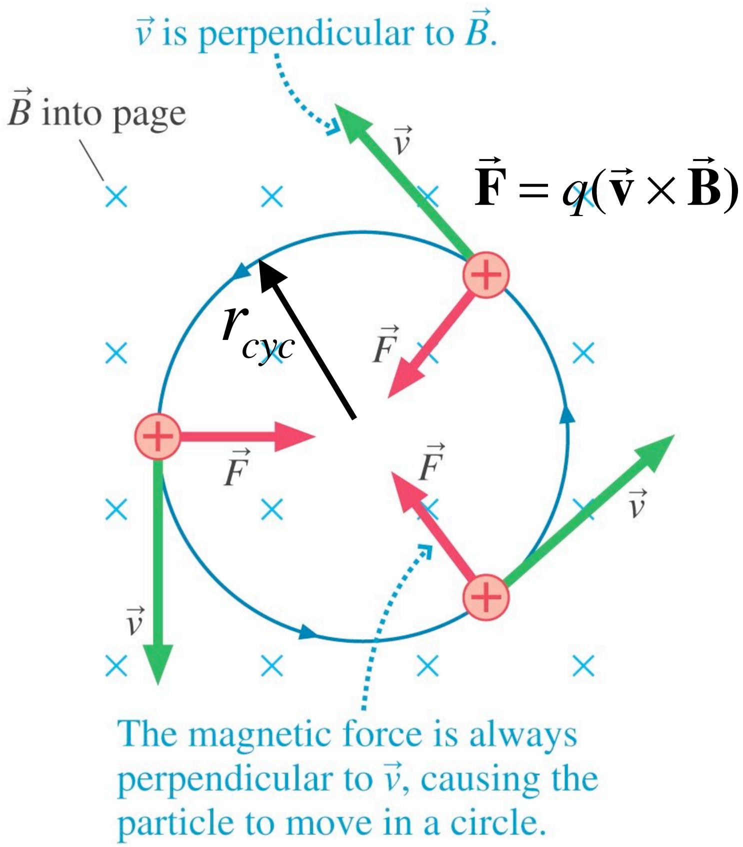
$$\vec{\mathbf{F}} = q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$





# Cyclotron Motion






Newton's Law $\quad m\vec{\mathbf{a}} = \vec{\mathbf{F}}$

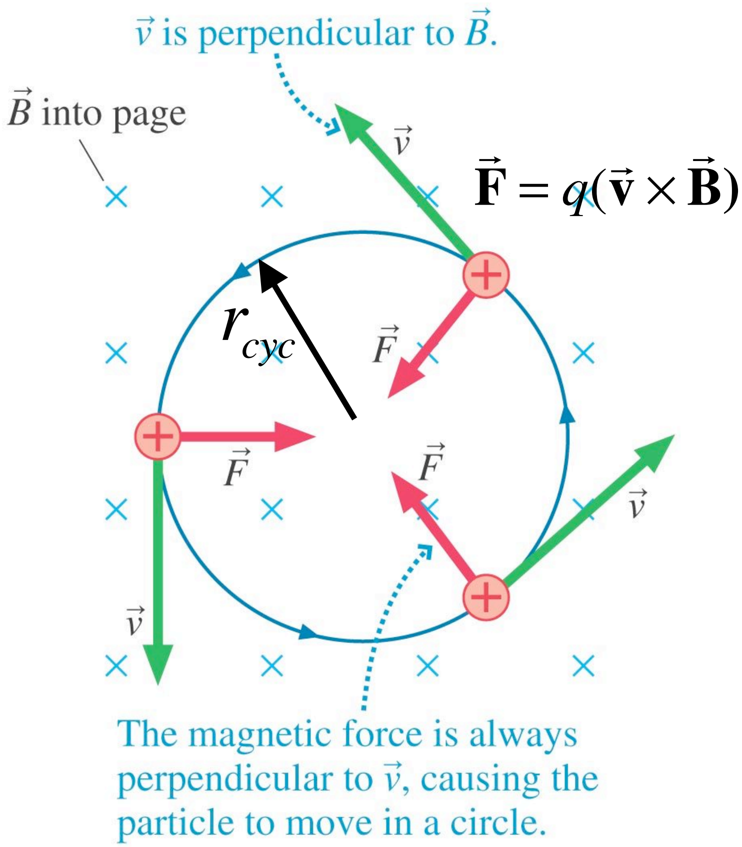
Both **a** and **F** directed toward center of circle

$$\left|\vec{\mathbf{a}}\right| = \frac{v^2}{r_{cyc}} \qquad \left|\vec{\mathbf{F}}\right| = qvB$$

Equate

$$\Omega = \frac{v}{r_{cyc}} = \frac{qB}{m} = 2\pi f_{cyc}$$

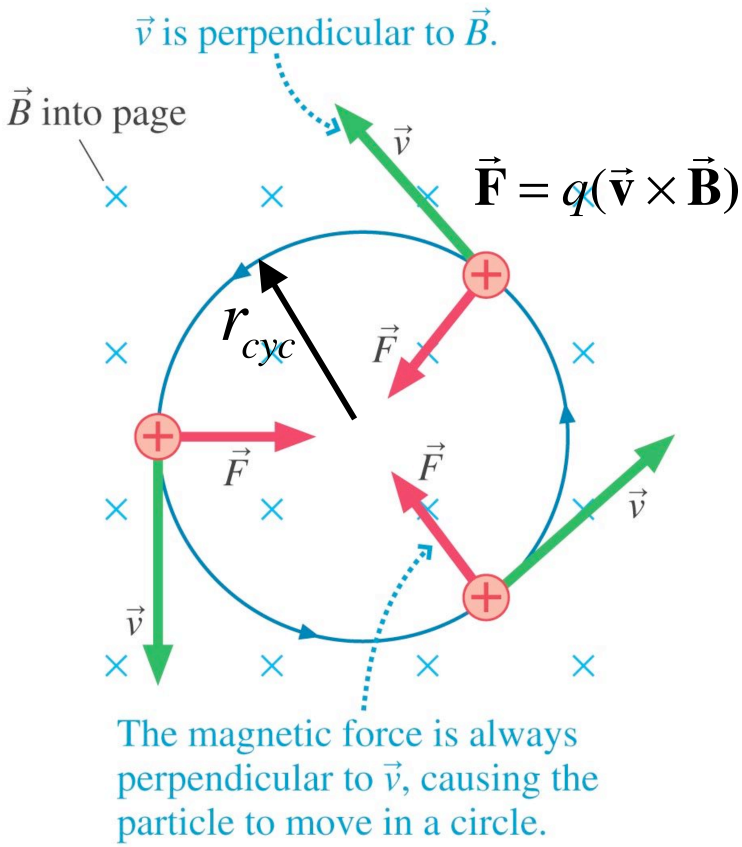
$$r_{cyc} = \frac{v}{\Omega} = \frac{v}{2\pi f_{cyc}}$$





In this picture B is pointing into the page and q is positive. What changes if q is negative and B points out of the page

A. Rotation changes to counter-clockwise
B. There is no change
C. Why do things always have to change?






In this picture B is pointing into the page and q is positive. What changes if q is negative and B points out of the page

A. Rotation changes to counter-clockwise
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# Some Examples

$$f_{cyc} = \frac{qB}{2\pi m} \qquad r_{cyc} = \frac{v}{2\pi f_{cyc}}$$

Ionosphere at equator

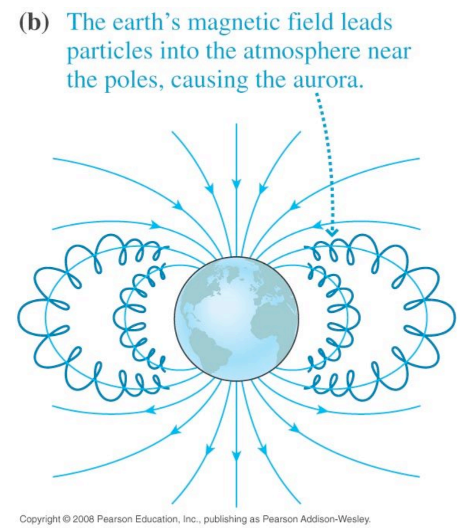
$$B \simeq 3.5 \times 10^{-5}\, T$$

$$T_e \simeq 300^\circ K$$

$$v \simeq 6 \times 10^4\, m/s$$

$$f_{cyc} \simeq 10^6\, Hz$$

$$r_{cyc} \simeq .01 m$$




ITER fusion exp

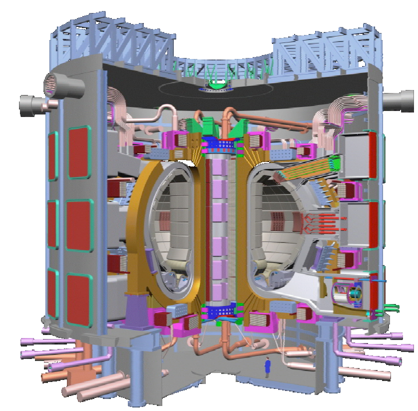
$$B \simeq 6T$$

$$T_e \simeq 10^8 \; {}^\circ K$$

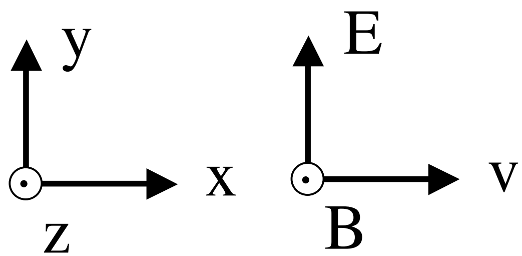
$$v \simeq .2c$$

$$f_{cyc} \simeq 1.67 \quad 10^{11}\, Hz$$

$$r_{cyc} \simeq 5.7 \times 10^{-5}\, m$$



# Motion of a charged particle in crossed electric and magnetic fields




Try these

$$\vec{\mathbf{v}} = v_x \mathbf{i}$$

$$\vec{\mathbf{E}} = E_y \mathbf{j}$$

$$\vec{\mathbf{B}} = B_z \mathbf{k}$$

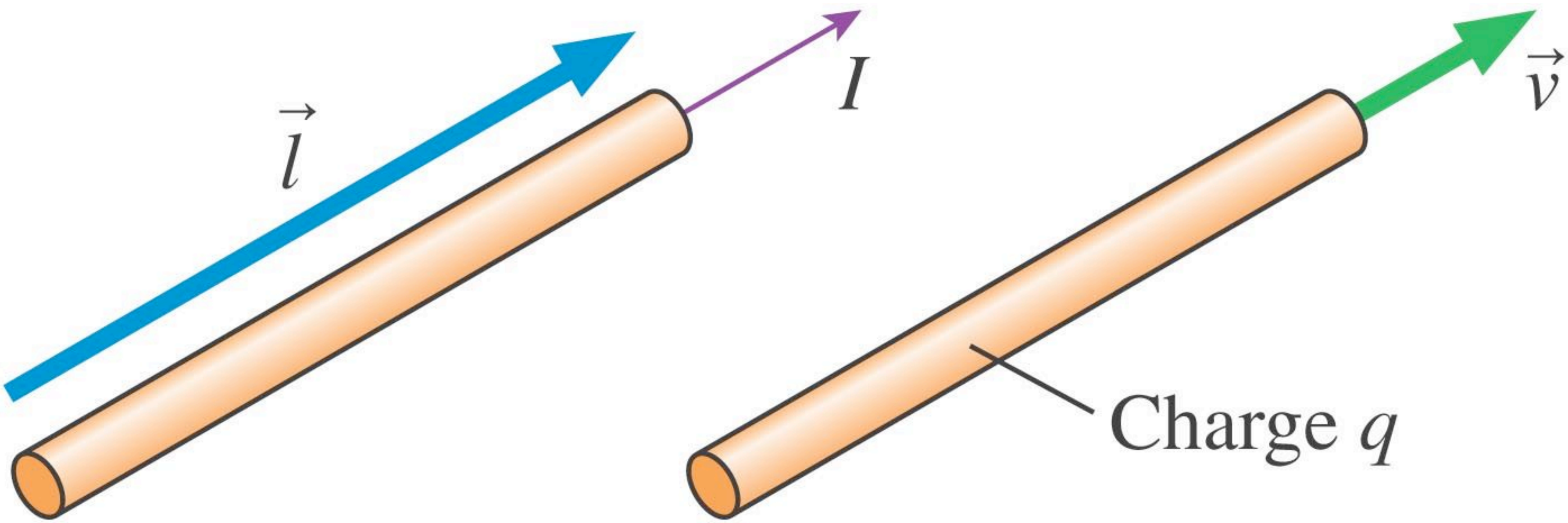
$$m\frac{d\vec{\mathbf{v}}}{dt} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right) = \mathbf{j}q\left(E_y - v_x B_z\right)$$

If $v_x = E_y/B_z$ force is zero and no acceleration

Particle will move at a constant velocity in a direction perpendicular to both E and B

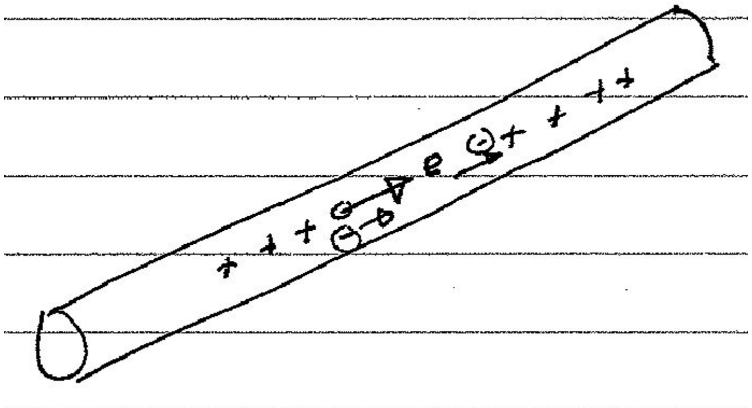
Where are you most likely to encounter E cross B motion?

