Chapter 35

Electromagnetic Fields and Waves

Galilean Relativity   Why do E and B depend on the observer?
Maxwell's displacement current (it isn't a real current)
Electromagnetic Waves

Chapter 35. Electromagnetic Fields and Waves

Topics:

- *E* or *B*? It Depends on Your Perspective
- The Field Laws Thus Far
- The Displacement Current
- Maxwell's Equations
- Electromagnetic Waves
- Properties of Electromagnetic Waves
- Polarization

Preview of what is coming

General Principles

Maxwell's Equations

These equations govern electromagnetic fields:

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$	Gauss's law
$\oint \vec{B} \cdot d\vec{A} = 0$	Gauss's law for magnetism
$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$	Faraday's law
$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} + \boxed{\epsilon_0 \mu_0 \frac{d\Phi_e}{dt}}$	Ampère-Maxwell law

Maxwell's equations tell us that:

An electric field can be created by

- Charged particles
- A changing magnetic field

A magnetic field can be created by

- A current
- A changing electric field

New!
Gives rise to Electromagnetic waves.

Lorentz Force

This force law governs the interaction of charged particles with electromagnetic fields:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

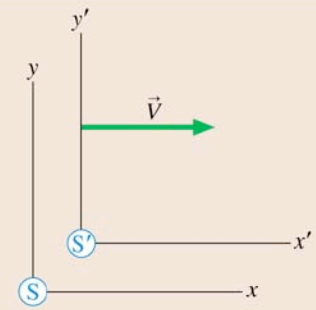
- An electric field exerts a force on any charged particle.
- A magnetic field exerts a force on a moving charged particle.

Field Transformations

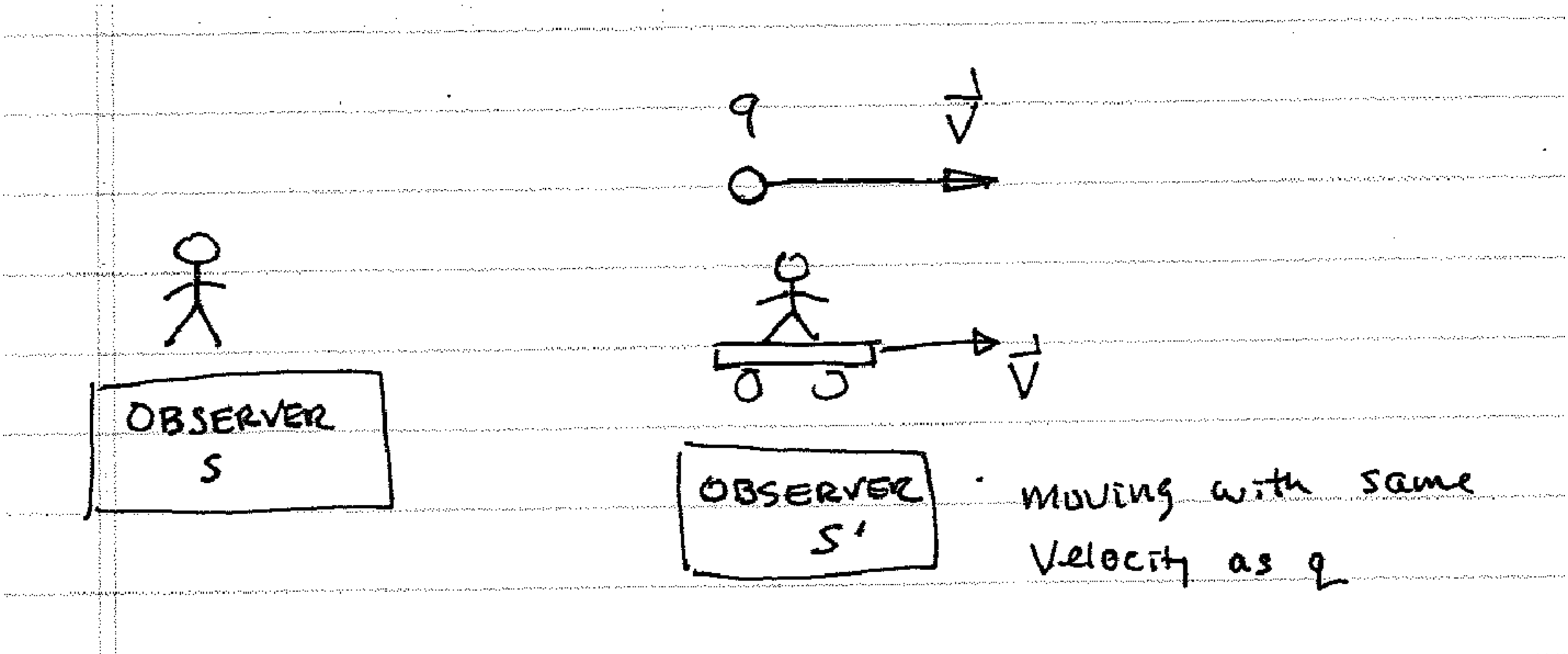
Fields measured in frame S to be $\vec{E}$ and $\vec{B}$ are found in frame S′ to be

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E}$$



Moving observers do not agree on the values of the Electric and magnetic fields.



Observer S says: $\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ q makes E and B

Observer S' says: $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ v=0 for him, q makes E

How can both be right?

Option A: There is a preferred reference frame (for example S). The laws only apply in the preferred frame. But, which frame?

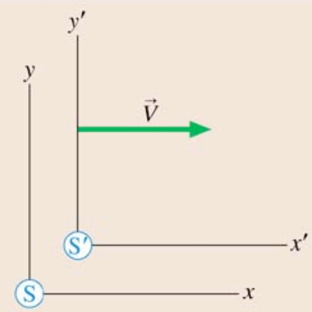
Option B: No frame is preferred. The Laws apply in all frames. The electric and magnetic fields have different values for different observers.

Field Transformations

Fields measured in frame S to be $\vec{E}$ and $\vec{B}$ are found in frame S′ to be

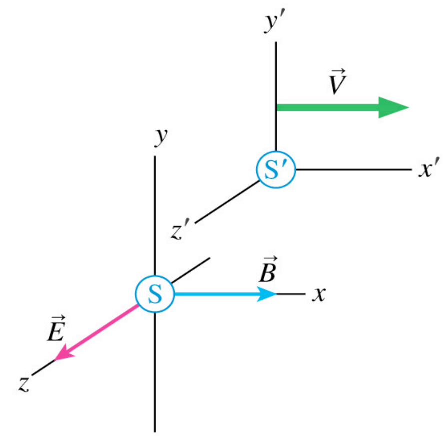
$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

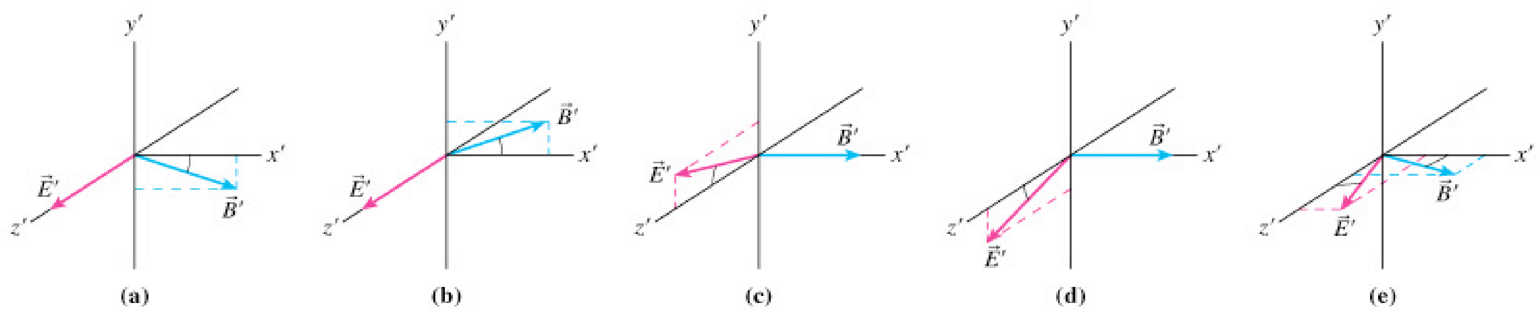
$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E}$$



Extended Option B: No frame is preferred. The Laws apply in all frames. All observers agree that light travels with speed c. Einstein's postulates →Special Relativity

Which diagram shows the fields in frame S´?





Field Transformations

Fields measured in frame S to be $\vec{E}$ and $\vec{B}$ are found in frame S′ to be

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

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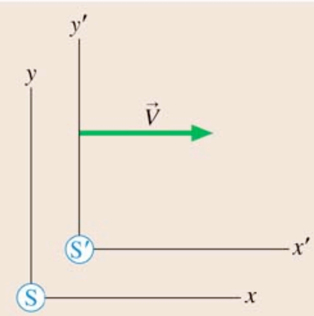


What we know about fields - so far

Integrals over closed surfaces

Gauss' Law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$

Gauss' Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

Integrals around closed loops

Faraday's Law:

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt}\Phi_{m-through}$$

Ampere's Law:

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

Integrals around closed loops

Faraday's Law:

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt}\mathbf{\Phi}_{m-through}$$

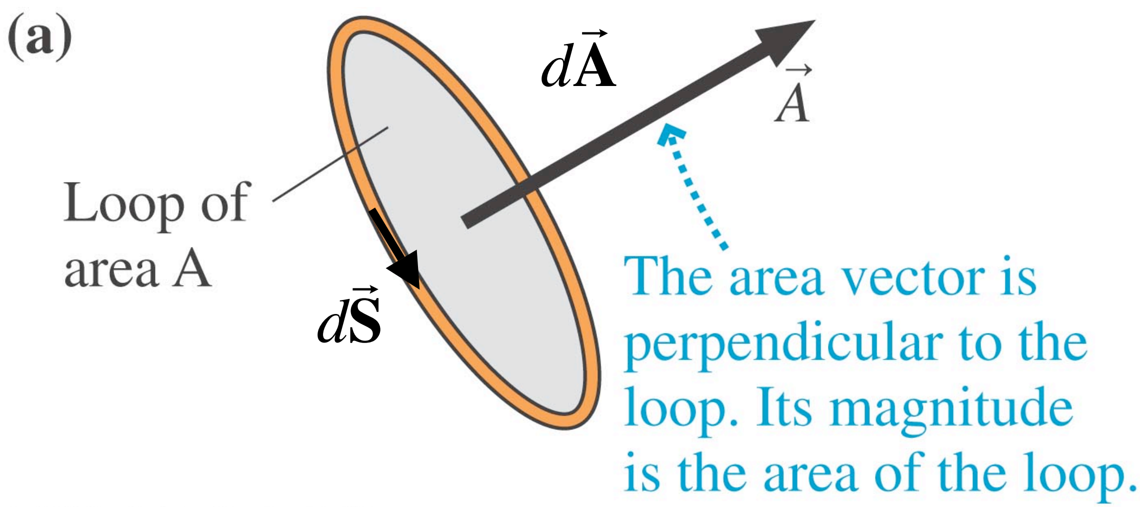
Ampere's Law:

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

Faraday's Law for Stationary Loops

$$EMF = \oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\int_{Area} \frac{\partial}{\partial t} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

