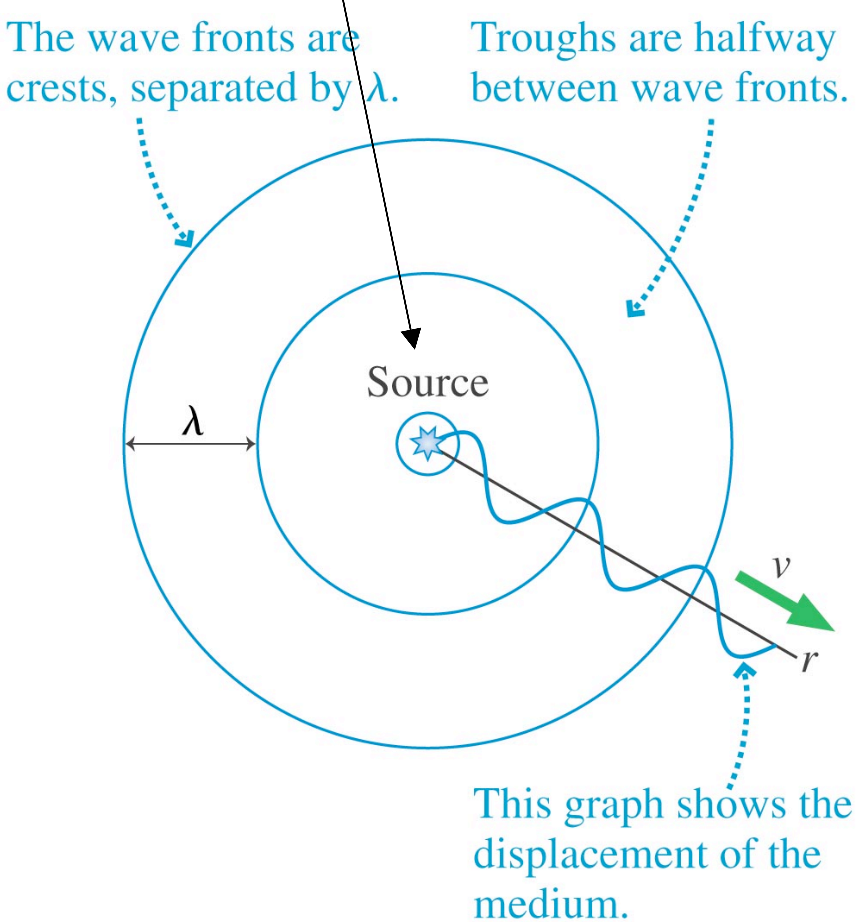
Chapters 21 & 22

Interference and Wave Optics

Waves that are coherent can add/cancel

Patterns of strong and weak intensity

Single Spherical Source



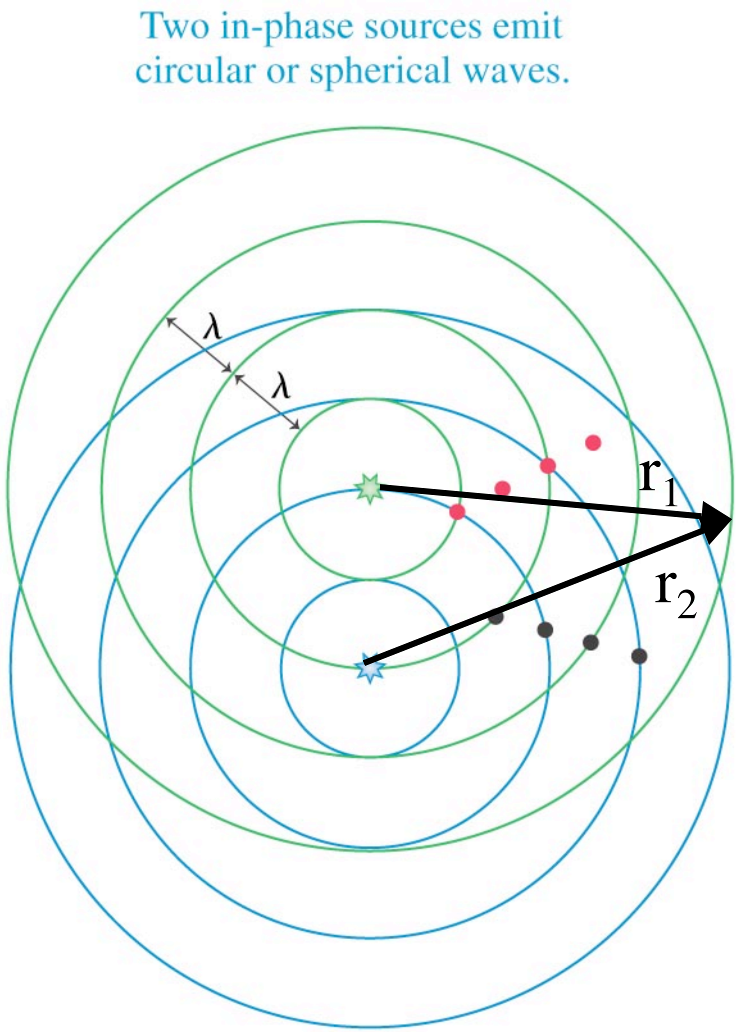
Approximate Electric Field:

$$E(r,t) = A(r)\cos(kr - \omega t + \theta)$$

Field depends on distance from source and time.

Typically $A(r) \sim 1/r$

Most important dependence is in the cosine



- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference. A crest is aligned with a trough of another wave.

Two sources that have exactly the same frequency. "Coherent"

$$E(r,t) = A(r_1)\cos(kr_1 - \omega t + \phi_1)$$

$$+ A(r_2)\cos(kr_2 - \omega t + \phi_2)$$

Sources will interfere constructively when

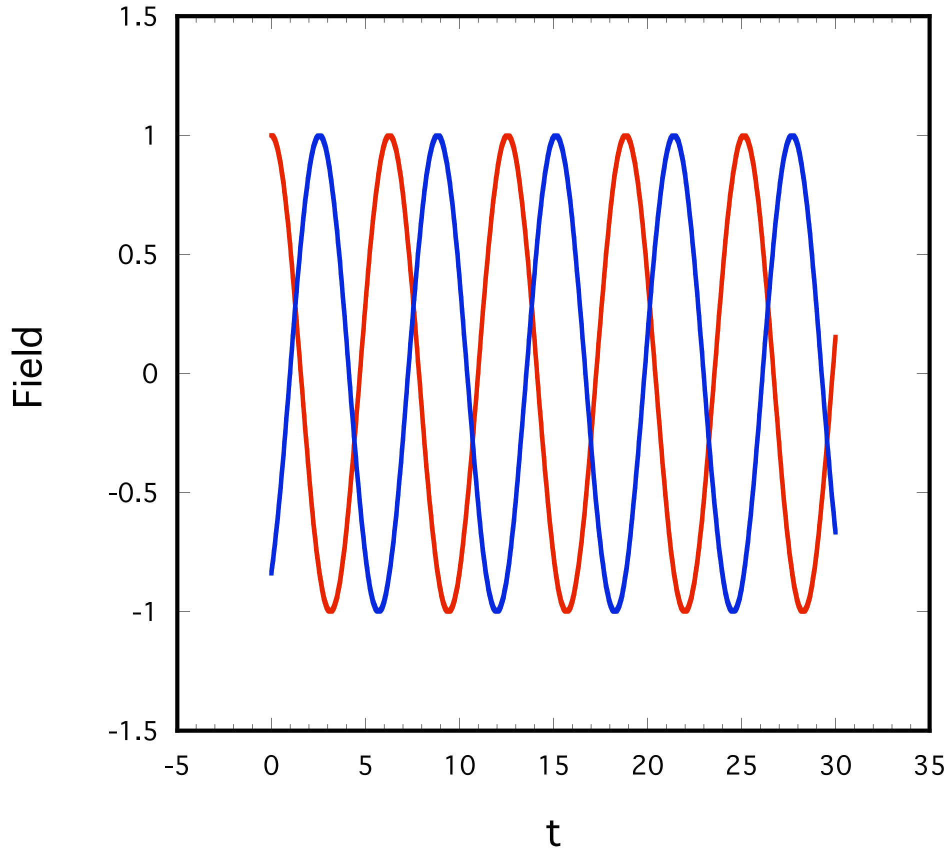
$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = 2\pi m$$
$$m = 0, 1, 2, \ldots$$

Sources will interfere destructively when

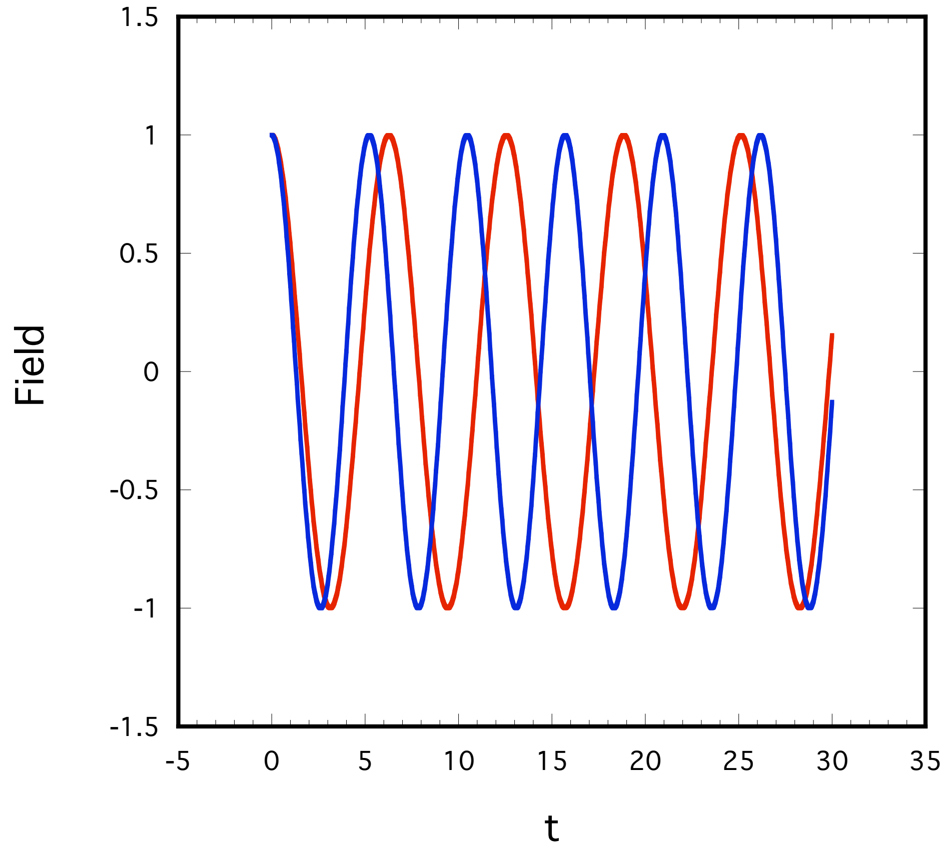
$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = 2\pi\left(m + \frac{1}{2}\right)$$

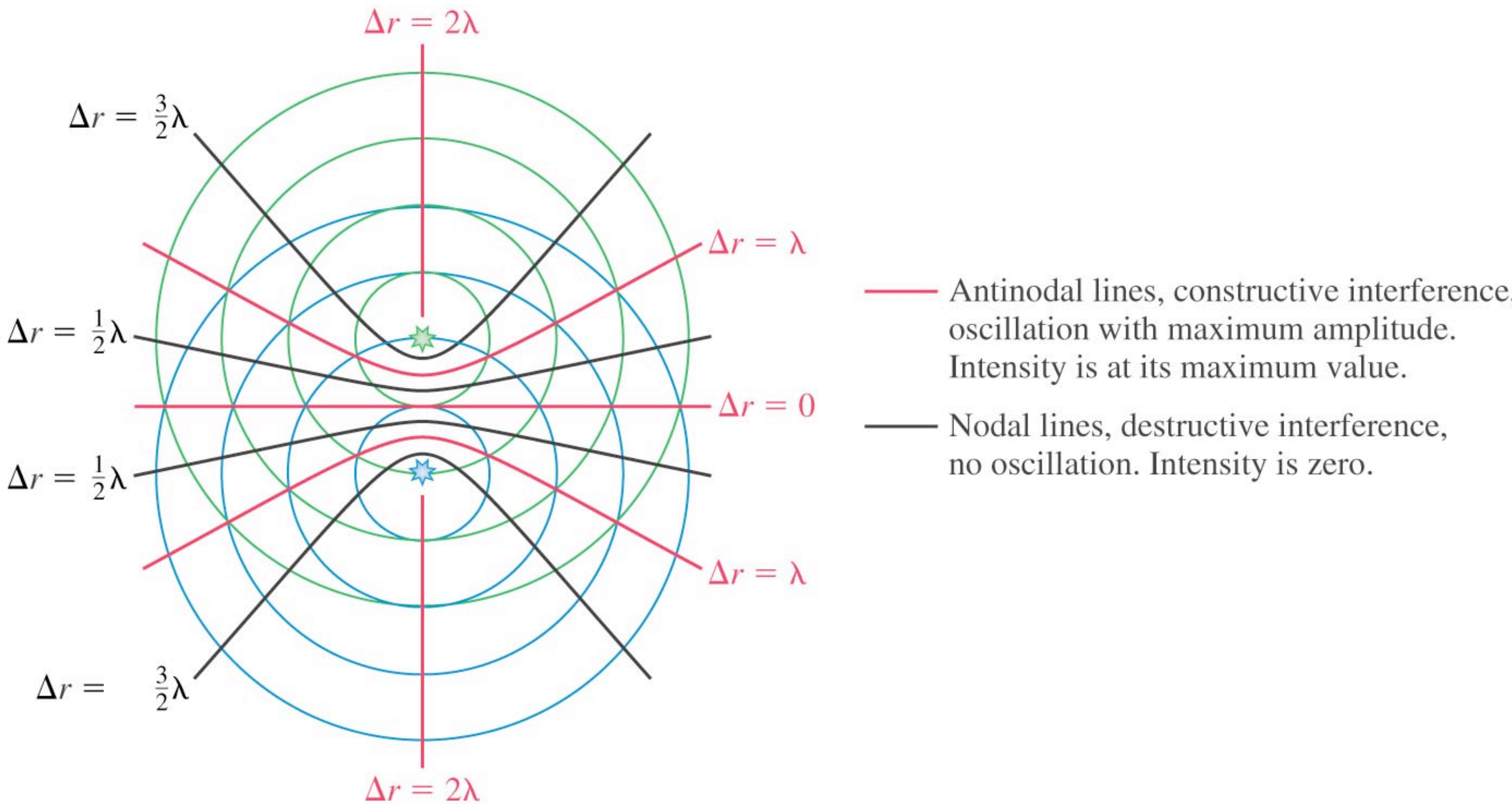
Incoherent vs Out of Phase

<u>Coherent, but out of phase</u>.
Two signals have the same frequency, but one leads or lags the other.