A. Your doctor's office
B. Your Kitchen
C. The bridge of the Starship Enterprise




A current consists of charge carriers $q$ moving with velocity $\vec{v}$.

# Current carrying wire



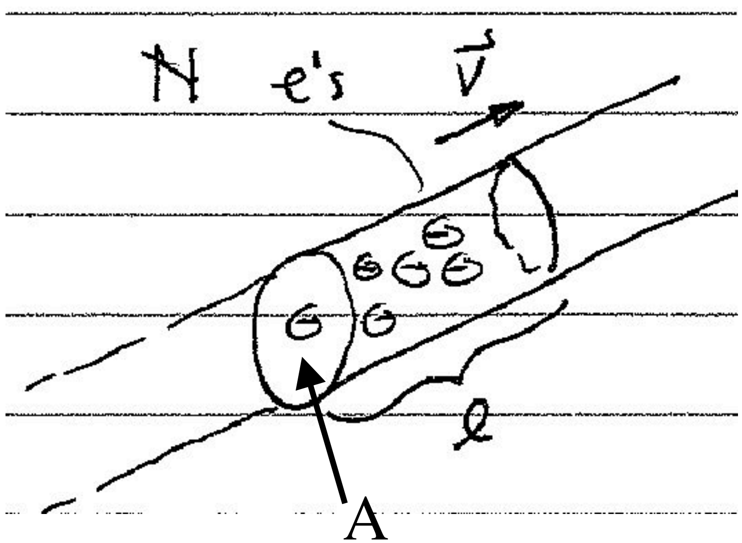
Inside the wire there are stationary charges and moving charges.

Metallic conductor
<u>Moving</u> - free electrons
<u>Stationary</u> - positive ions and bound electrons

In any length of wire the number of positive and negative charges is essentially the same (unless the wire is charged).

The wire can carry a current because the free electrons move.

# Current, number density and velocity




Number density, $n$, of free electrons is a property of the material.
For copper $n \simeq 1.1 \times 10^{29} \quad m^{-3}$

Consider a segment of length $l$, cross sectional area A

volume

The segment has N free electrons $N = n\,(lA)$

Suppose the free electrons have speed v. How long will it take all the electrons to leave the segment?

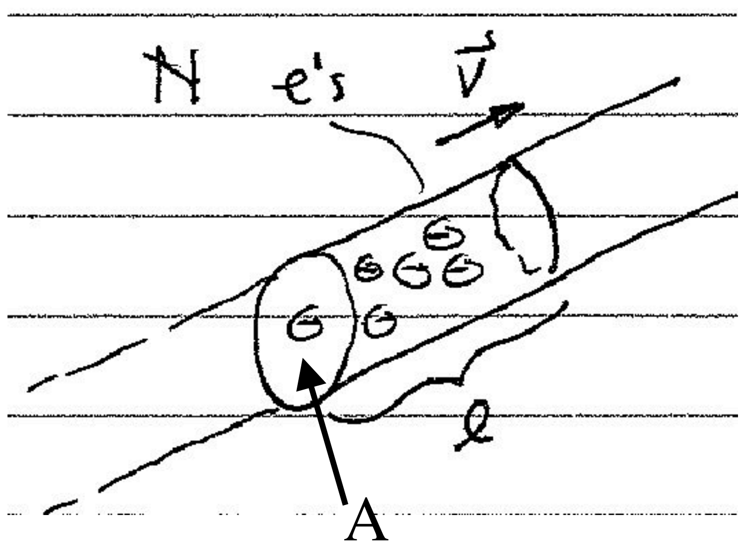
$$\Delta t = l / v$$

# Current

During a time interval $\Delta t = l/v$, a net charge $Q = -eN$ flows through any cross section of the wire.

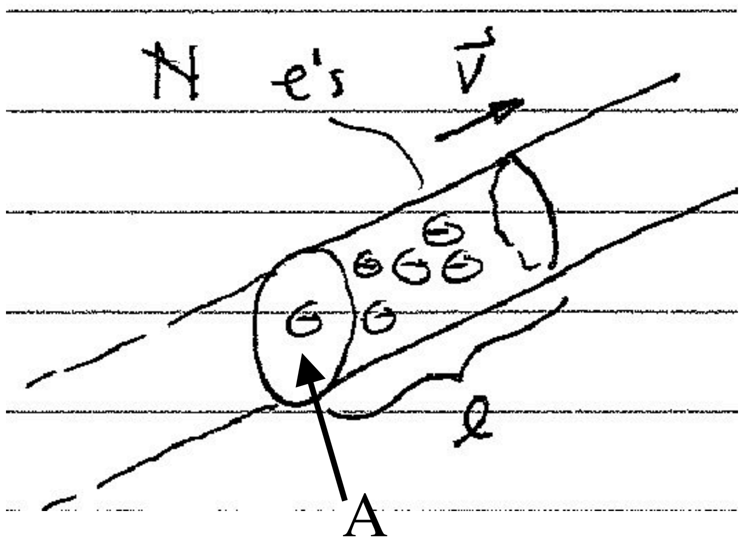
The wire is thus carrying I

$$I = \frac{|Q|}{\Delta t} = \frac{eNv}{l}$$

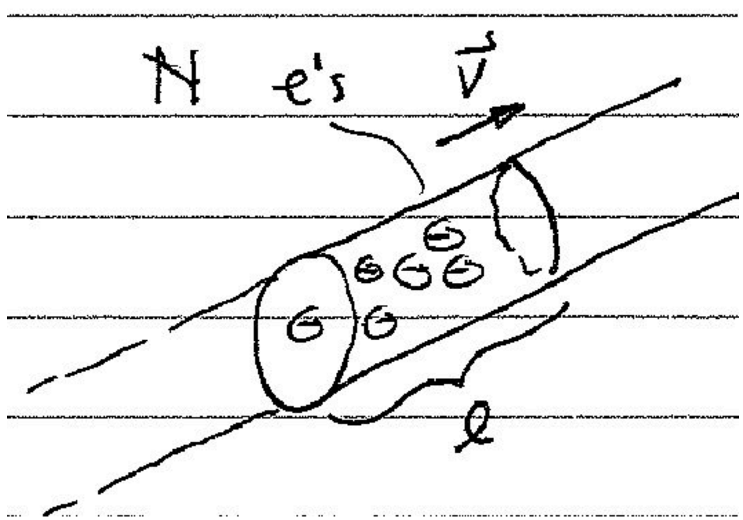



The current is flowing in a direction opposite to that of v.

# How big is v for typical currents in copper?




A. Close to the speed of light
B. Supersonic
C. About as fast as a Prius
D. Glacial




What is the force on those N electrons?

Force on a single electron

$$\vec{\mathbf{F}} = -e\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Force on N electrons

$$\vec{\mathbf{F}} = -eN\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

Now use $\quad eN\left|\vec{\mathbf{v}}\right| = I\,l$

Lets make *l* a vector $\quad -eN\vec{\mathbf{v}} = I\,\vec{l}$

points parallel to the wire in the direction of the current when I is positive

Force on wire segment:

$$\vec{\mathbf{F}}_{\mathbf{wire}} = I\,\vec{l} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_{\mathbf{wire}} = I\,\vec{l} \times \vec{\mathbf{B}}$$

Comments:

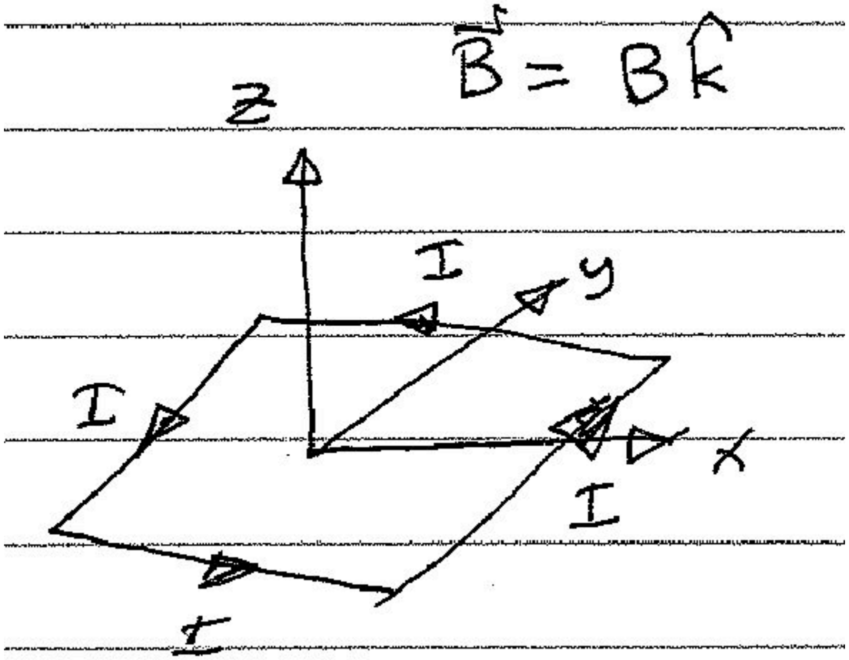
Force is perpendicular to both B and *l*

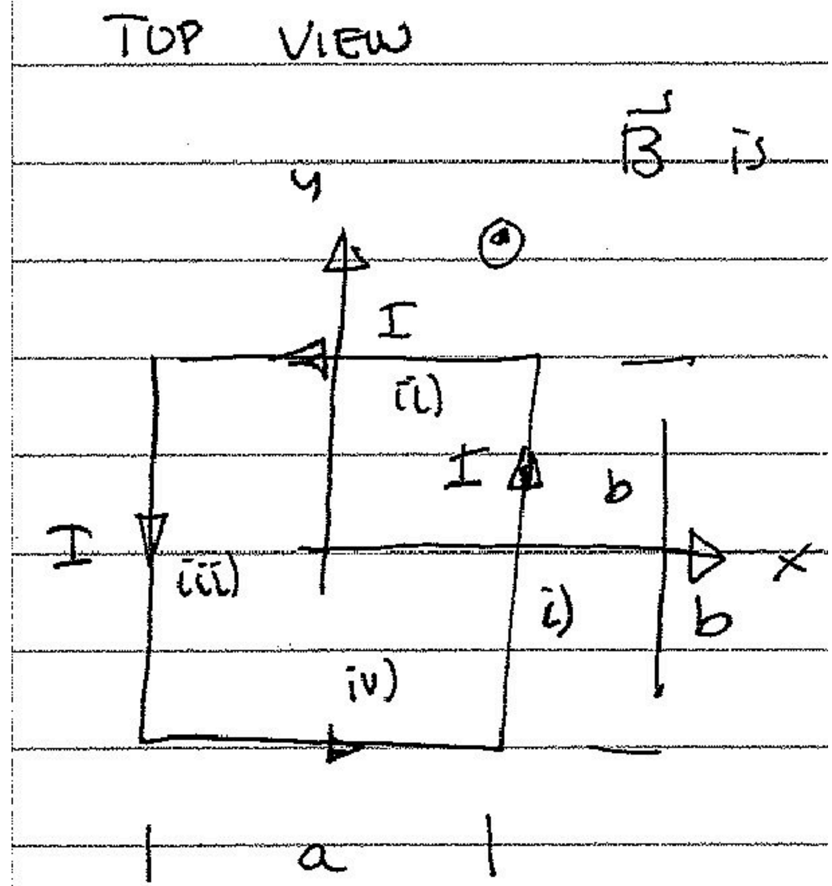
Force is proportional to I, B, and length of line segment