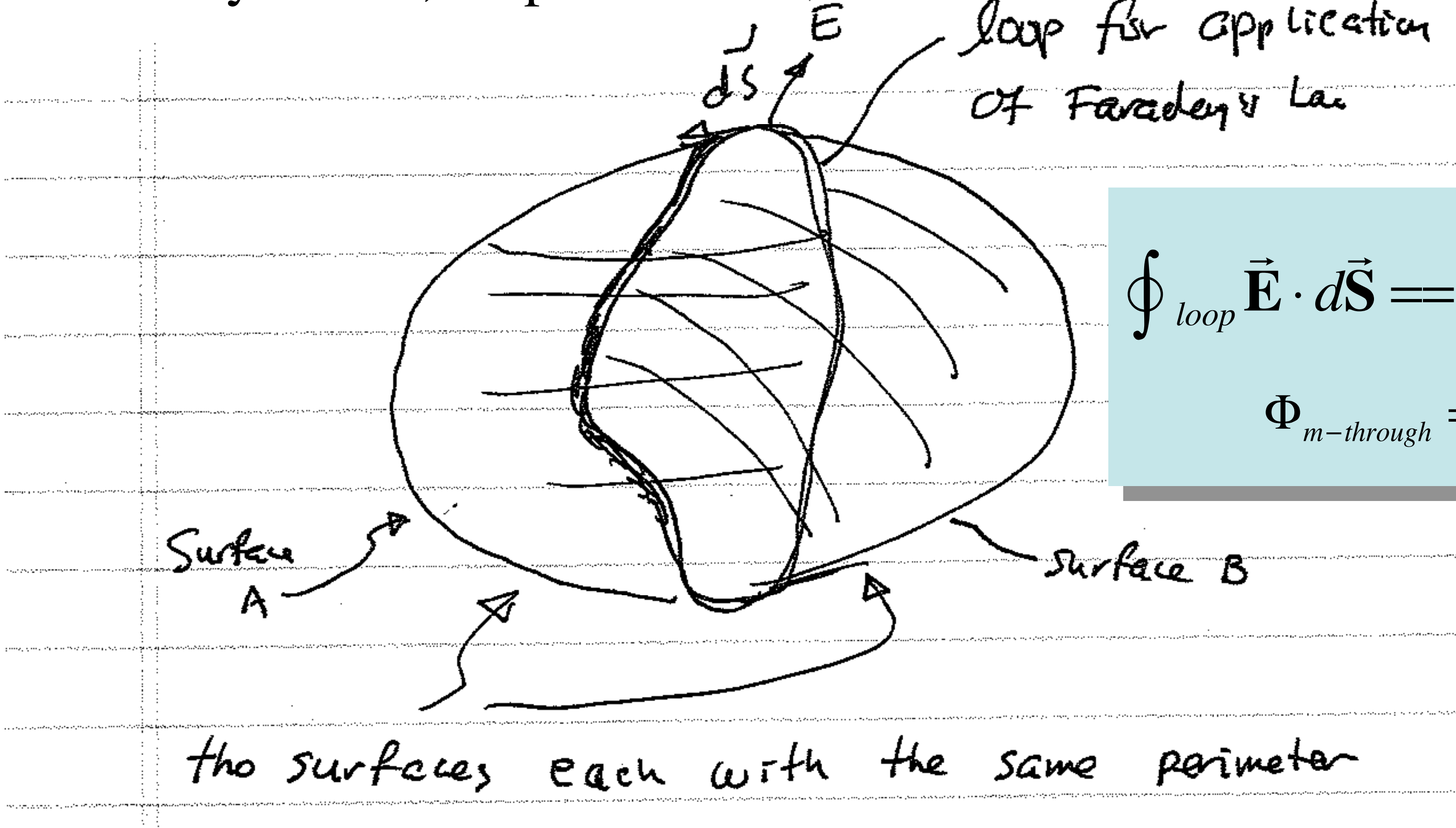
related by right hand rule



Which area should I pick to evaluate the flux?

A. The minimal area as shown
B. It doesn't matter

Faraday's Law, no problem



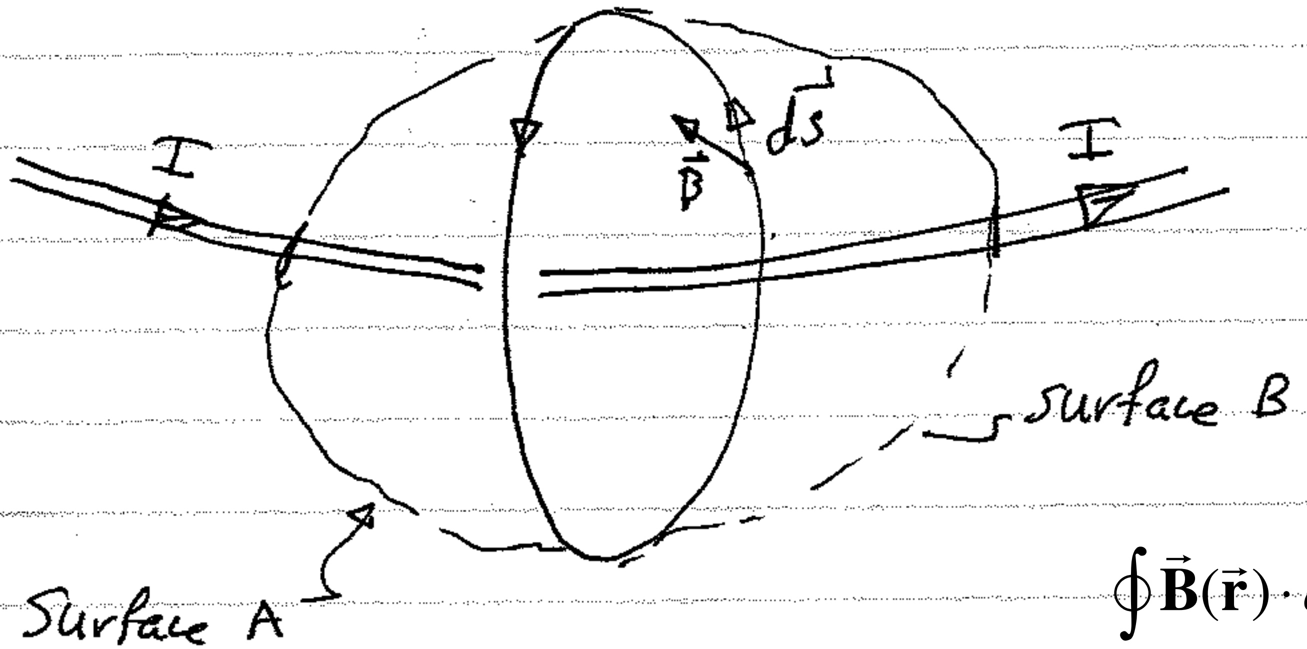
$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt}\Phi_{m-through}$$

$$\Phi_{m-through} = \int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Because for a closed surface $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

$$\Phi_{m-through} = \int_{Surface-A} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int_{Surface-B} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

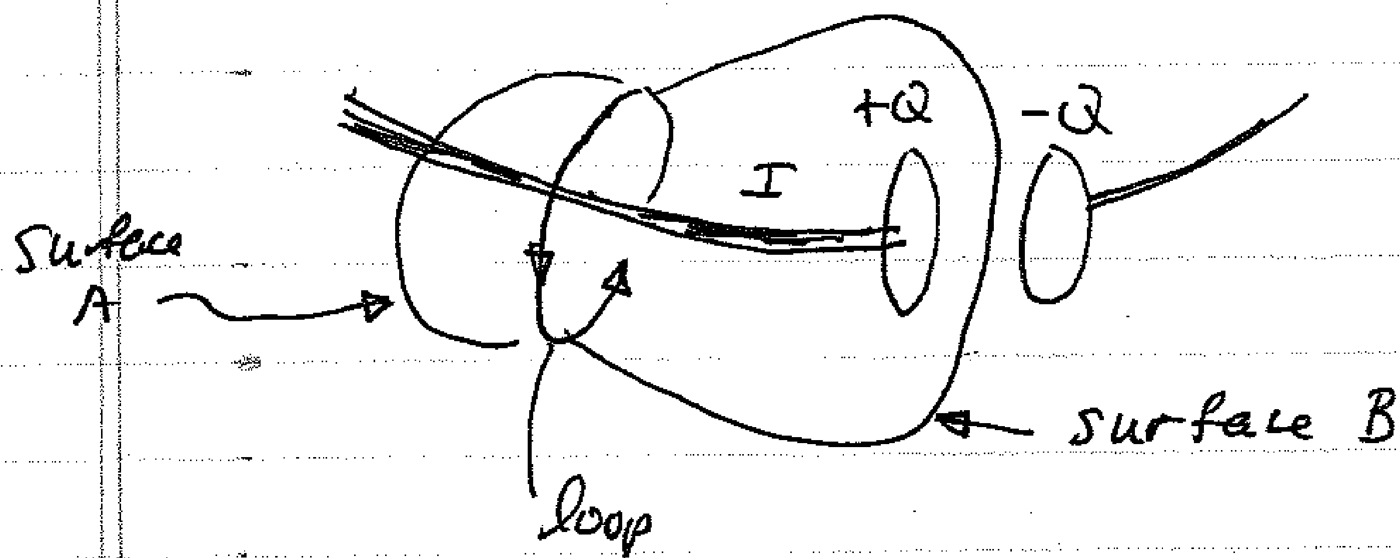
What About Ampere's Law



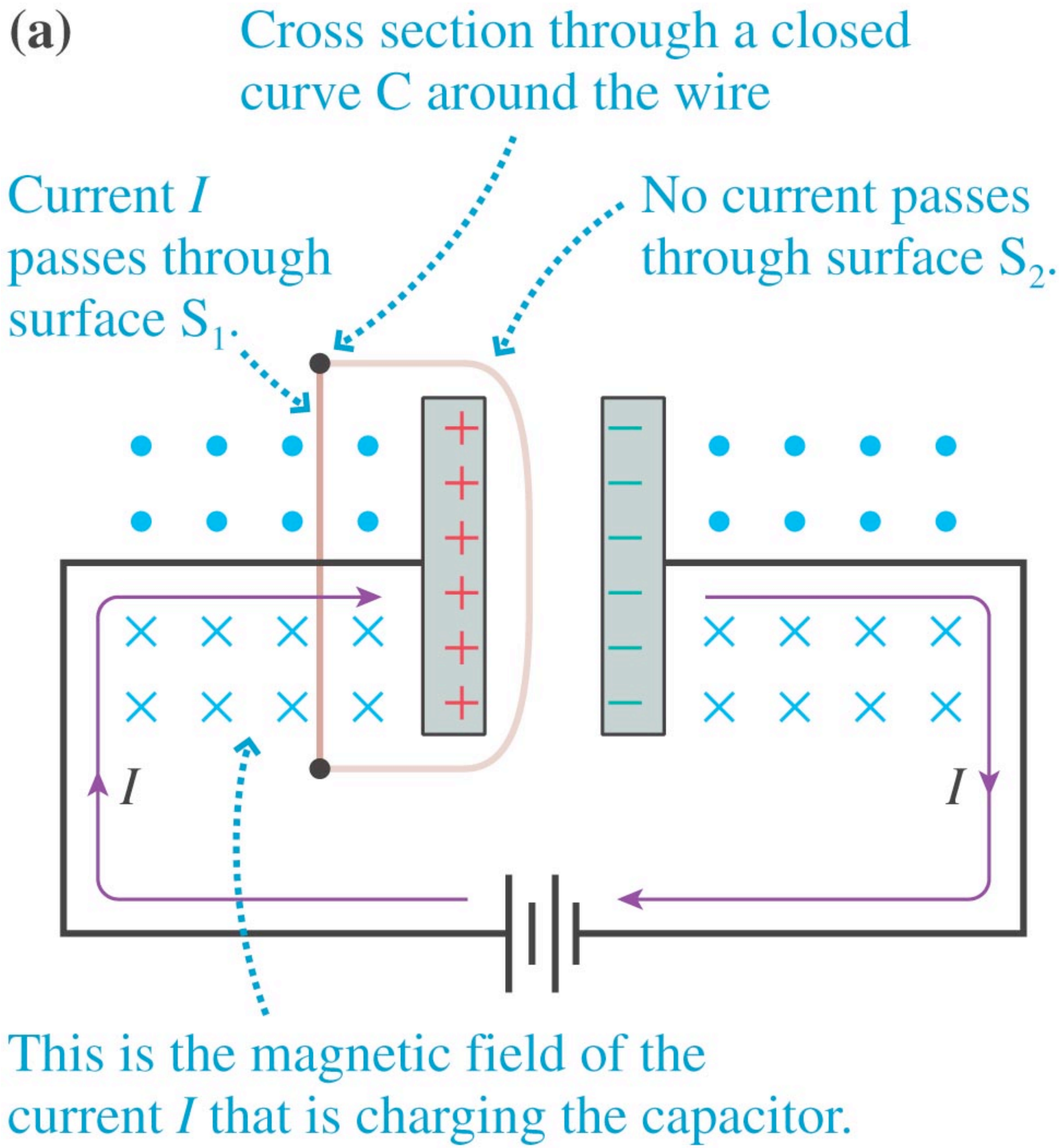
$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 I_{through}$$