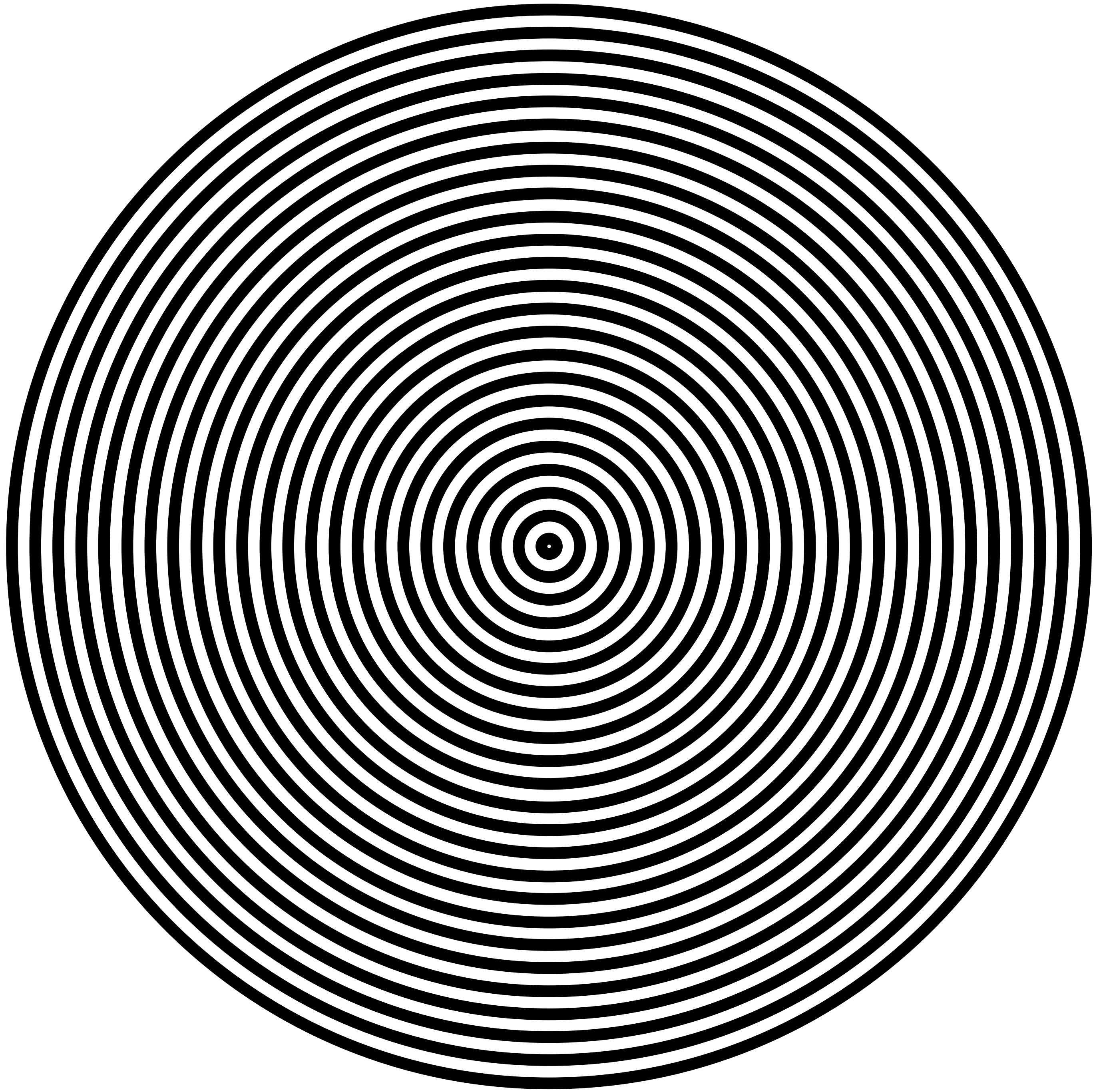


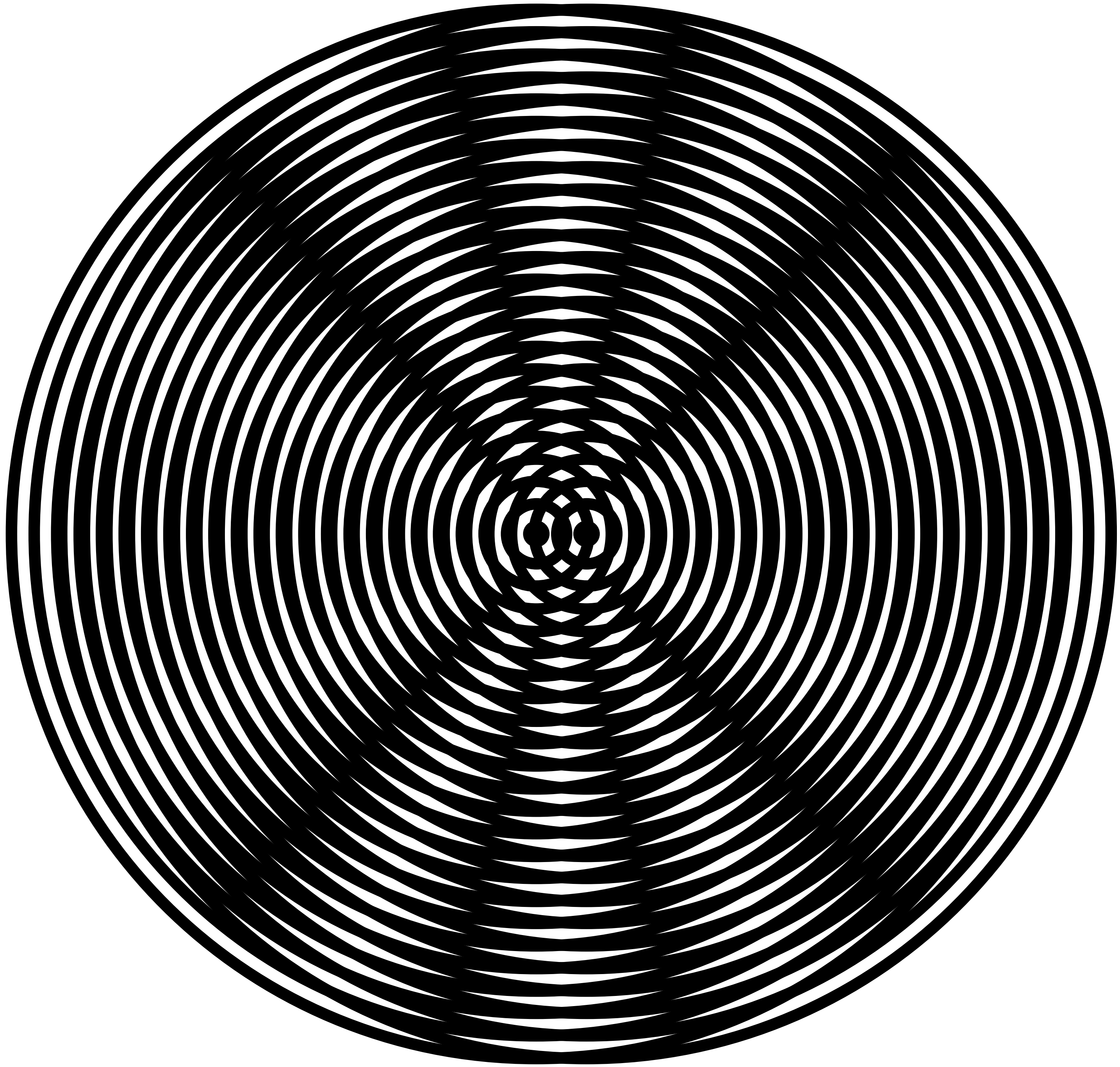
<u>Incoherent</u>
Two signals have different frequencies. Sometimes the same sign, sometimes opposite signs.





$\Delta r = 2\lambda$
$\Delta r = \frac{3}{2}\lambda$
$\Delta r = \lambda$
$\Delta r = \frac{1}{2}\lambda$
$\Delta r = 0$
$\Delta r = \frac{1}{2}\lambda$
$\Delta r = \lambda$
$\Delta r = \frac{3}{2}\lambda$
$\Delta r = 2\lambda$
Antinodal lines, constructive interference, oscillation with maximum amplitude. Intensity is at its maximum value.
Nodal lines, destructive interference, no oscillation. Intensity is zero.





Field and Intensity far from sources*

$$E(r,t) = A(r_1)\cos(kr_1 - \omega t) + A(r_2)\cos(kr_2 - \omega t)$$

suppose $A(r_1) = A(r_2)$

$$E(r,t) \simeq 2A\cos(\frac{k\Delta r}{2})\cos(k\bar{r} - \omega t)$$

$$\Delta r = r_1 - r_2$$

$$\bar{r} = \frac{r_1 + r_2}{2}$$

Field amplitude depends on space

Field oscillates in time.

Trigonometry

$$\cos(A) + \cos(B)$$
$$= 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$$

*Special case $\phi_1 = \phi_2 = 0$

Average intensity depends in difference in distance to sources, Δr

Field

$$E(r,t) \simeq 2A\cos(\frac{k\Delta r}{2})\cos(k\bar{r} - \omega t)$$

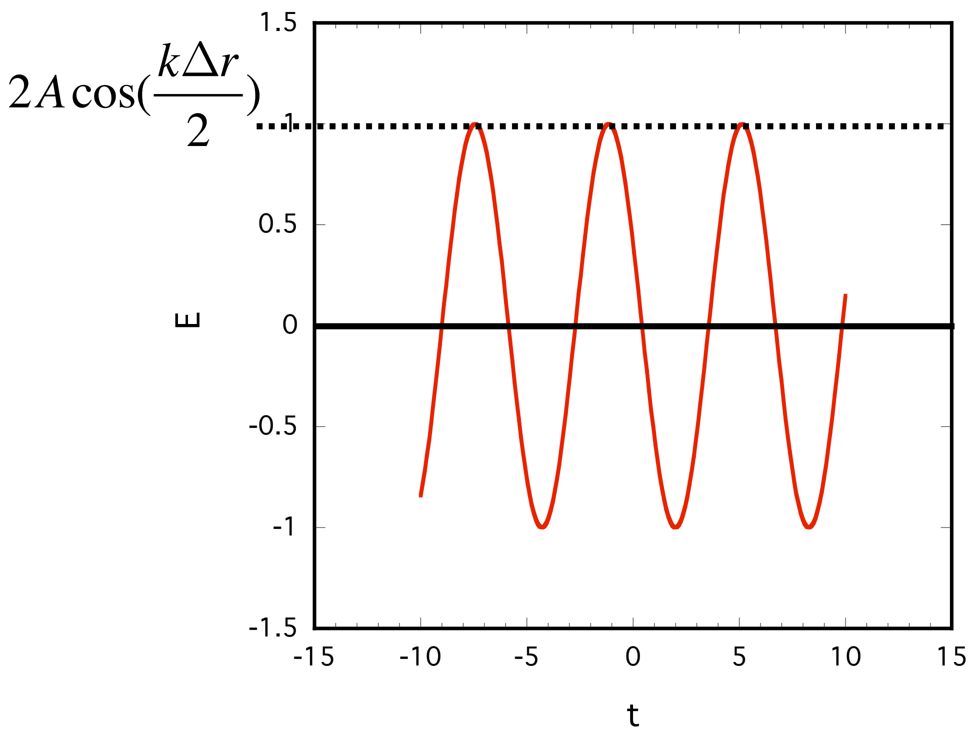
$$\Delta r = r_1 - r_2$$

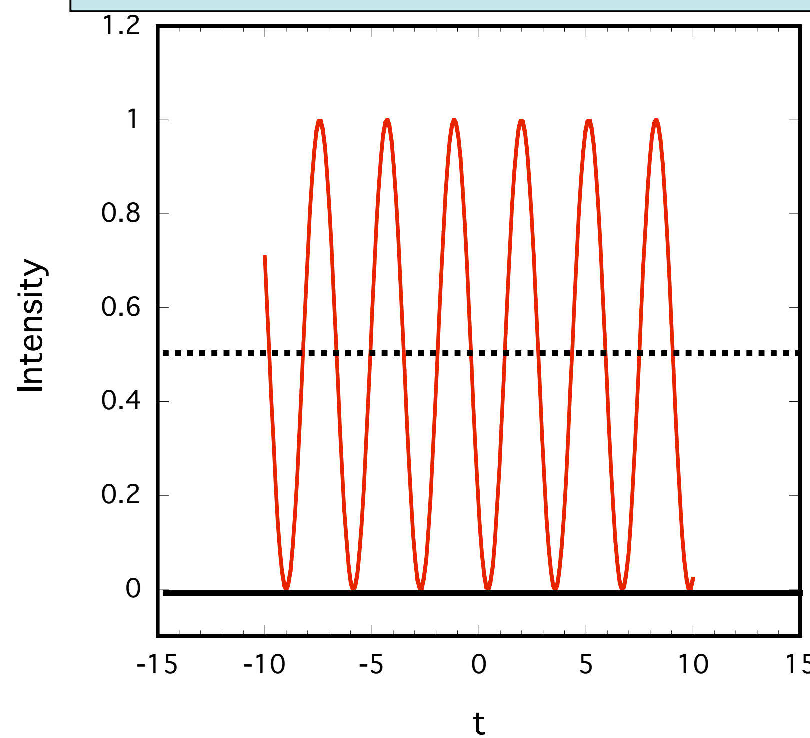
$$\bar{r} = \frac{r_1 + r_2}{2}$$

Intensity

$$I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

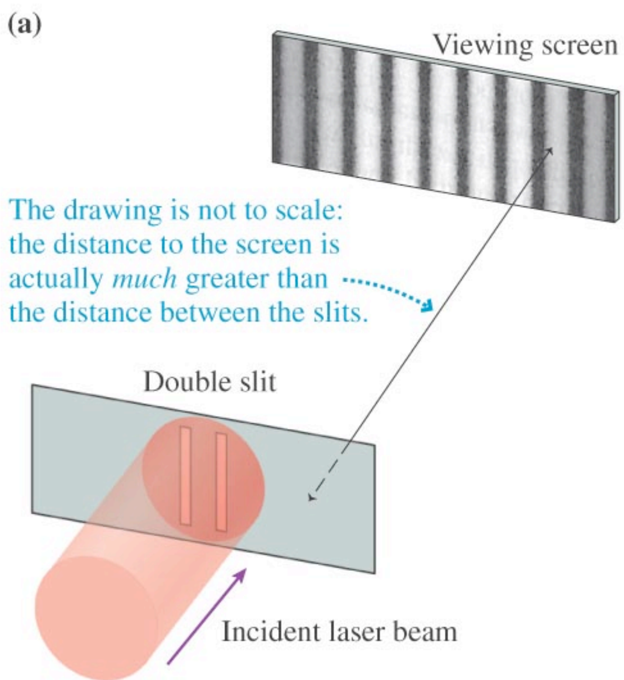
$$I_{ave} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|2A\cos(\frac{k\Delta r}{2})\right|^2$$

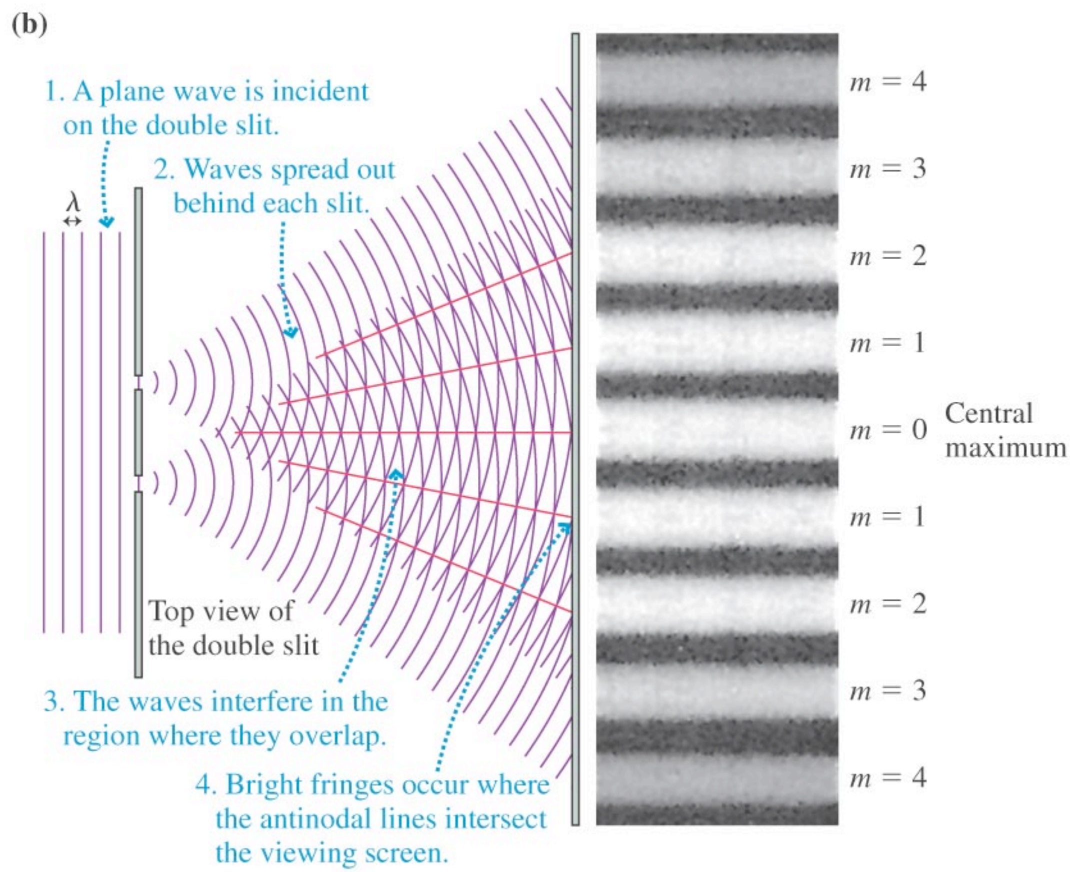


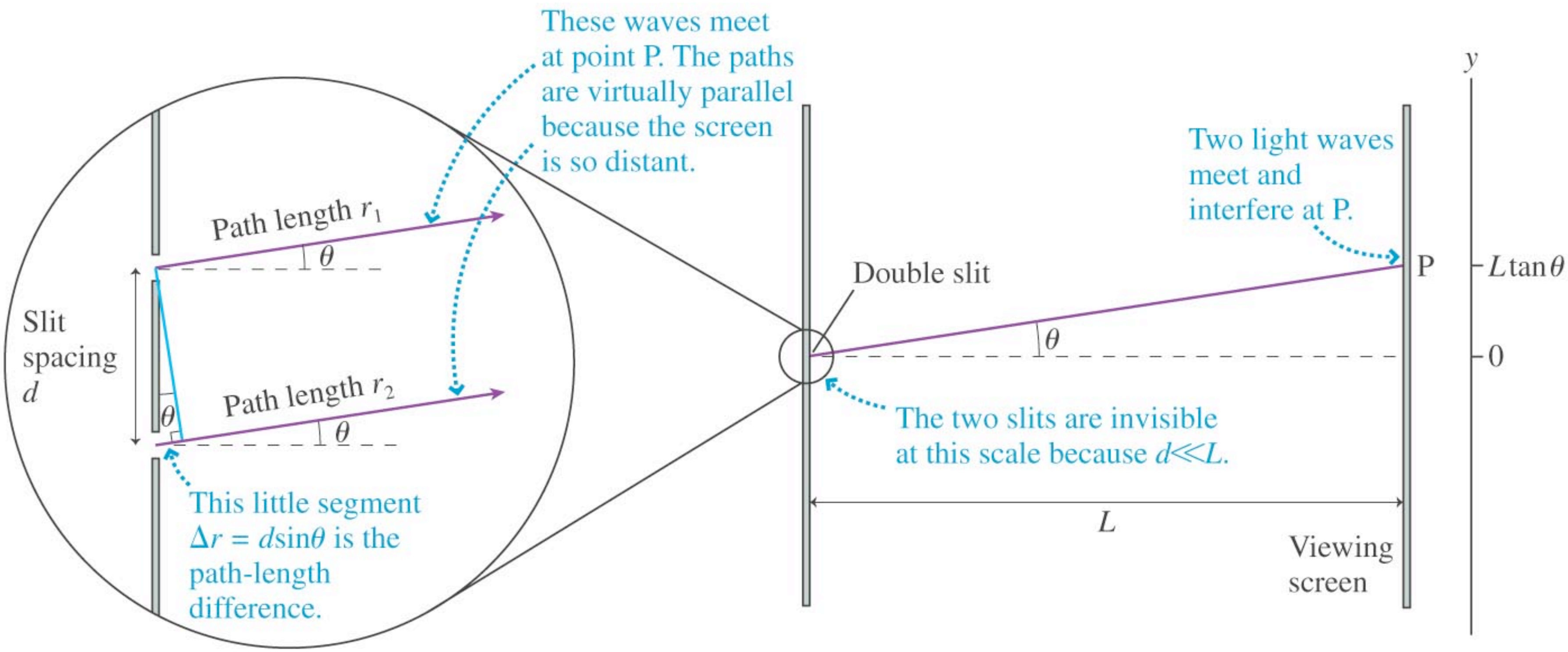


Interference of light

Coherence because sources are at exactly the same frequency







Sources will interfere constructively when

$$(kr_1 + \phi_1) - (kr_2 + \phi_2) = k\Delta r = 2\pi m$$

Phases same because source comes from a single incident plane wave

$$k\Delta r = kd\sin\theta = 2\pi m$$

$$\sin\theta_m \approx \theta_m = m\lambda / d$$

$$m = 0, 1, 2, ...$$

Dark fringes $\sin\theta_m \approx \theta_m = \left(m + \frac{1}{2}\right)\lambda / d$

Intensity on a distant screen

$$I = \sqrt{\frac{\varepsilon_0}{\mu_0}}|\mathbf{E}|^2$$

$$I_{ave} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|2A\cos(\frac{k\Delta r}{2})\right|^2$$

$$k\Delta r = kd\sin\theta \simeq kd\,\theta = \frac{2\pi}{\lambda}d\,\frac{y}{L}$$