Superposition: To find the total force on a wire you must break it into segments and sum up the contributions from each segment

$$\vec{\mathbf{F}}_{\mathbf{total}} = I \sum_{segments-i} \vec{l}_i \times \vec{\mathbf{B}}(\vec{\mathbf{r}}_{\mathbf{i}})$$

# What is the force on a rectangular loop?




what is force on each segment?

i) To the right $\hat{i}$ IbB

ii) up $\hat{j}$ IaB

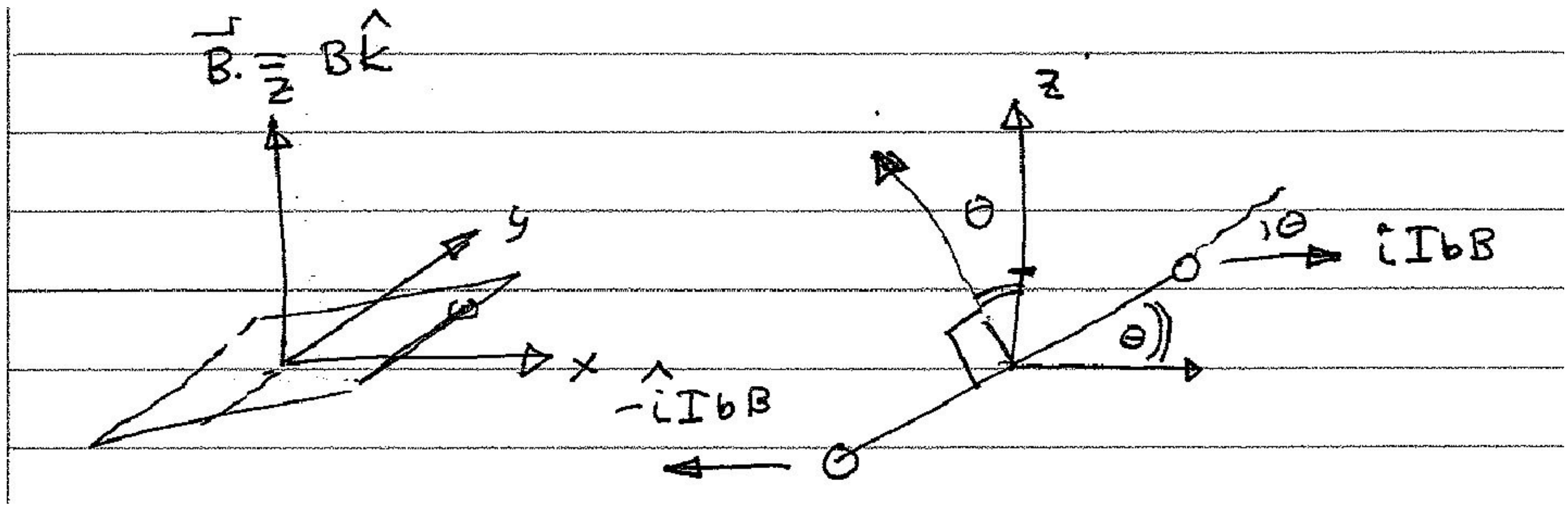
iii) To the left $-\hat{i}$ IbB

iv) down $-\hat{j}$ IaB

Net force is zero

$$\vec{F} = \hat{i}\,(IbB - IbB) + \hat{j}\,(IaB - IaB) = 0$$

# Torque on a loop

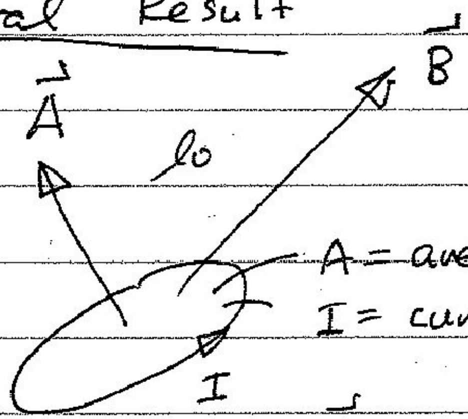



Plane of loop is tilted by angle θ with respect to direction of B.
The loop wants to move so that its axis and B are aligned.

Definition of torque $\vec{\tau} = \sum_{sides} \vec{r} \times \vec{F}$

$$\tau_y = 2\left(\frac{a}{2}\right) IbB\sin\theta = i(ab)B\sin\theta$$

## General Result



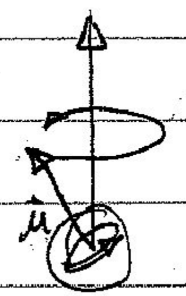

A = area of loop

I = current

$\vec{A}$ = vector in direction determined by Right hand rule

$$\vec{\mu} = I\vec{A}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$




loop precesses like a top

Nuclear magnetic resonance   Basis of MRI magnetic resonance imaging

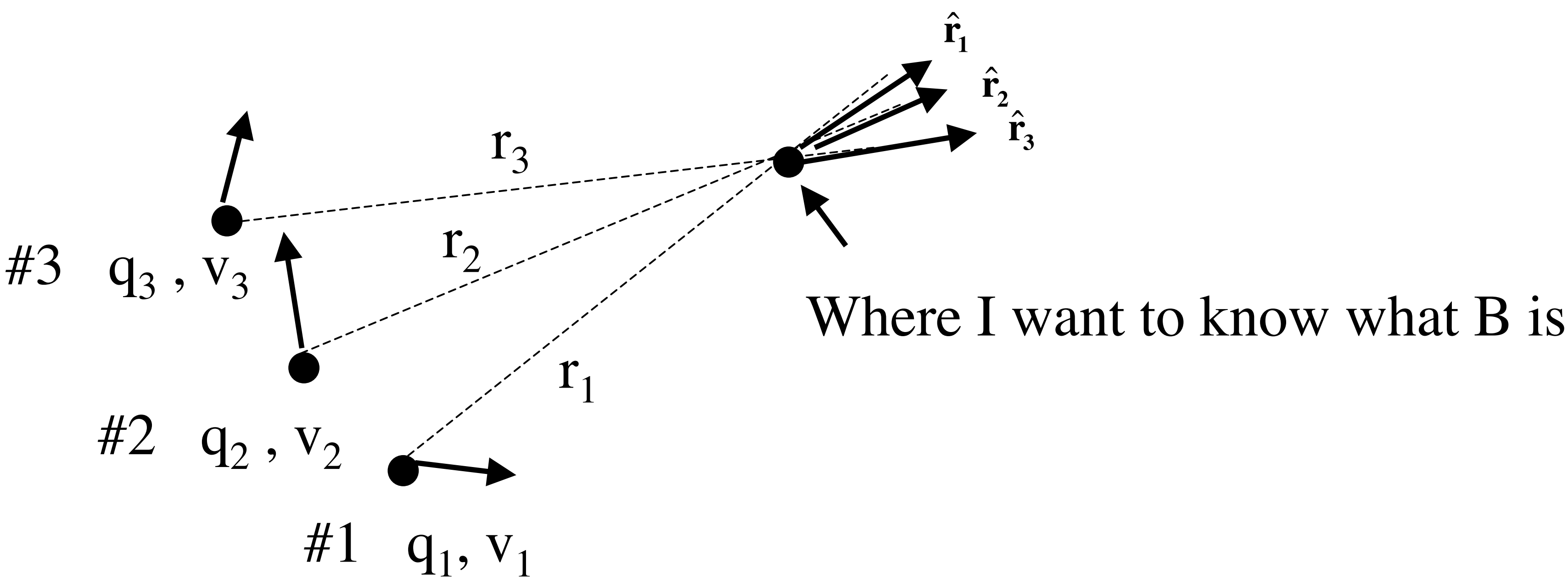
Magnetic Field due to a current

Magnetic Field due to a single charge

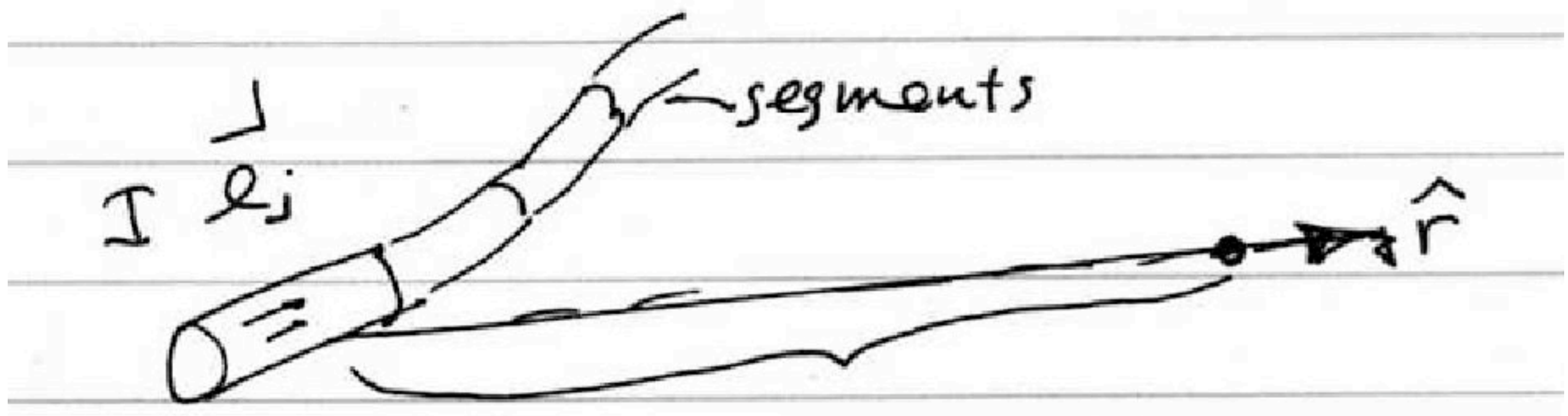
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

If many charges use superposition

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\sum_{charges-i}\frac{q_i\vec{\mathbf{v}}_\mathbf{i} \times \hat{\mathbf{r}}_\mathbf{i}}{r_i^2}$$



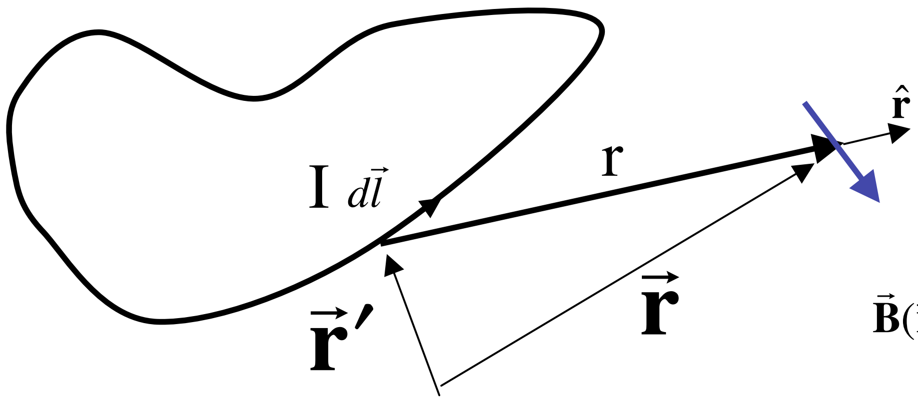
For moving charges in a wire, first sum over charges in each segment, then sum over segments




$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \sum_{sgments-j} \left( \sum_{\substack{charges\ in \\ each\ segment-i}} \frac{q_i \vec{\mathbf{v}}_{\mathbf{i}} \times \hat{\mathbf{r}}_{\mathbf{i}}}{r_i^2} \right) = \frac{\mu_0}{4\pi} \sum_{sgments-j} I \frac{\vec{l}_{\mathbf{j}} \times \hat{\mathbf{r}}_{\mathbf{j}}}{r_j^2}$$

$$\sum_{\substack{charges\ in \\ each\ segment-i}} \frac{q_i \vec{\mathbf{v}}_{\mathbf{i}} \times \hat{\mathbf{r}}_{\mathbf{i}}}{r_i^2} = I \frac{\vec{l}_{\mathbf{j}} \times \hat{\mathbf{r}}_{\mathbf{j}}}{r_j^2}$$