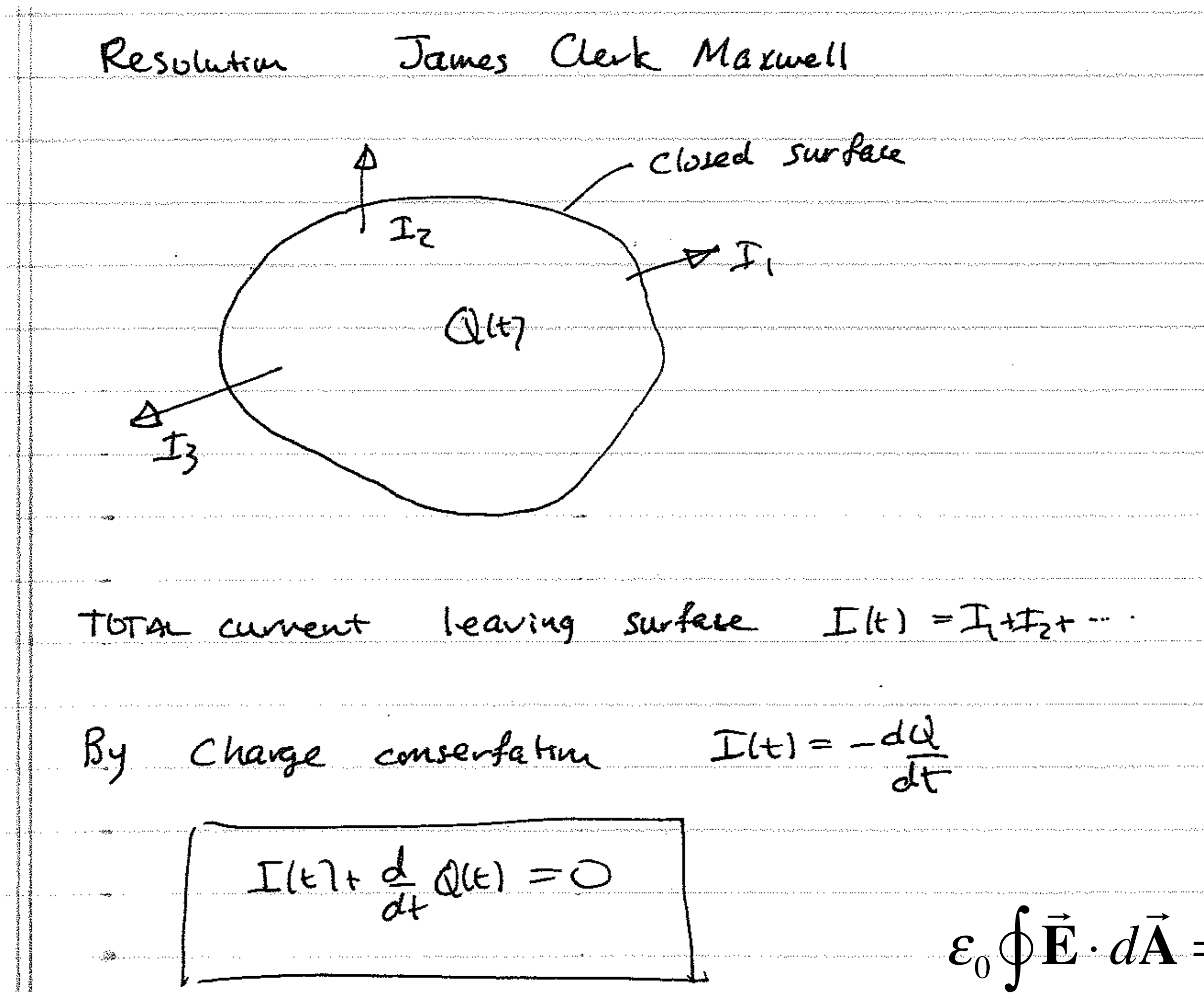
As long as net current leaving surface is zero, again no problem

However, consider this case



Current through surface B is zero

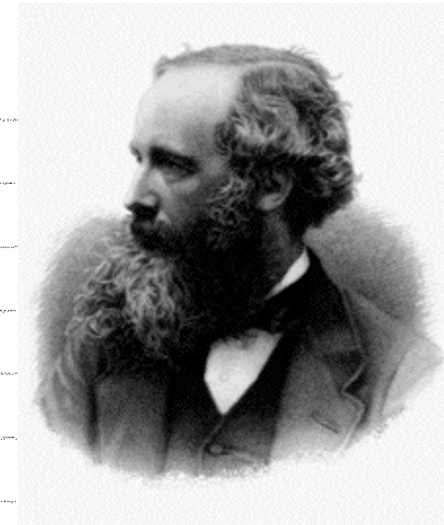

(a)
Cross section through a closed curve C around the wire
Current I passes through surface S_1.
No current passes through surface S_2.
I
I
This is the magnetic field of the current I that is charging the capacitor.


Resolution James Clerk Maxwell

TOTAL current leaving surface $I(t) = I_1 + I_2 + \cdots$

By charge conservation $I(t) = -\dfrac{dQ}{dt}$

$$I(t) + \frac{d}{dt} Q(t) = 0$$


James Clerk Maxwell
(1831–1879)

en.wikipedia.org/wiki/James_Clerk_Maxwell

$$\varepsilon_0 \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q_{in}$$

Thus, for any closed surface $$I_{through} + \varepsilon_0 \frac{d}{dt} \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

Maxwell said to add this term

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{\text{through}} + I_{\text{disp}}) = \mu_0 \left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt} \right) \qquad (35.22)$$

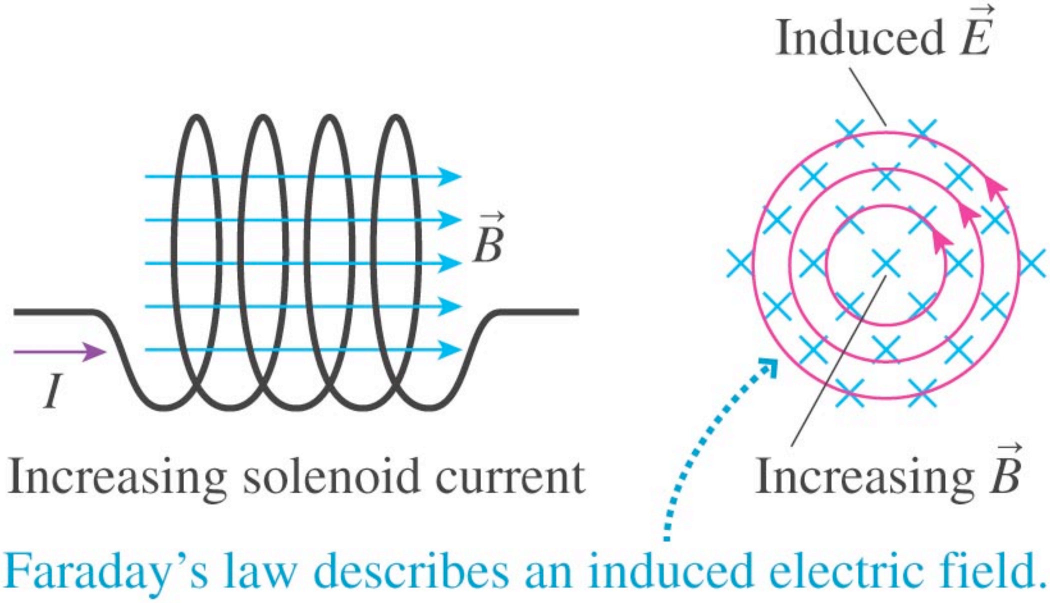
$$\Phi_e = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

The new term is called the displacement current.

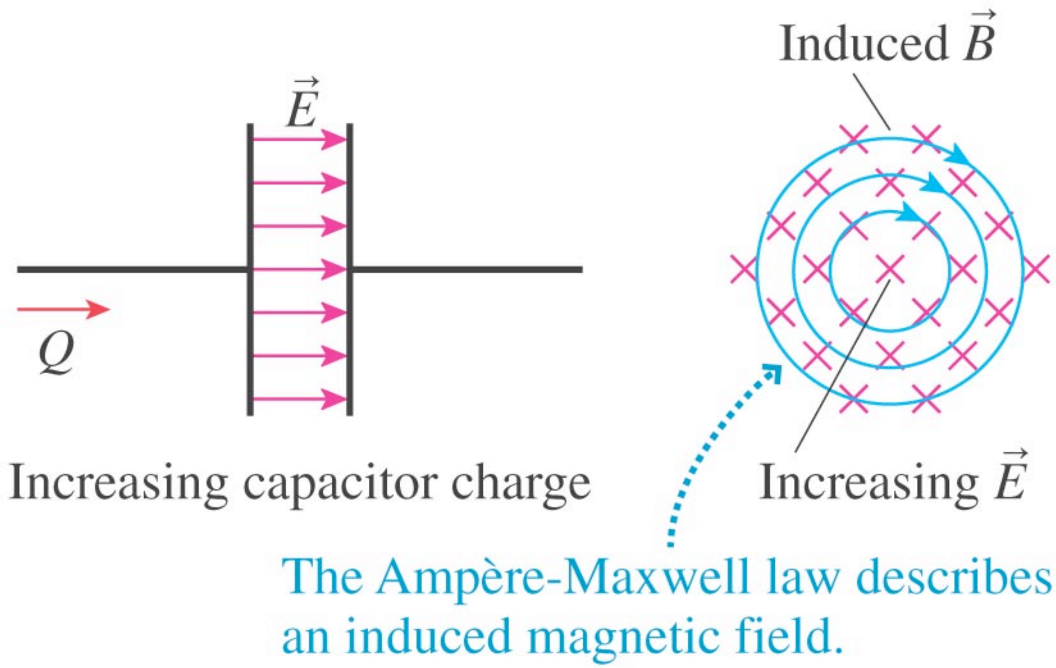
It is an unfortunate name, because it is not a current.

Maxwell is known for all of the following except:

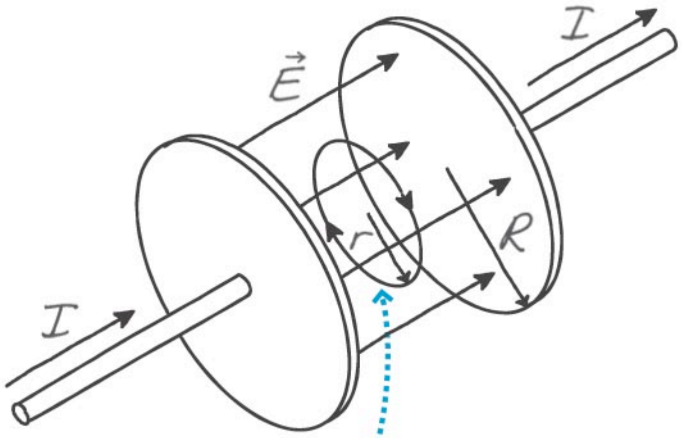
A. Proposing the displacement current and unifying electromagnetism and light.
B. Developing the statistical theory of gases.
C. Having a silver hammer with which he would bludgeon people.
D. Studying color perception and proposing the basis for color photography.



Induced $\vec{E}$
$\vec{B}$
I
Increasing solenoid current
Increasing $\vec{B}$
Faraday's law describes an induced electric field.



Induced $\vec{B}$
$\vec{E}$
Q
Increasing capacitor charge
Increasing $\vec{E}$
The Ampère-Maxwell law describes an induced magnetic field.



The magnetic field line is a circle concentric with the capacitor. The electric flux through this circle is $\pi r^2 E$.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = B_\theta(r) 2\pi r$$

$$\Phi_e = \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \pi r^2 E_z$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0 (I_{through} + \varepsilon_0 \frac{d\Phi_e}{dt})$$

$$B_\theta(r) = \frac{\mu_0 \varepsilon_0 r}{2} \frac{\partial E_z}{\partial t}$$

Recall from Faraday: $$E_\theta(r) = -\frac{r}{2} \frac{\partial B_z}{\partial t}$$

Faraday: time varying B makes an E

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt}\int_{\mathbf{S}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

example

$$E_\theta(r) = -\frac{r}{2}\frac{\partial B_z}{\partial t}$$

Ampere-Maxwell: time varying E makes a B

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0\varepsilon_0 \frac{d}{dt}\int_{S} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

example

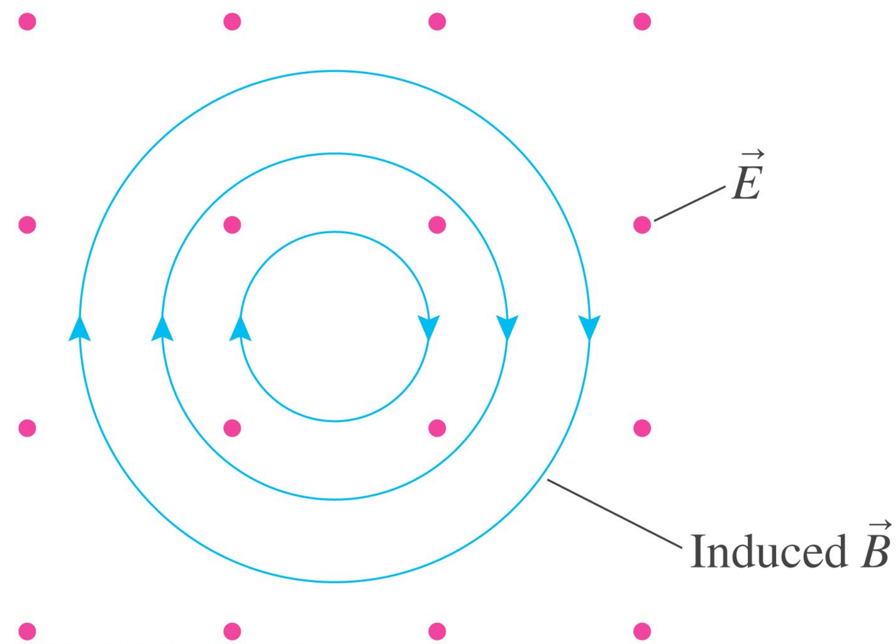
$$B_\theta(r) = \frac{\mu_0\varepsilon_0 r}{2}\frac{\partial E_z}{\partial t}$$

Put together, fields can sustain themselves - Electromagnetic Waves

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Based on the arrows, the electric field coming out of the page is