Fringe spacing

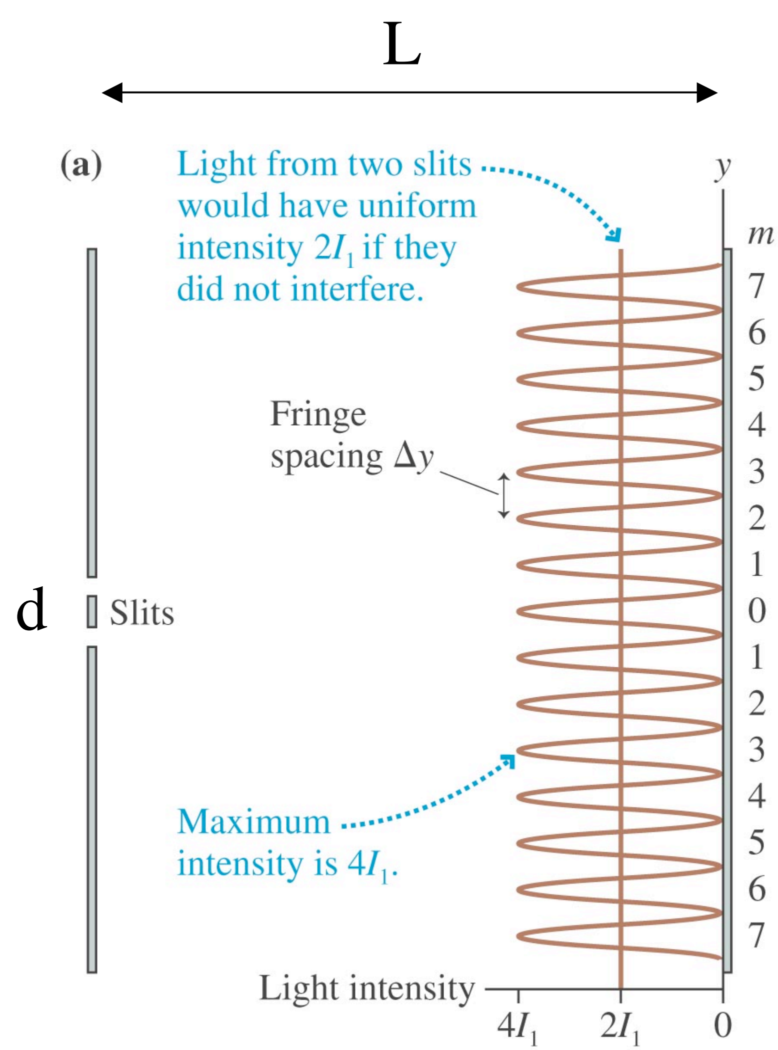
$$\Delta y = \frac{L\lambda}{d}$$

Intensity from a single source

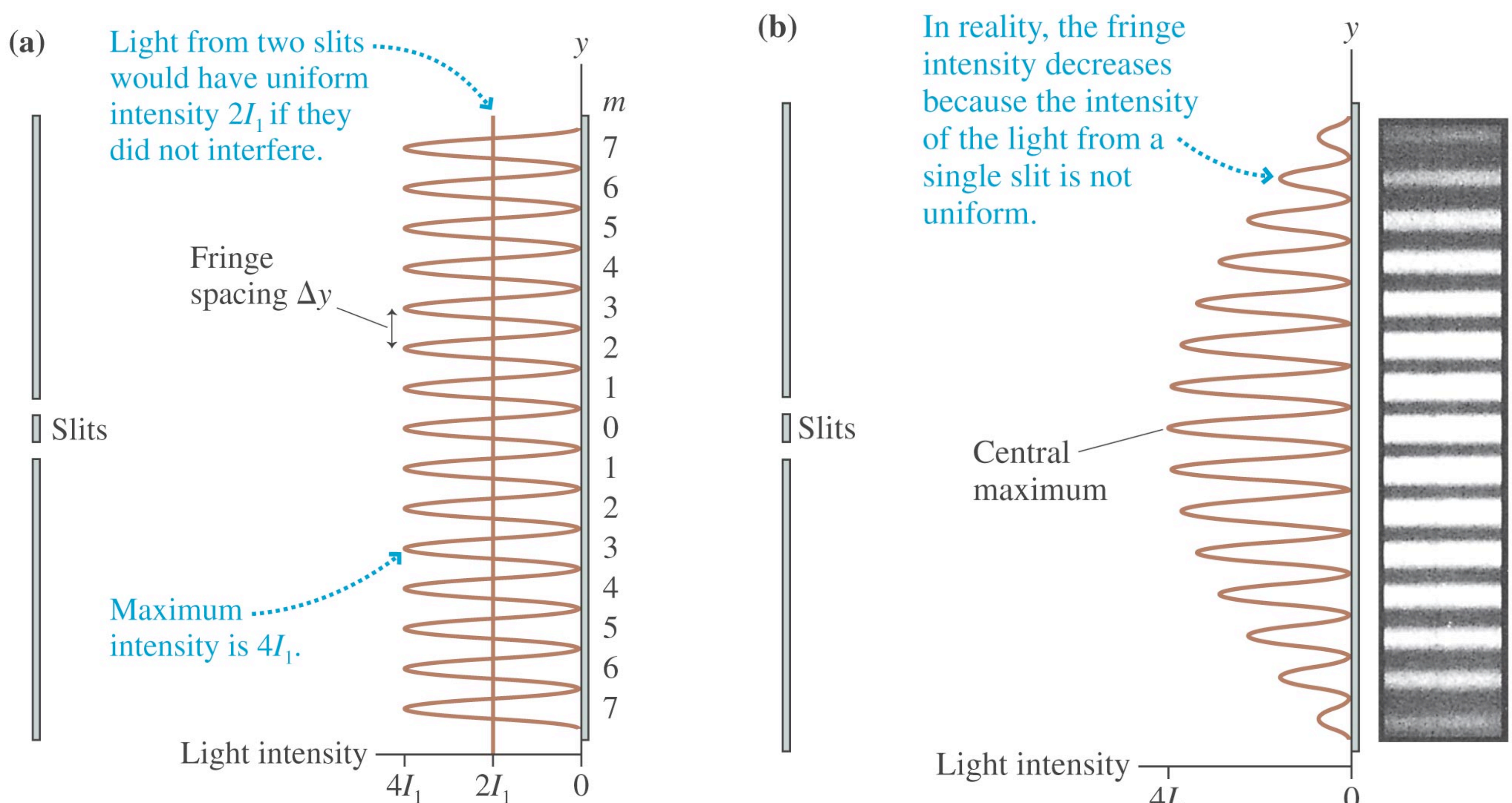
$$I_1 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|A|^2$$

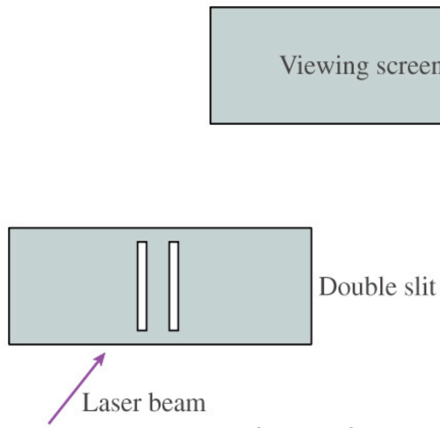
Maximum Intensity at fringe

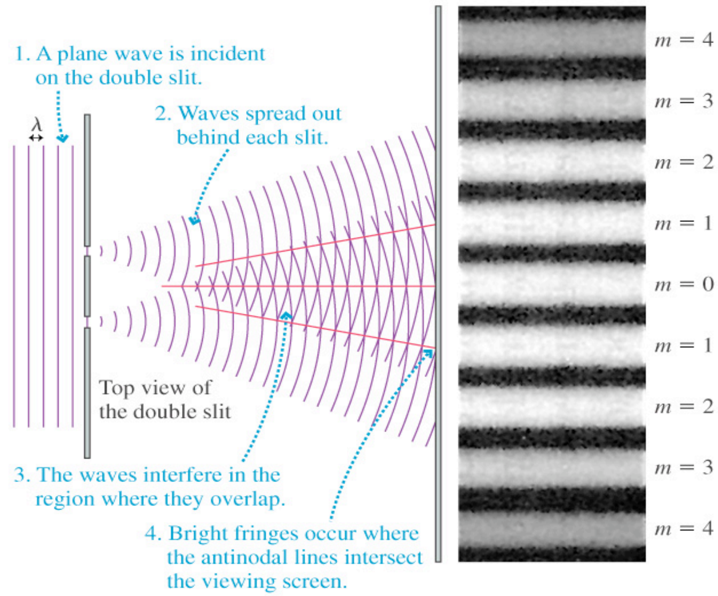
$$I_{fringe} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,|2A|^2 = 2I_1$$



Real pattern affected by slit opening width and distance to screen





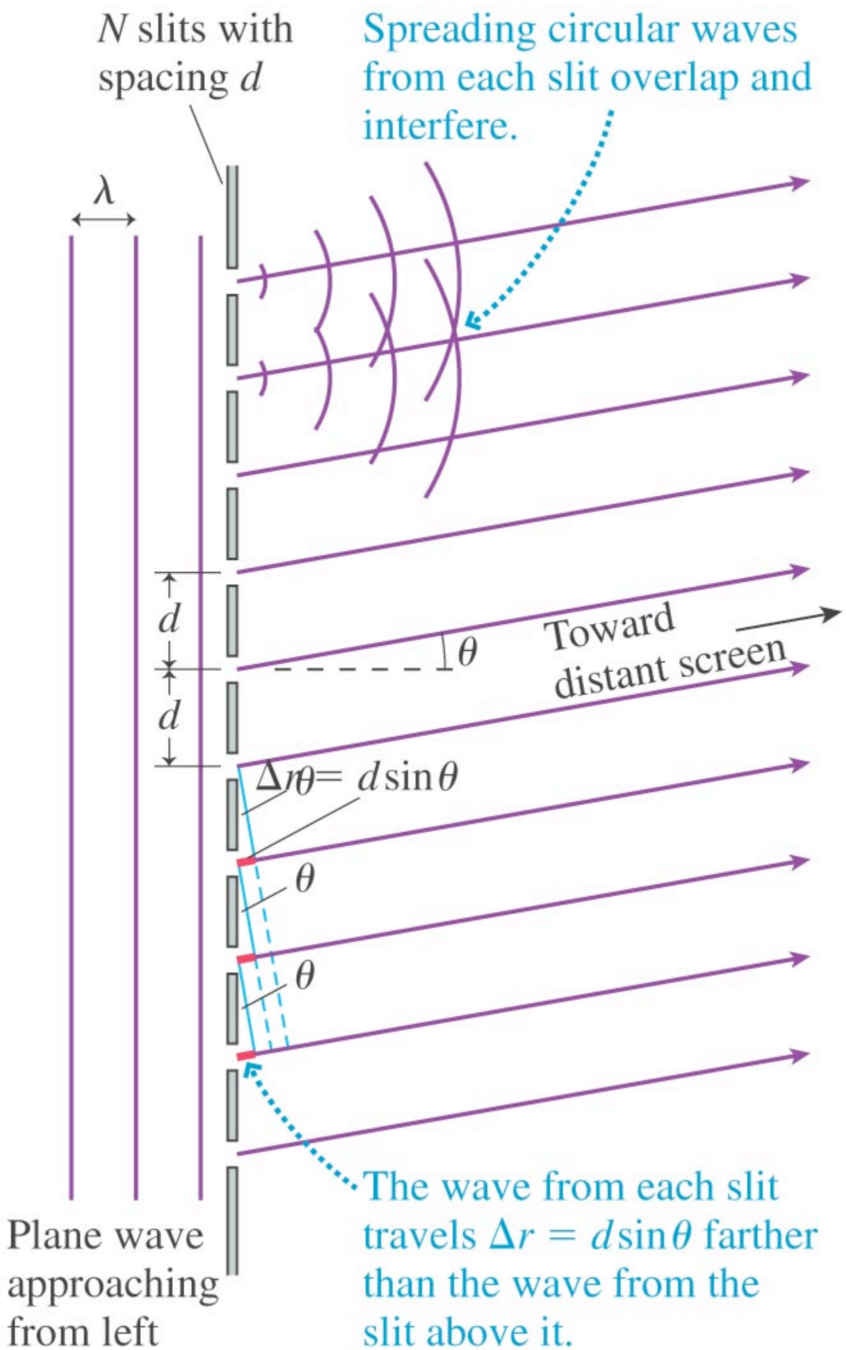


Suppose the viewing screen in the figure is moved closer to the double slit. What happens to the interference fringes?

A. They fade out and disappear.
B. They get out of focus.
C. They get brighter and closer together.
D. They get brighter and farther apart.
E. They get brighter but otherwise do not change.

Light of wavelength λ_1 illuminates a double slit, and interference fringes are observed on a screen behind the slits. When the wavelength is changed to λ_2, the fringes get closer together. How large is λ_2 relative to λ_1?

A. λ_2 is smaller than λ_1.
B. λ_2 is larger than λ_1.
C. Cannot be determined from this information.



Diffraction Grating

N slits, sharpens bright fringes

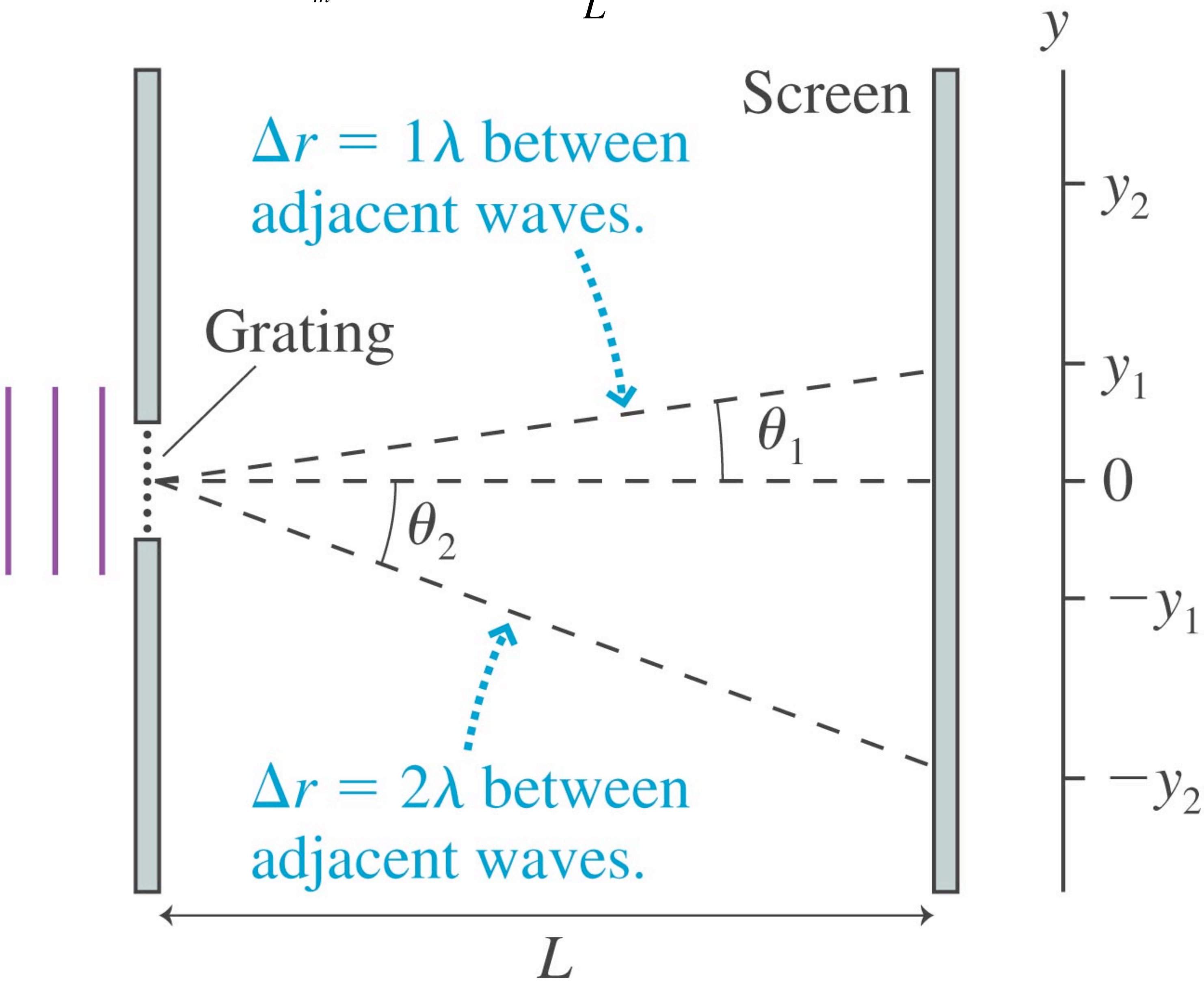
Bright fringes at same angle as for double slit

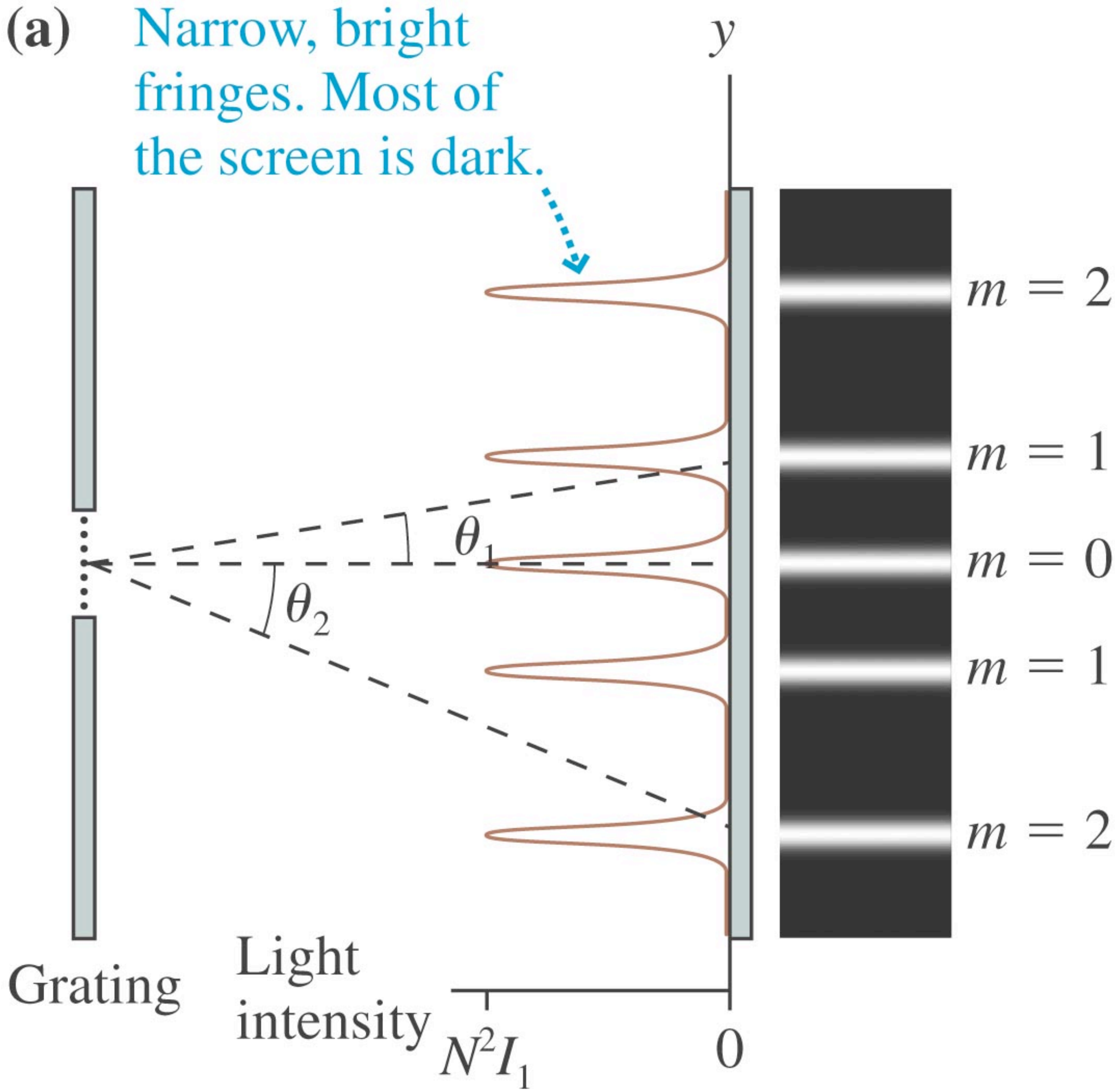
$$\sin\theta_m = m\lambda / d$$

$$m = 0, 1, 2, \ldots$$

Location of Fringes on distant screen

$$\sin\theta_m = m\lambda / d \qquad \frac{y_m}{L} = \tan\theta_m$$




(a)
Narrow, bright fringes. Most of the screen is dark.
y
m = 2
m = 1
m = 0
m = 1
m = 2
θ1
θ2
Grating
Light intensity
N²I1
0

Intensity on a distant screen $$I = \sqrt{\frac{\varepsilon_0}{\mu_0}} |\mathbf{E}|^2$$