Summing over segments - integrating along curve




Biot Savart law

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} \sum_{sgments-j} I \frac{\vec{l}_{\mathrm{j}} \times \hat{\mathbf{r}}_{\mathrm{j}}}{r_j^{\,2}} \rightarrow \frac{\mu_0}{4\pi} \oint I \frac{d\vec{l} \times \hat{\mathbf{r}}}{r^2}$$

Integral expression looks simple but…..you have to keep track of two position vectors

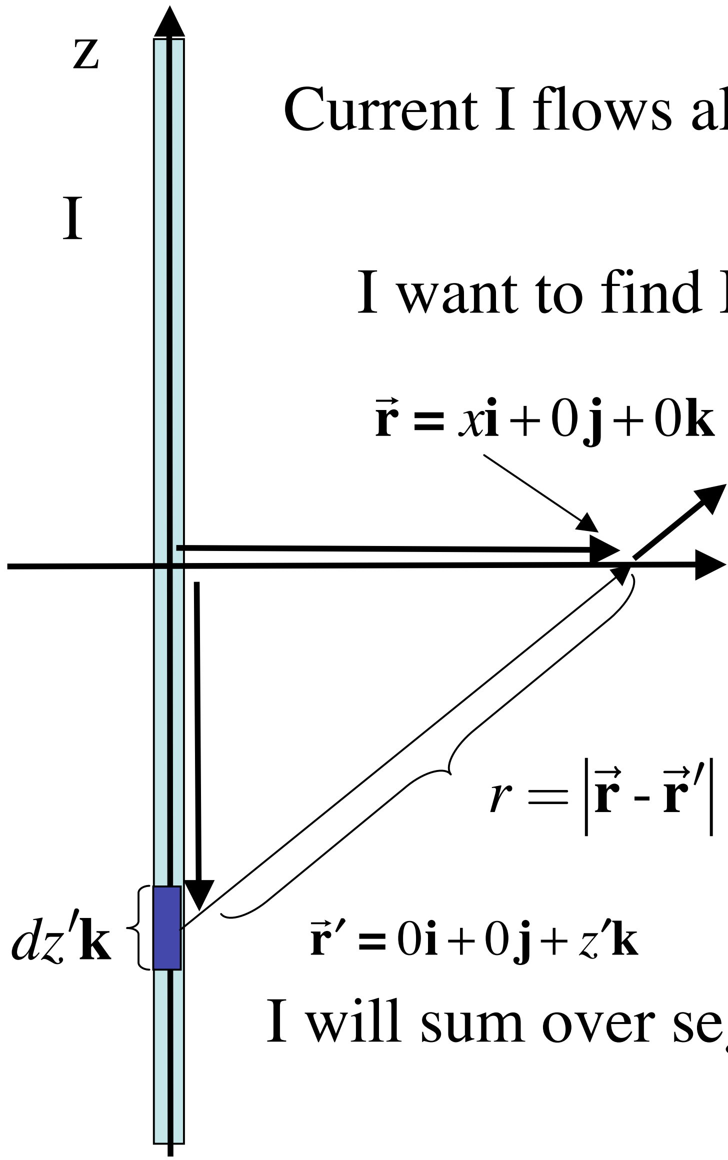
$\vec{\mathbf{r}}$ which is where you want to know B

$\vec{\mathbf{r}}'$ which is the location of the line segment that is contributing to B. This is what you integrate over.

# Magnetic field due to an infinitely long wire

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int I \frac{d\vec{l} \times \hat{\mathbf{r}}}{r^2}$$

Current I flows along z axis




I want to find B at the point

$$\vec{\mathbf{r}} = x\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|} = \frac{x\mathbf{i} - z'\mathbf{k}}{\sqrt{x^2 + z'^2}}$$

$$r = \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right| = \sqrt{x^2 + z'^2}$$

$$d\vec{l} = dz'\mathbf{k}$$

$$\vec{\mathbf{r}}' = 0\mathbf{i} + 0\mathbf{j} + z'\mathbf{k}$$

I will sum over segments at points

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) == \frac{\mu_0}{4\pi}\int I\,\frac{d\vec{l}\times\hat{\mathbf{r}}}{r^2}$$

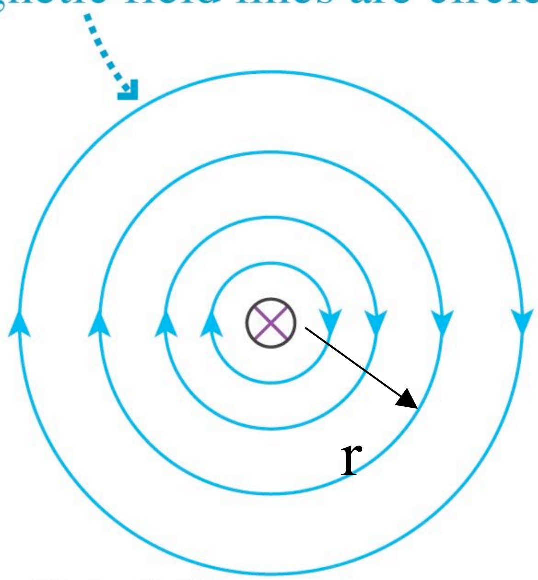
$$\left\{\begin{aligned} d\vec{l} &= dz'\mathbf{k} \\ \hat{\mathbf{r}} &= \frac{\vec{\mathbf{r}}-\vec{\mathbf{r}}'}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right|} = \frac{x\mathbf{i}-z'\mathbf{k}}{\sqrt{x^2+z'^2}} \\ r &= \left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right| = \sqrt{x^2+z'^2} \end{aligned}\right.$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) == \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{dz'\left(x\,\overset{\mathbf{j}}{\mathbf{k}\times\mathbf{i}} - z'\overset{0}{\mathbf{k}\times\mathbf{k}}\right)}{\left(x^2+z'^2\right)^{3/2}}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) == \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{dz'\left(x\,\mathbf{j}\right)}{\left(x^2+z'^2\right)^{3/2}} = \frac{\mu_0 I}{2\pi x}$$

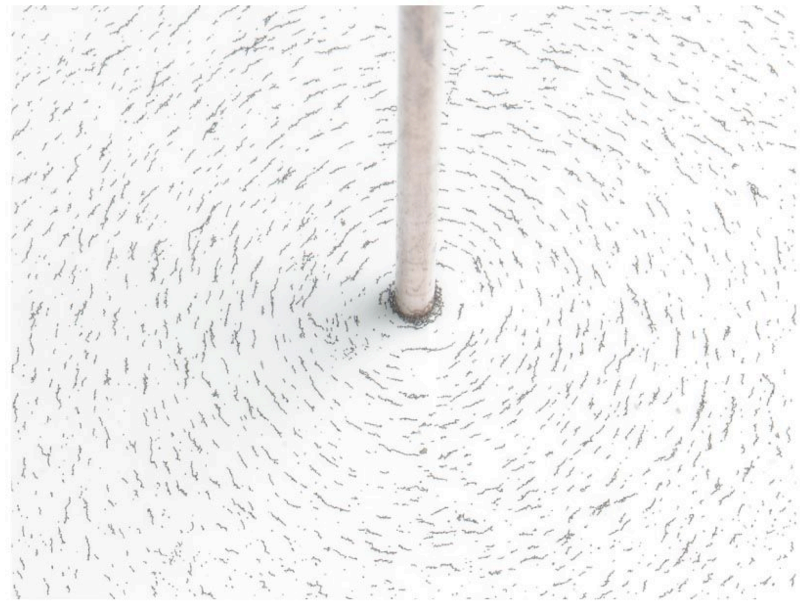
See lecture notes

**(b)** Magnetic field lines are circles.






**(c)**



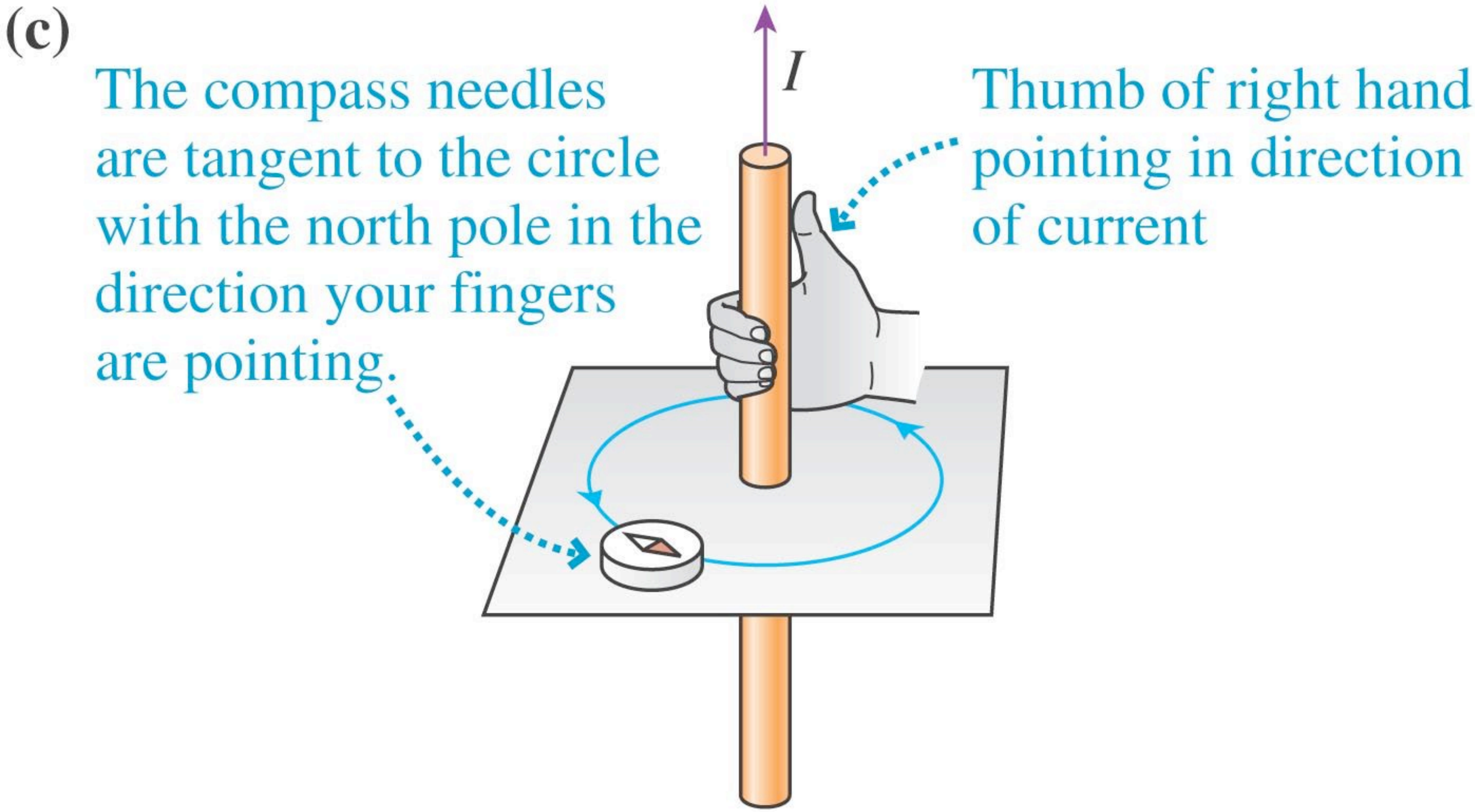
The magnetic field is revealed by the pattern of iron filings around the current-carrying wire.