A. increasing
B. decreasing
C. not changing
D. undetermined



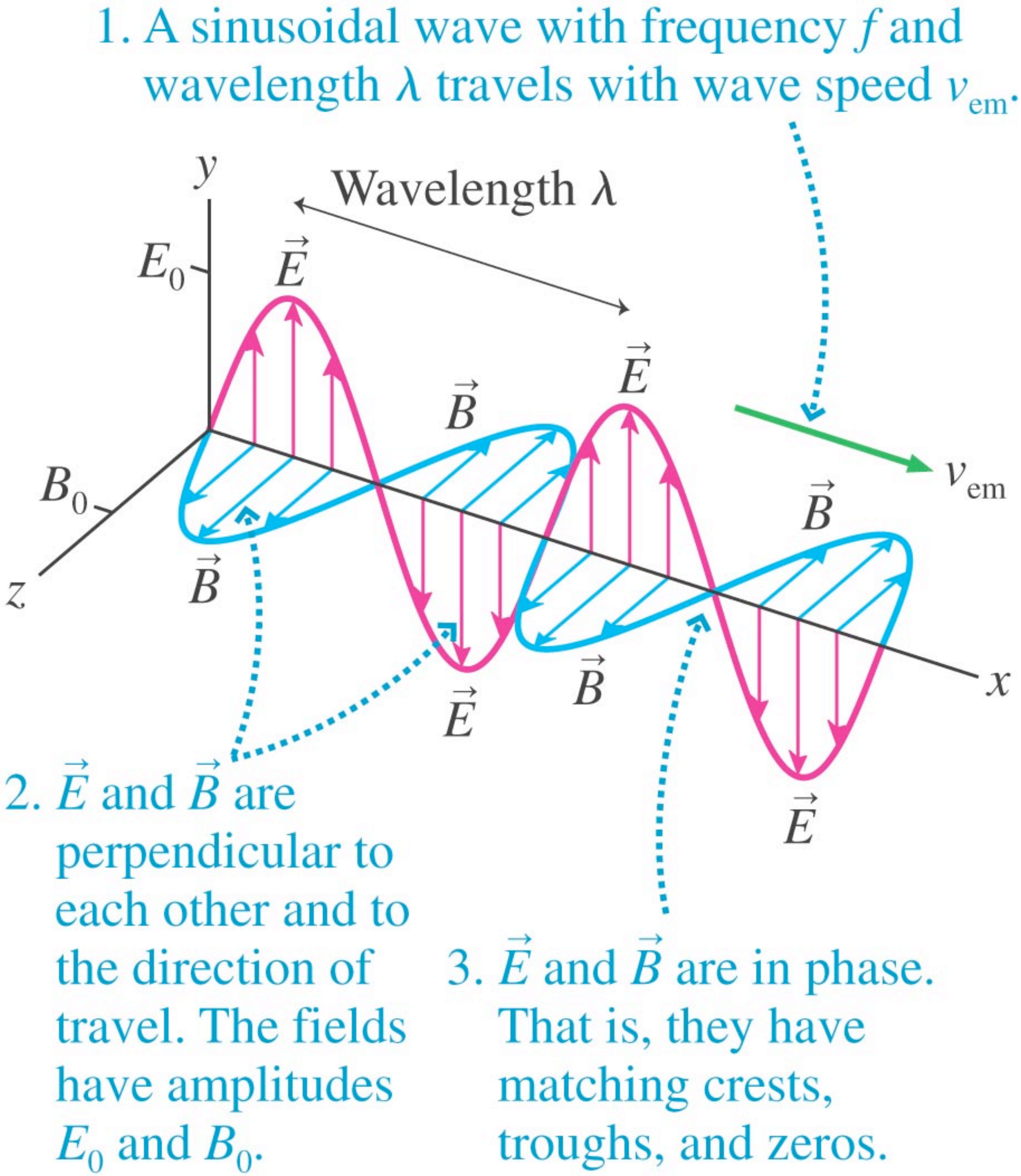
Properties of electromagnetic waves:

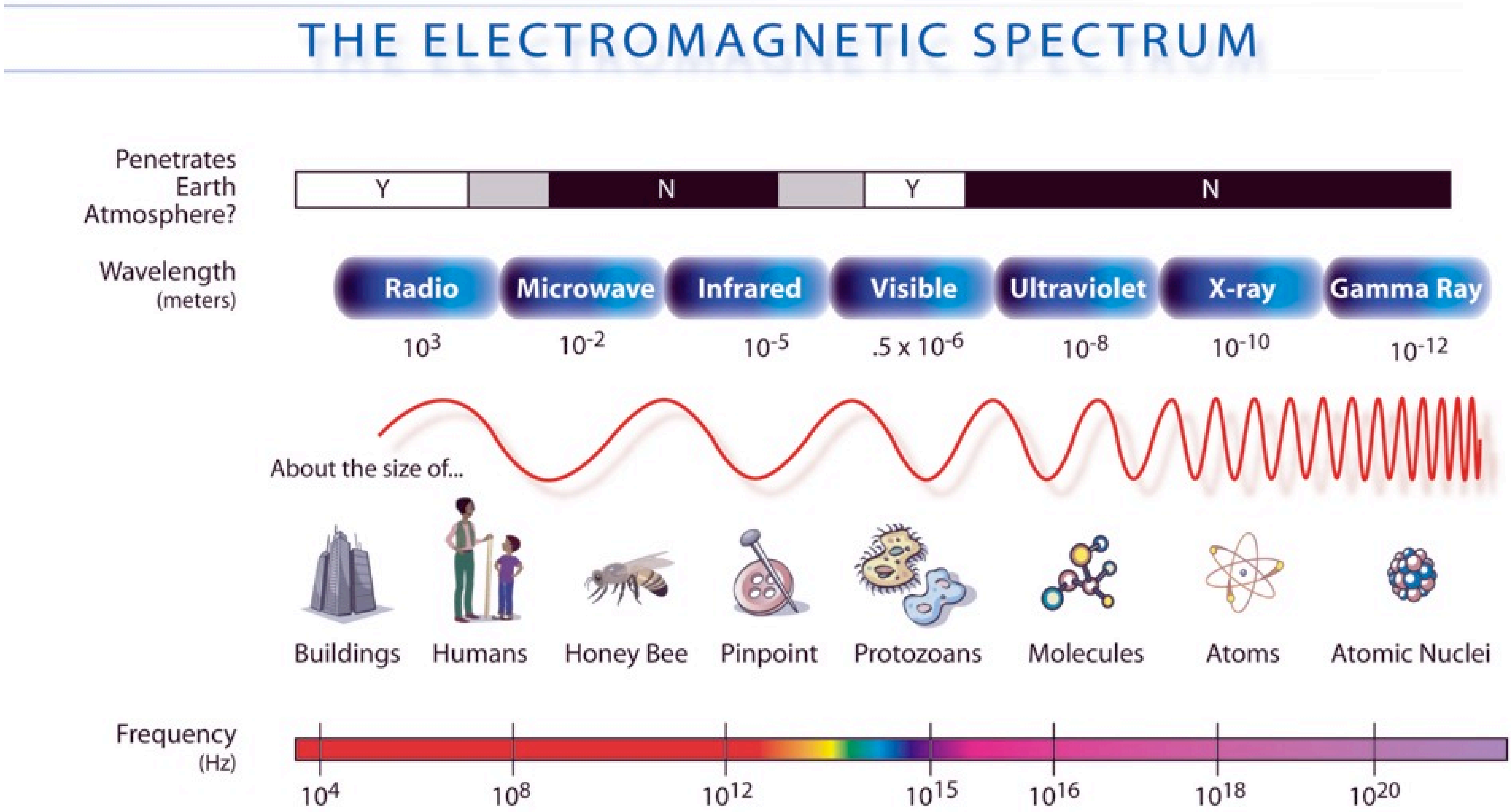
Waves propagate through vacuum (no medium is required like sound waves)

All frequencies have the same propagation speed, c.

Electric and magnetic fields are oriented transverse to the direction of propagation. (transverse waves)

Waves carry both energy and momentum.


1. A sinusoidal wave with frequency f and wavelength λ travels with wave speed v_{em}.
Wavelength λ
y
E_0
$\vec{E}$
$\vec{B}$
B_0
z
x
v_{em}
2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.
3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.



mynasadata.larc.nasa.gov/ ElectroMag.html

35.5 - Electromagnetic Waves

1. Assume that no currents or charges exist nearby, Q=0 I=0
2. Try a solution where:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_y(x,t)\mathbf{j}$$ Only y-component, depends only on x,t

$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_z(x,t)\mathbf{k}$$ Only z-component, depends only on x,t

3. Show that this satisfies Maxwell Equations provided the following is true:

$$-\frac{\partial B_z(x,t)}{\partial t} = \frac{\partial E_y(x,t)}{\partial x}$$ Faraday

$$-\frac{\partial B_z(x,t)}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y(x,t)}{\partial t}$$ Ampere-Maxwell

4. Show that the solution of these are waves with speed c.

Maxwell's Equations in Vacuum
$Q_{in} = 0$, $I_{through} = 0$

Integrals over closed surfaces

Gauss' Law: $$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

Gauss' Law: $$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Integrals around closed loops

Faraday's Law:

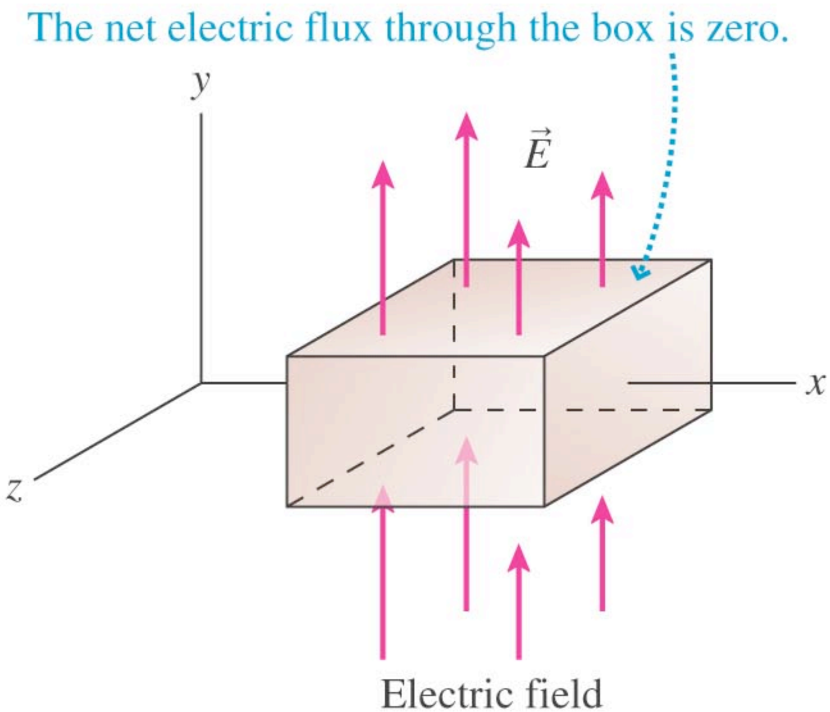
$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\Phi_{m-through}$$

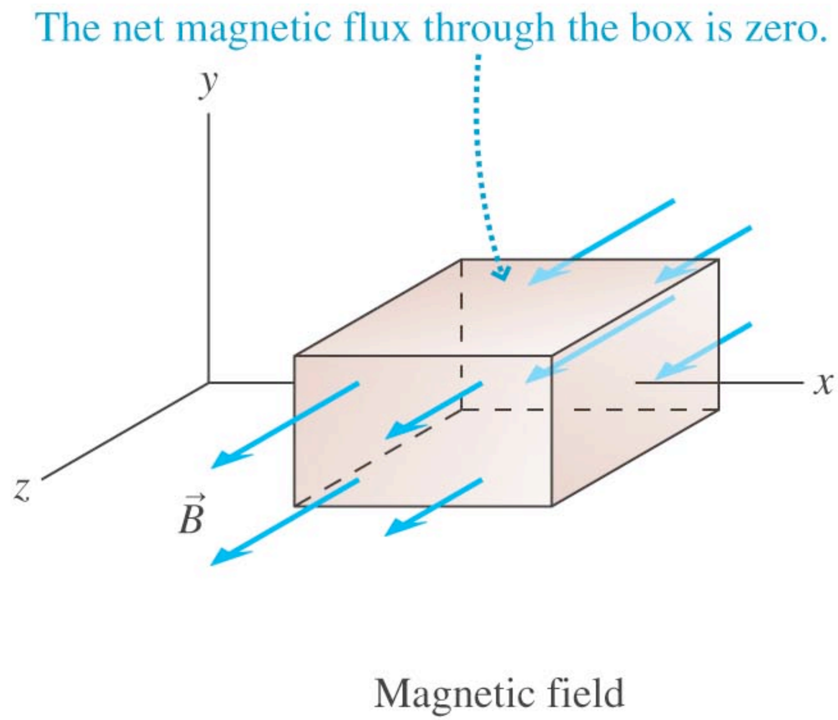
Ampere's Law:

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt}\Phi_{e-through}$$

Do our proposed solutions satisfy the two Gauss' Law Maxwell Equations?