Average over time $$I_{ave} = \frac{1}{2} I$$

Intensity from a single slit

amplitude from a single slit

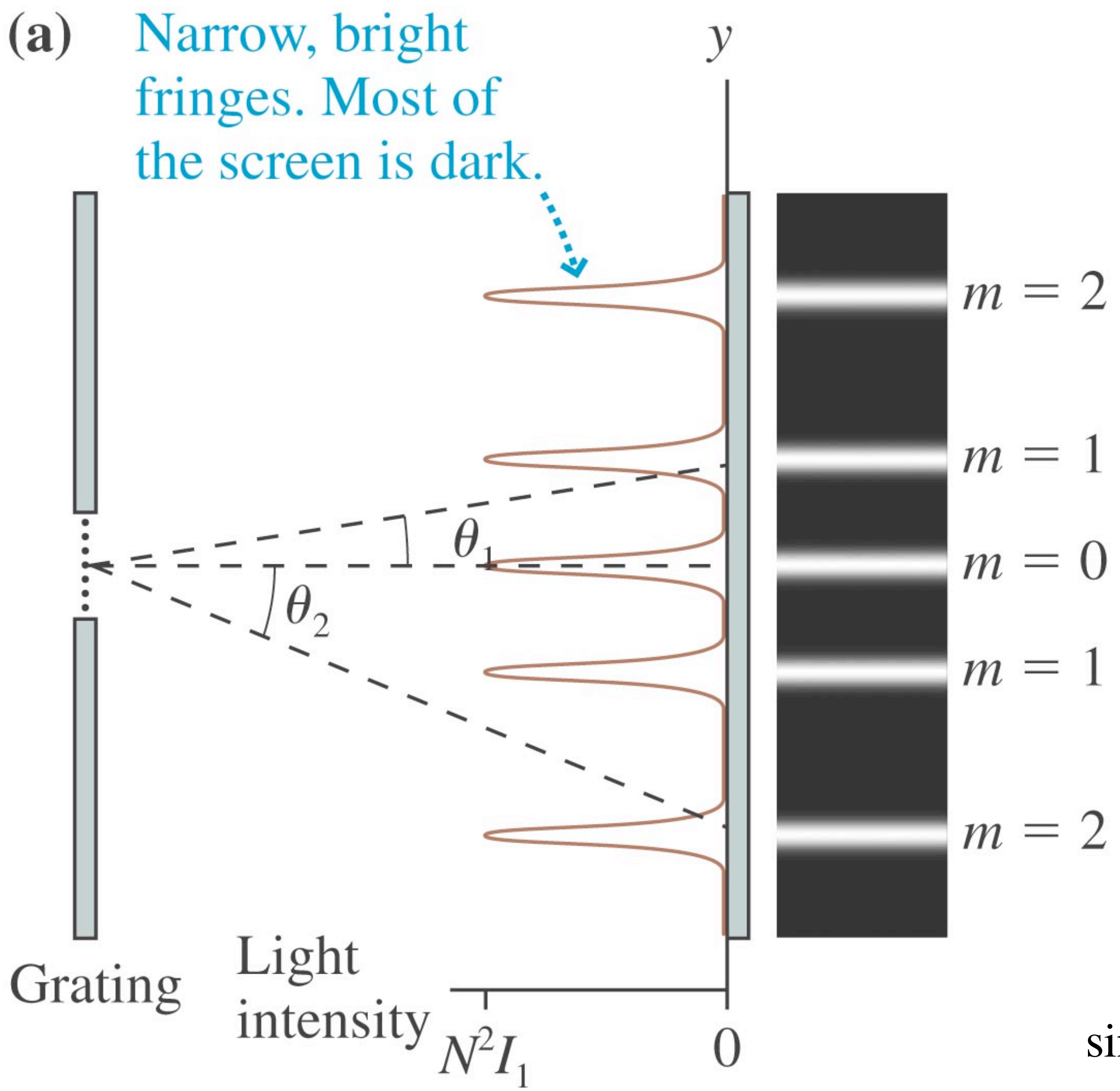
$$I_1 = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, |A|^2$$

At the bright fringe N slits interfere constructively

$$I_{fringe} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, |NA|^2 = N^2 I_1$$

Spatial average of intensity must correspond to sum of N slits

$$I_{SA} = N I_1$$



$$\frac{I_{fringe}}{I_{SA}} = N$$

width of fringe

$$\frac{fringe\ width}{fringe\ spacing} = \frac{1}{N}$$

$$\sin\theta_m = m\lambda / d \qquad \frac{y_m}{L} = \tan\theta_m$$

Measuring Light Spectra

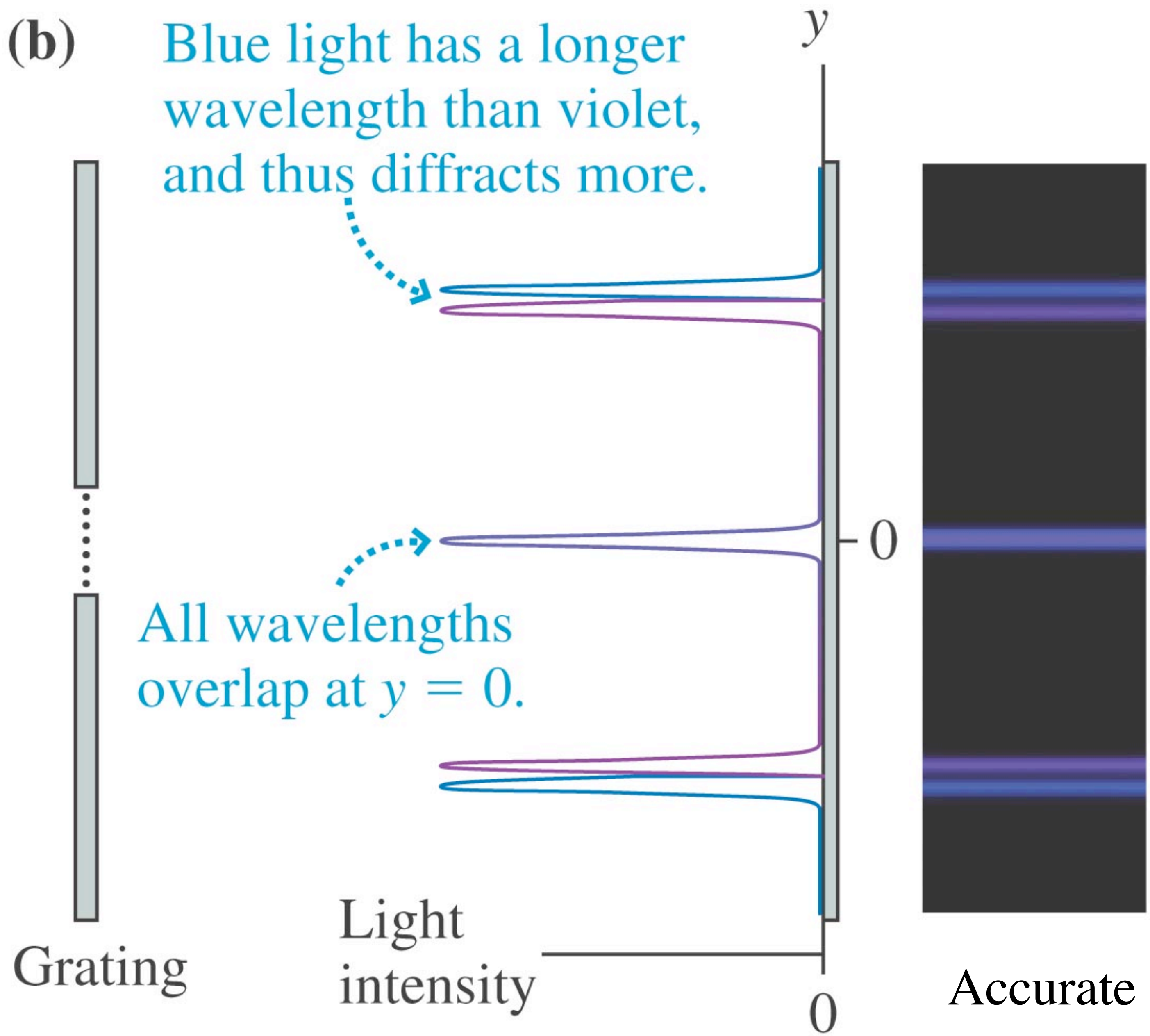
Light usually contains a superposition of many frequencies.

The amount of each frequency is called its spectrum.

Knowing the components of the spectrum tells us about the source of light.

Composition of stars is known by measuring the spectrum of their light.

$$\sin\theta_m = m\lambda / d \qquad \frac{y_m}{L} = \tan\theta_m$$

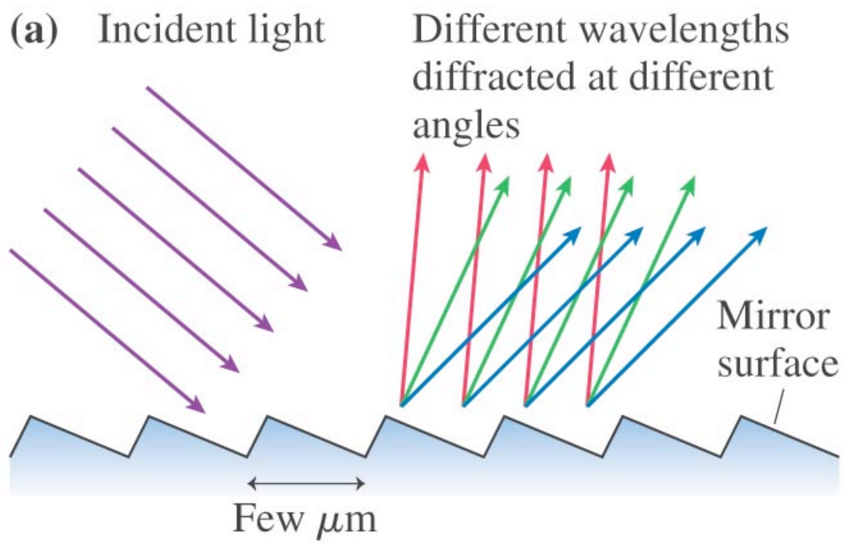


Accurate resolution of spectrum requires many lines

White light passes through a diffraction grating and forms rainbow patterns on a screen behind the grating. For each rainbow,

A. the red side is farthest from the center of the screen, the violet side is closest to the center.
B. the red side is closest to the center of the screen, the violet side is farthest from the center.
C. the red side is on the left, the violet side on the right.
D. the red side is on the right, the violet side on the left.

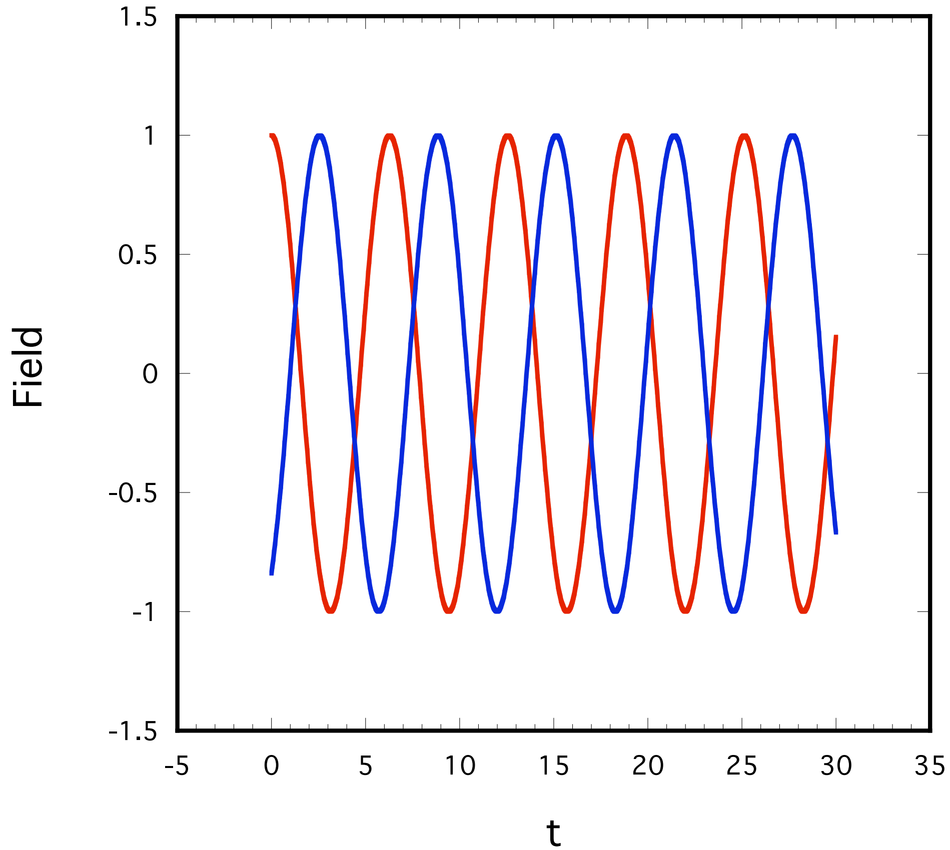
Reflection Grating



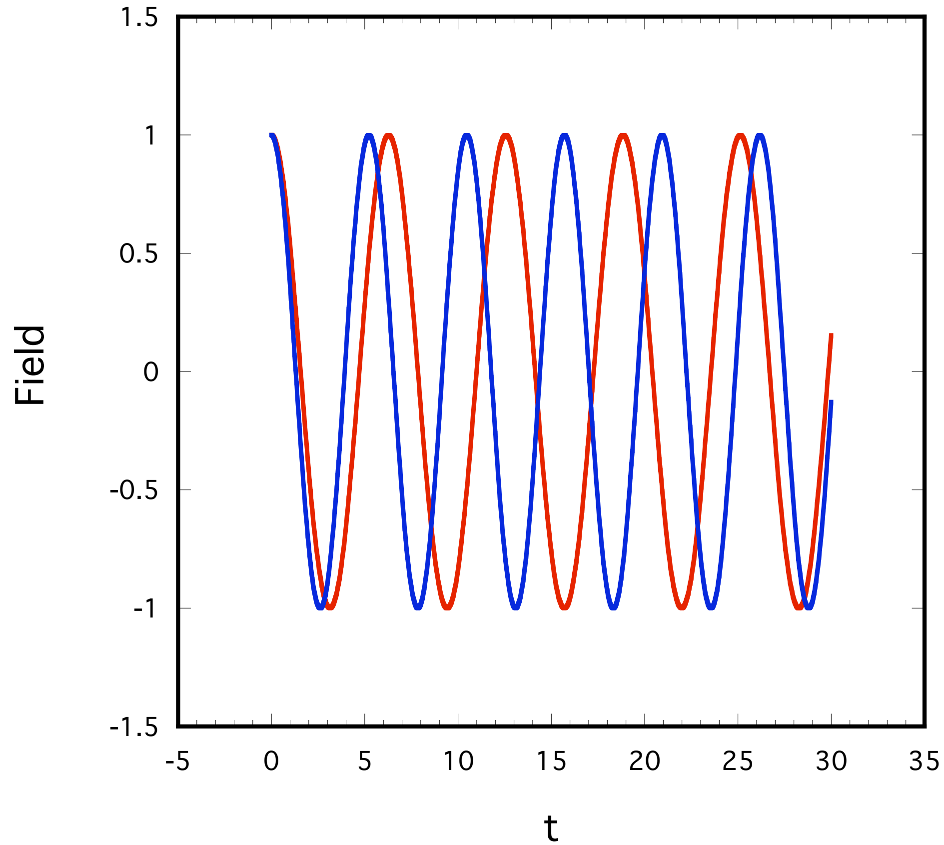
A reflection grating can be made by cutting parallel grooves in a mirror surface. These can be very precise, for scientific use, or mass produced in plastic.

Incoherent vs Out of Phase

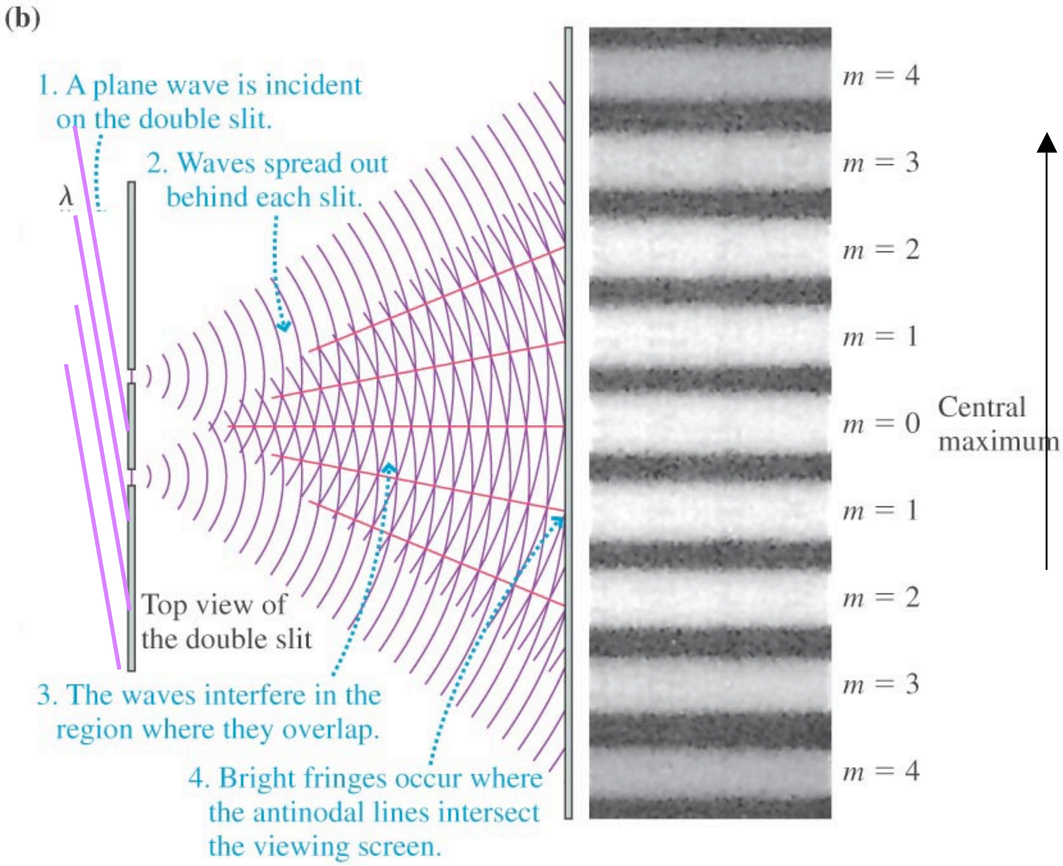
Coherent, but out of phase.
Two signals have the same frequency, but one leads or lags the other.



Incoherent
Two signals have different frequencies. Sometimes the same sign, sometimes opposite signs.



Fields in slits are coherent
but out of phase



Diffraction pattern shifts

Field and Intensity far from sources

$$E(r,t) = A(r_1)\cos(kr_1 - \omega t + \phi_1) + A(r_2)\cos(kr_2 - \omega t + \phi_2)$$

Trigonometry

$$\cos(A) + \cos(B)$$
$$= 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$$

suppose $A(r_1) = A(r_2)$

$$E(r,t) \simeq 2A\cos(\frac{k\Delta r}{2} + \frac{\phi_1 - \phi_2}{2})\cos(k\bar{r} - \omega t + \frac{\phi_1 + \phi_2}{2})$$

$$\Delta r = r_1 - r_2$$

$$\bar{r} = \frac{r_1 + r_2}{2}$$

Field amplitude depends on space

Field oscillates in time.