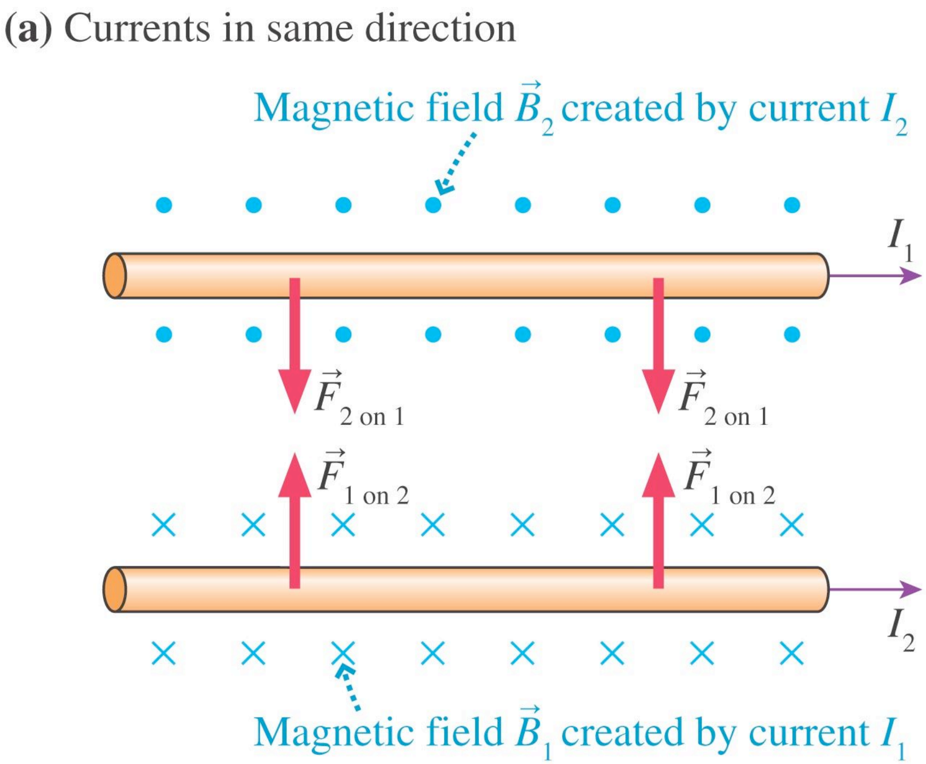
$$\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{2\pi r}$$

compare with E-field for a line charge

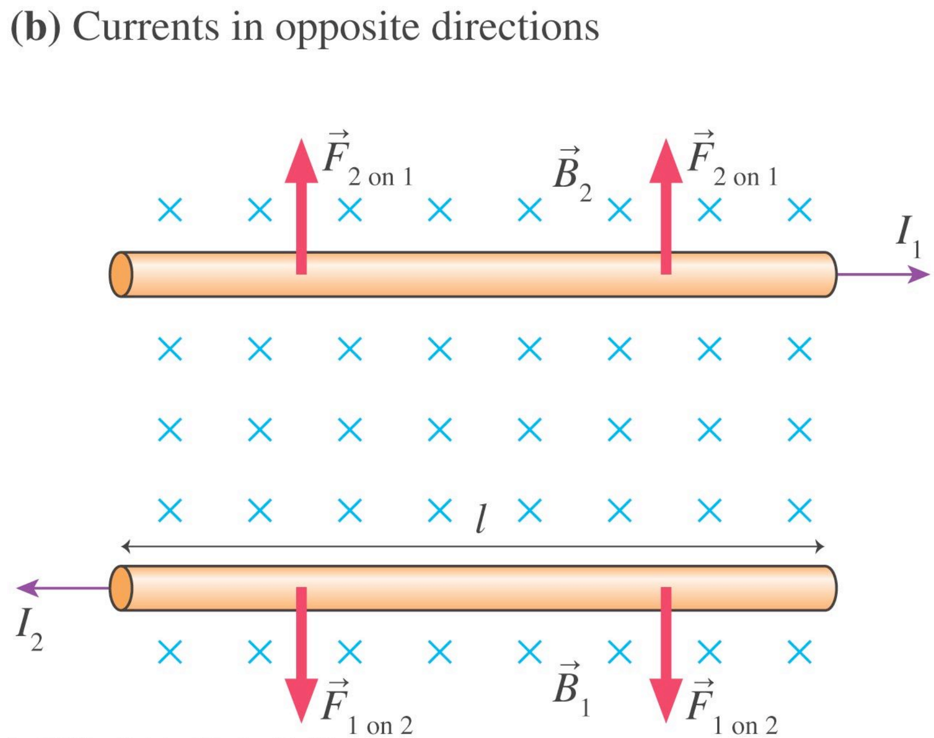
$$\left|\vec{\mathbf{E}}\right| = \frac{\lambda}{2\pi\varepsilon_0 r}$$


(c)
The compass needles are tangent to the circle with the north pole in the direction your fingers are pointing.
I
Thumb of right hand pointing in direction of current

# Forces on Parallel Wires









$$F_{2on1} = I\,\vec{l}_1 \times \vec{B}_2(1)$$

Evaluated at location of #1

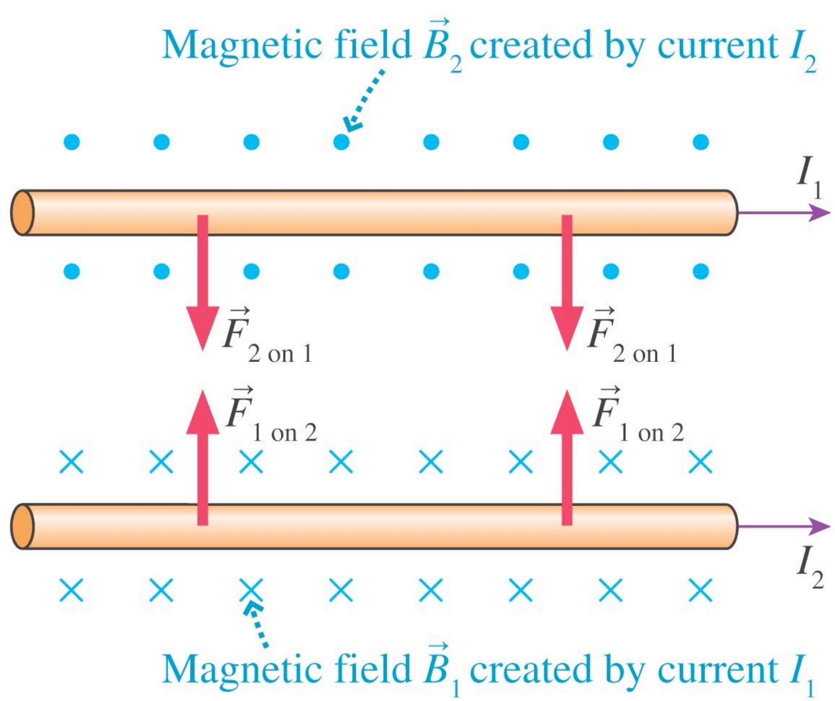
Magnetic field due to #2

$$F_{2on1} = I\,\vec{l}_1 \times \vec{B}_2(1)$$

Evaluated at location of #1

Magnetic field due to #2

(a) Currents in same direction

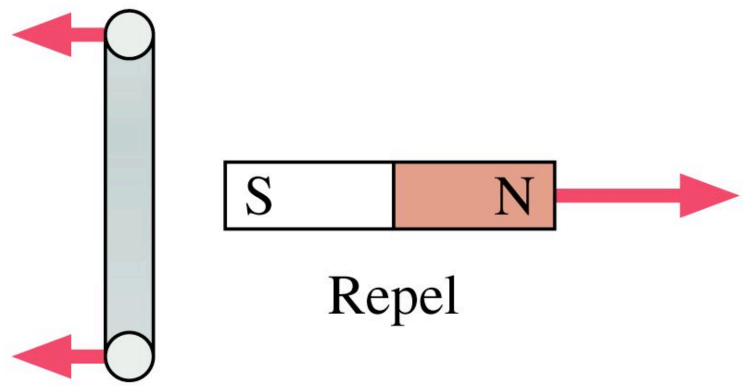




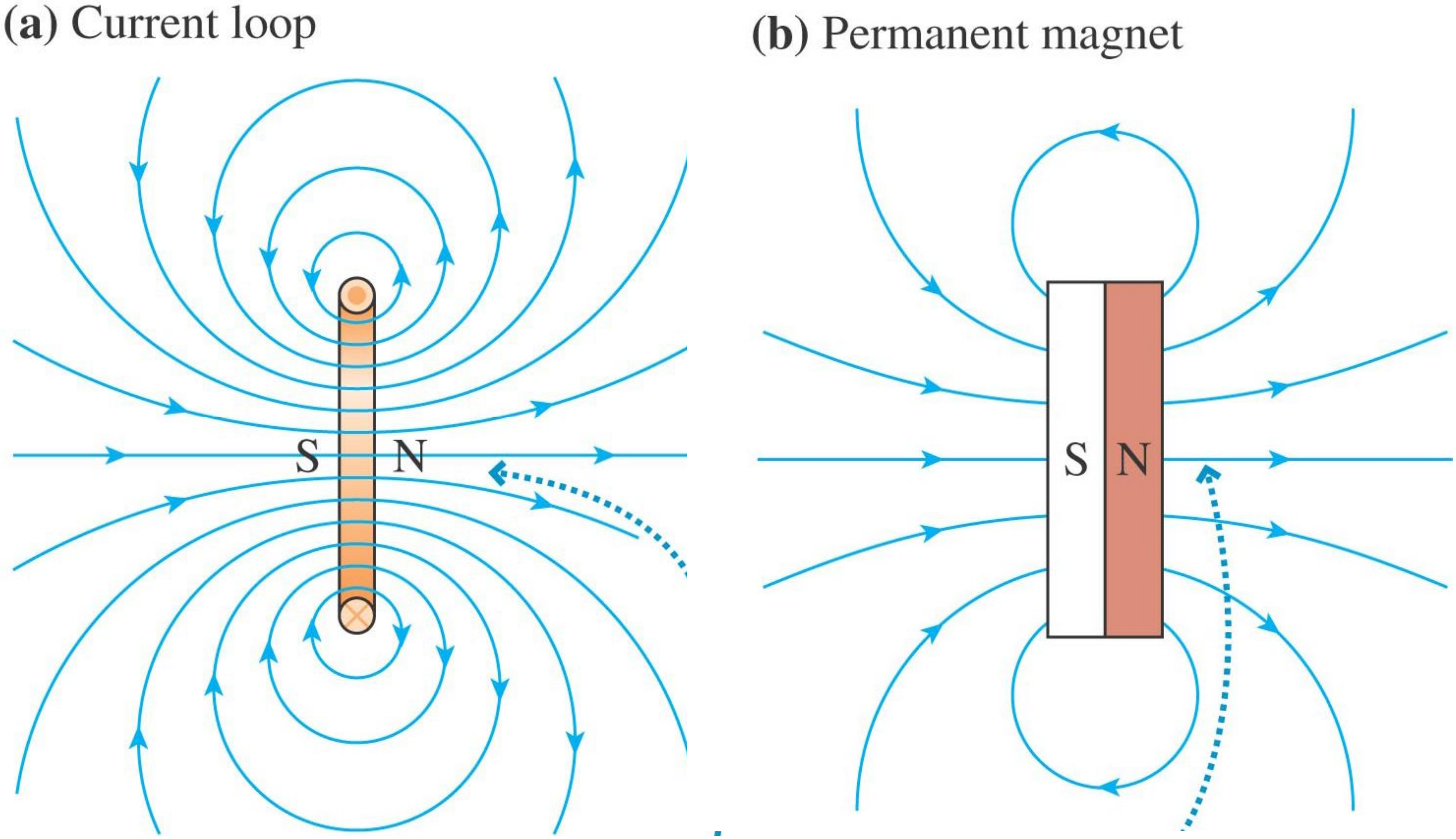

$$\left|\vec{B}_2(1)\right| = \frac{\mu_0 I_2}{2\pi d}$$

Direction out of page for positive

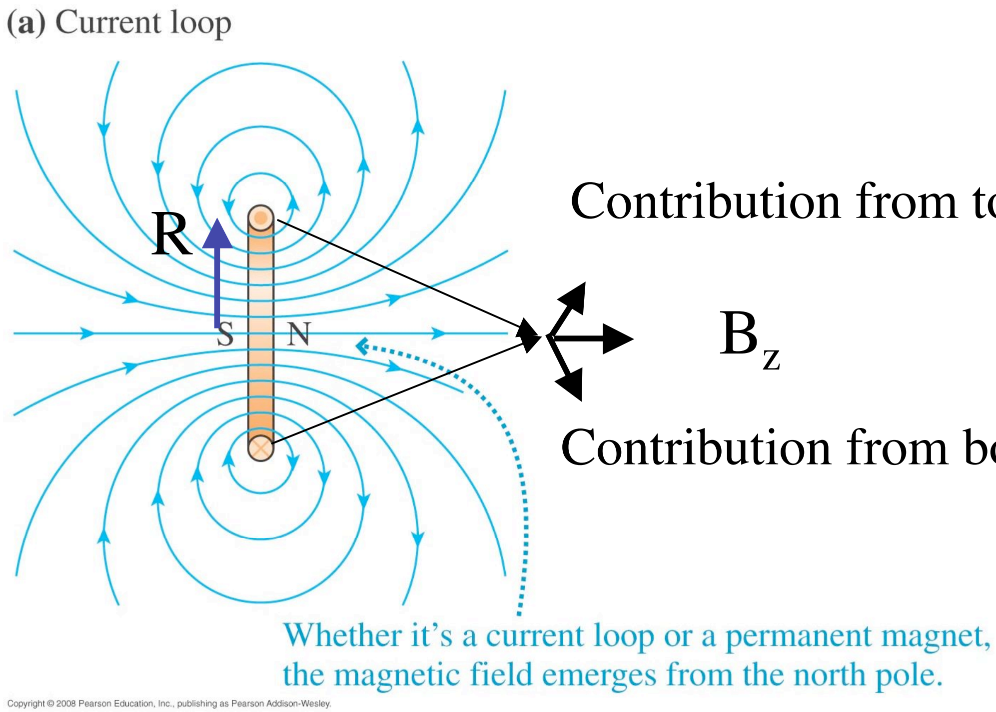
# What is the current direction in the loop?