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_y(x,t)\mathbf{j} \qquad \vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_z(x,t)\mathbf{k}$$



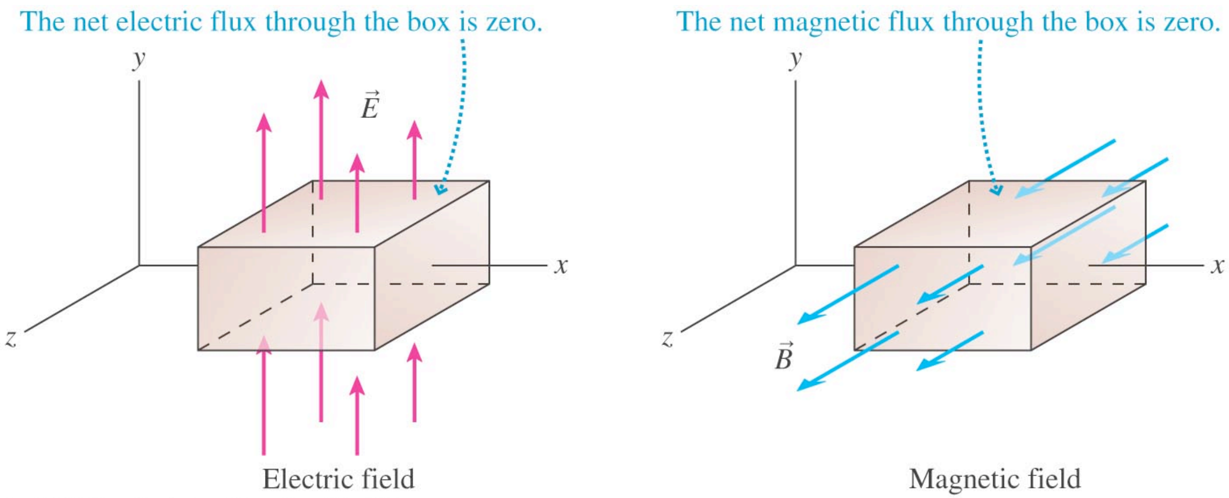


$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0 \qquad \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Ans: Yes, net flux entering any volume is zero

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_y(x,t)\mathbf{j}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_z(x,t)\mathbf{k}$$



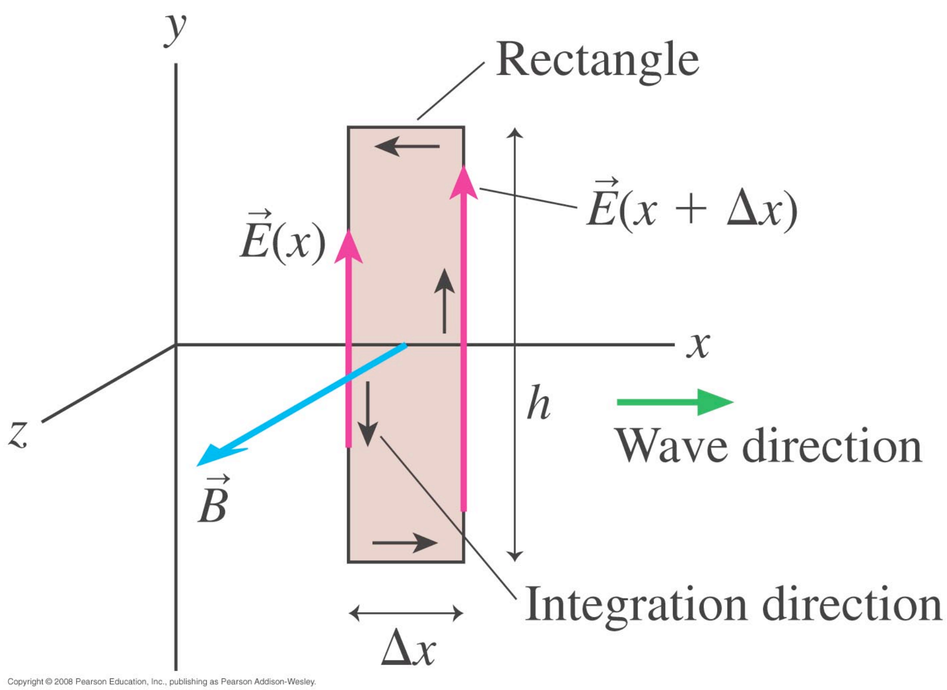
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

True or False? We showed that GL is satisfied for the surfaces of rectangular volumes. But it would not be satisfied for other shapes.

A. True
B. False

What about the two loop integral equations?



Faraday's Law

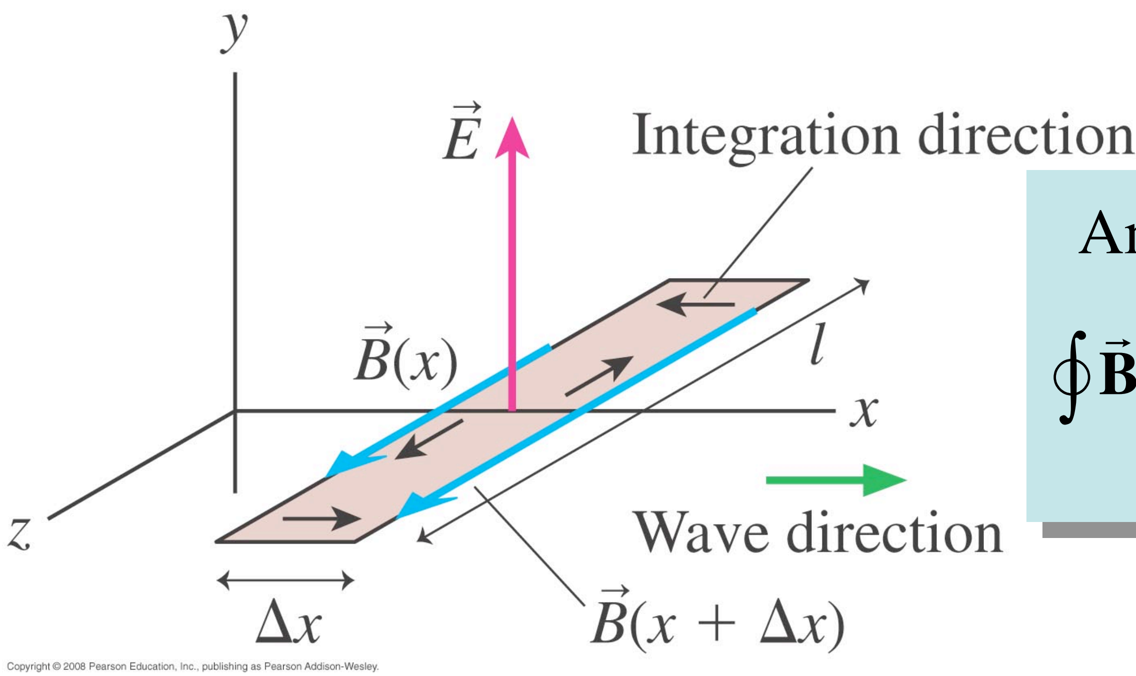
$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d}{dt} \int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Apply Faraday's law to loop

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = h\left[E_y(x+\Delta x) - E_y(x)\right] \simeq h\Delta x \frac{\partial E_y(x)}{\partial x}$$

$$\int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = h\Delta x B_z(x,t)$$

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$



Ampere-Maxwell Law:

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int_{Surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Apply Ampere-Maxwell law to loop:

$$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = h\left[-B_z(x+\Delta x) + B_z(x)\right] \simeq -h\Delta x \frac{\partial B_z(x)}{\partial x}$$

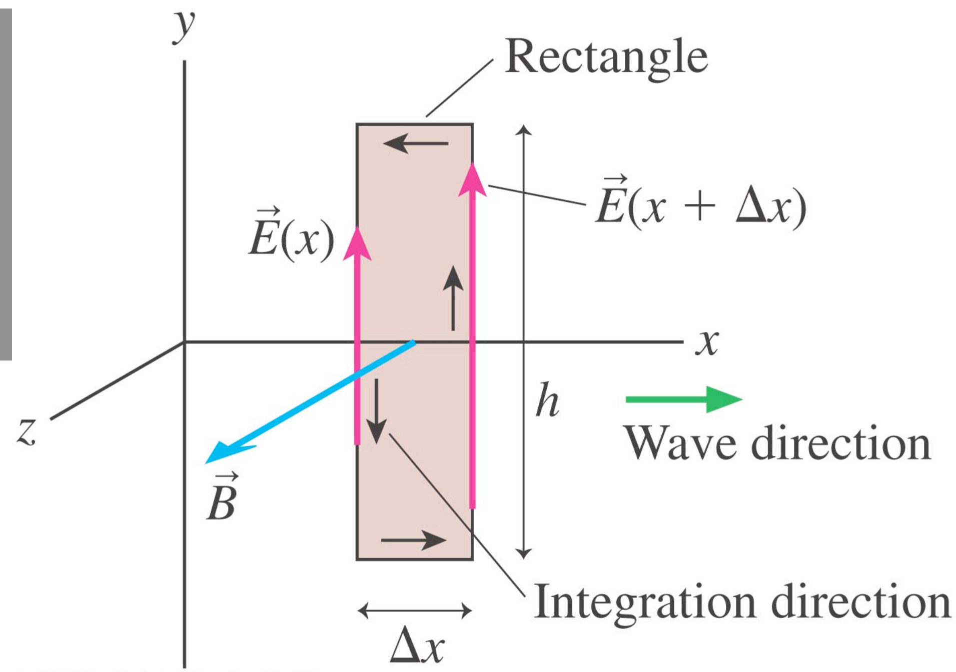
$$\int_{Surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} \simeq h\Delta x E_y(x,t)$$

$$-\frac{\partial B_z(x,t)}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y(x,t)}{\partial t}$$

Faraday's Law:

$$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\frac{d}{dt}\Phi_{m-through}$$

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$



Should I apply Faraday's law to a loop lying in the y-z plane?

A. No satisfying FL for one loop is enough
B. Yes, but the result would still be $\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$
C. Yes, but the result would be 0=0.

Combine to get the Wave Equation:

$$\#1 \quad -\frac{\partial B_z(x,t)}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y(x,t)}{\partial t} \qquad \#2 \quad \frac{\partial E_y(x,t)}{\partial x} = -\boxed{\frac{\partial B_z(x,t)}{\partial t}}$$

Differentiate #1 w.r.t. time

$$-\frac{\partial}{\partial x}\boxed{\frac{\partial B_z(x,t)}{\partial t}} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

Use #2 to eliminate B_z

$$\boxed{\frac{\partial^2 E_y(x,t)}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}}$$

The Wave Equation

Solution of the Wave equation

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

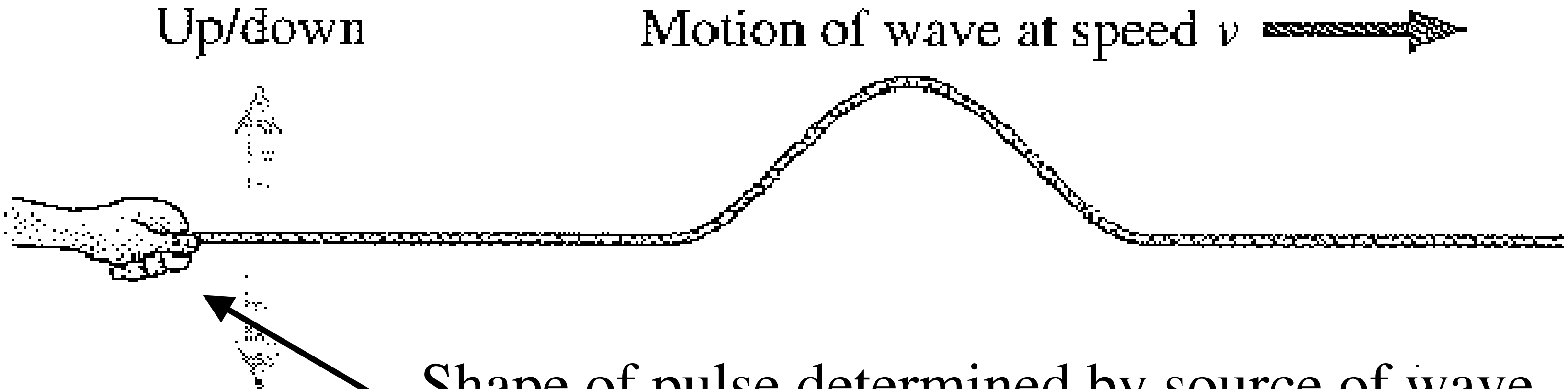
$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

Where $f_{+,-}$ are any two functions you like,
and

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

v_{em} is a property of space. $v_{em} = 2.9979 \times 10^8\ m/s$

$f_{+,-}$ Represent forward and backward propagating wave (pulses). They depend on how the waves were launched