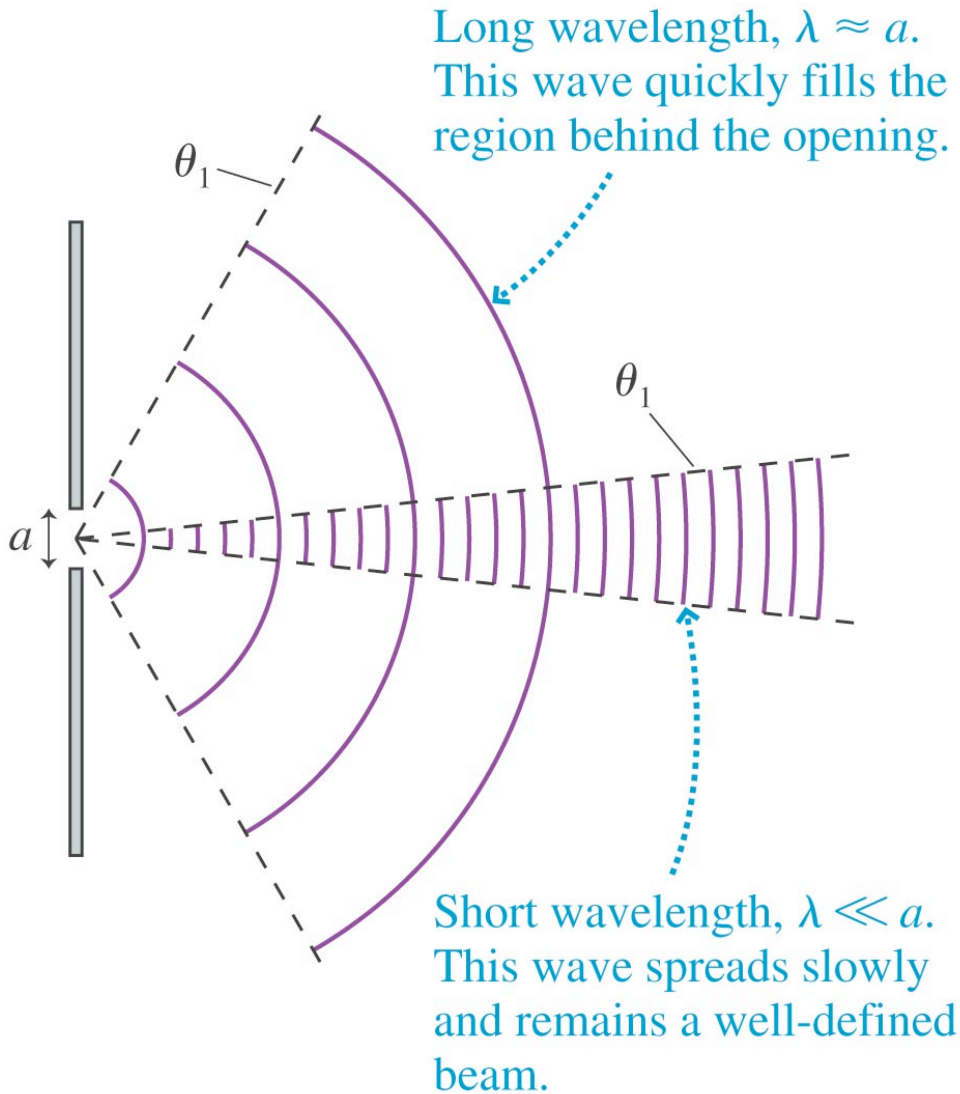
$$\Delta r = d\sin\theta$$

Constructive interference when

$$\frac{k\Delta r}{2} + \frac{\phi_1 - \phi_2}{2} = m\pi$$

$$d\sin\theta = \lambda\left(m - \frac{\phi_1 - \phi_2}{2\pi}\right)$$

Propagation of wave fronts from a slit with a nonzero width



Sources are not points.

How do we describe spreading of waves?

Ans. Just solve Maxwell's equations. (wave equation)

That is not always so easy.

In the past, not possible.

In the distant past equations were not known.

Huygen's Principle

1. Each point on a wave front is the source of a spherical wavelet that spreads out at the wave speed.

2. At a later time, the shape of the wave front is the line tangent to all the wavelets.

Huygen's (1629-1695) Principle

1. Each point on a wave front is the source of a spherical wavelet that spreads out at the wave speed.

2. At a later time, the shape of the wave front is the line tangent to all the wavelets.

R. Plant

Not the same person.

www.guerrillacandy.com/.../

C. Huygens

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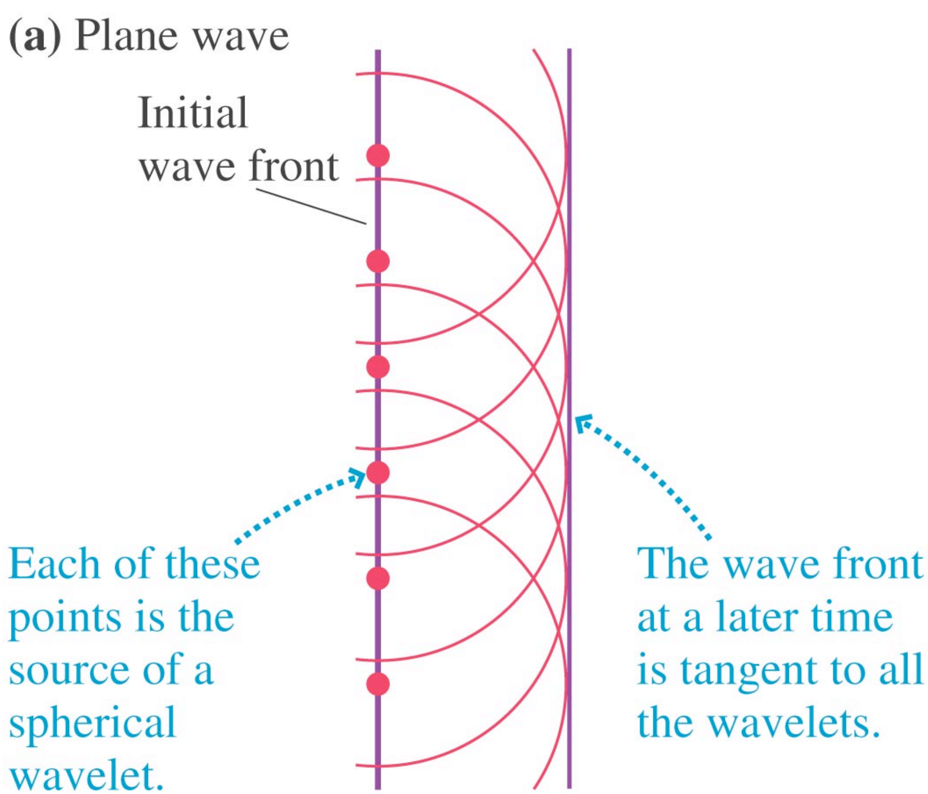
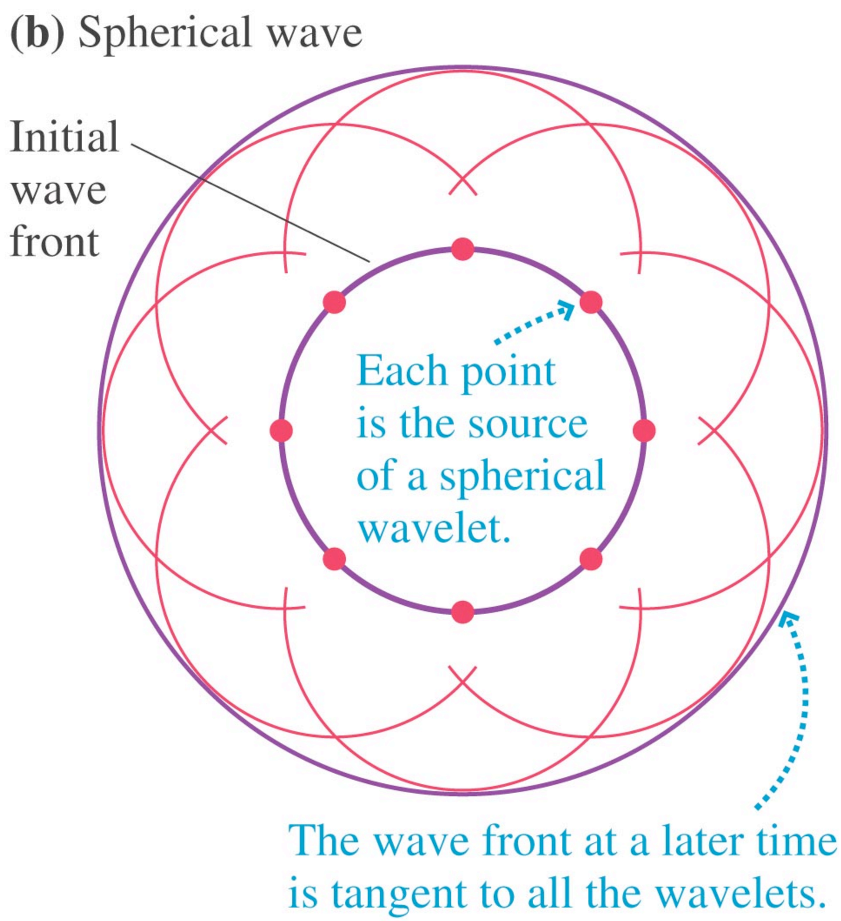
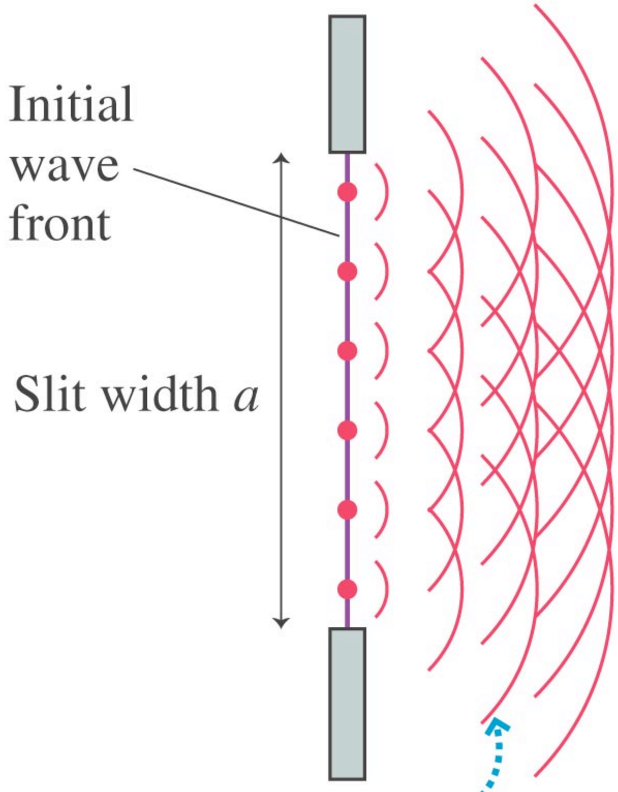
Huygens Principle:

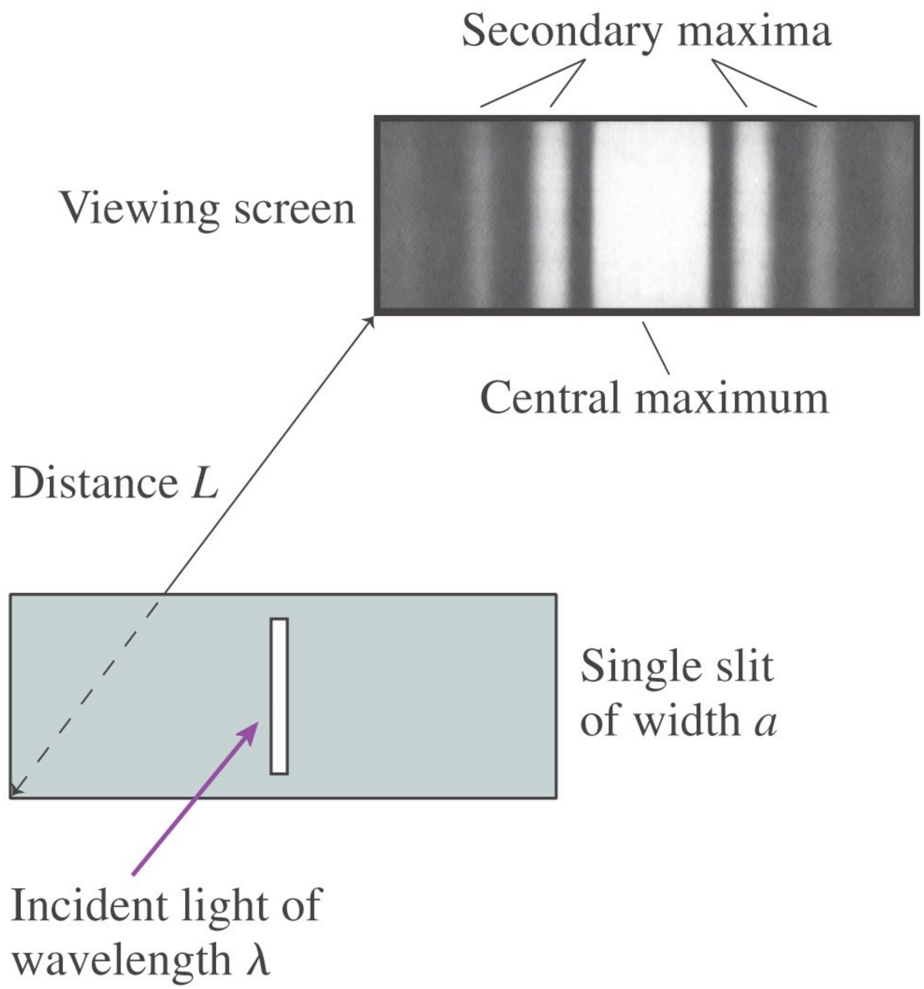




(a) Greatly magnified view of slit



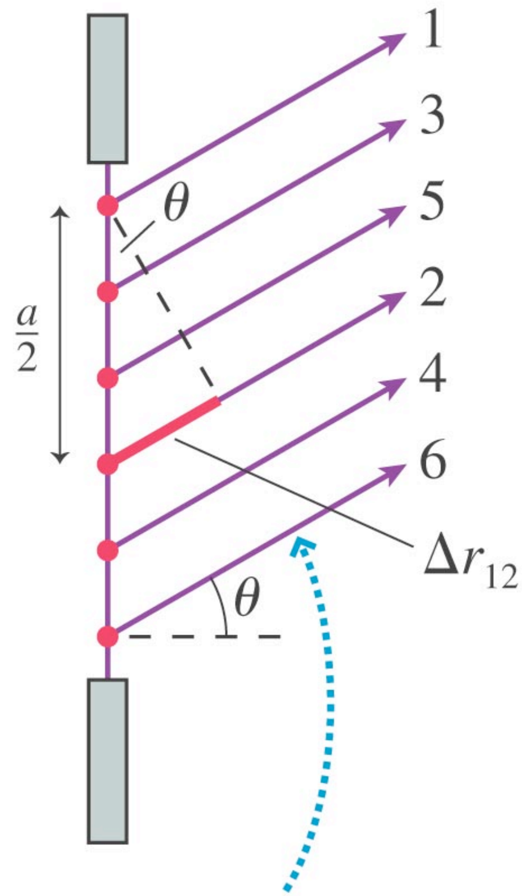
The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.



When is there perfect destructive interference?

(c)

Each point on the wave front is paired with another point distance $a/2$ away.



These wavelets all meet on the screen at angle θ. Wavelet 2 travels distance $\Delta r_{12} = (a/2)\sin\theta$ farther than wavelet 1.

Destructive when

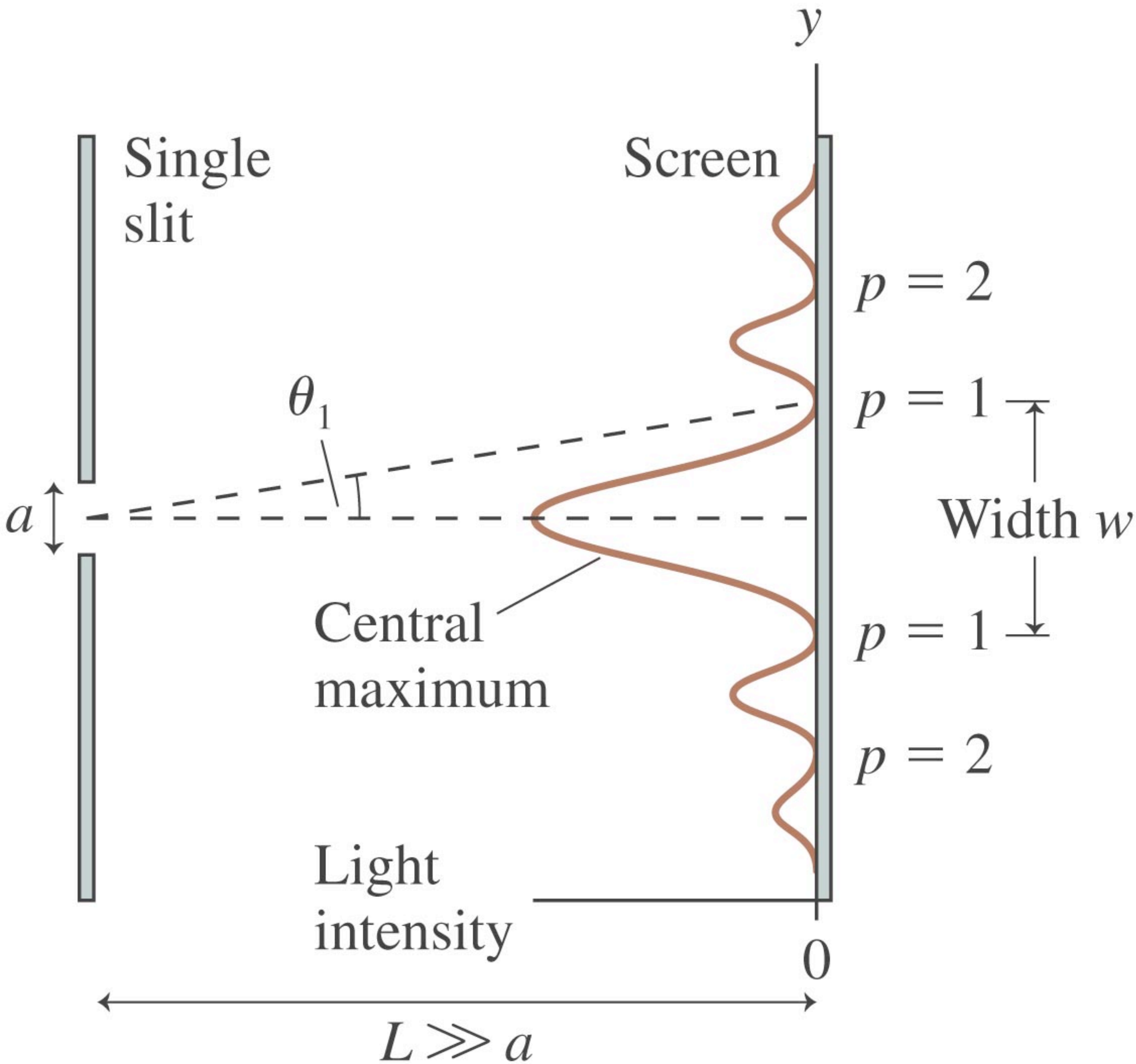
$$\Delta r_{12} = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

1 cancels 2
3 cancels 4
5 cancels 6
Etc.

Also:

$$\frac{a}{2p}\sin\theta_p = \frac{\lambda}{2}$$

$$p = 1, 2, 3.....$$



y
Single
slit
Screen
p = 2
θ_1
p = 1
a
Width w
Central
maximum
p = 1
p = 2
Light
intensity
0
$L \gg a$

We can calculate the pattern from a single slit!