A. Out of the page at the top of the loop, into the page at the bottom.
B. Out of the page at the bottom of the loop, into the page at the top.


(a) Current loop
S
N
(b) Permanent magnet
S
N

# Magnetic field due to a loop of current




See derivation in book

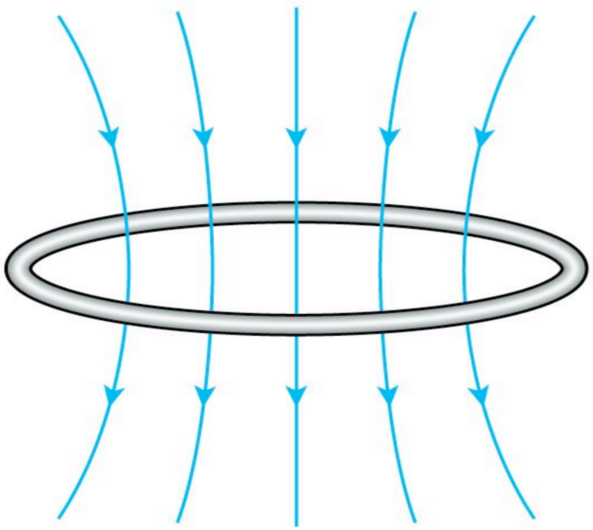
$$\left|\vec{\mathbf{B}}(r=0,z)\right| = \frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+Z^2\right)^{3/2}}$$

Note: if z >> R

$$\left|\vec{\mathbf{B}}(r=0,z)\right| \simeq \frac{\mu_0 I}{2}\frac{R^2}{Z^3} = \frac{\mu_0}{2\pi}\frac{(IA)}{Z^3}$$

Magnetic moment, Independent of loop shape

**What is the current direction in this loop? And which side of the loop is the north pole?**



A. To the right in front, north pole on bottom
B. To the left in front; north pole on bottom
C. To the right in front, north pole on top
D. To the left in front; north pole on top

Coulombs Law implies Gauss' Law and conservative E-field

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

Gauss' Law: $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

$$0 = \oint \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}}$$

But also, Gauss' law and conservative E-field imply Coulombs law

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

Biot-Savart Law implies Gauss' Law and Amperes Law

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0 q_1}{4\pi r^2}\vec{\mathbf{v}} \times \hat{\mathbf{r}}$$

Gauss' Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

Ampere's Law

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

But also, Gauss' law and Ampere's Law imply the Biot - Savart law

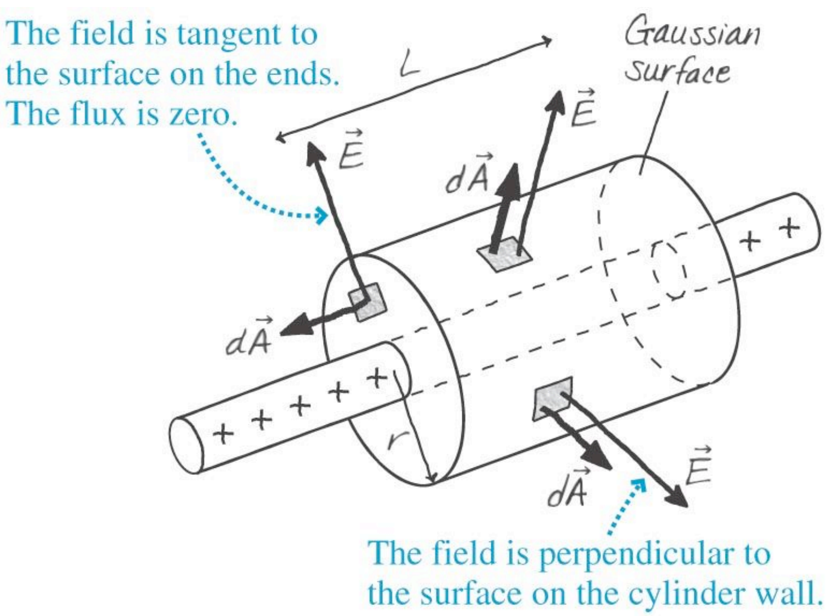
$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0 q_1}{4\pi r^2}\vec{\mathbf{v}} \times \hat{\mathbf{r}}$$

# Consequences of Coulomb's Law

Gauss' Law: $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

Electric flux leaving a closed surface is equal to charge enclosed/$\varepsilon_o$

Always true for any closed surface! Surface does not need to correspond to any physical surface. It can exist only in your head.

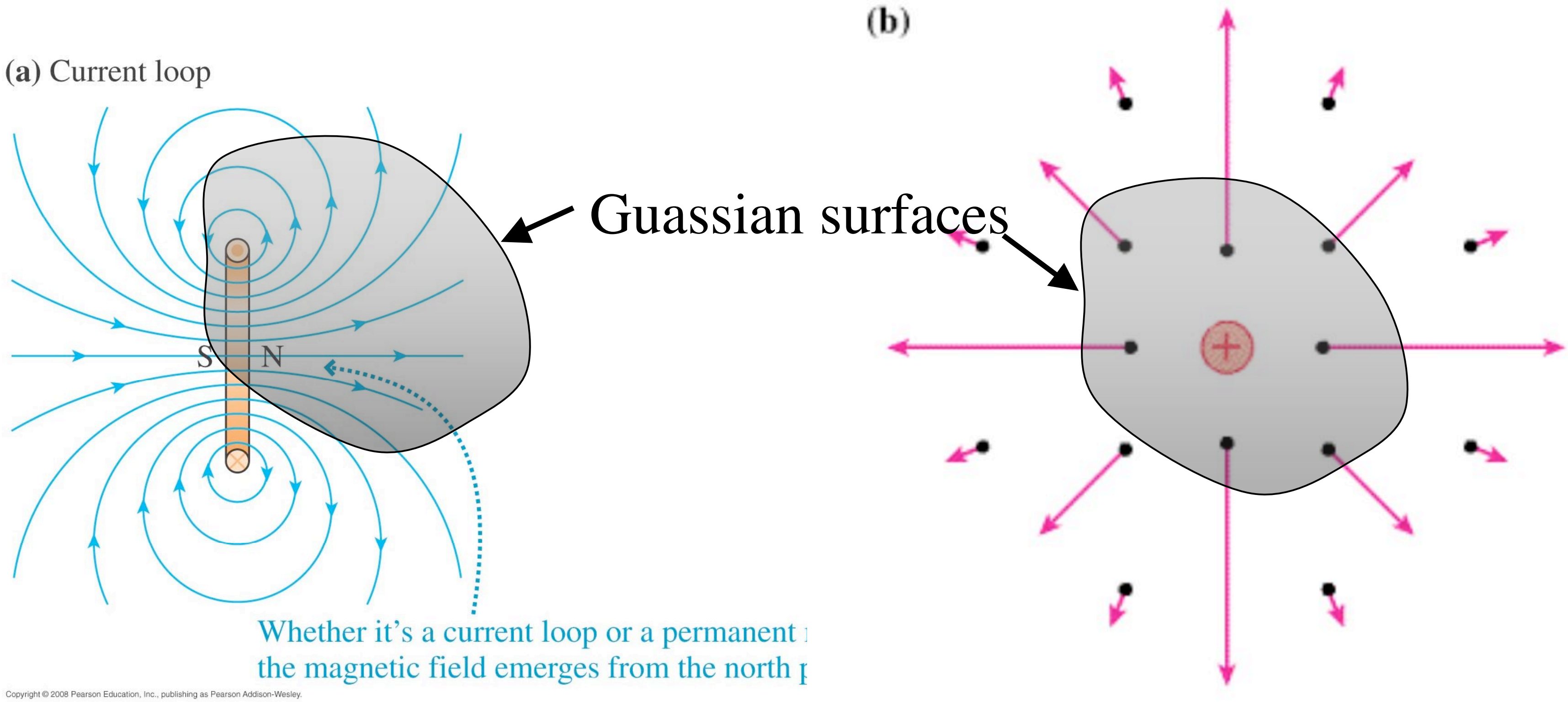





Surface used to calculate electric field near a line charge

Magnetic field due to a single loop of current

Electric field due to a single charge




$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

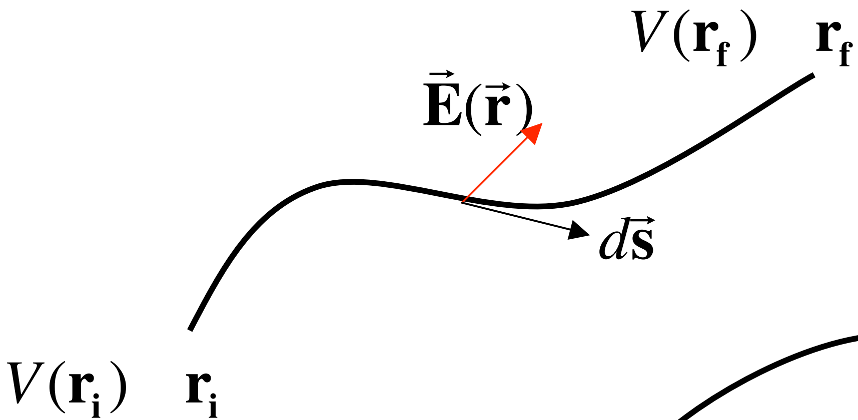
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$

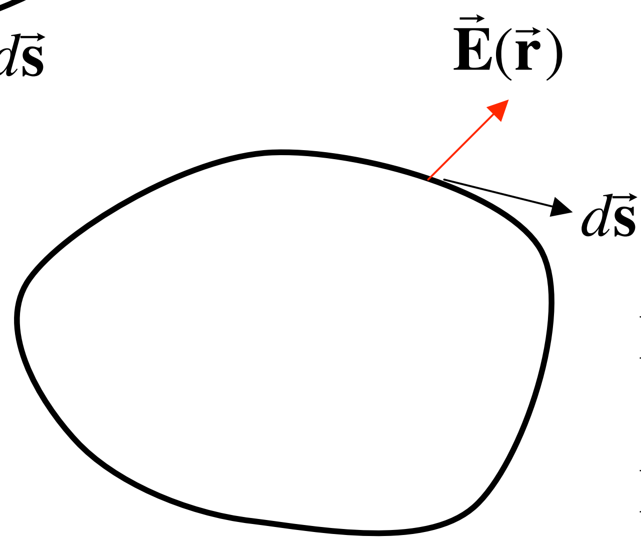
# Further consequences of CL

Existence of a potential

$$V(\mathbf{r}_f) - V(\mathbf{r}_i) = -\int_{r_i}^{r_f} \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}}$$

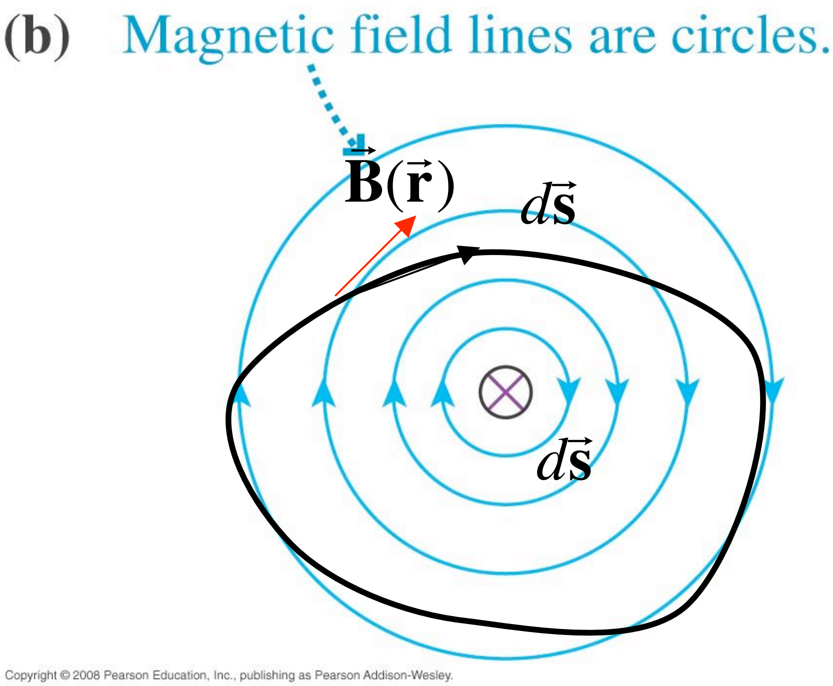
Difference in potential is <u>independent</u> of path between $r_i$ and $r_f$