Shape of pulse determined by source of wave.

$$E_y(x,t) = f_+(x - v_{em}t)$$

Speed of pulse determined by medium

$$v_{em} = 1/\sqrt{\mu_0 \varepsilon_0}$$

What is the magnetic field of the wave?

$$E_y(x,t) = f_+(x - v_{em}t) + f_-(x + v_{em}t)$$

Magnetic field can be found from either equation #1 or #2

#2 $$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

This gives:

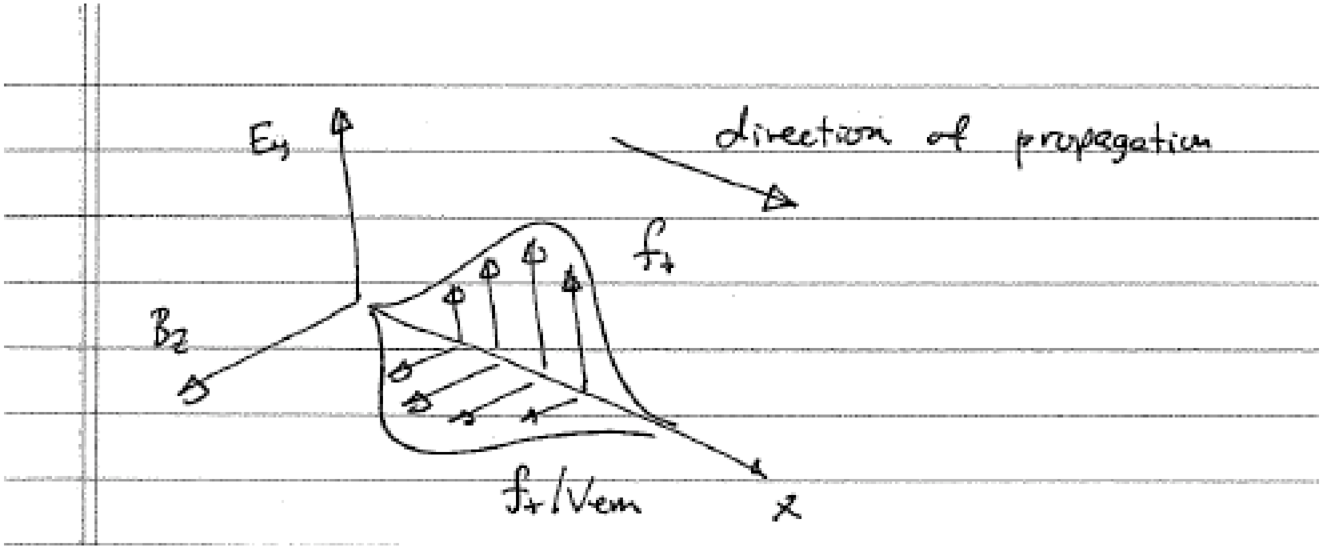
$$B_z(x,t) = \frac{1}{v_{em}}\left(f_+(x - v_{em}t) - f_-(x + v_{em}t)\right)$$

Notice minus sign

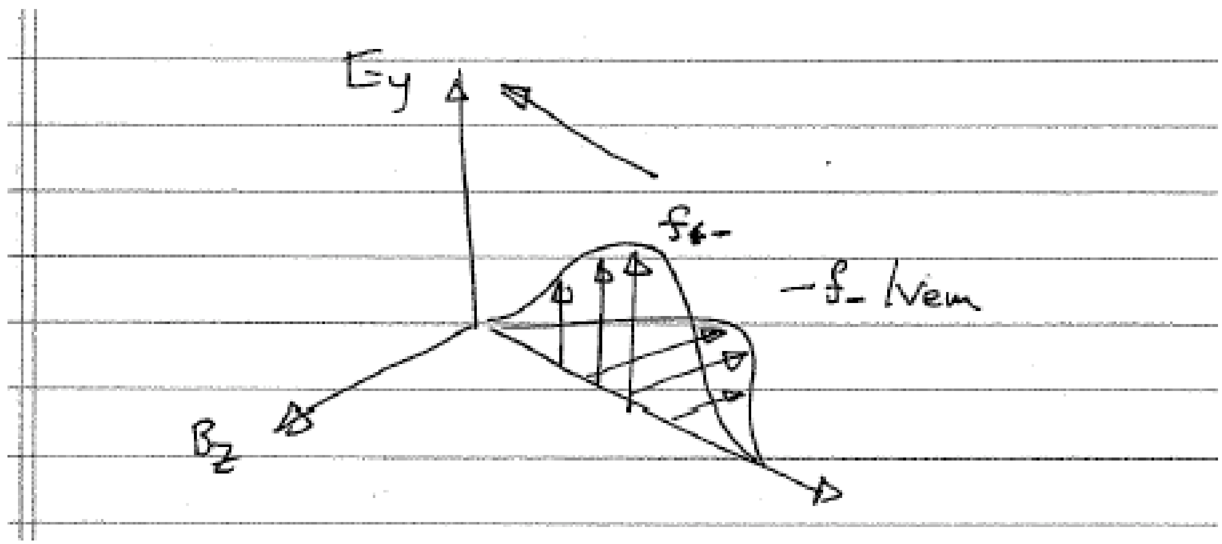
E and B fields in waves and Right Hand Rule:

f+ solution

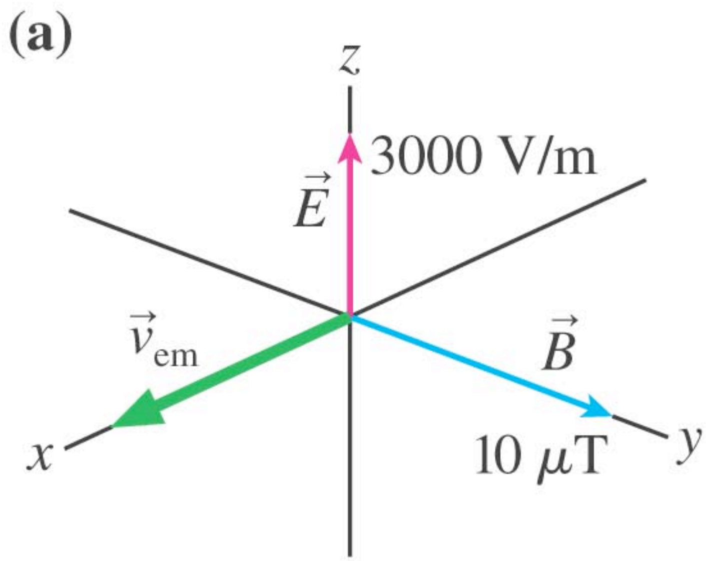
Wave propagates in **E** x **B** direction



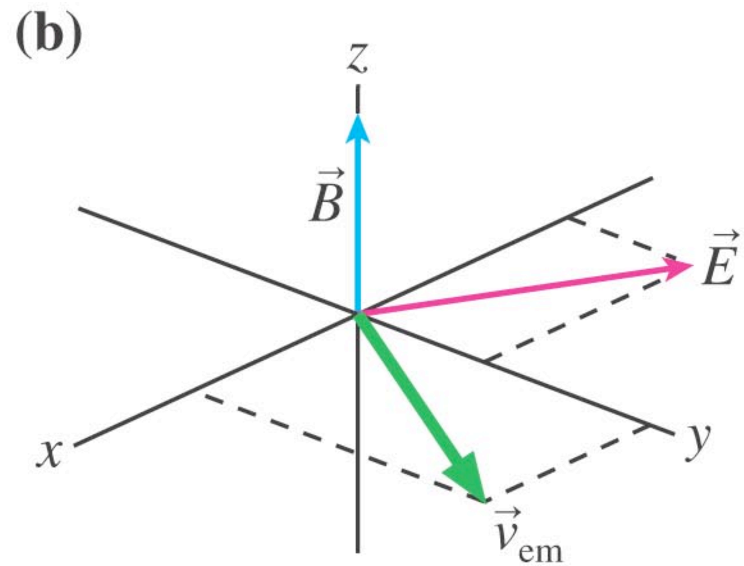
f- solution



Do the pictures below depict possible electromagnetic waves?



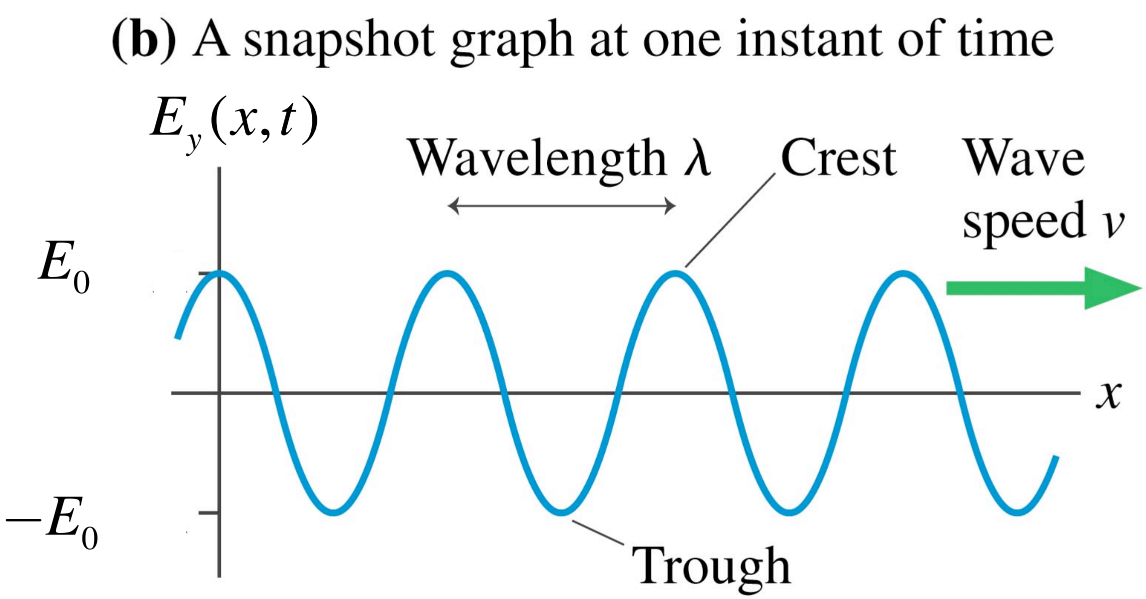
A. Yes
B. No



A. Yes
B. No

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$



Wavenumber and wavelength

$$k = 2\pi / \lambda$$

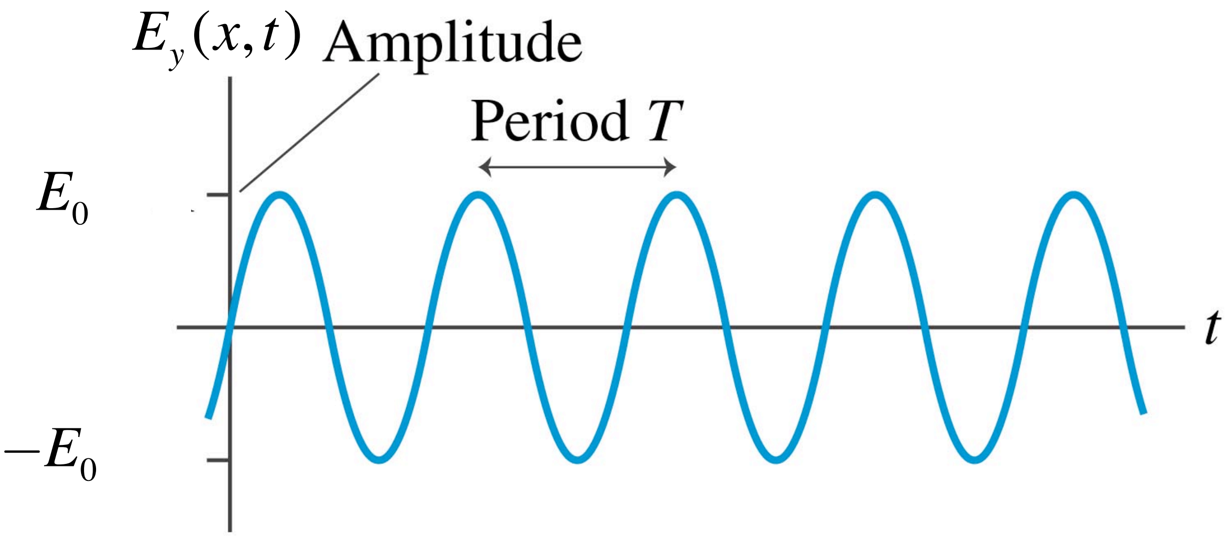
$$\lambda = 2\pi / k$$

These two contain the same information

Special Case Sinusoidal Waves

$$E_y(x,t) = f_+(x - v_{em}t) = E_0 \cos\left[k(x - v_{em}t)\right]$$

(a) A history graph at one point in space



$$2\pi = k v_{em} T$$

Introduce

$$\omega = 2\pi / T$$

$$f = 1/T$$

Different ways of saying the same thing:

$$\omega / k = v_{em} \qquad f\lambda = v_{em}$$

Energy Density and Intensity of EM Waves

Energy density associated with electric and magnetic fields

$$u_E = \frac{\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2}{2} \qquad u_B = \frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}$$