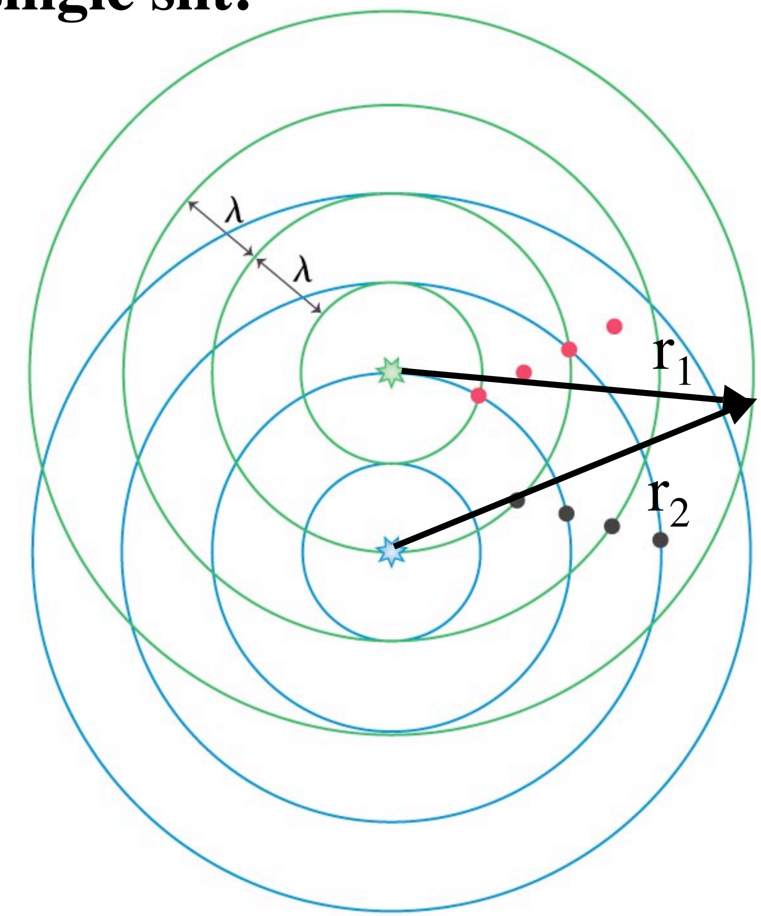
Field from two point sources

$$E(r,t) = A(r_1)\cos(kr_1 - \omega t) + A(r_2)\cos(kr_2 - \omega t)$$

Field from many point sources

$$E(r,t) = \sum_i A(r_i)\cos(kr_i - \omega t)$$

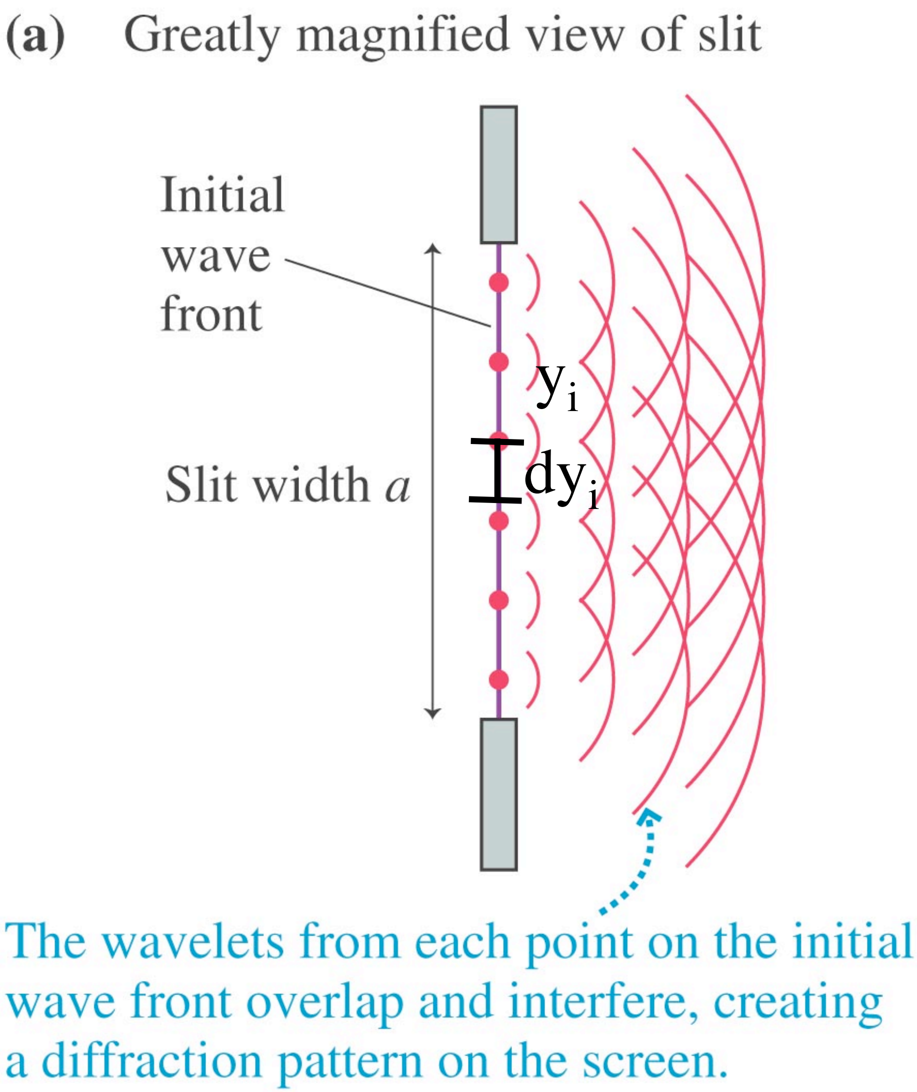
Two in-phase sources emit circular or spherical waves.



• Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.

• Points of destructive interference. A crest is aligned with a trough of another wave.

Field from a continuous distribution of point sources - Integrate!



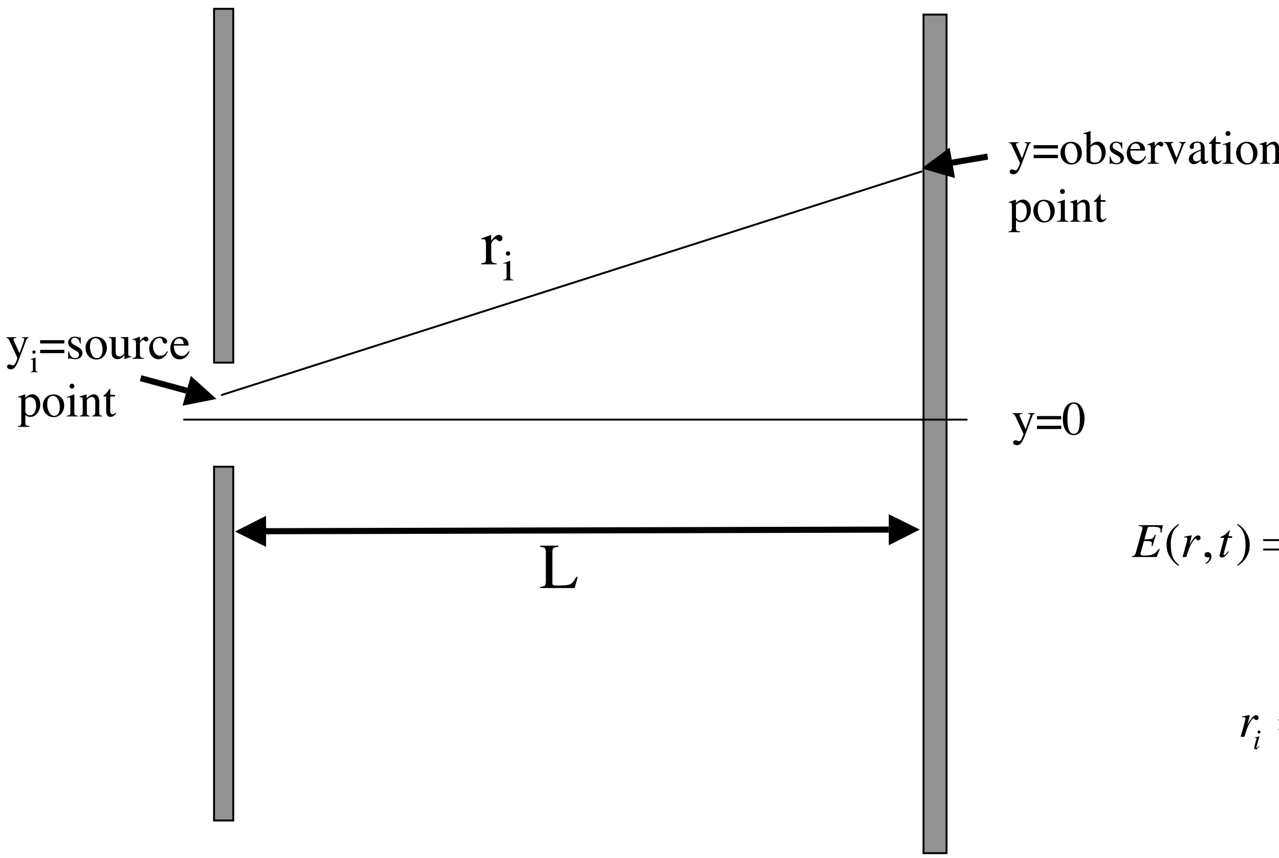
$$E(r,t) = \sum_i A(r_i)\cos(kr_i - \omega t)$$

⇓

$$E(r,t) = \int_{-a/2}^{a/2} \frac{dy_i}{a} A(r_i)\cos(kr_i - \omega t)$$

Replace sum by integral

Distance from source to observation point



$$E(r,t) = \int_{-a/2}^{a/2} \frac{dy_i}{a} A\cos(kr_i - \omega t)$$

$$r_i = \sqrt{L^2 + (y - y_i)^2}$$

Still can't do integral. Must make an approximation, $|y_i| << L, y$

$$r_i \simeq \sqrt{L^2 + y^2} - \frac{yy_i}{\sqrt{L^2 + y^2}}$$

Result

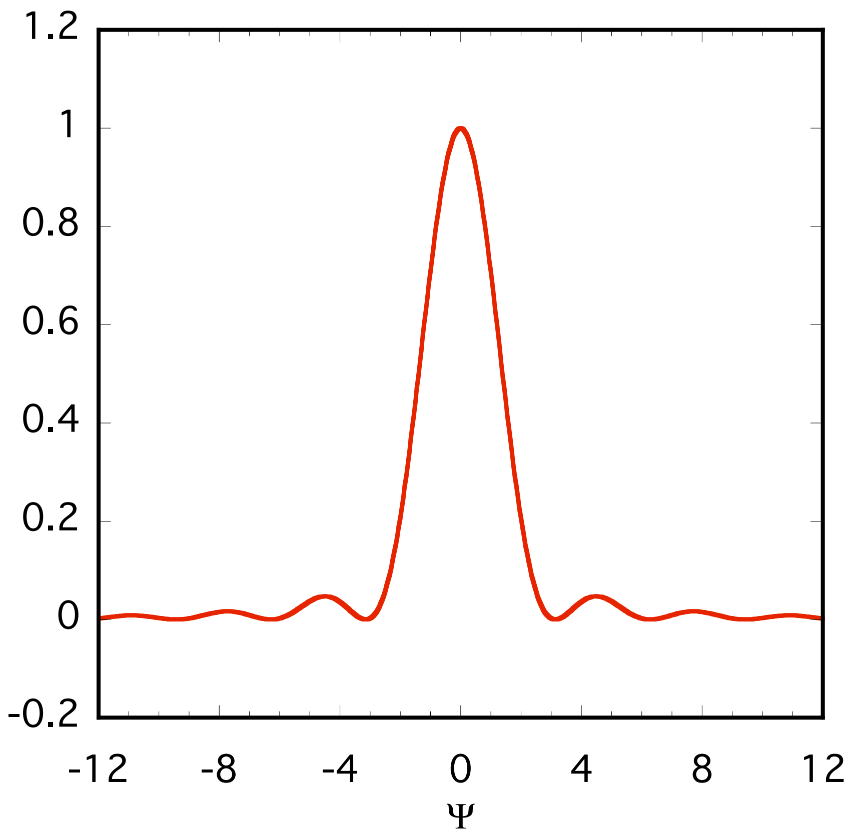
$$\bar{r} = \sqrt{L^2 + y^2}$$

$$E(r,t) = \int_{-a/2}^{a/2} \frac{dy_i}{a} A\cos(kr_i - \omega t) \simeq A\,\frac{\sin(\Psi)}{\Psi}\cos(k\bar{r} - \omega t)$$

$$\Psi = \frac{kay}{2\bar{r}}$$

Time average intensity

$$I_{ave} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left|A\,\frac{\sin(\Psi)}{\Psi}\right|^2$$



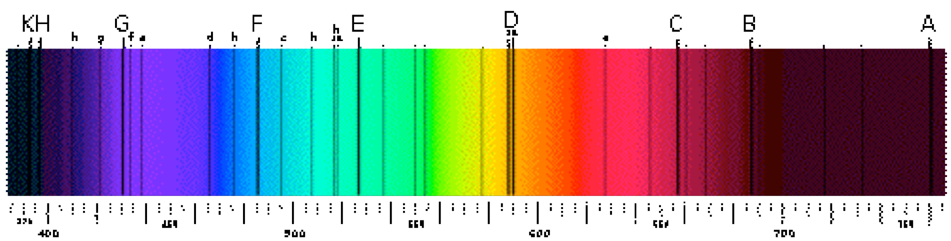
Intensity zero when

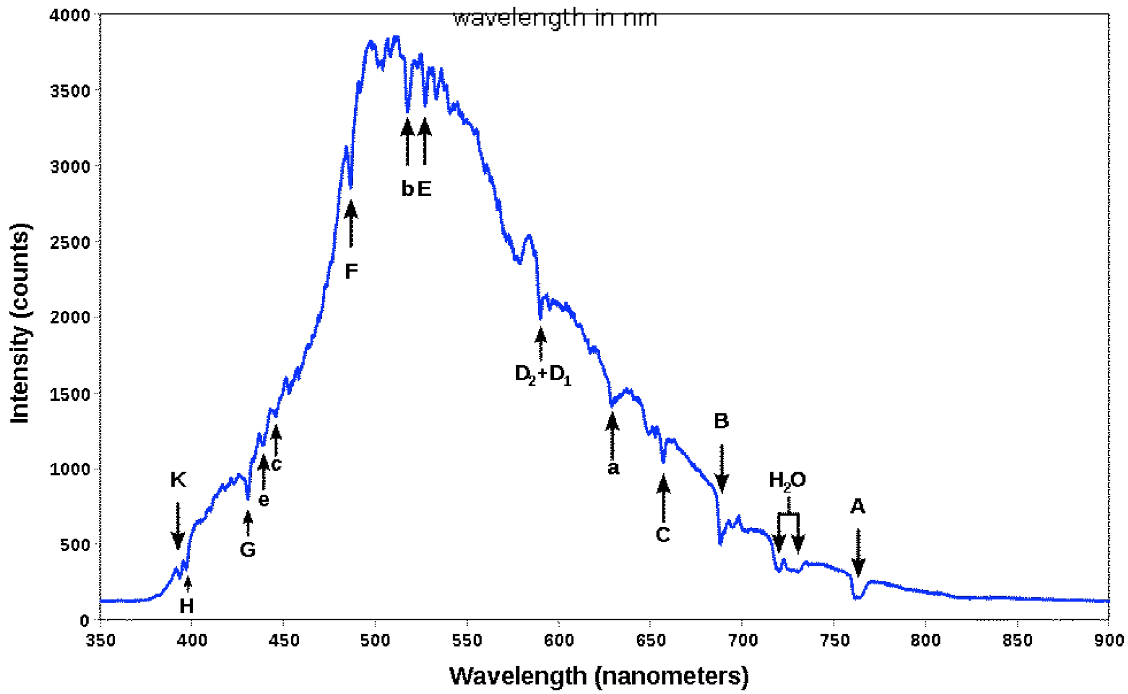
$$\Psi = p\pi$$

$$p = \pm 1, 2, 3, \ldots$$

$$\sin\theta = \frac{y}{\bar{r}} = \frac{p\lambda}{a}$$

Fraunhofer Approximation
Named in honor of Fraunhofer
Fraunhofer lines
Absorption lines in sunlight





Joseph von Faunhofer

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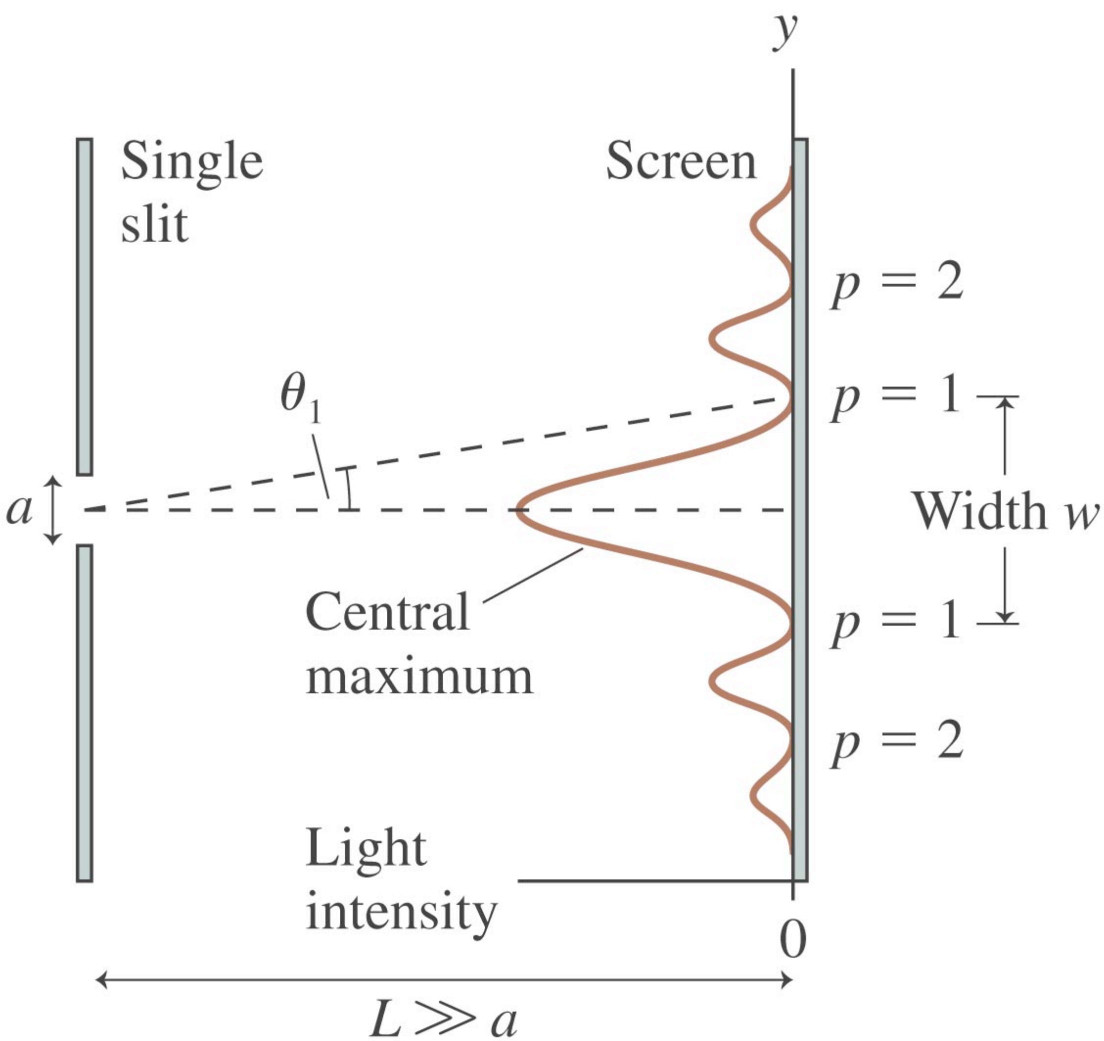
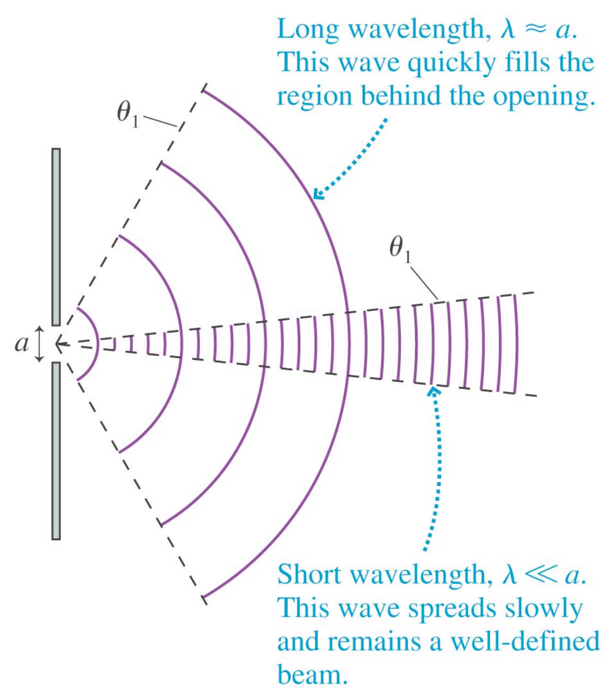
Width of Central Maximum