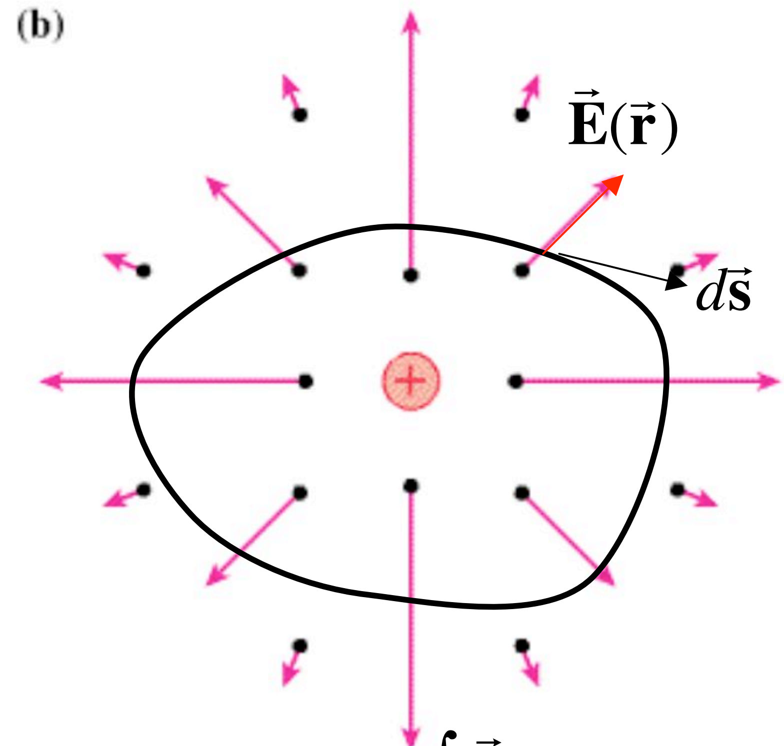




$$0 = \oint \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}}$$

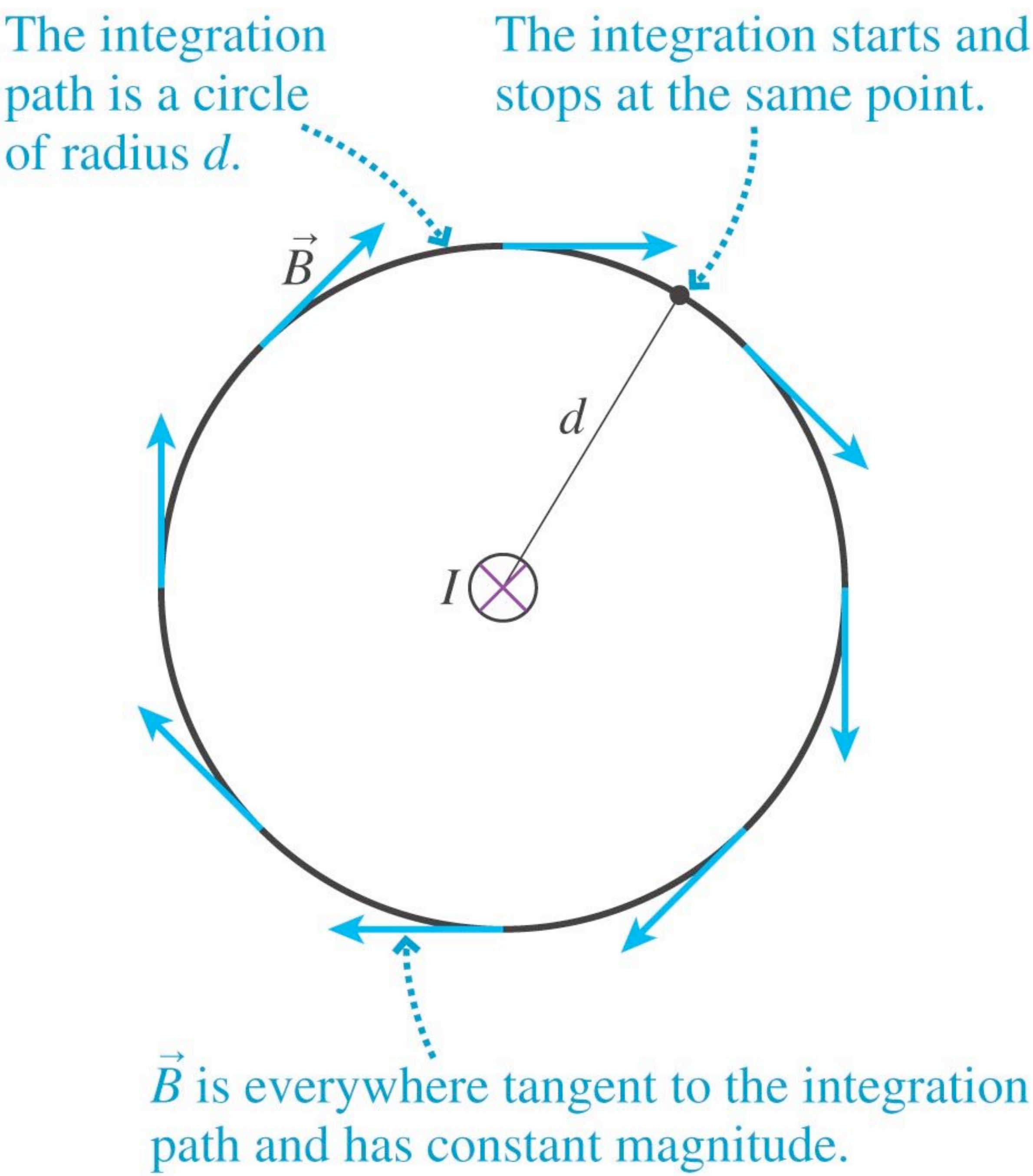
Line integral around any closed loop is zero.
Field is “conservative”