For a wave: $$\left|\vec{\mathbf{B}}\right| = \frac{1}{v_{em}}\left|\vec{\mathbf{E}}\right| = \sqrt{\varepsilon_0\mu_0}\left|\vec{\mathbf{E}}\right|$$

Thus:

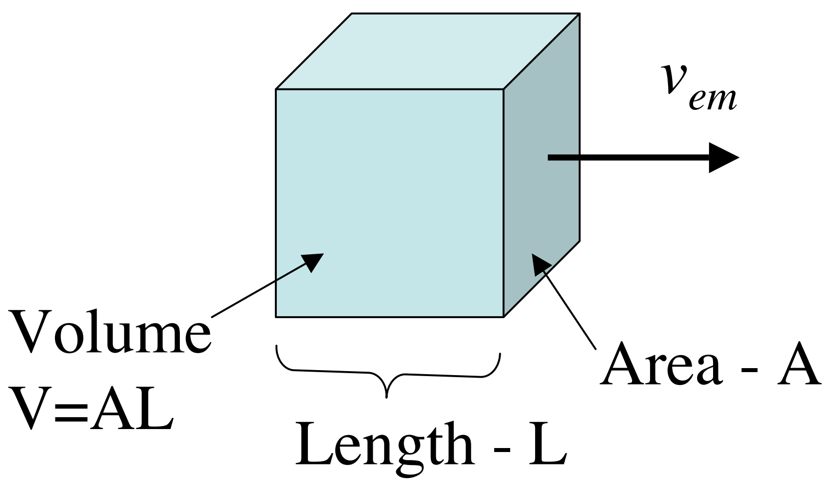
$$u_E = u_B$$ Units: J/m^3

Energy density in electric and magnetic fields are equal for a wave in vacuum.

Wave Intensity - Power/area

Energy density inside cube

$$u_E = \frac{\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2}{2} = u_B = \frac{\left|\vec{\mathbf{B}}\right|^2}{2\mu_0}$$



In time $\Delta t = L/v_{em}$ an amount of energy

$$U = V(u_E + u_B) = AL\varepsilon_0 \left|\vec{\mathbf{E}}\right|^2$$

comes through the area A.

Intensity

I=Power/Area

$$I = \frac{U}{\Delta t A} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \left|\vec{\mathbf{E}}\right|^2$$

Poynting Vector

The power per unit area flowing in a given direction

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

$$\left|\vec{\mathbf{S}}\right| = I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

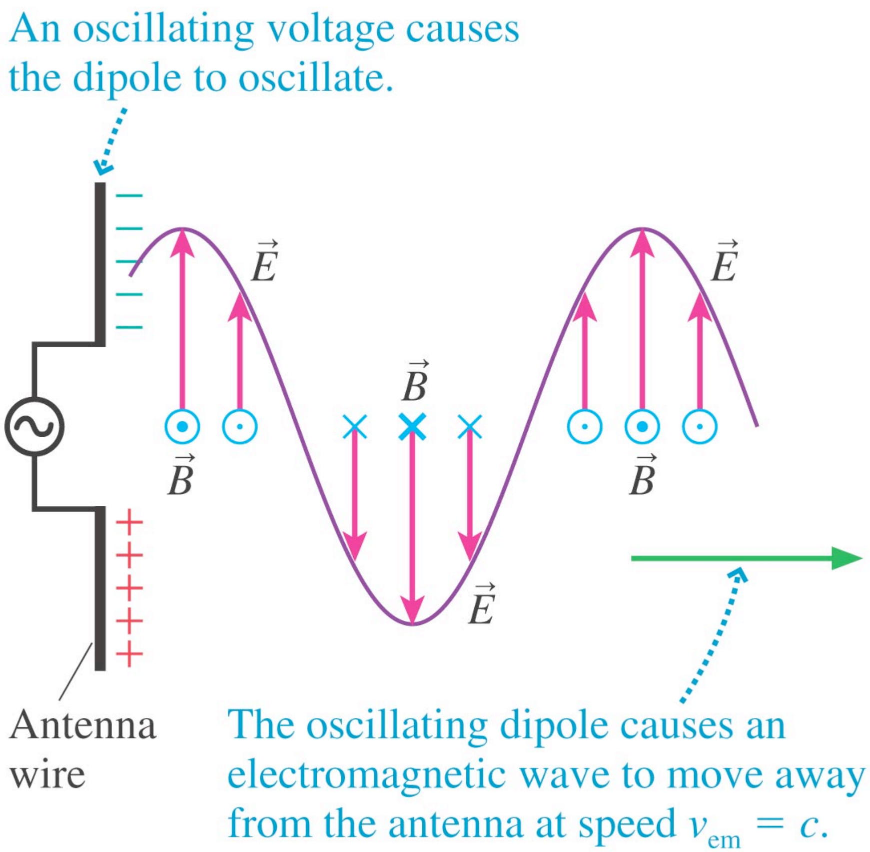
What are the units of $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ Ans: Ohms

I - W/m^2, E - V/m

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

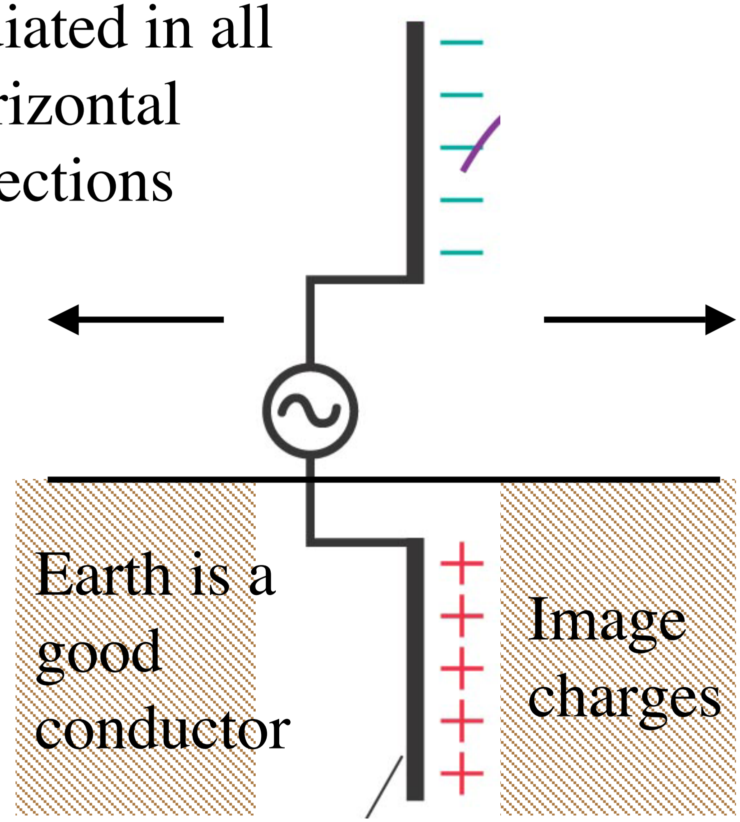
Antennas

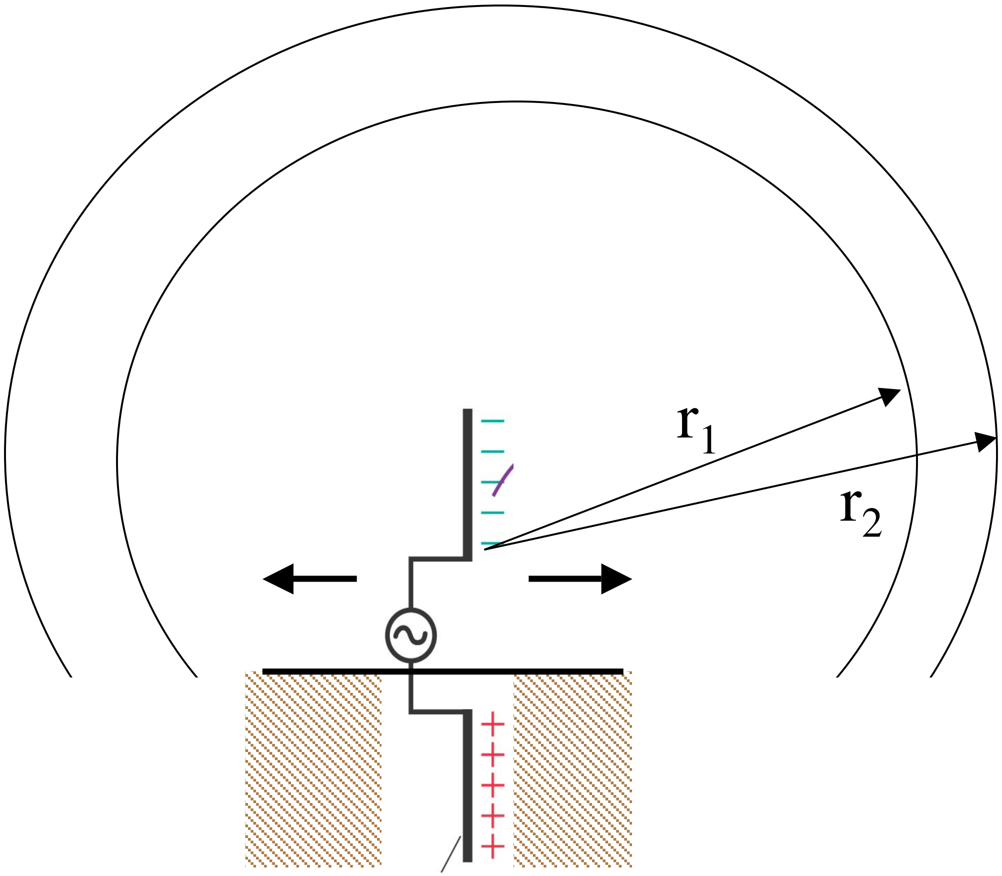
Simple dipole



AM radio transmitter

Power radiated in all horizontal directions





$$\left|\vec{\mathbf{S}}\right| = I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

To a good approximation what ever power goes the surface 1 also goes through surface 2.

Therefore:

A. $\left|\vec{\mathbf{S}}_1\right| = \left|\vec{\mathbf{S}}_2\right|$

B. $\dfrac{\left|\vec{\mathbf{S}}_1\right|}{r_1^2} = \dfrac{\left|\vec{\mathbf{S}}_2\right|}{r_2^2}$

C. $r_1^2\left|\vec{\mathbf{S}}_1\right| = r_2^2\left|\vec{\mathbf{S}}_2\right|$

D. $r_1\left|\vec{\mathbf{S}}_1\right| = r_2\left|\vec{\mathbf{S}}_2\right|$

The amplitude of the oscillating electric field at your cell phone is 4.0 μV/m when you are 10 km east of the broadcast antenna. What is the electric field amplitude when you are 20 km east of the antenna?