$$\frac{w}{r} = \frac{2\lambda}{a}$$

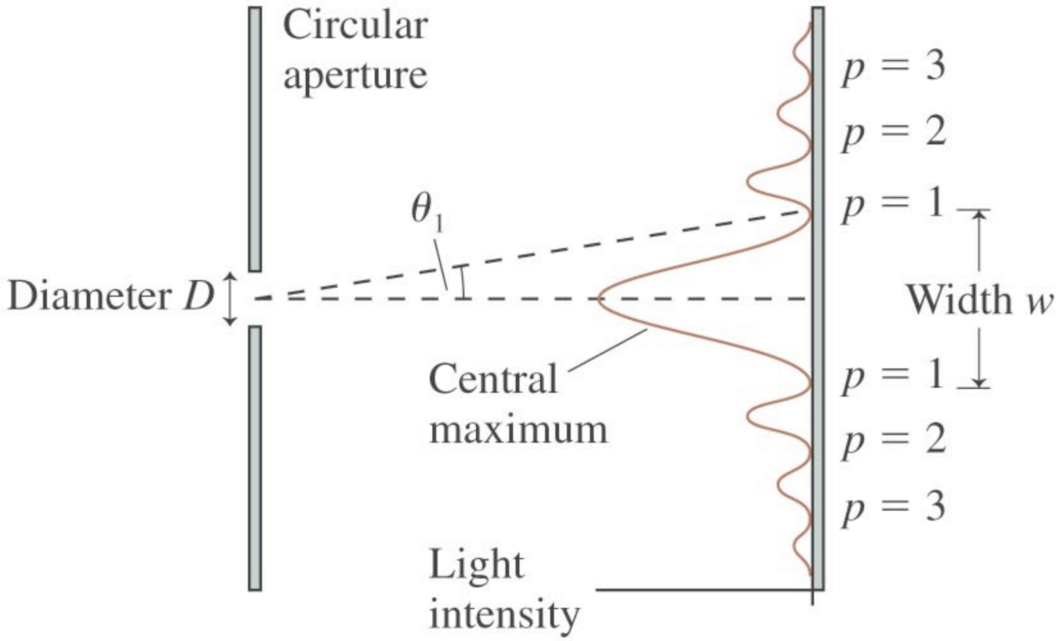


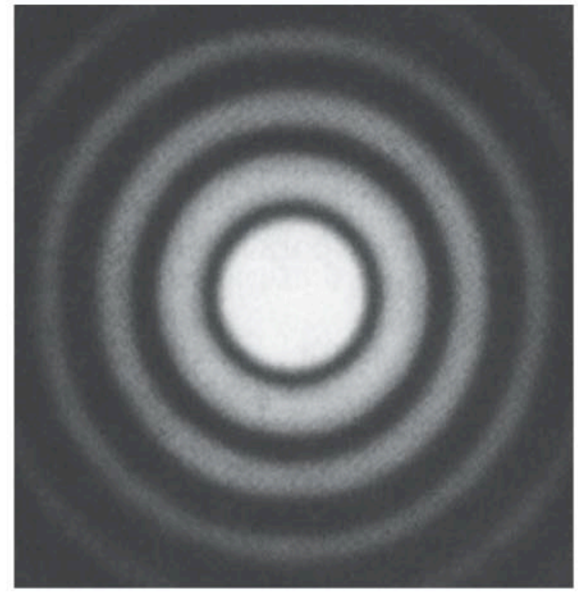
What increases w?

1. Increase distance from slit.
2. Increase wavelength
3. Decrease size of slit



Circular aperture diffraction

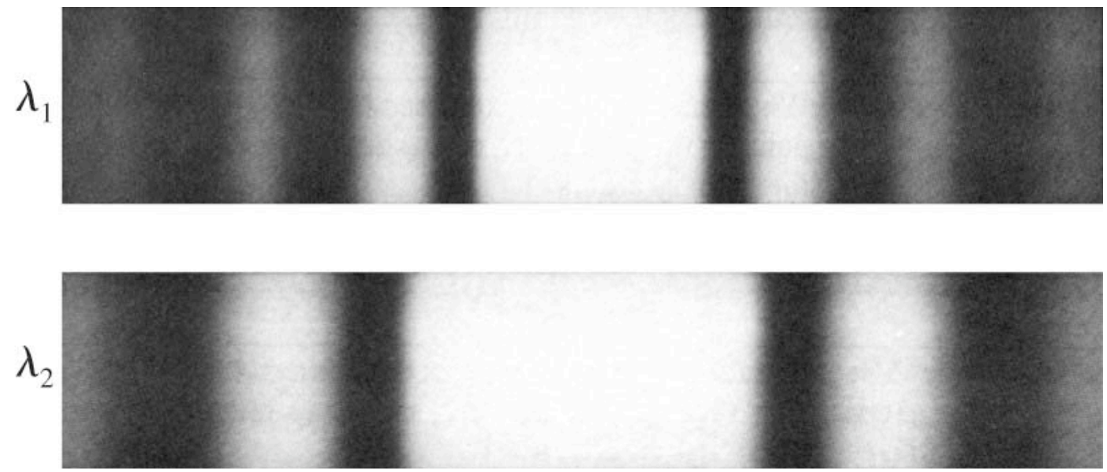




Width of central maximum

$$\frac{w}{L} = \frac{2.44\lambda}{D}$$

The figure shows two single-slit diffraction patterns. The distance between the slit and the viewing screen is the same in both cases. Which of the following could be true?



A. The wavelengths are the same for both; $a_1 > a_2$.

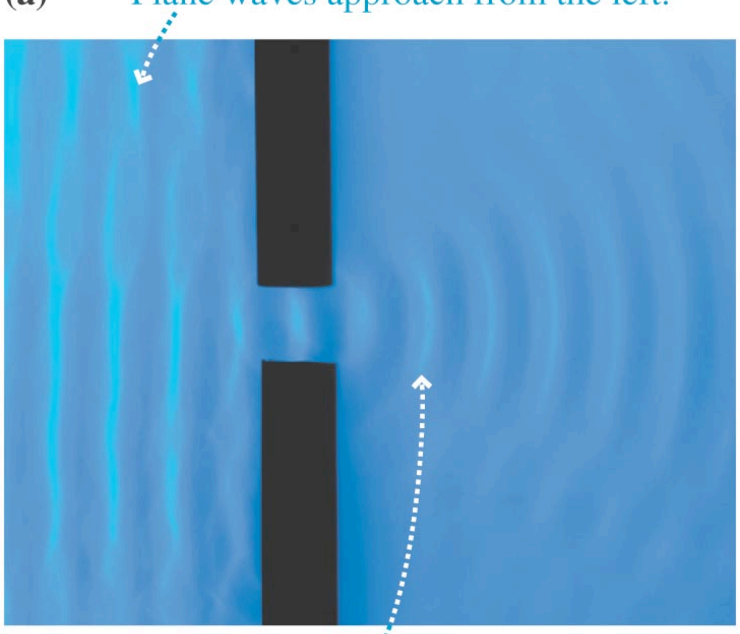
B. The wavelengths are the same for both; $a_2 > a_1$.

C. The slits and the wavelengths are the same for both; $p_1 > p_2$.

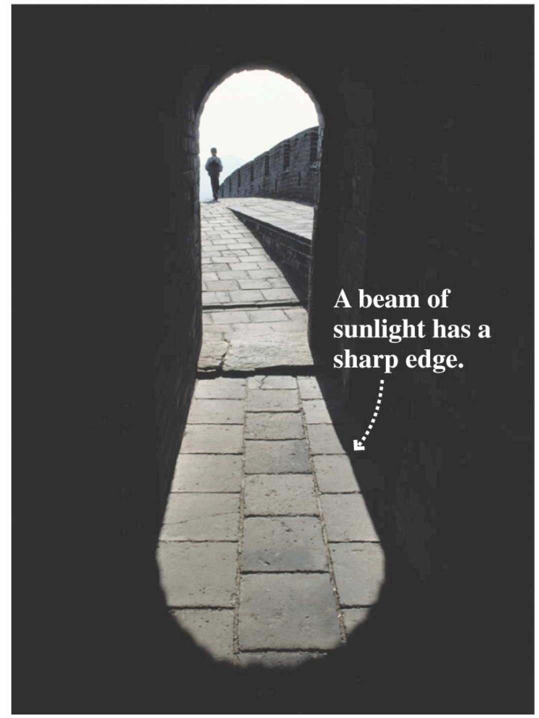
D. The slits and the wavelengths are the same for both; $p_2 > p_1$.

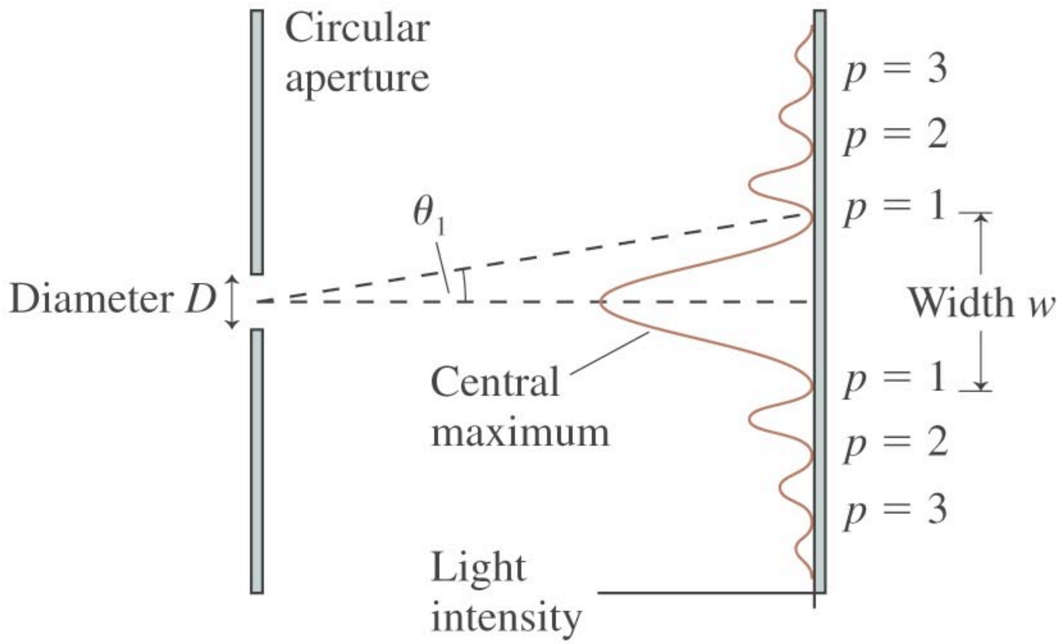
Wave Picture vs Ray Picture

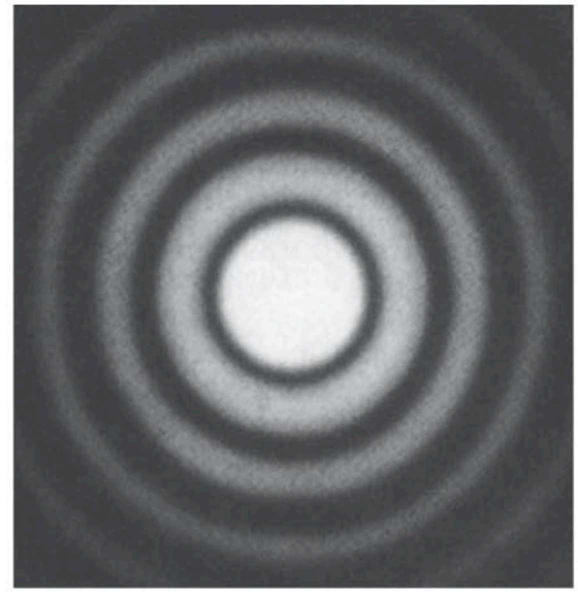
(a)



(b)







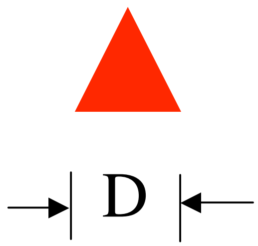
If D >> w, ray picture is OK
If D <= w, wave picture is needed

$$\frac{w}{L} = \frac{2.44\lambda}{D}$$

Critical size: $D_c = w$ ⇨ $D_c = \sqrt{2.44\lambda L}$

If product of wave length and distance to big, wave picture necessary.

Distant object

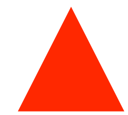


Example suppose object is on surface of sun

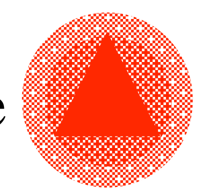
$$L = 1.5 \times 10^{11} m$$

$$\lambda = 500 nm = 5 \times 10^{-7} m$$

$$D_c = \sqrt{2.44 \lambda L} = 427 m$$

When will you see  ?

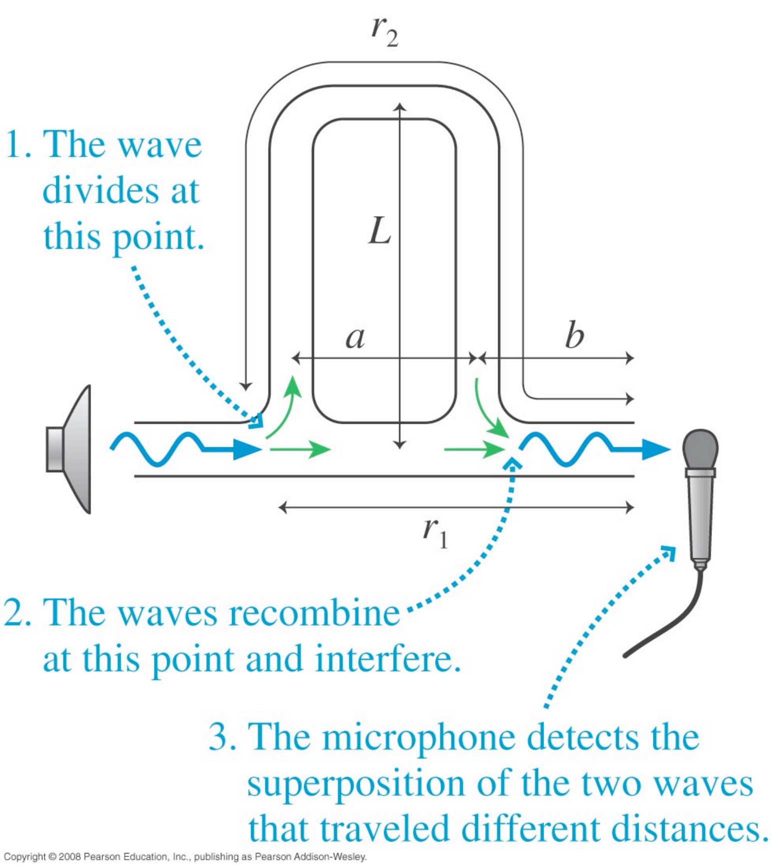
$$D > D_c = \sqrt{2.44 \lambda L}$$

When will you see  ?

Diffraction blurs image

$$D \le D_c = \sqrt{2.44 \lambda L}$$

Interferometer



If I vary L

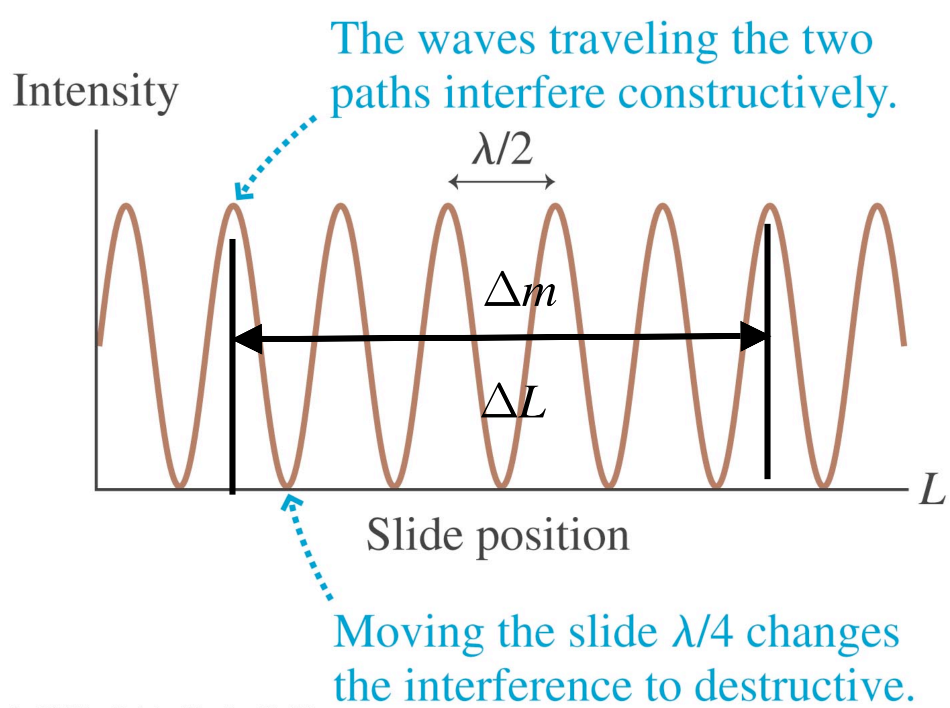
$$\Delta m = \frac{\Delta L}{\lambda / 2}$$