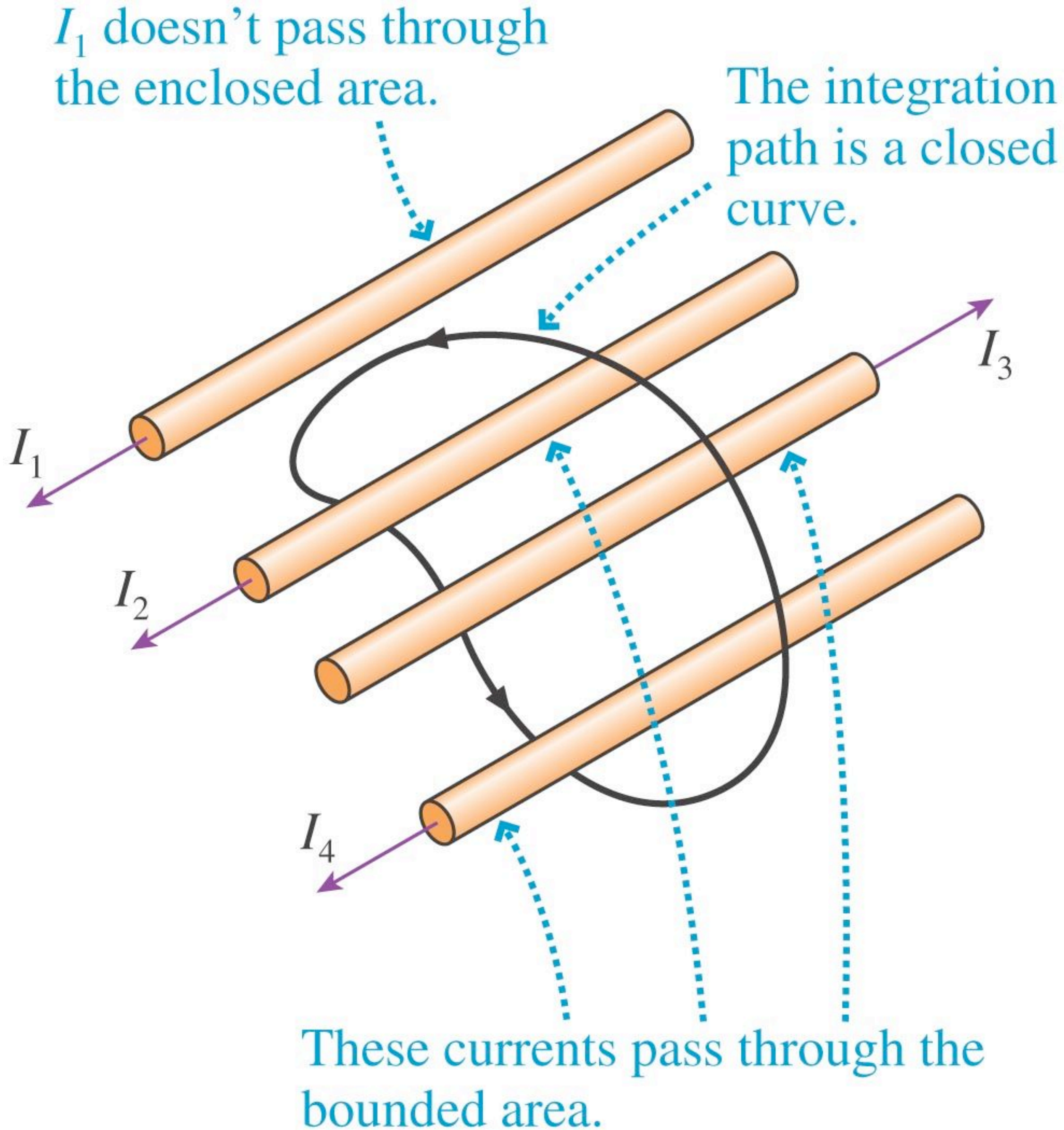


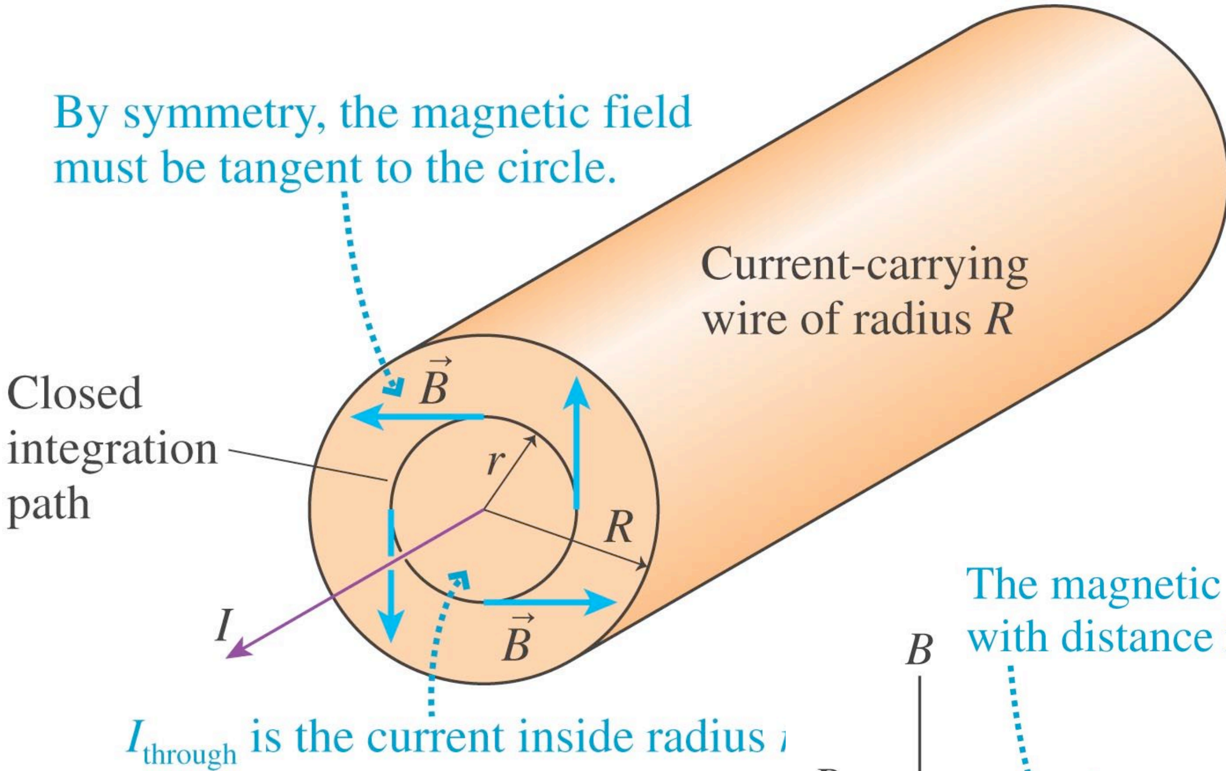

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I$$




$$0 = \oint \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}}$$

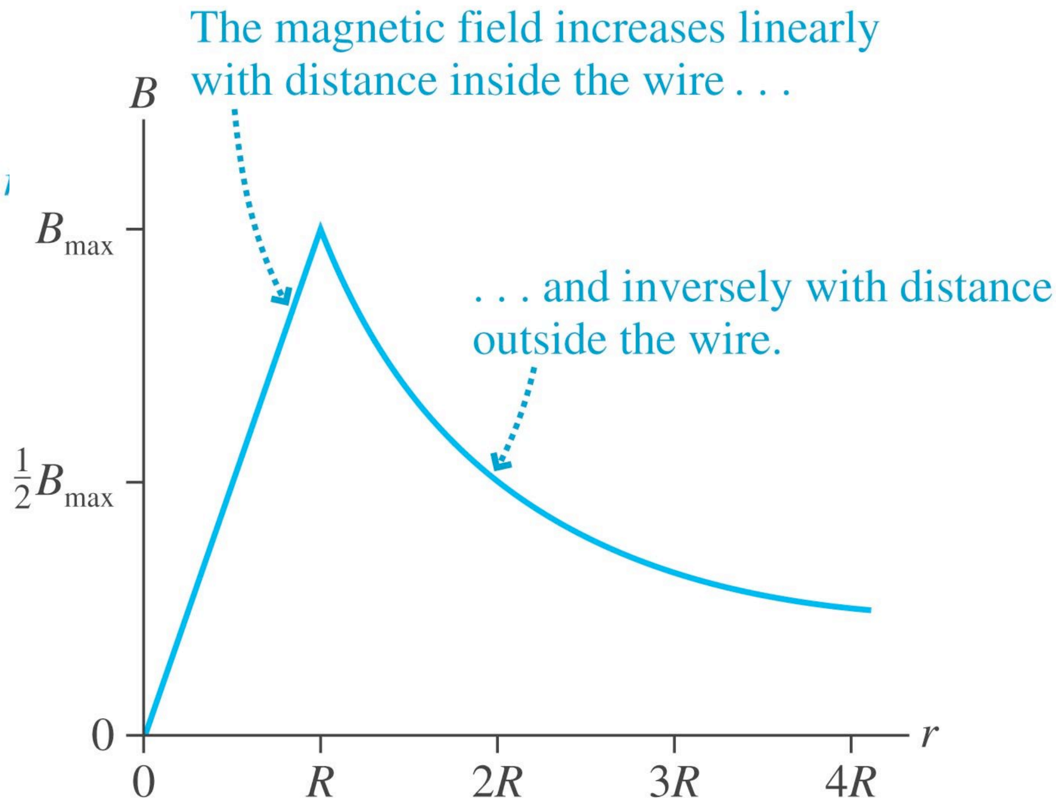

The integration path is a circle of radius $d$.
The integration starts and stops at the same point.
$\vec{B}$
$d$
$I$
$\vec{B}$ is everywhere tangent to the integration path and has constant magnitude.


$I_1$ doesn't pass through the enclosed area.
The integration path is a closed curve.
$I_3$
$I_1$
$I_2$
$I_4$
These currents pass through the bounded area.



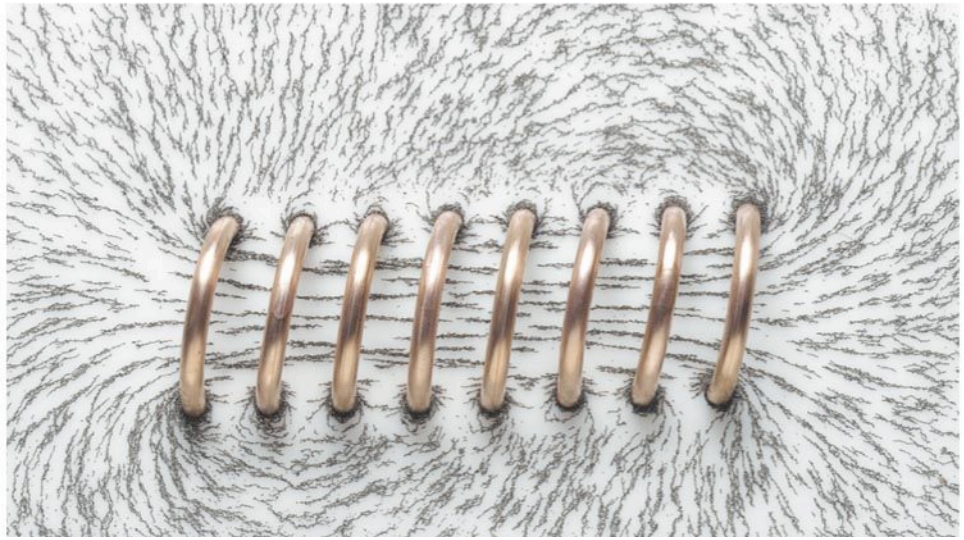
By symmetry, the magnetic field must be tangent to the circle.
Current-carrying wire of radius $R$
Closed integration path
$\vec{B}$
$r$
$R$
$I$
$\vec{B}$
$I_{\text{through}}$ is the current inside radius $r$



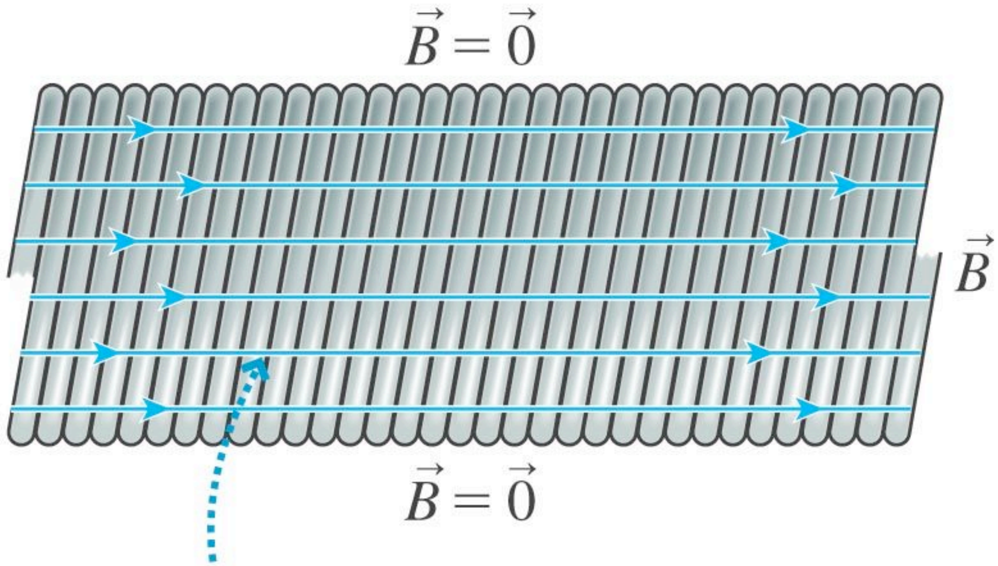



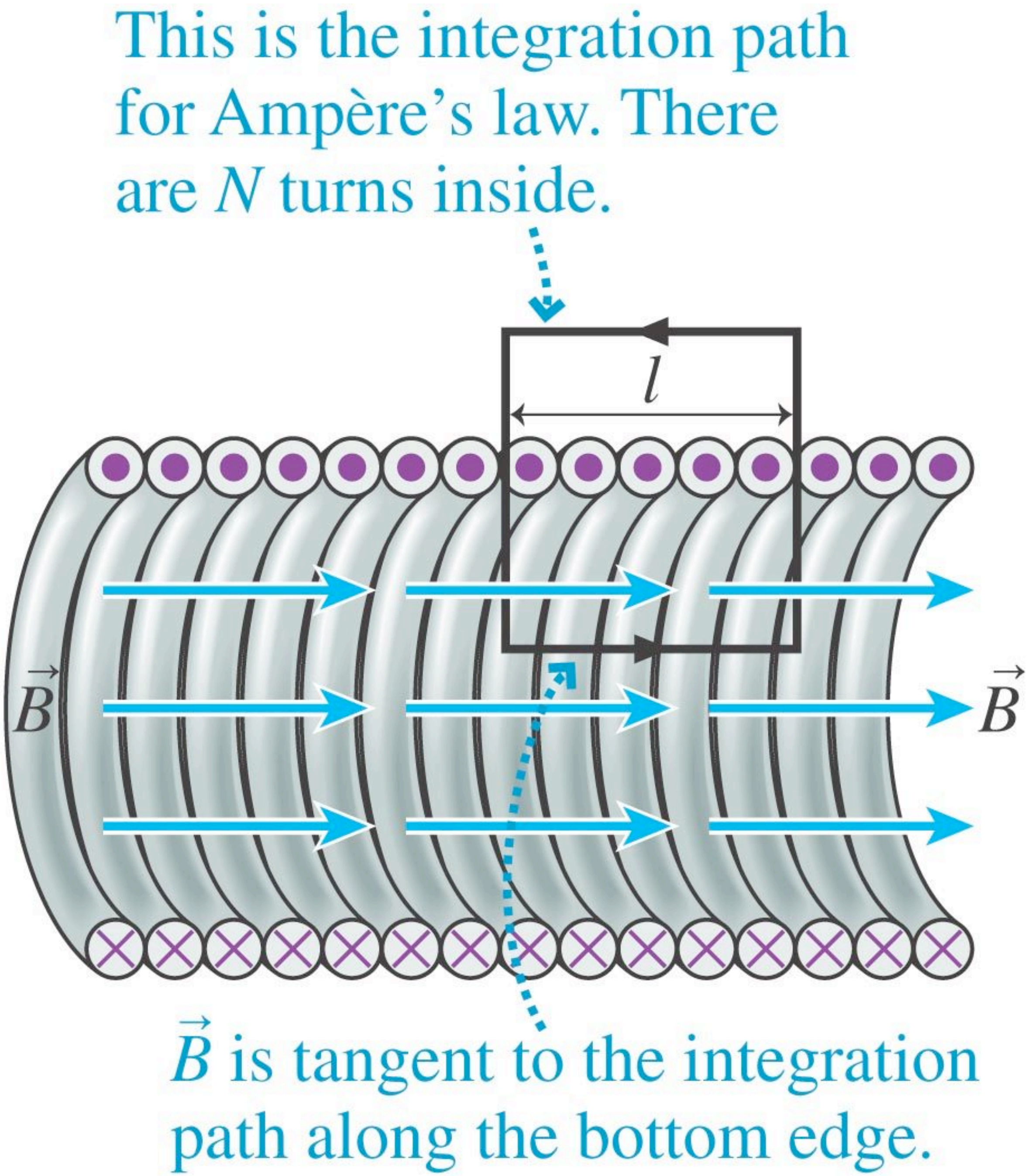
The magnetic field increases linearly with distance inside the wire . . .
$B$
$B_{\text{max}}$
. . . and inversely with distance outside the wire.
$\frac{1}{2}B_{\text{max}}$
0
$r$
0
$R$
$2R$
$3R$
$4R$

(a) A short solenoid



(b)








$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = Bl$$

$$\mu_0 I_{through} = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{l} = \mu_0 (N/l) I$$

# turns per unit length

Coulombs Law implies Gauss' Law and conservative E-field

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Gauss' Law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$

$$0 = \oint \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}}$$

But also, Gauss' law and conservative E-field imply Coulombs law

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{q_1}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

Biot-Savart Law implies Gauss' Law and Amperes Law

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