A. 4.0 μV/m
B. 2.0 μV/m
C. 1.0 μV/m
D. There's not enough information to tell.

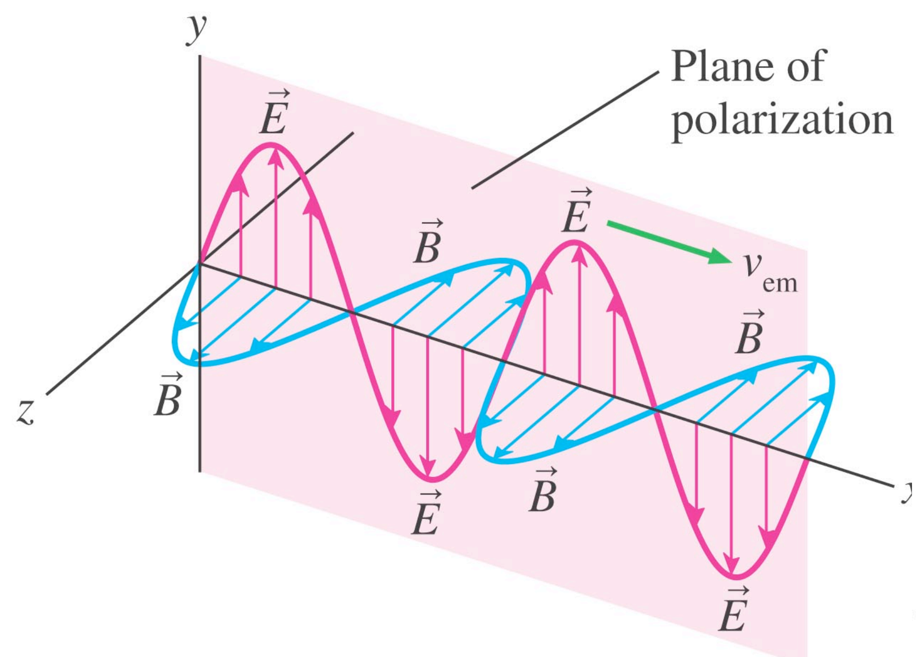
$$\left|\vec{\mathbf{S}}\right| = I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left|\vec{\mathbf{E}}\right|^2$$

Polarizations

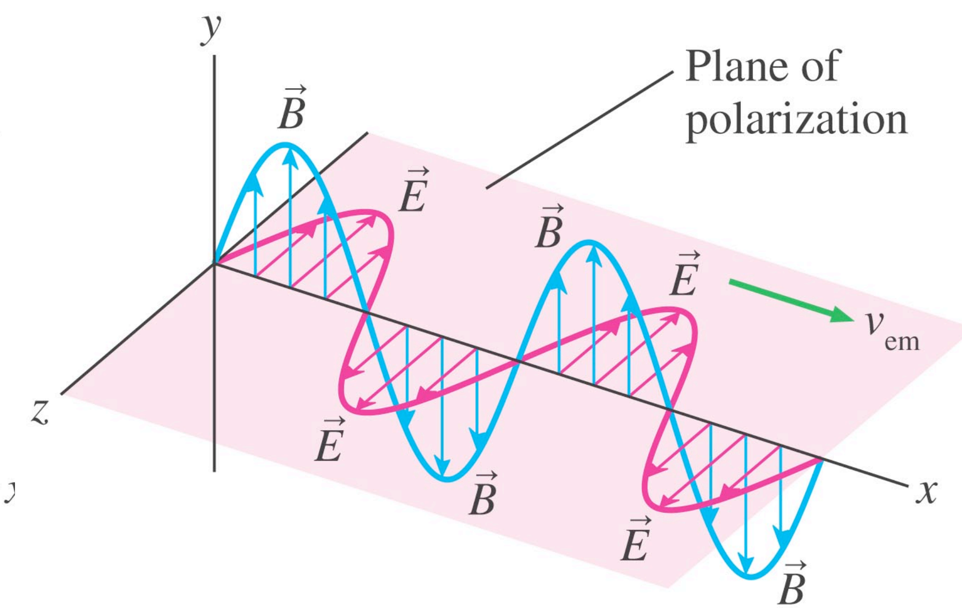
We picked this combination of fields: E_y - B_z

Could have picked this combination of fields: E_z - B_y

(a) Vertical polarization

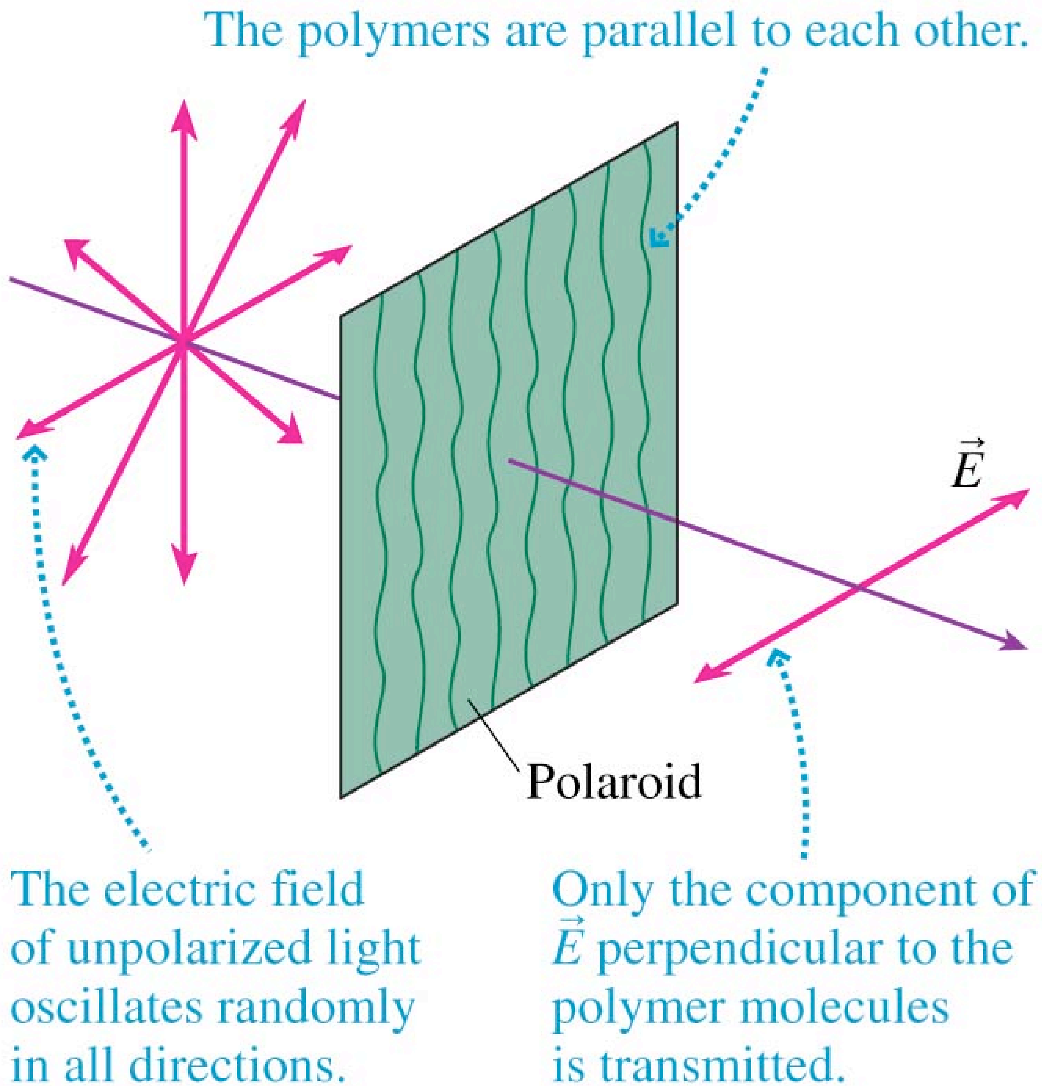


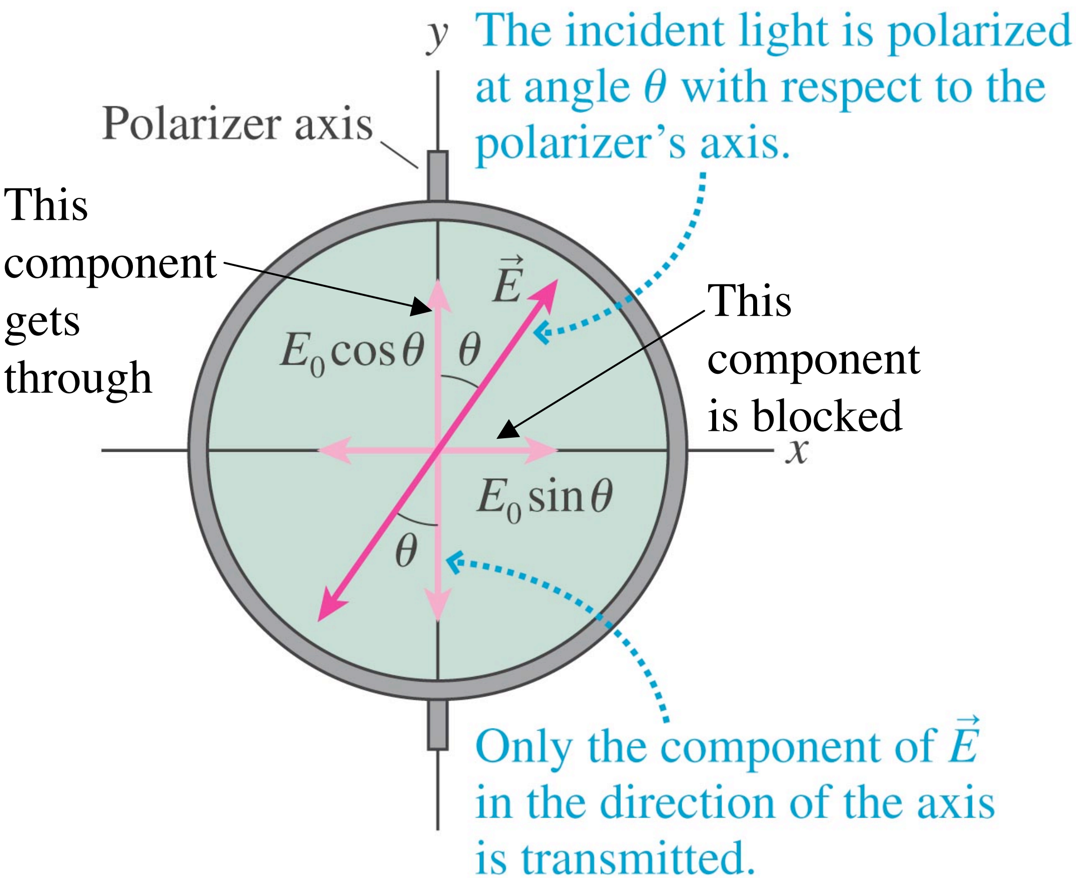
(b) Horizontal polarization

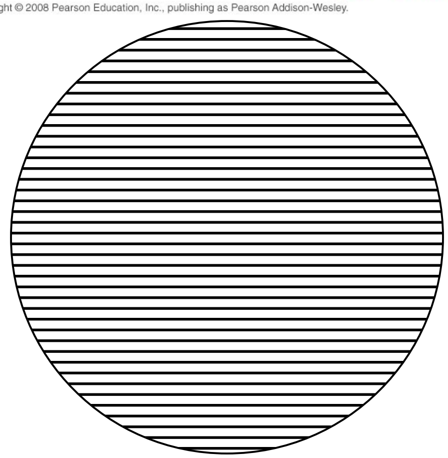


These are called plane polarized. Fields lie in plane

FIGURE 35.28 A polarizing filter.







Orientation of polymers

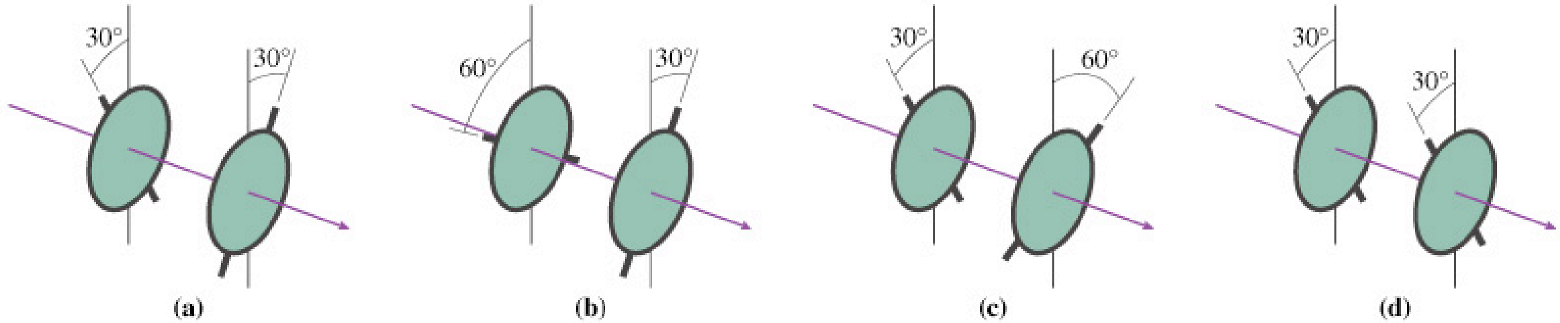
The wave that passes through the polarizer has an electric field amplitude

$$\left|\vec{\mathbf{E}}\right|_{out} = \left|\cos\theta\right|\left|\vec{\mathbf{E}}\right|_{in}$$

If input light is unpolarized

$$I_{out} = \left\langle \cos^2\theta \right\rangle I_{in} = \frac{1}{2} I_{in}$$

Malus's Law



Unpolarized light of equal intensity is incident on four pairs of polarizing filters. Rank in order, from largest to smallest, the intensities I_a to I_d transmitted through the second polarizer of each pair.

A. $I_a = I_d > I_b = I_c$
B. $I_b = I_c > I_a = I_d$
C. $I_d > I_a > I_b = I_c$
D. $I_b = I_c > I_a > I_d$
E. $I_d > I_a > I_b > I_c$

Integral versus Differential Versions of Maxwell's Equations

Integral	Differential
$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$	$\nabla \cdot \vec{\mathbf{E}} = \frac{\rho}{\varepsilon_0}$ (Charge density)
$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	$\nabla \cdot \vec{\mathbf{B}} = 0$
$\oint_{loop} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} == -\frac{d}{dt} \int_{Surface} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$	$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$
$\oint \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot d\vec{\mathbf{s}} = \mu_0 (I_{through} + \varepsilon_0 \frac{d}{dt} \int_{surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}})$	$\nabla \times \vec{\mathbf{B}} = \mu_0 (\vec{\mathbf{J}} + \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t})$ (Current density)