Sources will interfere <u>constructively</u> when

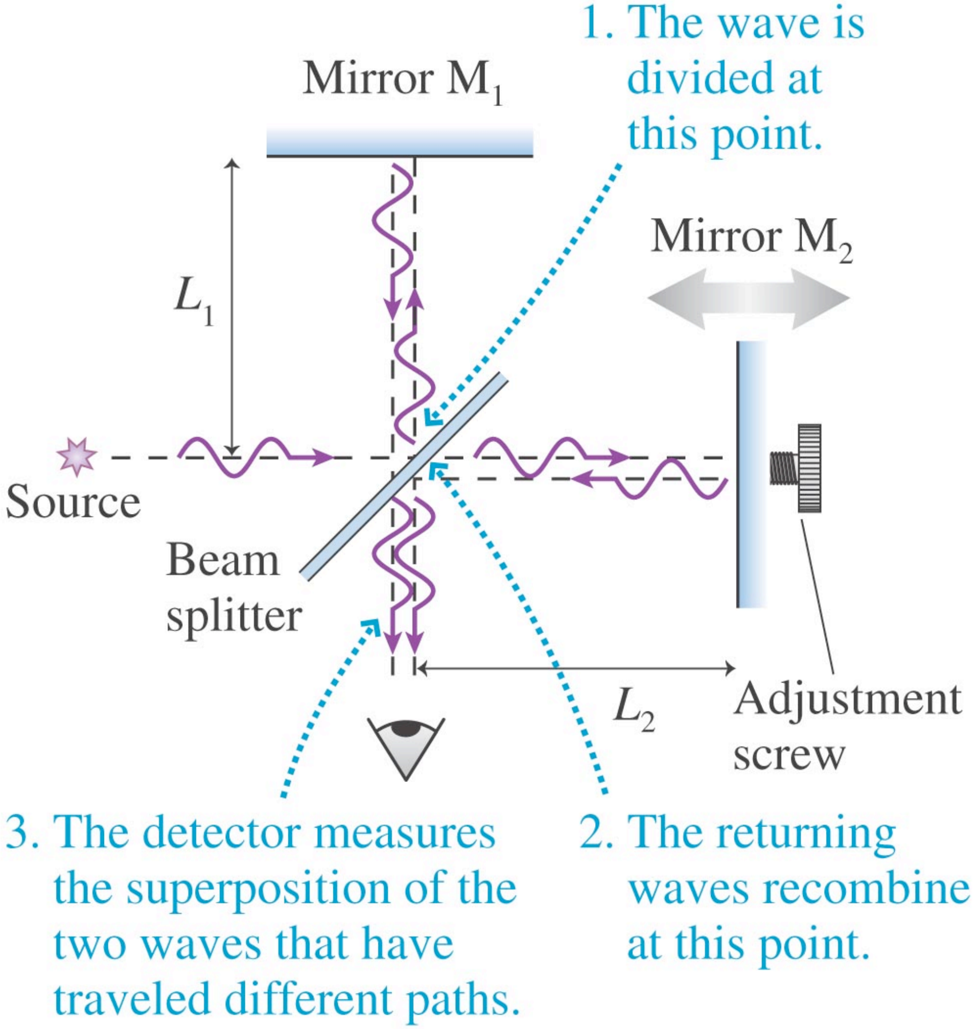
$$\Delta r = 2L = m\lambda \qquad m = 0, 1, 2, \ldots$$

Sources will interfere <u>destructively</u> when

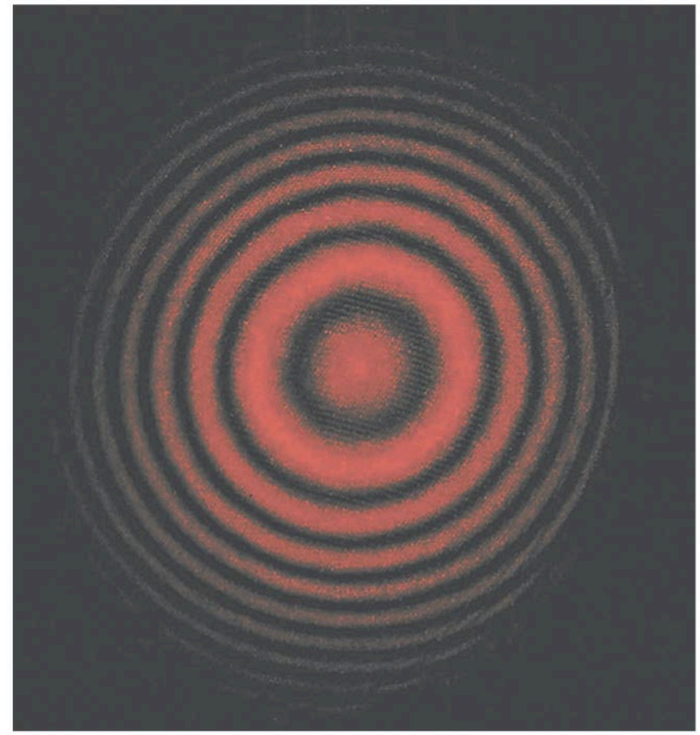
$$\Delta r = 2L = \left(m + \frac{1}{2}\right)\lambda$$



Michelson Interferometer



What is seen



As L2 is varied, central spot changes from dark to light, etc. Count changes = Δm

If I vary L_2 $$\Delta m = \frac{\Delta L_2}{\lambda / 2}$$

Using the interferometer Michelson and Morley showed that the speed of light is independent of the motion of the earth.

This implies that light is not supported by a medium, but propagates in vacuum.

Led to development of the special theory of relativity.

Albert Michelson
First US Nobel Science Prize Winner

Wikimedia Commons

Albert Michelson was the first US Nobel Science Prize Winner. The first US Nobel Prize winner was awarded the Peace Prize.

This American is known for saying:

A. Peace is at hand.
B. All we are saying, is give peace a chance.
C. There will be peace in the valley.
D. Speak softly, and carry a big stick.

A Michelson interferometer using light of wavelength λ has been adjusted to produce a bright spot at the center of the interference pattern. Mirror M_1 is then moved distance λ toward the beam splitter while M_2 is moved distance λ away from the beam splitter. How many bright-dark-bright fringe shifts are seen?

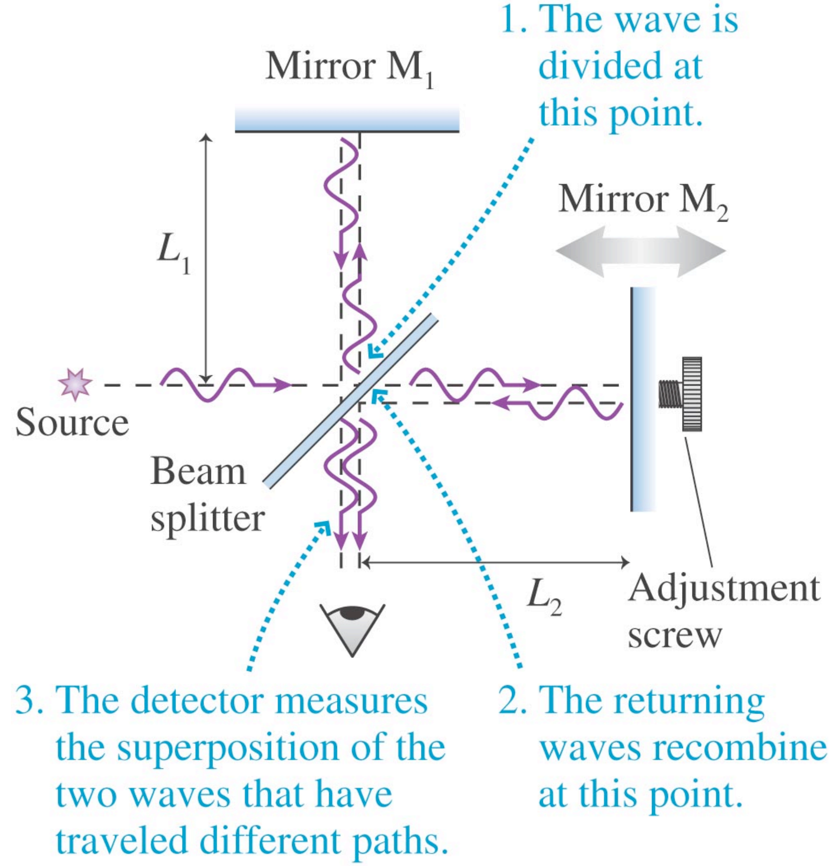
A. 4

B. 3

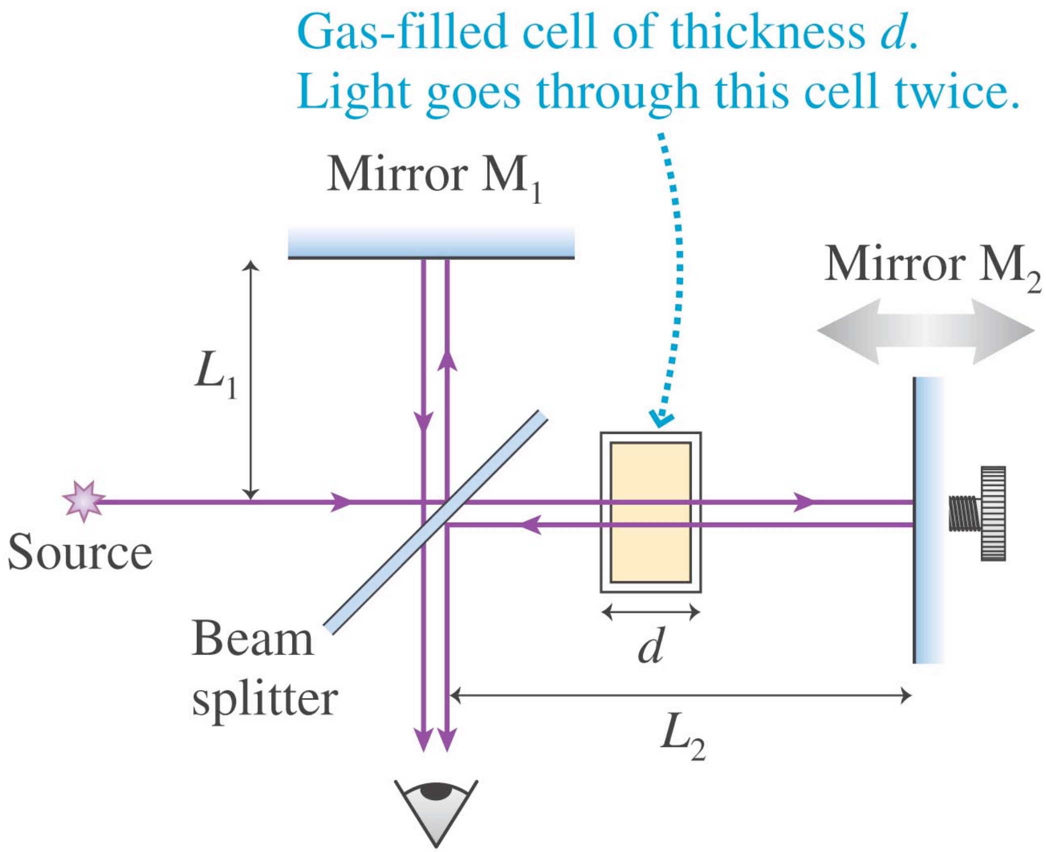
C. 2

D. 1

E. 0



Measuring Index of refraction



Number of wavelengths in cell when empty

$$m_1 = \frac{2d}{\lambda_{vac}}$$

Number of wavelengths in cell when full

$$m_2 = \frac{2d}{\lambda_{gas}} = \frac{2d}{\lambda_{vac} / n_{gas}}$$

Number of fringe shifts as cell fills up

$$\Delta m = m_2 - m_1 = \left(n_{gas} - 1\right)\frac{2d}{\lambda_{vac}}$$

EXAMPLE 22.9 Measuring the index of refraction

QUESTION:

EXAMPLE 22.9 Measuring the index of refraction

A Michelson interferometer uses a helium-neon laser with wavelength $\lambda_{vac} = 633$ nm. As a 4.00-cm-thick cell is slowly filled with a gas, 43 bright-dark-bright fringe shifts are seen and counted. What is the index of refraction of the gas at this wavelength?

EXAMPLE 22.9 Measuring the index of refraction

MODEL The gas increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift.

What do we know?

$$\Delta m = m_2 - m_1 = \left(n_{gas} - 1\right)\frac{2d}{\lambda_{vac}}$$

EXAMPLE 22.9 Measuring the index of refraction

SOLVE We can rearrange Equation 22.36 to find that the index of refraction is

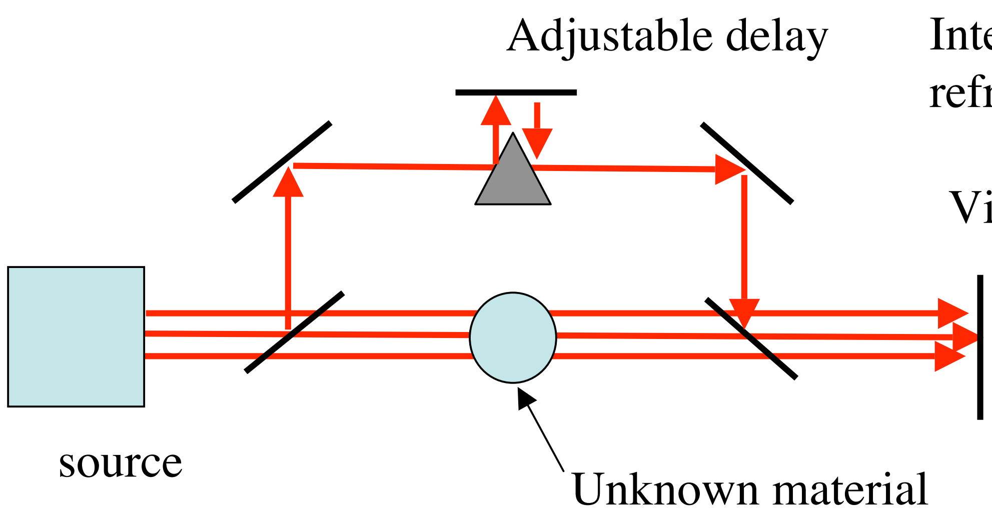
$$n = 1 + \frac{\lambda_{\text{vac}} \Delta m}{2d} = 1 + \frac{(6.33 \times 10^{-7}\ \text{m})(43)}{2(0.0400\ \text{m})} = 1.00034$$

$$\Delta m = m_2 - m_1 = \left(n_{gas} - 1\right)\frac{2d}{\lambda_{vac}}$$

EXAMPLE 22.9 Measuring the index of refraction

ASSESS This may seem like a six-significant-figure result, but it's really only two. What we're measuring is not n but $n - 1$. We know the fringe count to two significant figures, and that has allowed us to compute $n - 1 = \lambda_{vac}\Delta m/2d = 3.4 \times 10^{-4}$.

Mach-Zehnder Interferometer



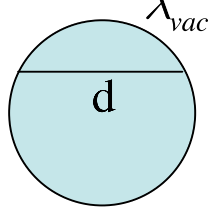
Interference depends on index of refraction of unknown

Viewing screen or camera

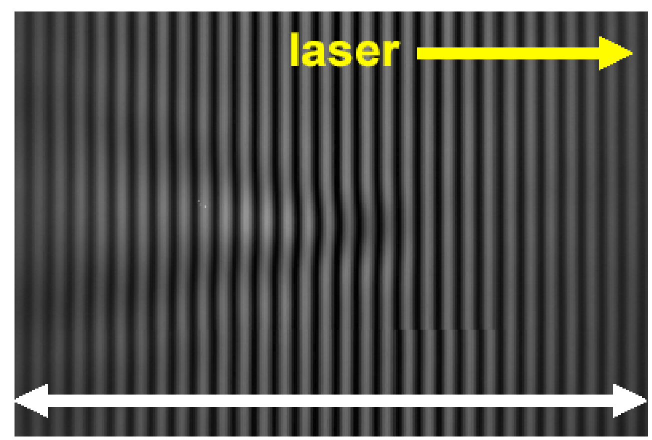
Imaging a profile of index change

displacement of interference fringes gives “line averaged” product

$$(n-1)\frac{2d}{\lambda_{vac}}$$



interferogram

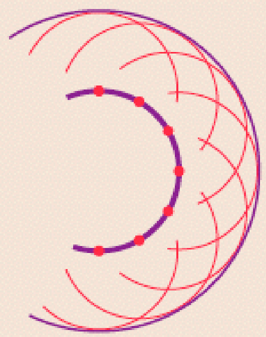


6.9 cm

Chapter 22. Summary Slides

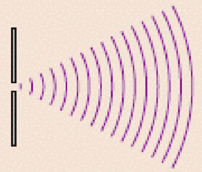
General Principles

Huygens' principle says that each point on a wave front is the source of a spherical wavelet. The wave front at a later time is tangent to all the wavelets.

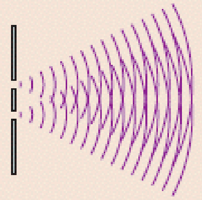


General Principles

Diffraction is the spreading of a wave after it passes through an opening.



Constructive and destructive interference are due to the overlap of two or more waves as they spread behind openings.



Important Concepts

The wave model of light considers light to be a wave propagating through space. Diffraction and interference are important.

The ray model of light considers light to travel in straight lines like little particles. Diffraction and interference are not important.

Diffraction is important when the width of the diffraction pattern of an aperture equals or exceeds the size of the aperture. For a circular aperture, the crossover between the ray and wave models occurs for an opening of diameter $D_c = \sqrt{2.44\lambda L}$.

In practice, $D_c \approx 1$ mm. Thus

- Use the wave model when light passes through openings <1 mm in size. Diffraction effects are usually important.
- Use the ray model when light passes through openings >1 mm in size. Diffraction is usually not important.

Applications

Single slit of width a.
A bright **central maximum** of width

$$w = \frac{2\lambda L}{a}$$

is flanked by weaker **secondary maxima.**
Dark fringes are located at angles such that

$$a \sin\theta_p = p\lambda \qquad p = 1, 2, 3, \ \ldots$$

If $\lambda/a \ll 1$, then from the small-angle approximation

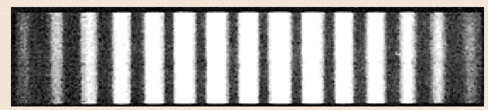
$$\theta_p = \frac{p\lambda}{a} \qquad y_p = \frac{p\lambda L}{a}$$

Applications

Interference due to wave-front division

Waves overlap as they spread out behind slits. Constructive interference occurs along antinodal lines. Bright fringes are seen where the antinodal lines intersect the viewing screen.

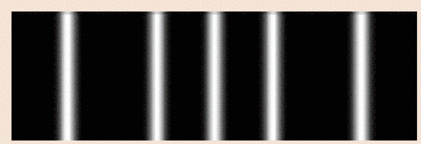
Double slit with separation d.
Equally spaced bright fringes are located at



$$\theta_m = \frac{m\lambda}{d} \qquad y_m = \frac{m\lambda L}{d} \qquad m = 0, 1, 2, \ldots$$

The **fringe spacing** is $\Delta y = \dfrac{\lambda L}{d}$

Diffraction grating with slit spacing d.
Very bright and narrow fringes are located at angles and positions

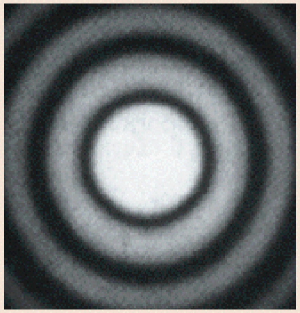


$$d \sin\theta_m = m\lambda \qquad y_m = L \tan\theta_m$$

Applications

Circular aperture of diameter D.
A bright central maximum of diameter

$$w = \frac{2.44\lambda L}{D}$$



is surrounded by circular secondary maxima.
The first dark fringe is located at

$$\theta_1 = \frac{1.22\lambda}{D} \qquad y_1 = \frac{1.22\lambda L}{D}$$

For an aperture of any shape, a smaller opening causes a more rapid spreading of the wave behind the opening.

Applications

Interference due to amplitude division

An interferometer divides a wave, lets the two waves travel different paths, then recombines them. Interference is constructive if one wave travels an integer number of wavelengths more or less than the other wave. The difference can be due to an actual path-length difference or to a different index of refraction.

Michelson interferometer

The number of bright-dark-bright fringe shifts as mirror M_2 moves distance ΔL_2 is

$$\Delta m = \frac{\Delta L_2}{\lambda/2}$$